INDUSTRIAL APPLICATION OF FLOTATION MODELS

PART I

by

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I am also indebted to the Johannesburg Consolidated Investments Corporation for the opportunity to extend my education in a practical environment.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

III. THE RUSTENBURG DEPOSIT AND ITS TREATMENT

Formation of the Deposit

The Bushvelt Igneous Complex is one of the world's most famous geological phenomena and consequently has been subject to a large amount of study since the 1900's. Formed over 2000 million years ago, the complex is of an oblong shape, its long axis stretching from somewhat east of Burgerfort, to Zeerust, RSA, in the west, a total length of 288 miles. On its north-south axis the complex begins just north of Pretoria and extends north for 153 miles.

The origin of the Bushvelt Complex is still a subject of some debate; originally believed to be the result of a differentiation of an 18000' thick magma, the most popular theory today is that proposed by F.C. Truter in 1955.[1] . Truter proposed a series of magmatic intrusions from five different points. This long continuous process of repeated intrusion is said to have given rise to the layered structure. Differentiation then plays a minor part in formation of the complex.

A second theory suggests the advent of a series of extensive sheets, each one being differentiated separately, thus the formation of regular layers without cross-cutting.[2] A relatively new theory which sounds far-fetched but is geologically supported, proposes the occurrence of three nearly simultaneous impacts from comet-like bodies which resulted in either the production of voluminous magma by fusion of crystal and mantle rocks or by a severe thinning of the earth's crust and later magmatic intrusion.[3]

Other theories have been proposed, including metasomatism (transformation) of the upper Transvaal sediments,[4] but none receives the support of the above three theories.

Mineralogy of the Deposit.

Regardless of the origin of the Bushvelt Igneous Complex, the structure and associated mineralogy has been well defined. E.R. Schmidt in 1948 made a good study of the Rustenburg area in his Master's Thesis.[5] In the Rustenburg area, the Merensky reef strikes eight degrees east of true north at a dip of eight to ten degrees. Along its plane the reef undulates and varies widely in width. Associated with the famous Merensky reef are several other mineral bearing bands, in descending order they are :

1. mottled anorthosite
2. hang-wall spotted gabbro
3. the Merensky reef
4. a small chrome band (not always present)
5. a 10 to 24" band of coarse felspatic pyroxenite
6. a large chrome band, ¾" or greater
7. a narrow band of pure anorthosite
8. spotted anorthosite gabbro

In normal mine terminology, groups 1, 2, 7 and 8 are referred to as norite, group 3 as Merensky reef and groups 4, 5 and 6 as reef.

At times, where the structural disturbances described and explained by Schmidt exist, some of the bands disappear momentarily. In dome structures the coarse felspathic pyroxenite band is absent near the top of the dome while in the circular depressions the larger chrome band is absent.

The Relationship between Values and Structure.

The PGM (platinum group metals) and sulphide minerals appear to be in association except in the chrome bands where

the PGM value outweighs sulphide value. PGM value is concentrated in the chrome bands and overlaying coarse felspatic pyroxenites; however sometimes PGM value is found in the spotted anorthosite gabbro when the band is high in sulphides. The Merensky Reef, the hanging-wall spotted gabbro and mottled anorthosite always contain relatively very small amounts of PGM and sulphides.

It has been management policy in the past to separate "waste" and "reef" in times of a high market demand for PGM, and when the market falls to re-process the waste material. Thus for increased production the spotted or mottled rock (primary norite) is hand-sorted by unskilled labour such that the majority of plant feed consists of the chrome bands and the coarse felspatic pyroxenite. Unfortunately during the course of this investigation, various portions of the "waste" material were processed with the run of mine ore. Varying minerologies have an effect on the flotation performance, as illustrated in fig. 1. Evaluation of plant performance of the various minerologies, in terms of strict model parameters would be an expensive and time-consuming operation.

Several minerological investigations of the RPM (Rustenburg Platinum Mine) ores have been reported for the benefit of the Johannesburg Consolidated Investments.[6] These reports have been studied and main points of mineral association are discussed below. No reference is made with regard to PGM minerals and their associations for two reasons: 1) all modelling work references total sulphide or flotation of various sulphide minerals, and 2) the information concerning PGM minerology and production is considered confidential by the company and cannot be publicised.

Sulphide Minerals and their Association.

Various sulphide minerals are present in the Rustenburg ore and are listed here in order of their relative abundance. The four major sulphide constituents are Pyrrhotite ($Fe_7\ S_8$), Pentlandite ($(Fe,Ni)_9\ S_8$), Chalcopyrite ($Cu\ Fe\ S_2$) and Pyrite ($Fe\ S_2$). Minor sulphides are Mackinawite (Fe S), Vallenite ($Cu_3\ Fe_4\ S_7$), Cubanite ($Cu\ Fe_2\ S_3$), Bornite ($5\ Cu_2\ S\ Fe_2\ S_3$) and Bismuthimite ($Bi_2\ S_3$).

The mineralogical researchers agree that sulphides crystallized during the later stages of magmatic differentiation, with the pyrrhotite probably crystallizing first. As a consequence, sulphides appear primarily as massive aggregates which have an interstitial relationship with the silicate minerals (primarily pyroxene and plagioclase feldspars). Normally they are moulded against the feldspar and/or pyroxene crystals; in the case of pyroxene, the sulphides often form very fine grained inter-growths along cleavage planes. These sulphide patches vary in size from one millimeter to several centimeters, however they are always accompanied by a number of very small sulphide flecks attached to the feldspar and pyroxene crystals. Thus while the majority of sulphide is liberated from gangue material at an early comminutation stage, a significant proportion of the sulphide remains attached to gangue materials even after fine comminutation.

The sulphide aggregates themselves have a complex nature and generally consist of varying proportions of the major sulphide minerals. Separation by grinding, of the various mineral species, particularly separation of pentlandite and chalcopyrite from pyrrhotite would require extremely fine grinding. The aggregates tend to have a specific structural nature, in which pyrrhotite forms a core intermittantly covered by pentlandite and then by chalcopyrite, with pyrite crystals

often evident in the pyrrhotite and pentlandite. (See Fig. 2).

Treatment of the Rustenburg Deposit

The regularity of the strike and dip of the reef in the Rustenburg area has made mine planning and development a rather simple problem, the only interference having come from the circular depressions and the domes. Like gold mining, the valuable material is contained in very small "reef" thus stoping is done in rather narrow bands following the dip of the reef. The typical mine stoping width is 28 inches, this includes the chrome band, approximately 19 inches of the overlaying pyroxenites and 9 inches of the underlaying norite. Large proportions of the norite are sorted on the stoping face and used as fill for supports. The ore is then transported by means of a number of haulage ways to the hoist where it is raised and loaded onto trains for transportation to the reduction works. The RPM reduction works may be broken into three sections, the modelling investigation applies to only one of these sections, the flotation section. The other sections, comminution and gravity concentration, may contribute to some of the problems encountered in the flotation modelling, and hence they are described briefly here.

From the mine the ore passes through jaw crushers reducing large lumps to minus six inch, from there the ore is passed over washing screens and the plus two inch is sorted for waste. The remaining coarse material is crushed in two stages to minus half inch for ball mill feed. The feed is stored in silos, where retention time will vary up to 60 hours depending upon production schedule. Fine comminutation is done in two-stage ball milling, all mills operating in close circuit with hydrocyclones.

Following the milling section is the gravity concentration

section. In this section the heavy "metallic" PGM values are trapped in corduroy cloth. Since a large amount of labour is employed here, considerable attempts have been made to by-pass this section, in fact for the bulk of the test work the corduroy section was not in operation.

Flotation follows the regular practice of normal sulphide flotation, all the sulphides are floated since the main economic value lies in recovering PGM metals not in the production of pure copper and/or nickel concentrates. The flotation circuit is a simple one, normal operation consists of a roughing and a cleaning section. The circuit was changed for certain tests so that a flotation model could be used to predict plant performance in tune with circuit design, potentially, modelling's largest advantage.

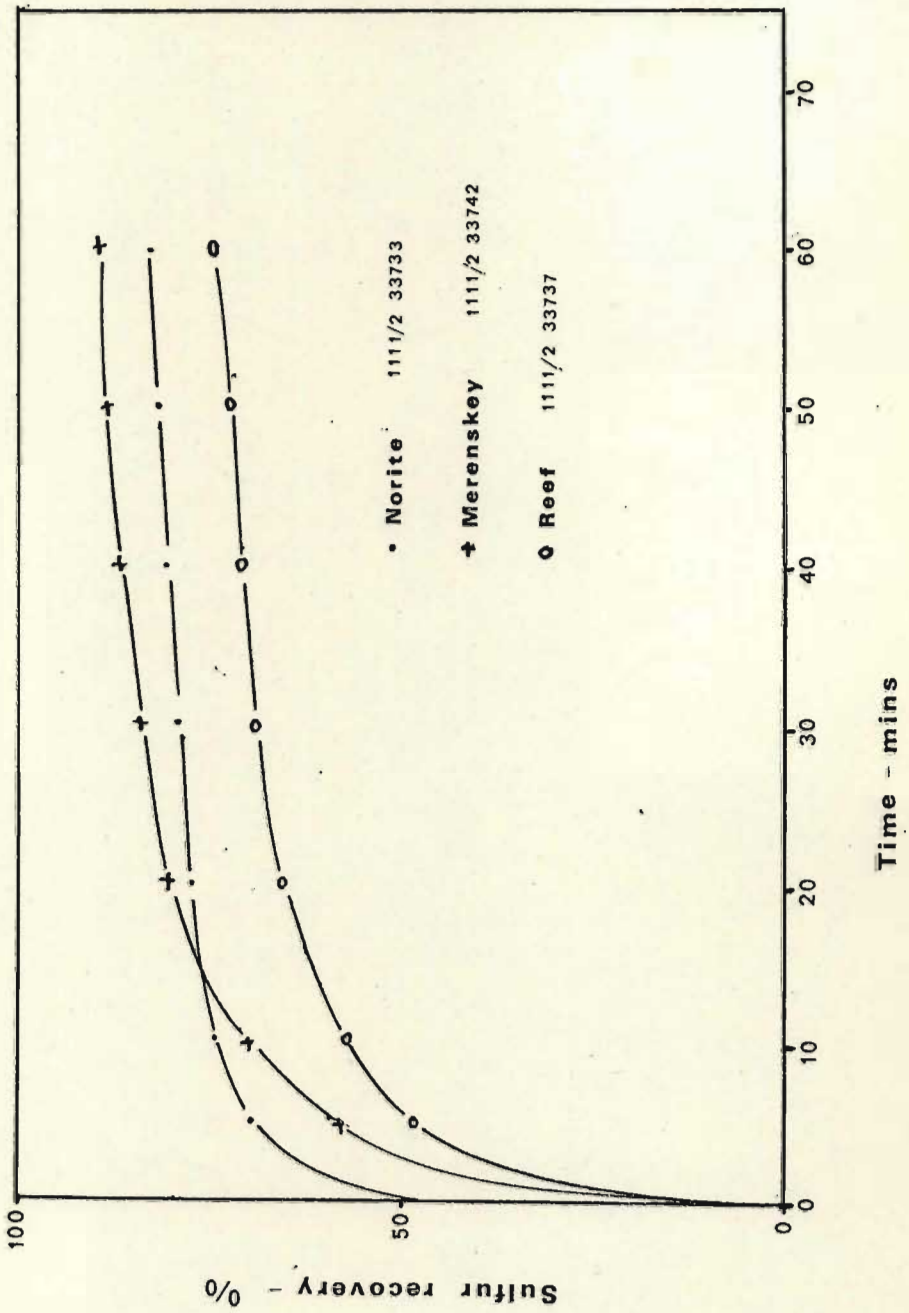
The reagents used in flotation are common to sulphide flotation. Xanthate is added as the collector, the mechanism of rendering the mineral surface hydrophobic is not completely understood, however it is assumed that the anionic portion of the molecule attaches to the mineral surface while the hydrophobic carbon-chain tails provide the water repellant surface. Xanthates are formed by reaction of alcohols with carbon disulphide, as follows :

$$KOH + ROH + CS_2 \longrightarrow \underset{S}{ROCSK} + H_2O$$

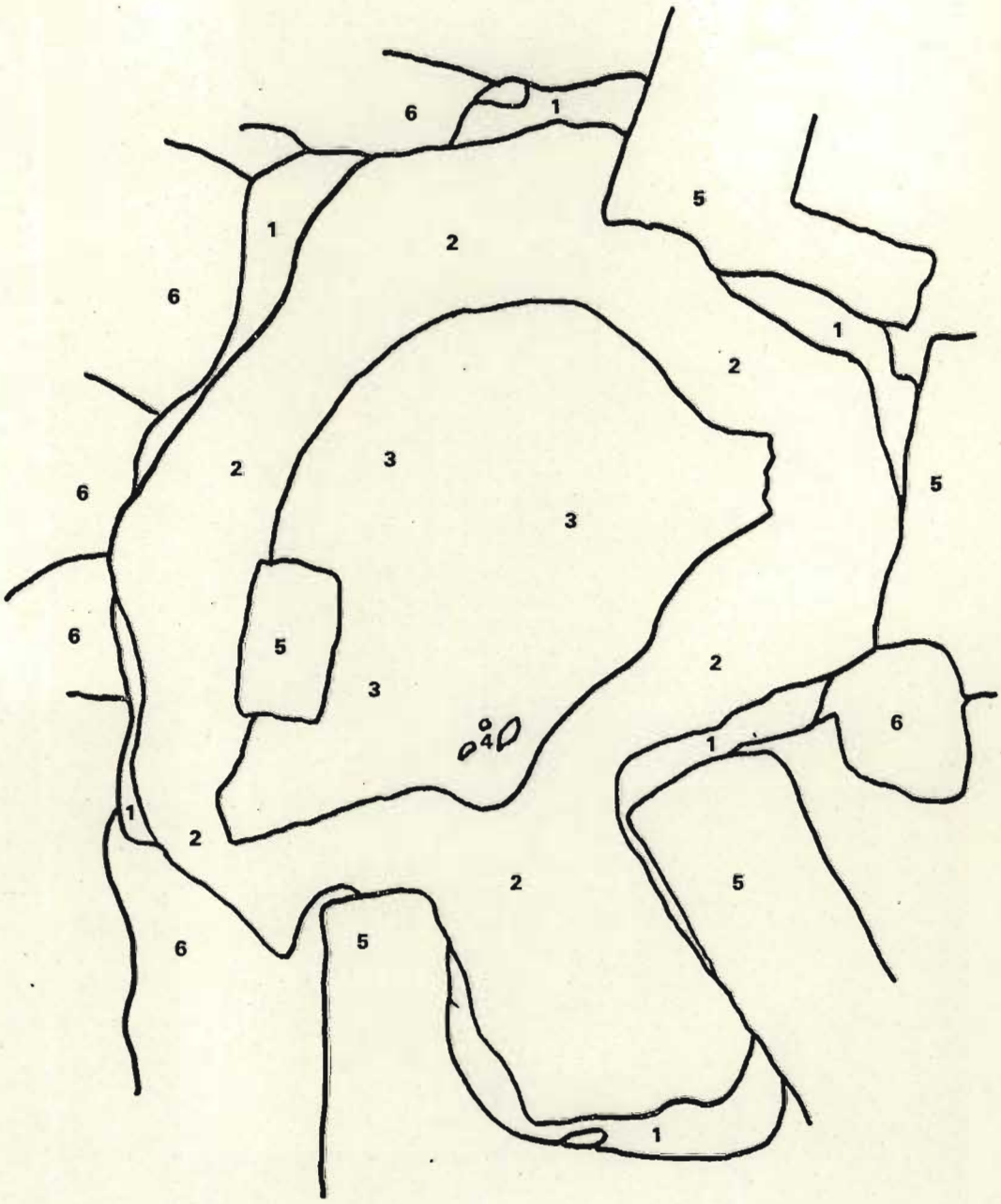
Copper sulphate is added as an activator, primarily for the pyrrhotite. Dextrin, an organic colloid used to depress slimes, is thought to be a general depressant for minerals through smearing particle surfaces. Two frothers are used, cresylic acid to promote a tough high recovery froth and Aerofroth 71 which gives a cleaner froth. Cresylic acid is an industrial waste product consisting primarily of the higher homologs of the phenols. The AF 71 is produced commercially

by American Cyanamid company and consists of six to nine carbon chain alcohols. The pH of the flotation circuit is not controlled although sulphide recovery would be improved through acid addition. The ore is buffered and the pH is consistently between 7,4 and 8,5.

The modelling investigation is concerned only with the very fundamentals of flotation. Reagent effects introduce a tremendously complicated system of variables and for this reason reagent consumption and additional points were not changed during the test work.



1. Flotation of various ores



1 Chalcopyrite

2 Pentlandite

3 Pyrrhotite

4 Pyrite

5 Pyroxenite

6 Feldspar

2. Typical sulfide conglomerate[6]

IV. PRESENT FLOTATION MODELLING STATUS

The flotation process has long been regarded as more art than science. In the past, only through an extended association with the practice of flotation has the mineral processing metallurgist come to grasp its multiplicities. While in the chemical, petroleum and steel industries, mathematical modelling and sophisticated control of full-scale production processes has been well established, the mineral processing industry has remained an area where 'seat of the pants' engineering is a necessity. Flotation practice and design are still matters of experience; judgements based on the barest of facts when coupled with variations in the market of most mineral products place management personnel in a position of near impossibility in regard to production decisions. For economic optimization of a mineral processing investment, a great deal more fundamental information about beneficiation processes is necessary. Accurate information about the production capabilities of flotation works is of value, for if management can be supplied with recovery and grade estimates for varying through-puts, they will be better able to react to market fluctuations, thus maximizing profit for a fixed investment. A great deal of research is being directed towards the creation of just such a production tool; a tool designed to understand the flotation process through its fundamental laws rather than through increasing individual familiarity.

This task of increasing the degree of science in the understanding of flotation is valid, for such depth of knowledge can only lead to economic benefit. Mathematics, being the language of the sciences, is the best means to expand the logic framework of flotation. Successful mathematical modelling of flotation will aide the mineral processing metallurgist in rationalizing his flotation experience and thereby provide him with a sharp tool of competency. In addition, successful modelling

will provide production management with a sound basis for reacting to market fluctuations in an economically favourable manner.

The general objective in modelling the flotation process, i.e. development of an accurate production and design tool, may be sub-divided into several requirements for the model. The mathematical formulation must accurately describe metal and gangue recoveries in batch and continuous flotation; it must be able to predict continuous operation from the data of a batch test; it must reasonably simulate the operation of various flotation net-works, and lastly, it must respond to size and throughput variations. If a mathematical model can perform the functions with a degree of accuracy and reliability then it has become a useful production and design tool.

Attempts to mathematically define the flotation process are not entirely new; the concept of flotation as a rate process was originally proposed by Zuniga in 1935.[7] From the simple beginning, attempts to model flotation have burgeoned into three general categories; probabilistic, chemical analogy and linear correlation. The probabilistic approach of Schuhmann[8] has been developed by a great many other researchers such as Tomlinson and Fleming[9], Sutherland[10], Evans[11], Eigeles[12], Bushell[13], and Arbiter and Harris[14], into a basically mechanistic and complex description of collesion, adhesion, retention and froth retention of particles in the flotation system. The chemical analogy models have received the attention of most researchers and also have developed through the simple single rate model[15], discretely[16] and continuously distributed rate models[17], into the complexity of the distributed constant model of King[18], in which bubble residence time, particle size, available bubble surface area, aeration rate and other factors are modelled.

It is the point of view of this thesis that attempts to

develop extremely accurate and mechanistic models of flotation, as a general procedure, are unrealizable. The large number of process parameters in flotation leads the growth of such models to a degree of complexity that becomes unwieldy and impractical. Several flotation sub-processes have been described mathematically[19], but an entire multitude of sub-processes defy the acquisition of accurate data as well as the subsequent mathematical formulation. If the objective of flotation modelling is the development of a production design tool, then research should be directed towards defining the limitations of the simpler models in industrial application. Only through such experimentation and feed-back can the practical goal be reached.

The purpose of this thesis then is to examine the applicability of four of the simpler chemical analogy models in the context of an industrial operation. All of the work was performed at the Frank Reduction section of Rustenburg Platinum Mines, Bleskop, Transvaal, Republic of South Africa. A model which can serve as a production and design tool for Rustenburg Platinum Mines would be beneficial, for the platinum market is historically variable. The platinum market is competitive and for this reason, disclosure of tonnages, grades, or recoveries in terms of platinum group metals (PGM) is restricted. The thesis deals only with the flotation of chalcopyrite, pentlandite and pyrrhotite; no reference is made to PGM flotation. With regard to the sulphide minerals and non-sulphide gangue, each model will be investigated as a production design tool. Any errors in the model's formulation will be examined and for the most successful model some improvements will be suggested. The degree of accuracy and reliability of each model will be illustrated and the potential worth of flotation modelling in general will be commented upon.

V. FUNDAMENTALS OF FLOTATION AND CHARACTERISTICS OF THE MODELS

In order to evaluate a mathematical model of flotation it is necessary to understand both the fundamental characteristics of flotation and the basic structure of the model. In this chapter, the fundamentals of flotation, the basic structure of the chemical analogy models and the expected limitations of the four models will be discussed. Many of the possible pitfalls of model testing will be illuminated and any inconsistencies between the model structure and flotation fundamentals will be examined.

The number of variables in flotation is extremely large as shown in Table I.[20] Simple mathematical models such as are to be examined here can only hope to account for several of the major flotation process characteristics. An effort to incorporate all flotation parameters involves a complexity, probably without increased accuracy, beyond imagination. Hence, the thesis is concerned with an examination of only the fundamental flotation variables and the ability of the model to simulate these fundamentals.

TABLE I.

FLOTATION VARIABLES

Uncontrolled	Controllable
Raw Material	
Nature of valuable mineral Secondary valuable minerals Associated gangue Soluble constituents of the ore Degree of oxidation of the ore Oxidation during mining Quantity of make-up water available	Soluble impurities in the water Presence of dissolved gases Degree of alkalinity or acidity of water used
Grinding and classification	
Fineness of particles and closeness of association Degree of hardness of minerals and gangue Relative time for different minerals in grinding circuit	Type of machine used Type of grinding media used Oxidation during grinding Chemicals added during grinding Pulp density during grinding Rapidity of circulation Time
Conditioning	
	Pulp density Chemicals added Order and form of adding the chemicals Temperature Mean time allowed Intensity of agitation
Flotation	
Particle density	Particle size Pulp density Temperature pH value Circulating load Time in the machine Geometry of the machine Power intensity Impeller speed Degree and type of aeration Height of froth Specific chemicals added to circuit

The Flotation Process

The flotation process is an industrial method for separating small mineral particles from one another. The separation is performed in a water suspension of the mineral particles concerned; air is dispersed throughout this suspension and by suitable chemical treatment of the slurry, certain mineral surfaces are rendered hydrophobic. The hydrophobic mineral particles adhere to the rising air bubbles and are removed in a soapy froth. Obviously the object is to choose the chemical conditioning stage such that the valuable mineral particles are removed from the non-valued minerals.

In industrial application, flotation occurs in a series of two port tanks, the concentrate froth is removed at a variable rate from the tank and the tailings flow continuously to the next unit. Often a number of recycle streams are employed in an effort to improve the grade of the final product. Through a series of flotation cells the tailings stream is successively depleted of its more floatable, air-avid material and the concentrates are enriched in floatable, usually valued minerals.

From the micro rather than the macro view point, the mechanism of flotation becomes considerably more complex. The flotation of each individual valuable mineral particle, involves the complex chemistry of activation and collector coating as well as the physical processes of particle-bubble collision, bubble-particle adhesion and particle removal in a stable froth. It was essentially these physical micro processes which concerned early researchers like Schuhmann[21] who established the definite probabilistic nature of flotation. One of the major characteristics is the bubble-particle collision and adhesion, often this is considered similar to a reaction between particles and bubbles. This probabilistic

or time dependent nature of flotation is a primary characteristic of any flotation process.

A second physical sub-process, the removal of particles in a stable froth, has been mentioned as a significant factor by Arbiter and Harris,[22] and will be later shown to have a tremendous effect upon recovery and grade in a flotation cell. This second fundamental characteristic of the flotation process may be viewed as interacting with the time dependency of flotation; as the froth removal rate increases so does the apparent rate of flotation.

The third and fourth primary characteristics of the flotation process are more concerned with the particles themselves rather than any physical sub-processes in flotation. Firstly, it must be realized that each particle within a flotation pulp is a complete individual. Even those particles which may be classed as part of a mineralogical or metal species vary greatly in size, shape and surface characteristics, it would be miraculous indeed if particles of such variety exhibited identical flotation properties. The fourth factor which is of tremendous importance in application of the chemical analogy flotation models is brought about by the use of recycle streams within the normal industrial flotation network. Due to the inclusion of several recycle streams many mineral particles may circulate within the circuit for a considerable time before being removed at either the final concentrate or final tailings streams. Because flotation is basically a surface phenomena, the floatability of such particles may be effected by oxidation, collector coating removal or attrition while within the circuit. The significance of this factor is yet to be determined but such a time dependency of the particles' flotation characteristics must be noted as a possibility.

Structure of the Chemical Analogy Models

Discussion of modelling structure will be limited to the structure of the four models investigated at Rustenburg Platinum Mines. Throughout this thesis the four models will be referred to as 1) the single rate model, 2) the rate plus non-floating model, 3) the gamma function model, and 4) the two-phase model. Each of these four models is based upon first-order kinetics; that is the process of flotation is assumed to be consistent with a first-order reaction process. In the case of the two-phase model, both the froth and pulp phases are assumed to be governed by first-order kinetics. The general equation for a first-order reaction is:

$$dC/dt = -kC$$

where 'C' is the reactant concentration and 'k' is the first-order rate constant.

All of the models investigated assume this simple description of flotation as a first-order process and assume that rate constant values for a particular group of particles remain constant throughout a flotation network. The models differ in their distribution of first-order rate constants for a mineralogical or metal species and in the number of phases which are mathematically incorporated in the model structure. The single phase models, that is, the single rate model, the rate plus non-floating model and the gamma function model assume that the froth phase may either be neglected or may be incorporated as a rate determining factor, rather than as a separate phase. The single rate model assumes that the flotation of a mineralogical or metal species may be described by a single rate constant value. The rate plus non-floating model recognises the existence of a non-floatable portion of material

within every mineralogical or metal species, thus it describes flotation in terms of two first-order rate constants, one of which is zero. The gamma function model recognises the inherently individual nature of the multitude of particles in a particular mineralogical or metal species and assumes a modified form of the gamma function as a continuous distribution of first-order rate constants. The continuous rate constant distribution is entirely defined by its mean and variance. The two-phase model investigated, incorporates the froth phase directly in its basic structure and acknowledges the obvious return of solids to the pulp from the froth phase.[23] The transfer of a mineralogical or metal species between the two phases is governed by two rate constants; one first-order constant for transfer from pulp to froth and one first-order constant for transfer from froth to pulp.

As with many processes, flotation may be performed either as a batch or as a continuous process. In any time dependent process these two cases of operation are analytically dissimilar. In the batch model of flotation each particle is susceptible to flotation conditions for an equal length of time; in continuous operation however, a distribution of solids residence times exists. A theoretical residence time distribution equivalant to 'perfect mixing' is often assumed. It will be later demonstrated that the assumption of such a distribution is reasonable at Rustenburg.

Possible Limitations in the Model Structure

In order to understand the flotation process and expand that understanding through mathematical modelling of the process, it is necessary to clearly expose the assumptions of any model and discuss the limitations and possible problems which might arise in the application of that model. In the

two previous sections, the primary characteristics of industrial flotation and the basic structure of the investigated flotation models were discussed. It now remains to examine the plausibility of the basic model structures in regard to the flotation process primary characteristics.

All first-order models have assumed that the rate constant for a particular particle class is time-invariant; both with respect to varying mineralogies in the feed to the flotation net-work, and throughout the recycling in an industrial circuit. The various mineralogies at Rustenburg Platinum Mines do not float with the same rate distribution as previously illustrated in figure 1. Varying proportions of these mineral bearing bands in the flotation feed will alter the rate constant distribution for any particular mineral or metal species. It has not been demonstrated that the individual particles within a flotation network maintain their floatability throughout the various re-cyclings and re-flotations in the normal industrial circuit. Even if the normal practice of staged reagent additions is ignored, the inevitable processes of attrition and surface oxidation must affect the floatability of particles in the system. In the reflotation of concentrates, the particles' floatability may well be enhanced by the previous attachment of bubbles to their surfaces. The extent of this modelling assumption is not yet determined. Obviously the seriousness of feed variation depends both on the frequency and amplitude of such changes; the maintenance of a particles' floatability may or may not be of significance with regard to the objectives of flotation modelling but it is necessary that it is realized that the first-order models do not cater for and could not cater for a variation in the rate constant with time without the addition of many complicating mathematical features.

The froth phase and froth removal rate will be later demonstrated to be a very significant factor in flotation. Arbiter and Harris[24] have correctly described the fundamentals in that the flotation process is essentially a two-phase system, a pulp phase and a froth phase. Without some means other than direct kinetic consideration of the froth phase, the three single phase models can be expected to be severely limited by the assumption of a single phase system.

In consideration of the very individual nature of particles in a flotation pulp, it must be realized that the discrete rate constant models can only approximate the real situation. The gamma function model possesses a distinct advantage over the other three models investigated as it assigns a continuous distribution of rates to any species. In a recycle system, small errors arising from the assumption of discrete rate constants may accumulate into large errors in the entire system.

Any discrete rate constant or distribution of rate constants is obviously reactant specific. Throughout the application of a model it is imperative that material representative of the feed to the flotation network is examined. It became clear early in the research work that standard laboratory batch testing of plant feed material was considerably different to batch testing of plant pulp samples. It cannot be expected that material which is crushed and ground in a laboratory batch ball mill will give rise to particles with flotation characteristics which are identical to the characteristics of the same material comminuted in several stages of plant milling and cyclone separation. Figures 3, 4, 5 and 6 are comparisons of laboratory batch tests and batch tests done on the plant feed stream which underline the possibility of gross discrepancies between essentially the same material prepared for flotation in the laboratory and in the plant. The major discrepancy

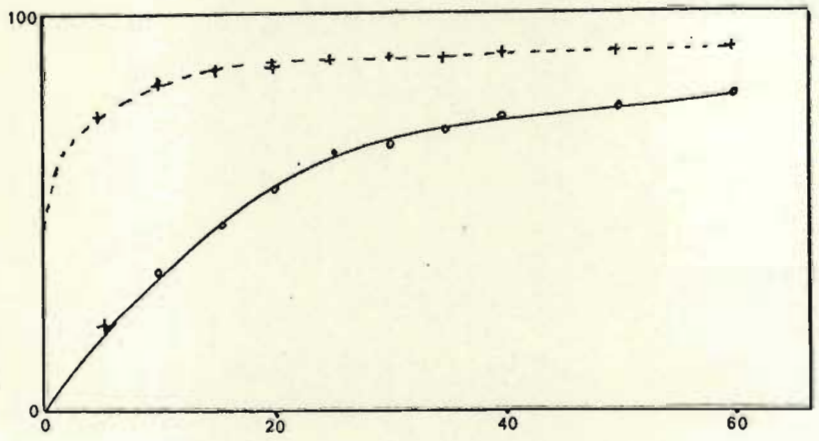
lies with the pyrrhotite mineral fraction. It is plausible that due to the ease of oxidation of the iron sulphide surfaces,[25] the higher plant milling temperatures have to a greater extent oxidized the pyrrhotite surfaces and reduced their floatability. Evidence of considerable iron sulphide oxidation is shown in figure 7, where acid addition has temporarily lowered the pulp pH and removed the oxide film on the sulphide surfaces, even after the pulp pH is returned to normal the iron sulphide flotation is enhanced. A second factor in the realization that flotation rate constants are reactant-specific became evident during research and is concerned with the quality of water used for flotation. Various ion species are known to adversely affect the flotation of sulphides[26] and no less can be expected as flotation is a surface phenomena. In illustration, figure 8 shows the effect of a change in plant water quality upon the final tailings value of the investigated concentrator. Due to the publication restrictions of the Rustenburg Platinum Mine, no actual value may be given to the final tailings grade, however, a considerable effect is evident.

These two previous illustrations serve only to illuminate the complexity of the flotation process and outline the necessity for careful consideration of metallurgical factors in application of any flotation model to an industrial plant.

It must also be born in mind with regard to first-order models as investigated at Rustenburg, that the rate constant or rate constant distribution for a mineral or metal species is equipment-specific. Such variables as impeller speed and design, air suction, frothing, surface to volume ratios, the means of froth removal and agitation intensity may have considerable effect upon the flotation rates of various minerals. As large scale plant cells are difficult to scale down to a reasonable size for batch operations, some discrepancies will obviously

exist between flotation equipment in batch and continuous testing work. The assumption that any batch test may be related to continuous operation through a residence time distribution may be seriously in error, and hence will contribute to the predictive quality of kinetic parameters regressed from batch testing.

The gamma function model is inherently the most powerful of the four investigated models for it incorporates the time-dependent nature of flotation; it recognizes the individual nature of the mineral particles and can easily simulate the complex industrial networks of most mineral processing plants. Provided that the froth phase can be included in any simulation as a factor which affects the rate constant distribution, it will provide the best basis for building a production design tool. Although a number of assumptions have been made in the application of all the models including the gamma function model, it must be remembered that the objective of the flotation modelling exercise is not the development of an every way complete description of flotation but rather the development of a simple model which can be used with reasonable accuracy and confidence as a tool for production and circuit design analysis of existing flotation networks.



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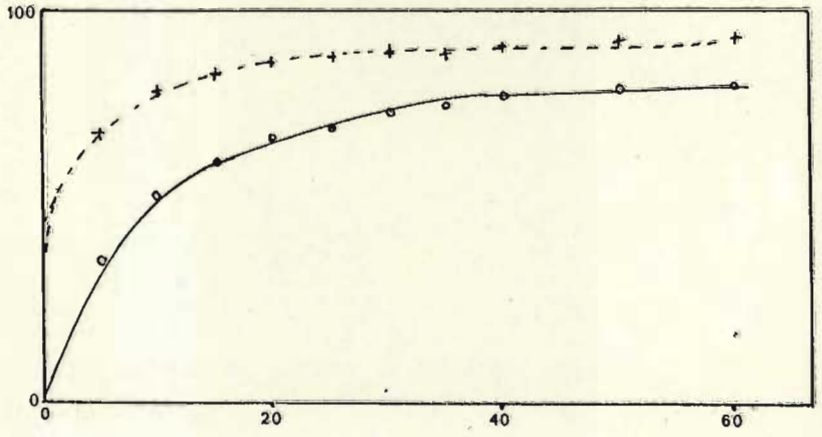
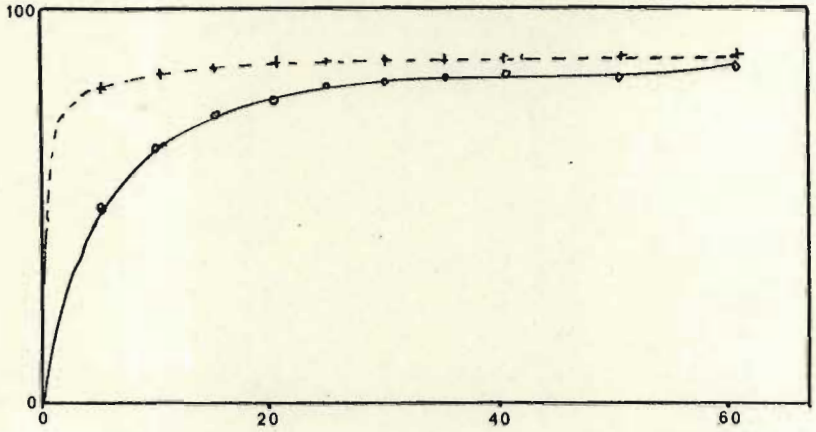


3. Sulfur recovery (%) versus flotation time (mins)



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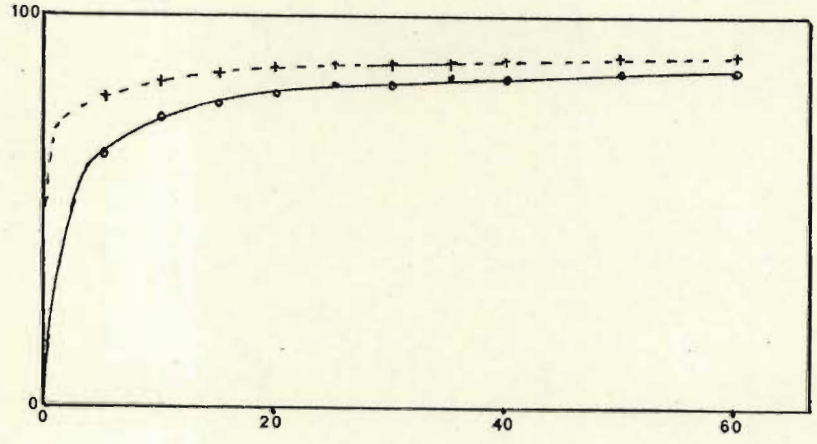
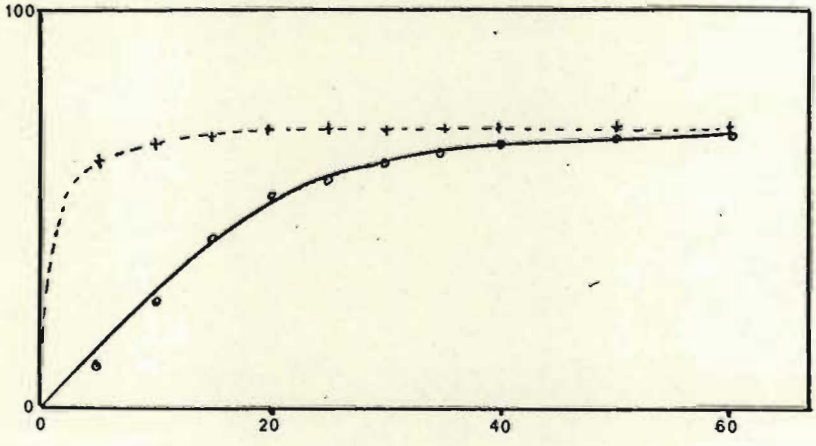


4. Chalcopyrite recovery (%) versus flotation time (mins)



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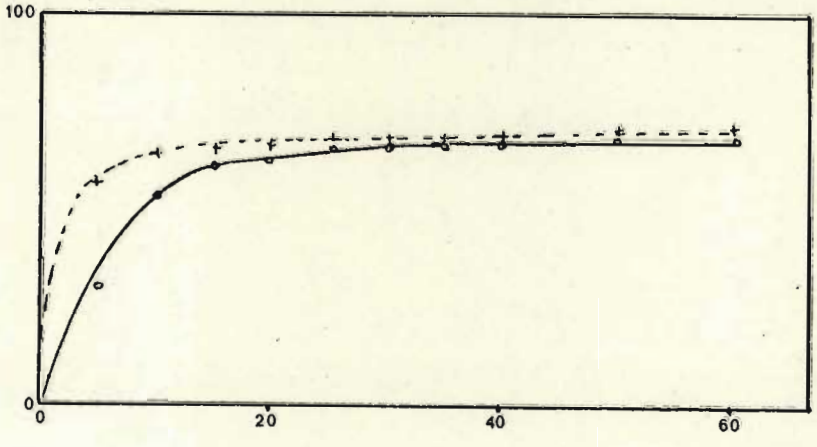
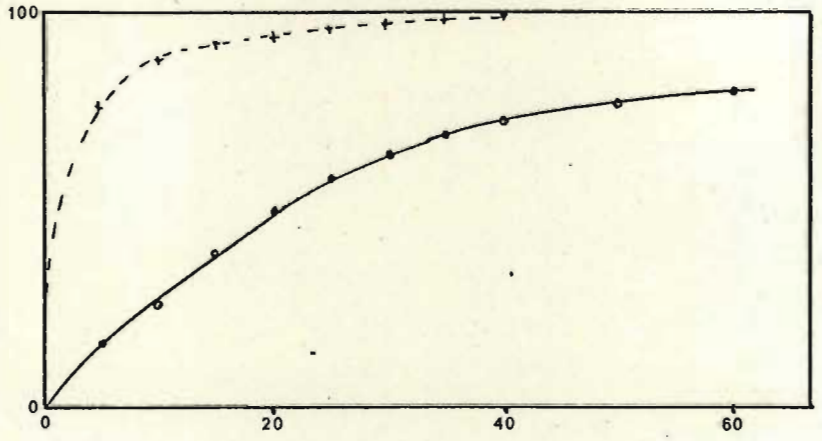
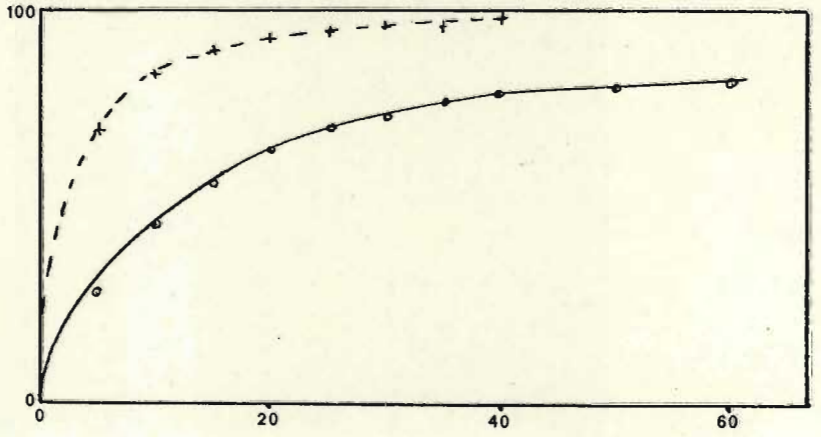


5. Pentlandite recovery (%) versus flotation time (mins)



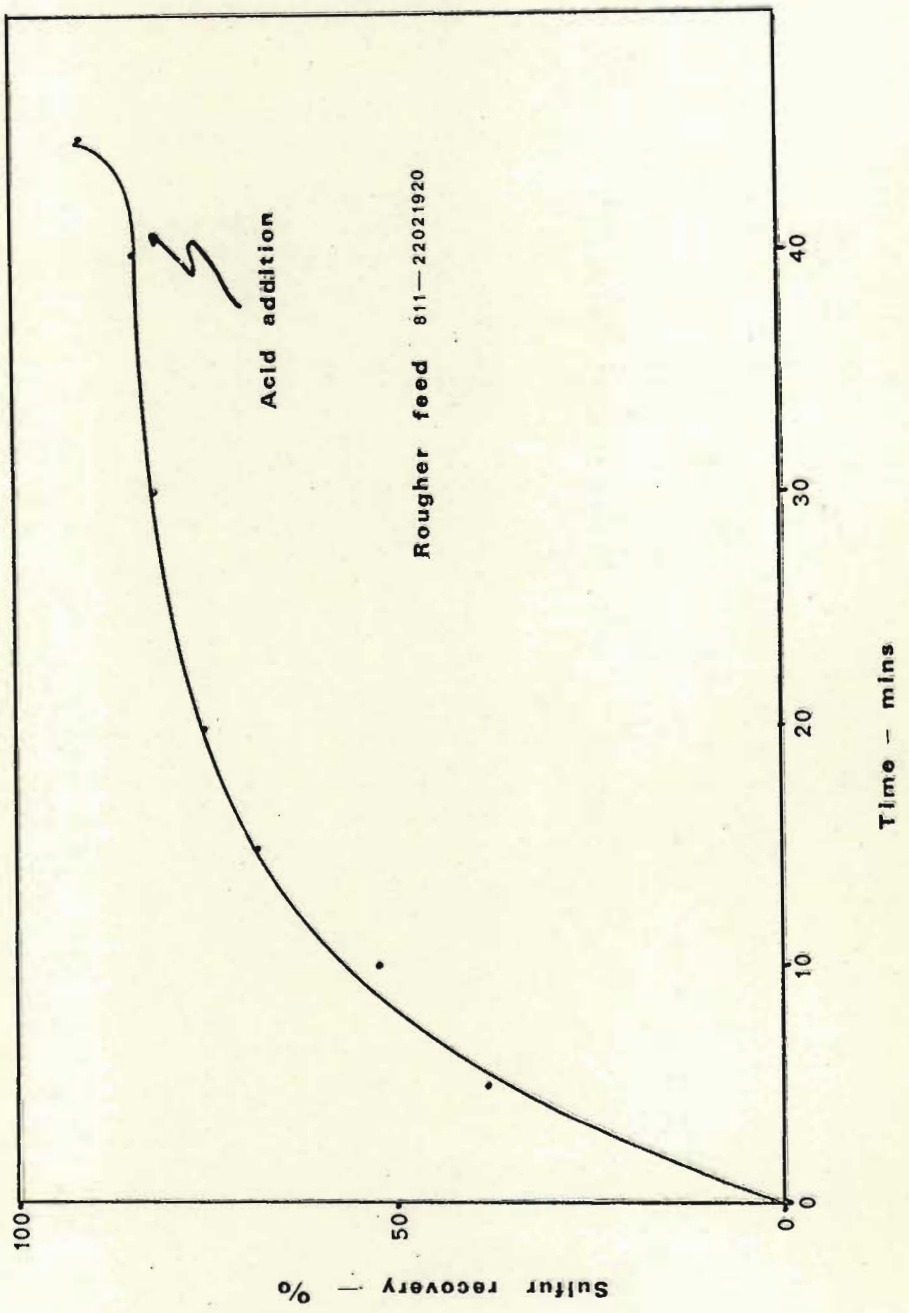
Lab batch 151 — 33409

Plant batch 151 23031 —

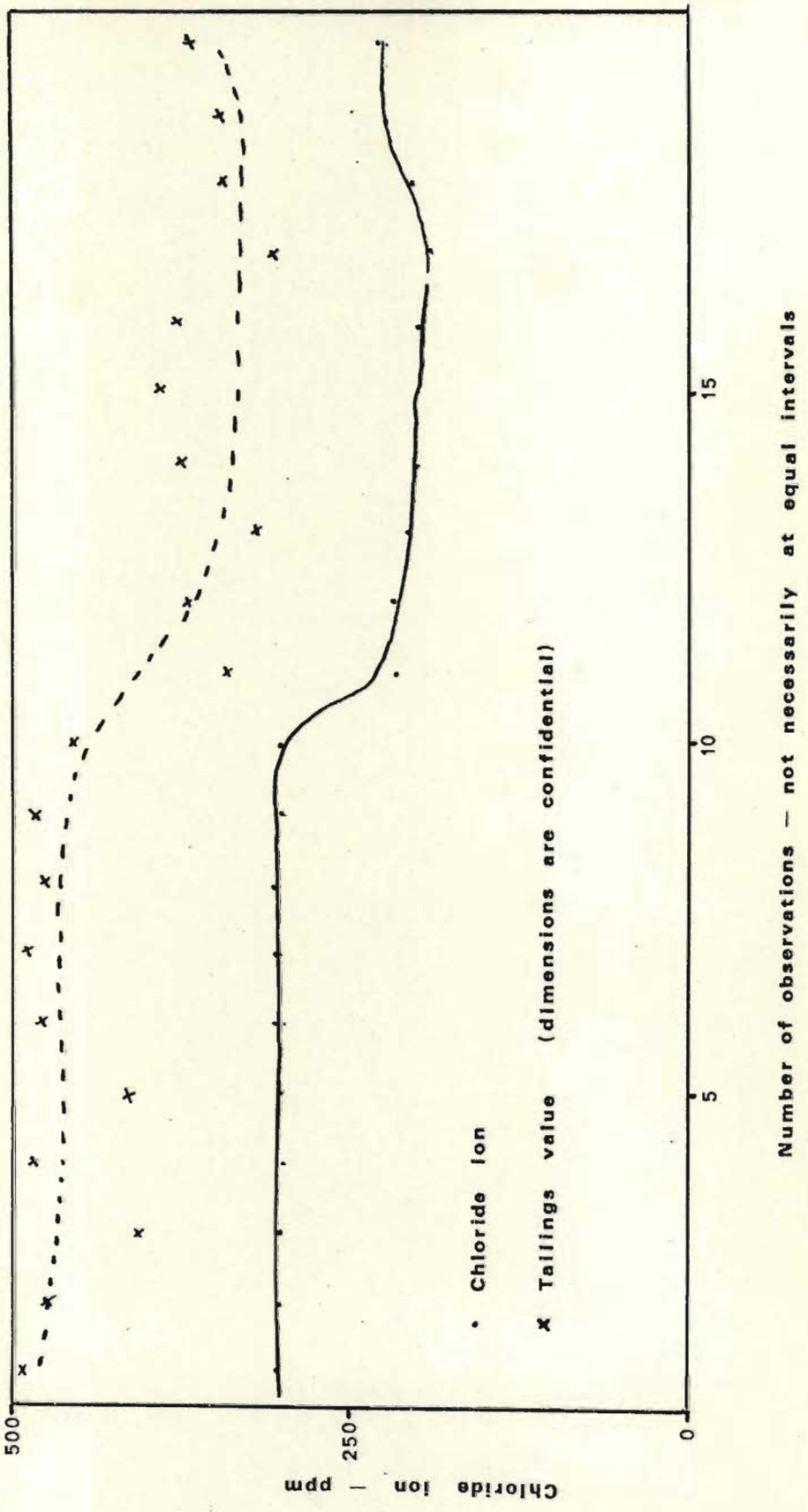


6. Pyrrhotite recovery (%)

versus flotation time (mins)



.7. Acid addition improves flotation



8. Effect of changes in mill water

VI. APPLICATION OF THE FOUR FLOTATION MODELS

In the analysis of the predictive quality of a number of flotation models with regard to industrial operation, several sub-goals may be readily defined. As an initial requirement of a valid flotation model it must be accurately descriptive of batch cell recovery vs. time curves. This is a relatively simple requirement and the majority of flotation models will fit batch data with reasonable accuracy.

The second more stringent test of a model's predictive capacity is also readily available in an operating plant. If the feed stream to an industrial flotation bank is sampled and floated in a batch cell, the kinetic parameters regressed from the batch curve may be used to predict the performance of the continuous bank. A recovery curve for the continuous operation may be obtained through sampling and compared to the model prediction. It is primarily in this testing that any problems arising from the fact that the kinetic parameters are equipment-specific will be evident.

One of the modelling assumptions which may be criticized is the time-invariant nature of the kinetic parameters. Considerable scope is available for testing this assumption in an operating plant, at Rustenburg this assumption was tested by performing batch tests on the rougher concentrates in connection with batch tests of the rougher feed. In theory, the batch flotation of the rougher feed will define the rougher plant operation and the consequent batch flotation of the rougher concentrate.

The fourth and most crucial test of the model's ability to serve as a production and design tool is based on the model's ability to predict the flotation operation in various circuit arrangements. A number of flotation networks were operated and

sampled for later comparison with the model predictions.

These four tests of the model applicability should well define any shortcomings in the model structure as well as illustrate the degree of confidence which may be placed in model simulations.

Throughout the thesis discussion, all raw data is referenced by a fourteen-digit code number; the raw data and full details of the code meaning can be found in the first section of part two of the thesis.

Measurement of the Residence Time Distribution

As previously discussed, in a continuously operating flotation cell each solid particle is subjected to a different flotation time depending upon its exact course of travel through the cell. This distribution of residence times which exists for the solids feed to the cell must be experimentally determined and mathematically formulated before the kinetic parameters of batch flotation may be applied to continuous operation. Jowett[27] has tested the assumption that a flotation cell operates as a perfect mixer and reported reasonable accuracy. Colborn[28] has reported a significant variation in residence time distributions for various size fractions. At Rustenburg, a liquid tracer test was performed on a bank of rougher cells and on a bank of cleaner cells. Sodium chloride was added to the head of the flotation bank in crystalline form; the tailings solution was then analyzed for chloride ion by titration with silver nitrate; potassium dichromate served as an indicator. Loveday[29] has presented the equation for the residence time distribution of a series of 'n' perfectly mixed cells

as follows :

$$E(t) = \frac{(nv/V)^n}{(n-1)!} \; t^{n-1} \; e^{-ntv/V}$$

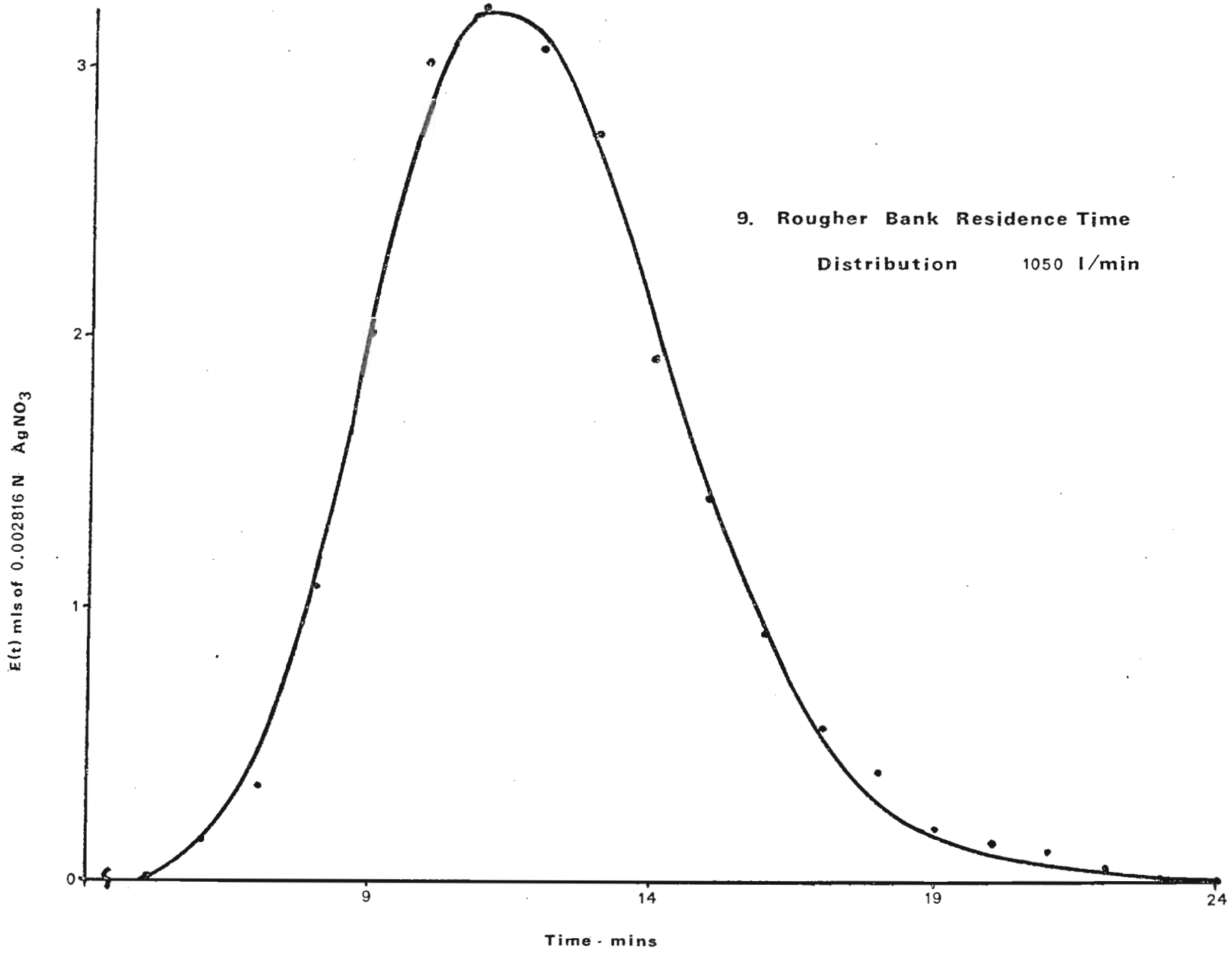
where 'V' is the effective volume of a single stirred unit and 'v' is the volumetric flowrate to the cells. Thus from the residence time distributions obtained, the effective volume of a single unit may be estimated. The residence time distribution obtained and the regressed fits of the perfectly mixed equation are shown in figures 9 and 10. Considerable discrepancy may exist between solid and liquid residence time distributions, particularly solids of various sizes. In addition to the liquid residence time tests performed at Rustenburg, solids radioactive tracer tests on four size fractions in the rougher bank were performed by personnel of the Atomic Energy Board.[30] Their results are summarized in Table II below :

Tracer	Regressed Effective Volume of Unit	Date
Cl^-	622 litres	7-1-74
+ 208 μ * (radioactive)	537	19-3-74
+ 104 μ * "	551	19-3-74
+ 53 μ * "	563	19-3-74
- 53 μ * "	571	19-3-74

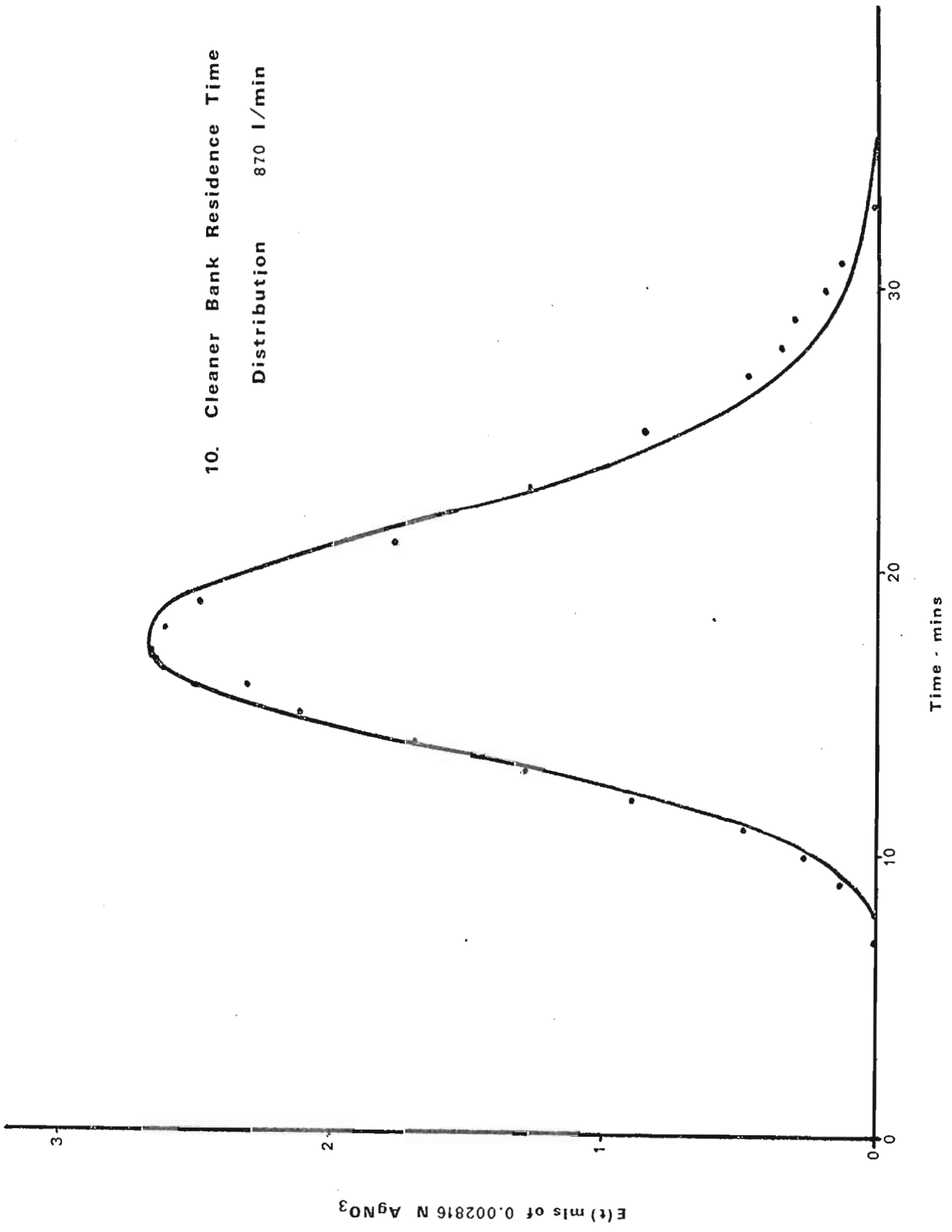
* performed by Atomic Energy Board personnel

Although the solids effective cell volumes are slightly lower than the liquid testing shown, there is little variation between size fractions. It is assumed that operating conditions of froth depth and froth removal rates can give rise to significant variations in the effective cell volumes hence the volumes of 622 l/cell were used throughout the thesis as the

effective perfectly mixed volume of any flotation cell. It is worthwhile to note that a considerable discrepancy exists between the effective cell colume and the cell volume calculated from its dimensions; the effective volume is approximately fifty-five percent of the manufacturer's dimensional volume.



9. Rougher Bank Residence Time Distribution 1050 l/min


10. Cleaner Bank Residence Time
Distribution 870 l/min
E(t) mls of 0.002816 N AgNO3
Time - mins
0
1
2
3
0
10
20
30

PERFORMANCE OF THE SINGLE PHASE MODELS

Each of the single phase models may be used in theory to describe batch flotation, continuous flotation, or flotation in a plant network. Because of their varying treatment of the distribution of first-order rate constants, the single rate model, the rate plus non-floating and the gamma function model describe each flotation circumstance with varying degrees of accuracy. Initially the first three sub-goals outlined for the analysis of the predictive quality of the single-phase models will be discussed separately for each model. The models may be expected to have reasonable descriptive capacity for batch flotation and a good measure of predictive capacity in a simple single bank continuous operation. The prediction of concentrate reflotation can be expected to illuminate problems in the model structures. The single phase models can also be expected to be severely limited in application to various frothing conditions.

The Single Rate Model

The single rate first-order irreversible model proposes that the flotation of a species is a single exponential function of time. Every particle in a designated species floats according to the same first-order rate constant.

A large number of batch tests were performed on various plant streams at Rustenburg; species were described through physical and chemical means. Several examples of the first-order model's ability to fit observed batch data for a species are given in Figures 11, 12 and 13. It is obvious from the figures that the single rate model is considerably lacking in its descriptive powers.

Theoretically if the flotation of a species may be

described by a single first-order rate constant, then the batch flotation of any metallurgical stream in a flotation network would give rise to the same regressed rate constant for a particular species. This is definitely not true as can be seen in Table III. The same flotation species in various streams floated in the same batch equipment gives rise to considerably different first-order constants.

The single rate model may also be applied to flotation in a series of continuous flotation cells; fits of the model to continuous operation concentration-time curves are shown in Figures 14 and 15. Again the same problems of inaccuracy are evident in the continuous flotation case.

It is obvious from a preliminary study of batch concentration curves that invariably some portion of any mineralogical or metal species remains non-floatable at long flotation times. Thus the single rate model could be improved by the mathematical recognition of this unfloatable fraction.

TABLE III. SIMPLE FIRST ORDER RATE CONSTANTS

MATERIAL AND TEST CODE	FLOTATION STREAM	RATE CONSTANT min^{-1}
SULPHUR		
31146210911711	Rougher Concentrate	0,08198
41146210911515	Cleaner Feed	0,08666
61144070911610	Cleaner Tailings	0,02905
61146210912019	" "	0,01934
51144070911850	Cleaner Middlings	0,1024 *
51146210911625	" "	0,04751 *
81144070911700	Rougher Feed	0,02379
81146210911912	" "	0,02511
NON-SULPHIDE		
32146210911711	Rougher Concentrate	0,01734
42146210911515	Cleaner Feed	0,02057
62144070911610	Cleaner Tailings	0,01088
6246210912019	" "	0,006152
52144070911850	Cleaner Middlings	0,05173 *
52146210911625	" "	0,02145 *
82144070911700	Rougher Feed	0,001691
82146210911912	" "	0,001031
COPPER		
431--110411445	Cleaner Feed	0,2094
831--130411435	Rougher Feed	0,1642
831--230411200	" "	0,1465

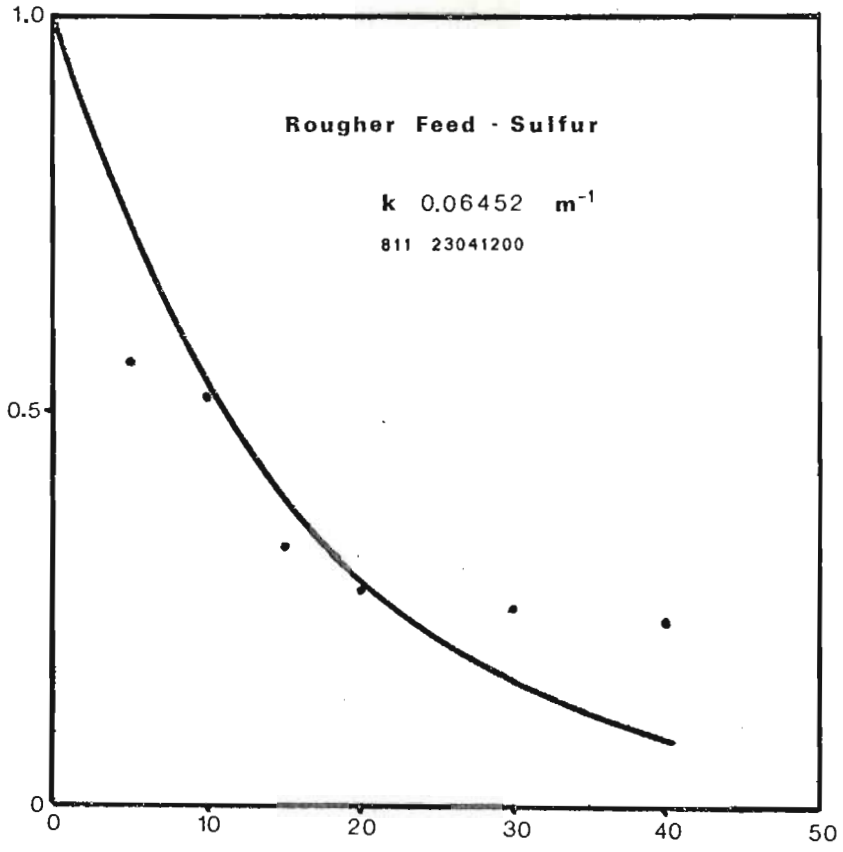
TABLE III. (Continued)

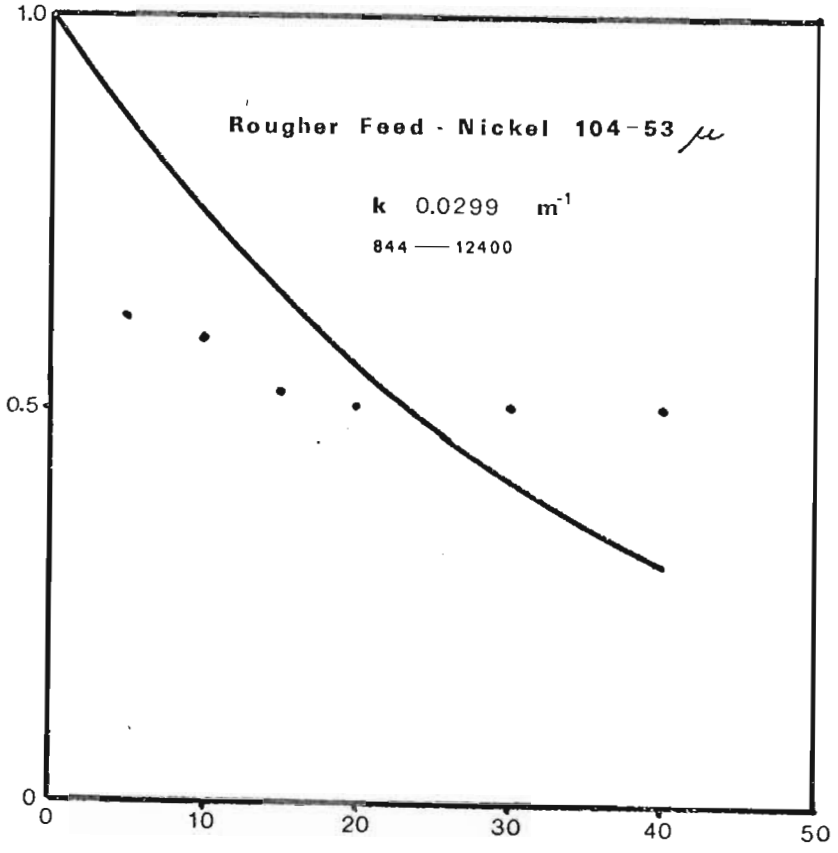
MATERIAL AND TEST CODE	FLOTATION STREAM	RATE CONSTANT min^{-1}
COPPER (208-104 microns)		
333------12400	Rougher Concentrate	0,3084
833------12400	Rougher Feed	0,03866
COPPER (104-53 microns)		
334------12400	Rougher Concentrate	0,4723
834------12400	Rougher Feed	0,1488
COPPER (- 53 microns)		
335------12400	Rougher Concentrate	0,4933
835------12400	Rougher Feed	0,04103
NICKEL		
441--110411445	Cleaner Feed	0,1108
841--130411435	Rougher Feed	0,05895
831--230411200	" "	0,06819
NICKEL (208-104 microns)		
343------12400	Rougher Concentrate	0,1171
843------12400	Rougher Feed	0,007798
NICKEL (104-53 microns)		
344------12400	Rougher Concentrate	0,2517
844------12400	Rougher Feed	0,02990

TABLE III. (Continued)

MATERIAL AND TEST CODE	FLOTATION STREAM	RATE CONSTANT min^{-1}
NICKEL (- 53 microns)		
345------12400	Rougher Concentrate	0,3073
845------12400	Rougher Feed	0,03843

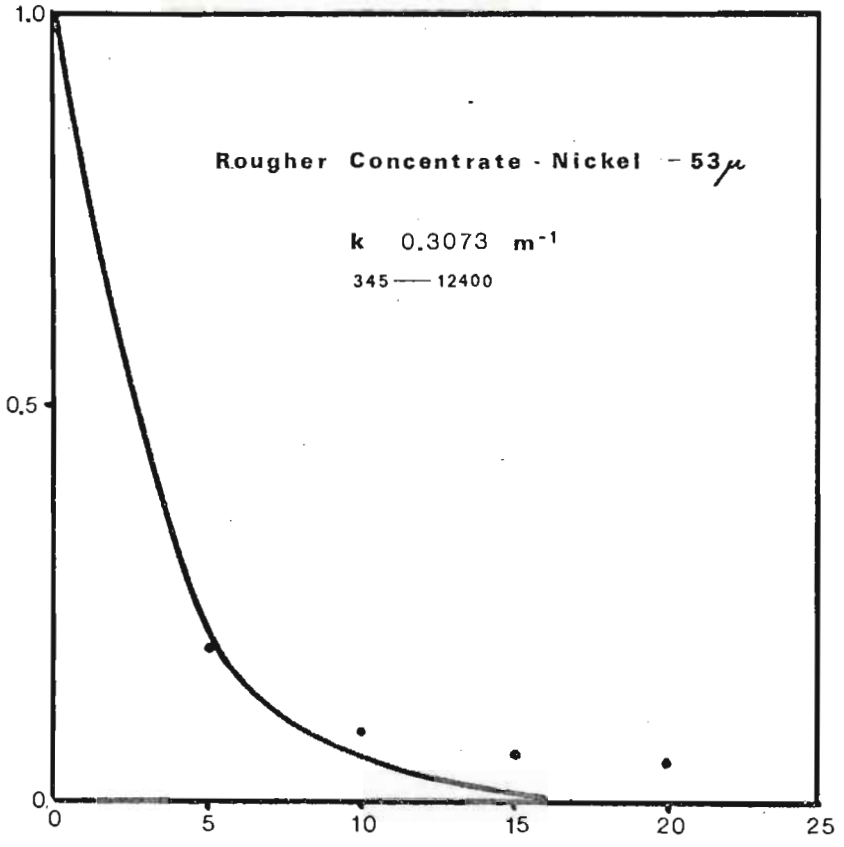
* These two tests were performed at greatly different froth removal rates.

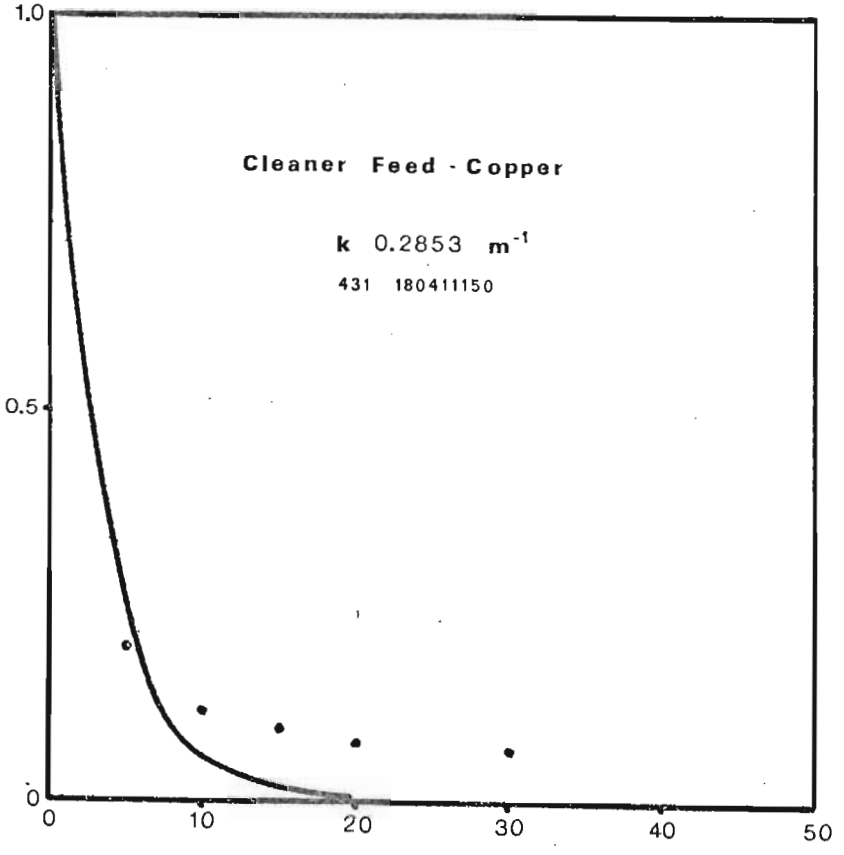




11. Batch Tests - Single Rate Model

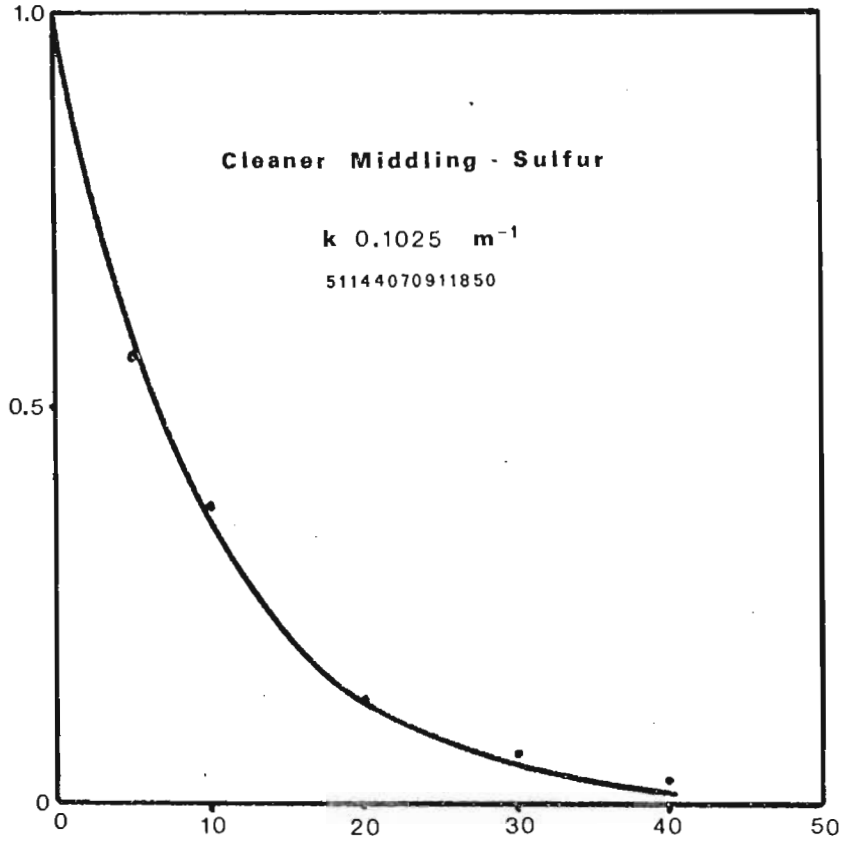
Fraction in Tailings vs Time - mins

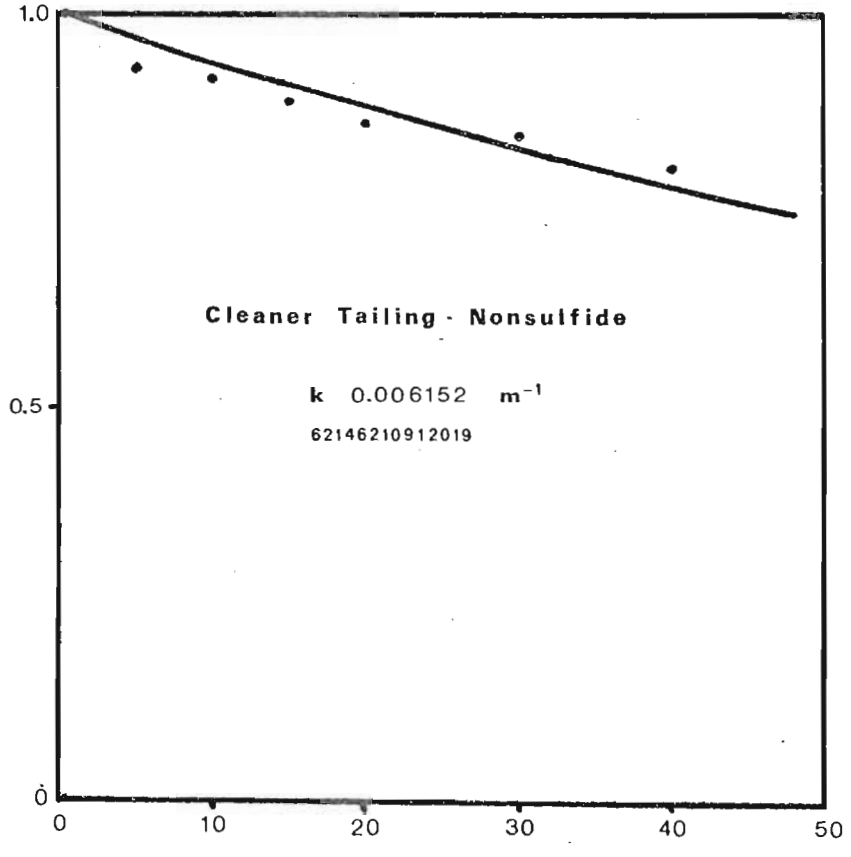




12. Batch Tests - Single Rate Model

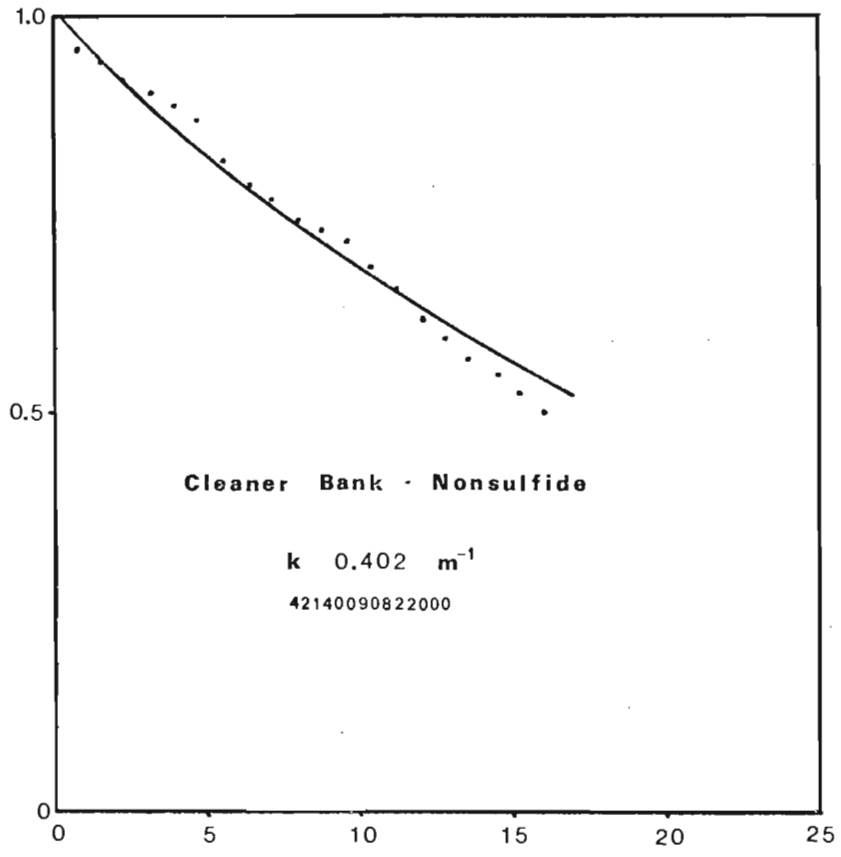
Fraction in Tailings vs Time - mins

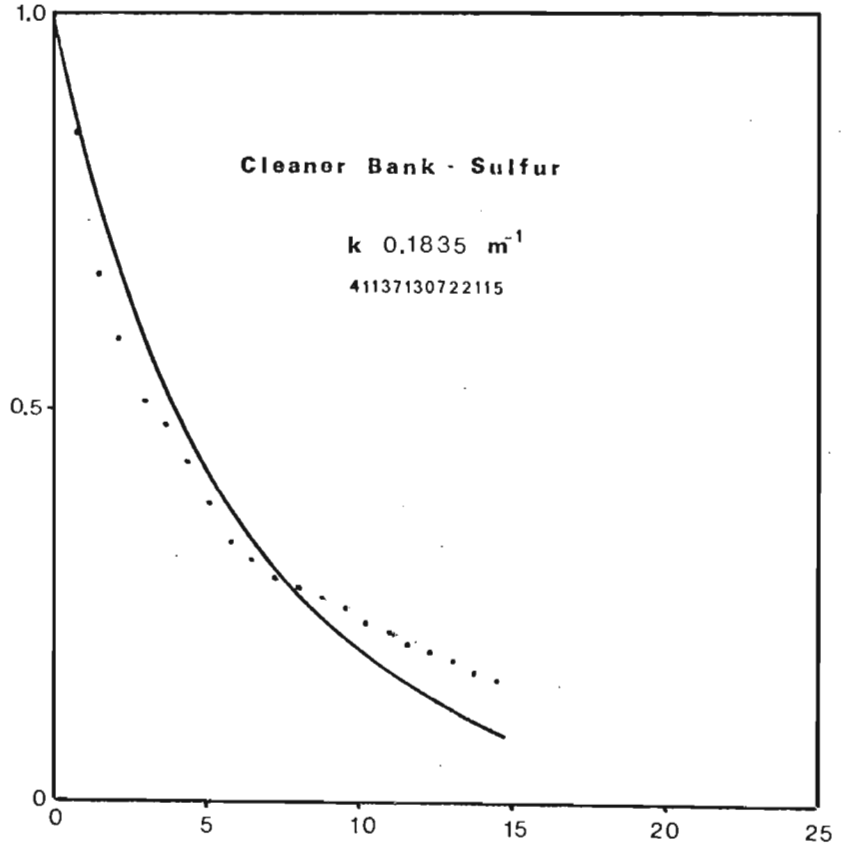




13. Batch Tests - Single Rate Model

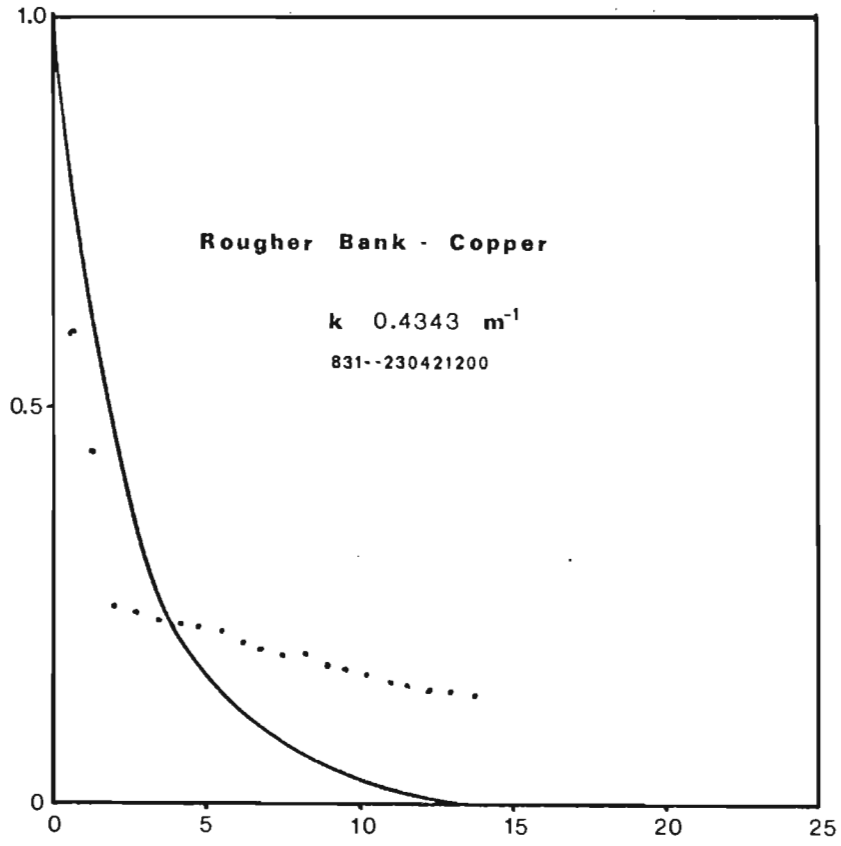
Fraction in Tailings vs Time - mins

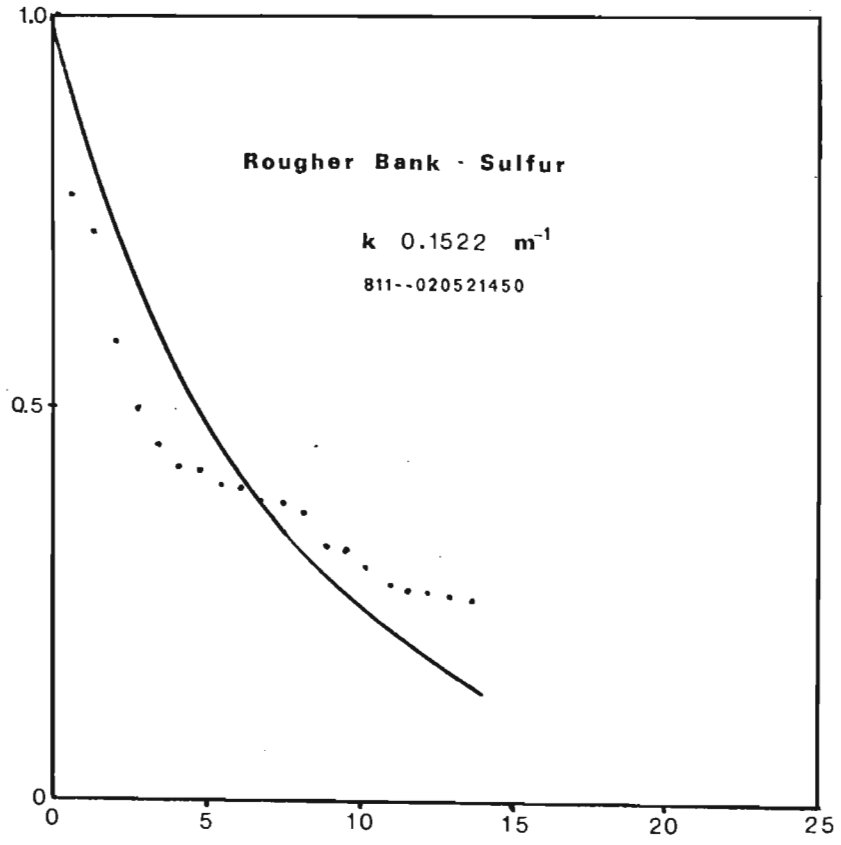




14. Continuous Tests - Single Rate Model

Fraction in Tailings vs Mean Time - mins





15. Continuous Tests - Single Rate Model

Fraction in Tailings vs Mean Time - mins

The Rate plus Non-floating Model

The rate plus non-floating first-order irreversible model analyzed at Rustenburg is a special case of the more general discretely distributed rate model proposed by Imaizumi and Inoue in 1965.[31] Difficulties with the single rate model gave rise to the understanding that flotation may well be better described by a series of rate constant values, each value governing the flotation of a fraction of the total mineralogical or metal species. Imaizumi and Inoue presented a graphical technique for estimating the first-order rate values and the fraction of material associated with that rate from a batch concentration versus time curve. Multiple linear regression may also be used to calculate the species 'k' distribution,[32] however, results vary with the discrete k values chosen and the method is highly sensitive to experimental error. The parameter estimation problems obviously become greater with increasing numbers of fractions and k values. The form of the discrete rate constant model applied at Rustenburg assumes that flotation of a species may be described by dividing the species into two fractions. One fraction has a rate constant of zero; it is non-floatable. The flotation of the remaining material may be described by a single first-order rate constant greater than zero.

Figures 16, 17 and 18 illustrate several fits of the rate plus non-floating model to the flotation of various species in a batch cell. In general the model describes batch flotation with reasonable accuracy, the non-floating fraction must be evident from the data however.

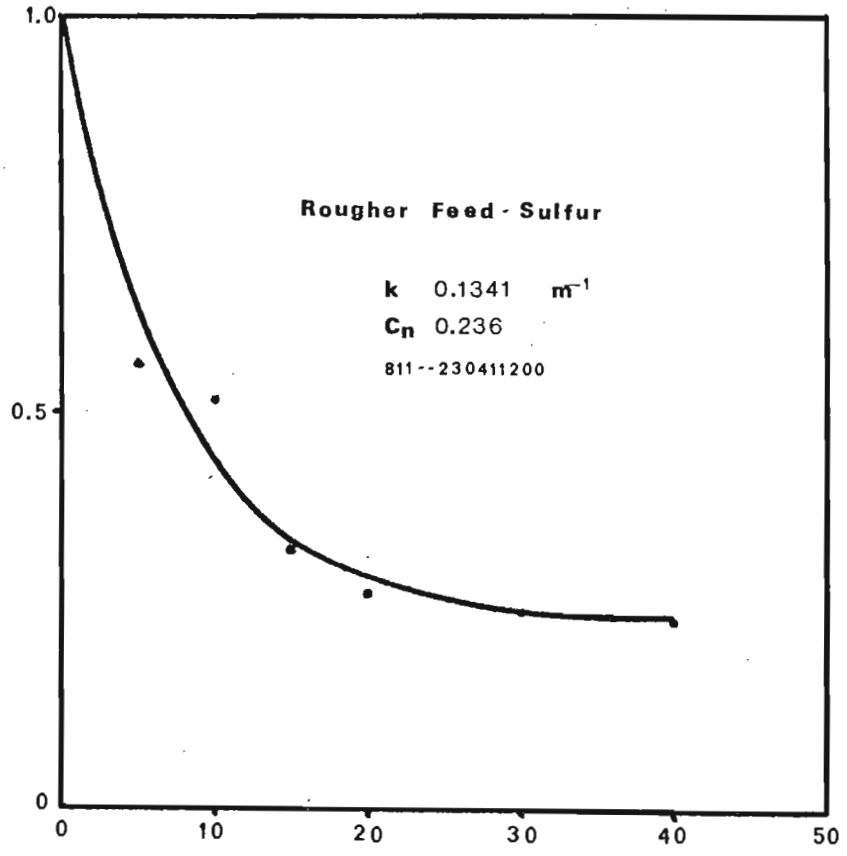
A number of continuous bank tests may also be used to test the flexibility of the rate plus non-floating model as shown in figures 19 and 20. The model structure in general is suitable but care must be exercised in using parameters regressed from

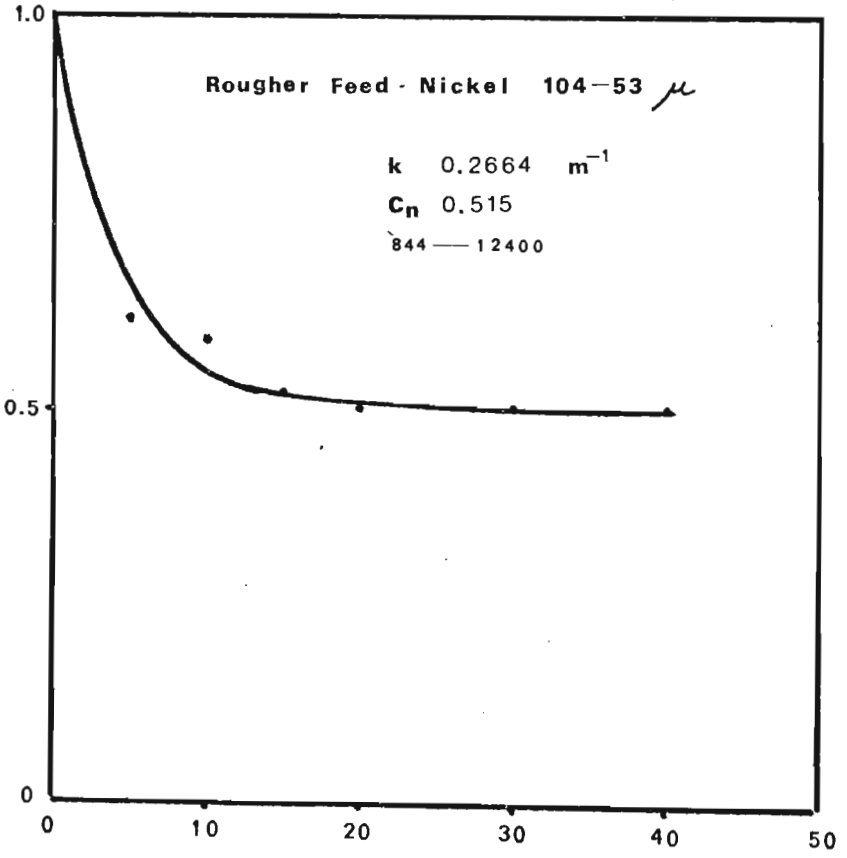
plant data. In an operating plant the flotation capacity is such that normal operation does not well define the fraction of a species with a rate constant of zero. Although the regression may fit the data well, no significance may be attached to the value of the non-floating fraction hence the consequent prediction of flotation recovery due to increases or decreases in plant capacity may be seriously in error.

A reasonable test of the rate plus non-floating model's predictive power may be analyzed through comparison of the operation of a single bank in the plant with the prediction based on kinetic parameters regressed from a batch test on the feed stream to that bank. The operation of a continuous bank may be predicted by the model through the residence time distribution. However, the model structure does not absorb any differences in machine characteristics such as impeller speed, bubble distribution, volume to surface ratios, etc. Figures 21, 22 and 23 illustrate the predictive capacity of the model and Table IV summarizes all the available data of this nature. Two problems are evident in predictions of this type; firstly, the froth removal rate during the batch test can greatly affect the accuracy of the prediction of continuous operation, this is particularly true of material which contains a high proportion of rapidly floating material as in the cleaner section at Rustenburg; secondly, the flotation of the total sulphide material appears to be equipment-specific in that predictions for the total sulphide material are seriously in error, on the other hand predictions for copper and nickel material are reasonably accurate. The sulphides at Rustenburg are primarily chalcopyrite, pentlandite and pyrrhotite; pyrrhotite is easily oxidized in the milling circuit thus its floatability may be more susceptable to agitation intensity and the scrubbing action in various types of flotation equipment and lead to the poor predictions for total sulphide material.

The assumption of the rate plus non-floating model that the first-order rate constants are time-invariant was tested by several pairs of batch tests on the rougher feed and rougher concentrate streams. In tests of this nature the froth was removed rapidly so as to minimize its effect on the kinetic parameters. For this testwork a sample of the feed to one of the four identical rougher banks was floated in a batch cell, immediately after this test was completed, a sample of the rougher concentrate stream (from all four rougher banks) was collected and floated in a batch cell. The regressed parameters of the rougher feed may be used to generate the theoretical parameters of the floated rougher concentrate material and this may be compared with the actual batch test flotation of the rougher concentrate stream. Figures 24 and 25 illustrate the ability of the model to predict the flotation of material in the rougher concentrate, theoretically this material will contain nothing of the non-floatable fraction in the rougher feed and will float at the same rate as the floatable fraction of the rougher feed. Table V contains a summary of test information for the rate plus non-floating model; the regressed values of the first-order rate constant and the fraction of non-floatable material are shown for both streams; also shown is the regressed rate constant for the concentrate stream where the non-floatable fraction is held at zero. Theoretically, these simple single rate constants of the rougher concentrate will equal the rate constant of the floatable fraction of the rougher feed. In general, the material floats at a faster rate in the rougher concentrate batch test than indicated by the rougher feed batch test, particularly so in the finer size fractions (less than 53 microns). As indicated in the table, the majority of tests were performed under normal plant conditions such that dextrin, a non-sulphide depressant, was added to the rougher concentrate before batch

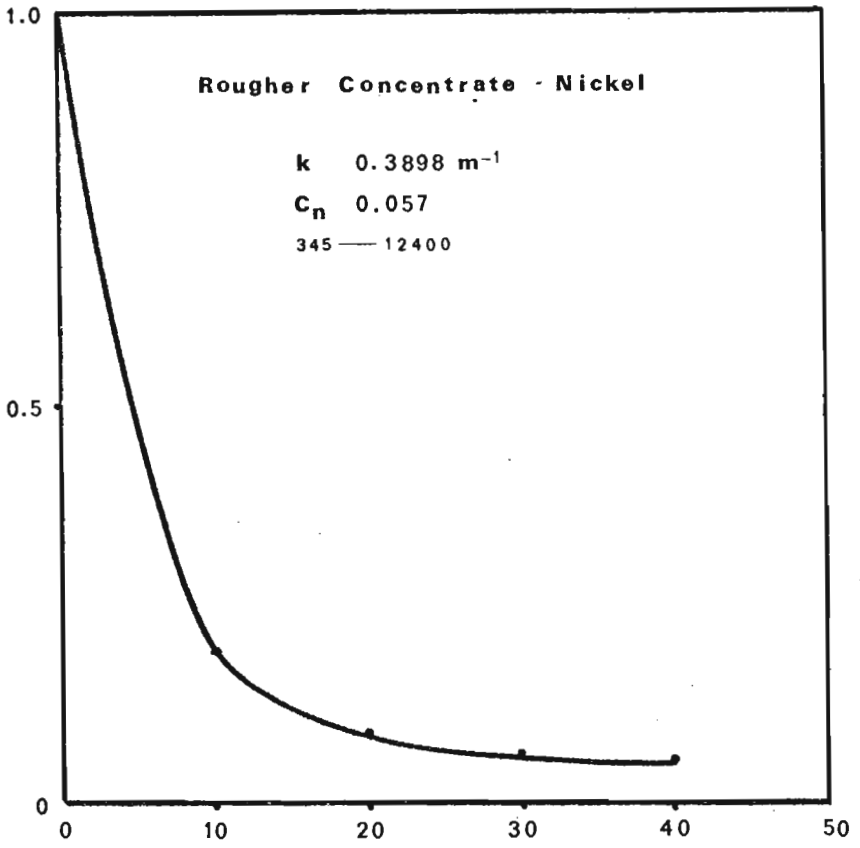
flotation. The one test performed without dextrin addition shows very similar results, however, and it must be concluded that dextrin addition does not significantly alter the floatability of the sulphide minerals.

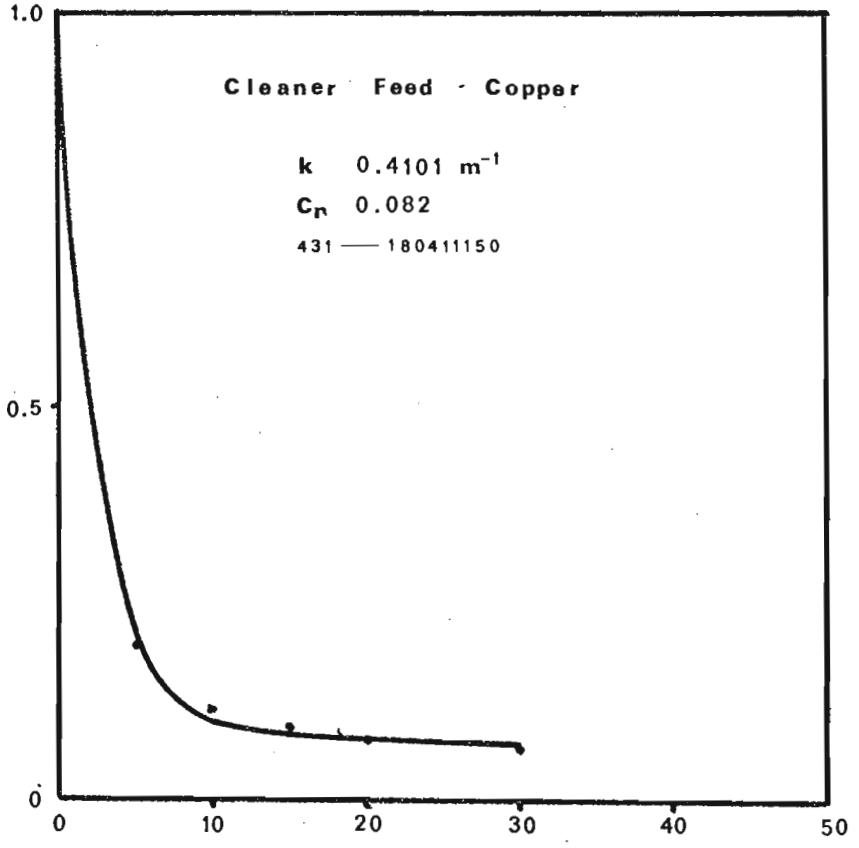




16. Batch Tests - Discrete Model

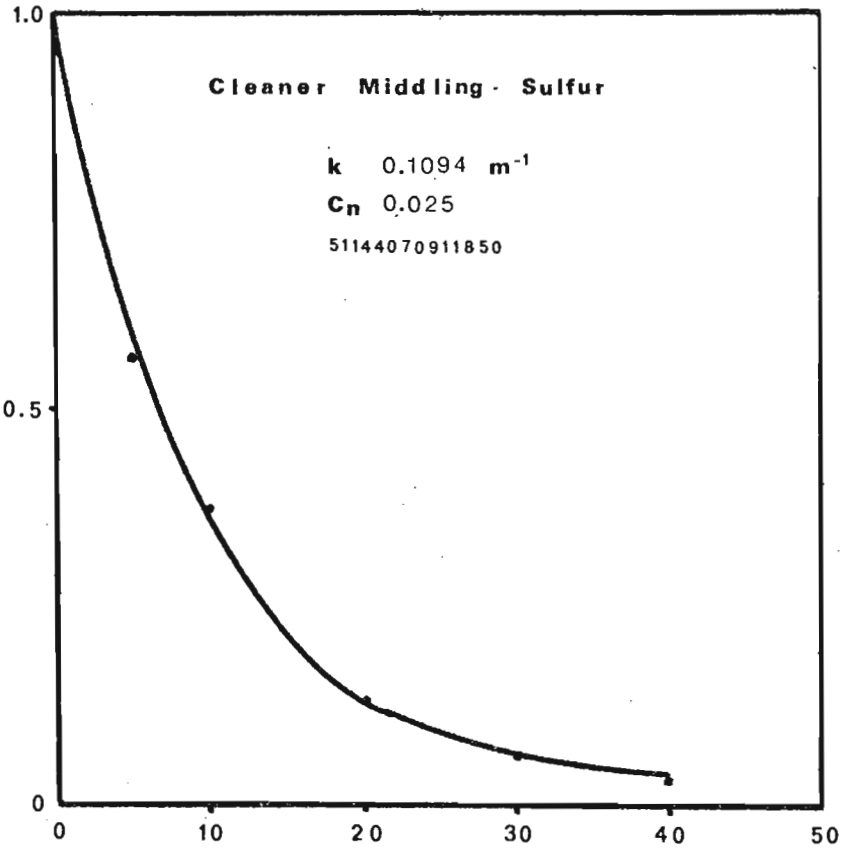
Fraction in Tailings vs Time - mins

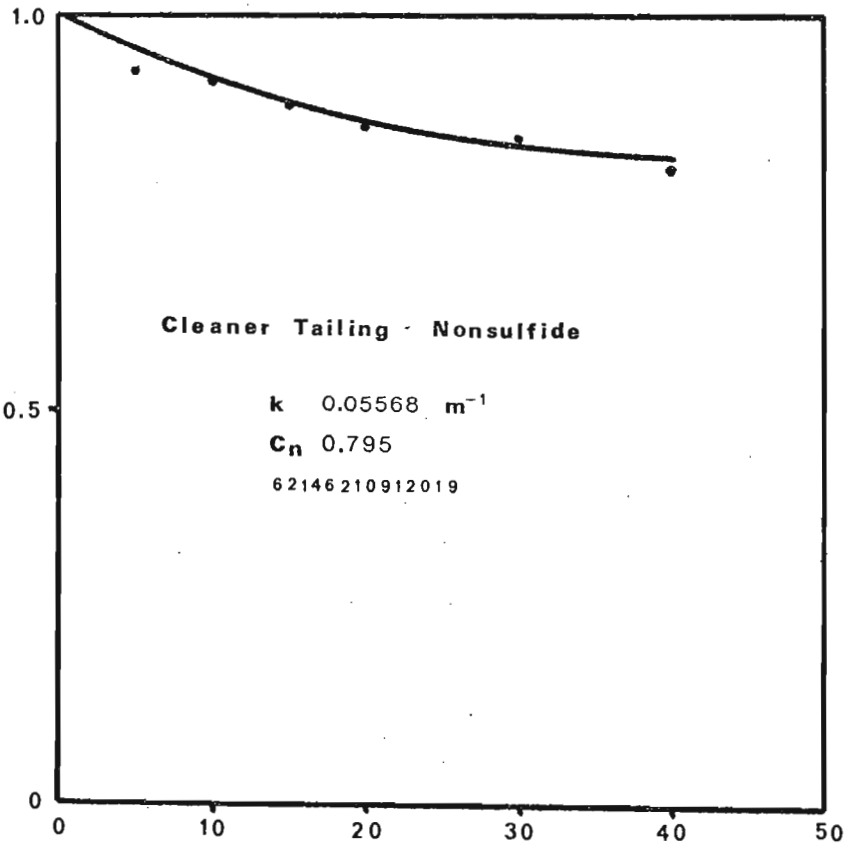




17. Batch Tests - Discrete Model

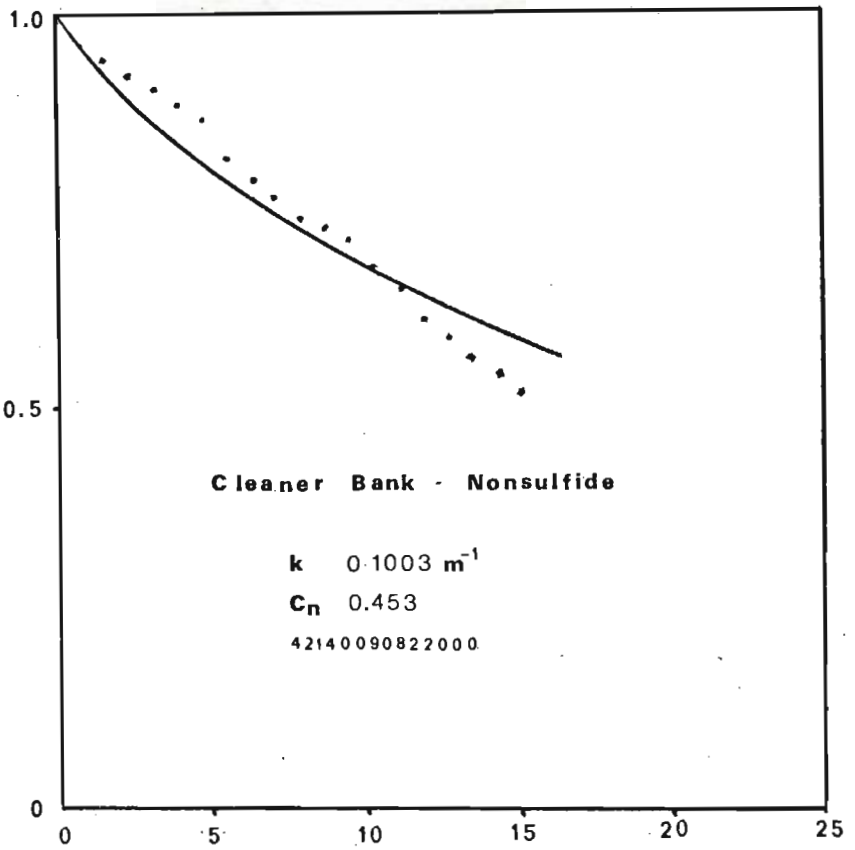
Fraction in Tailings vs Time - mins

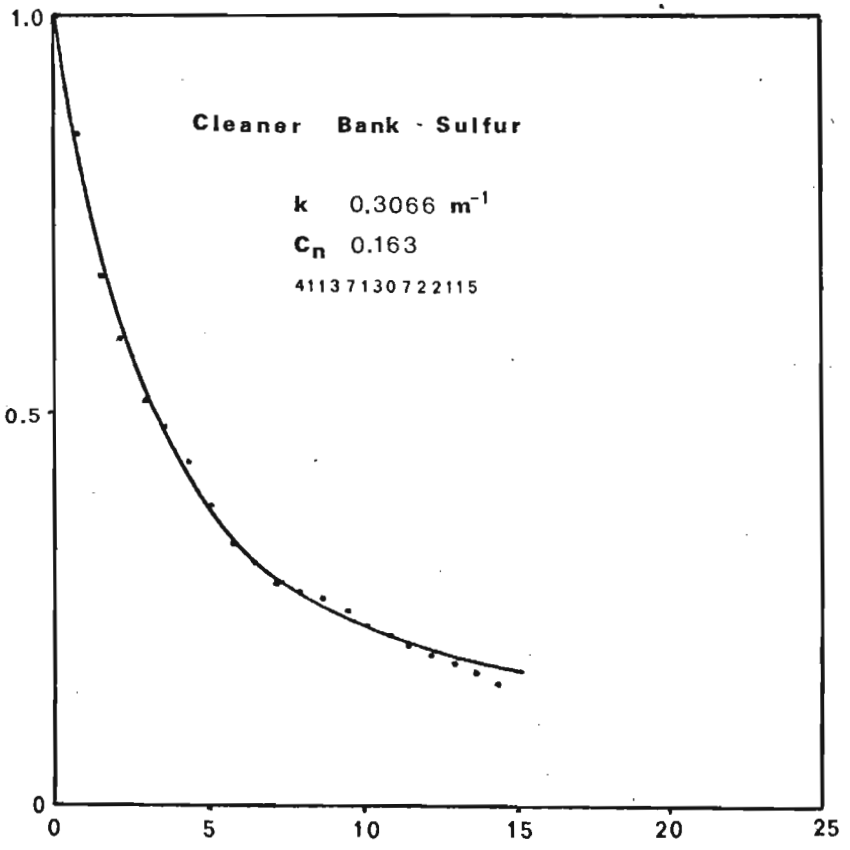




18. Batch Tests - Discrete Model

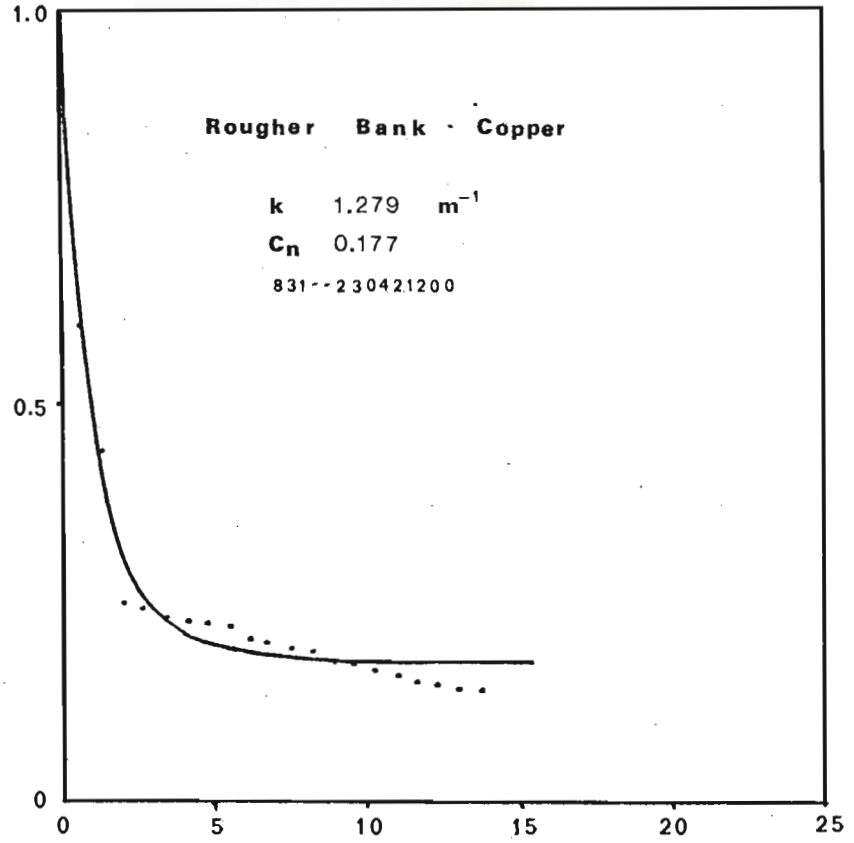
Fraction in Tailings vs Time - mins

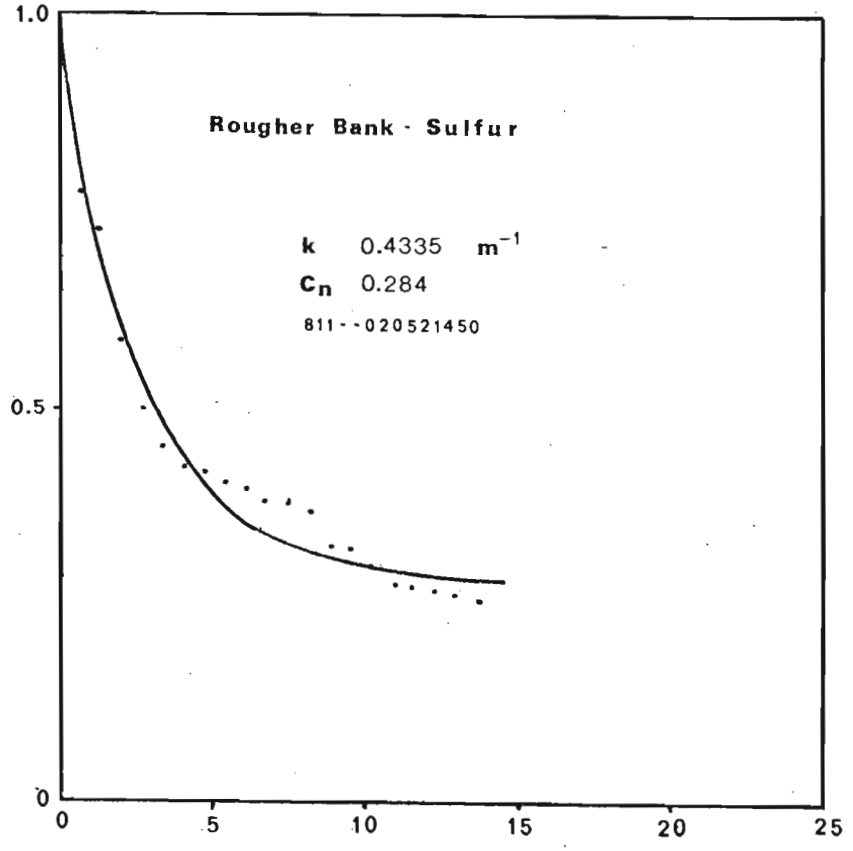




19. Continuous Tests - Discrete Model

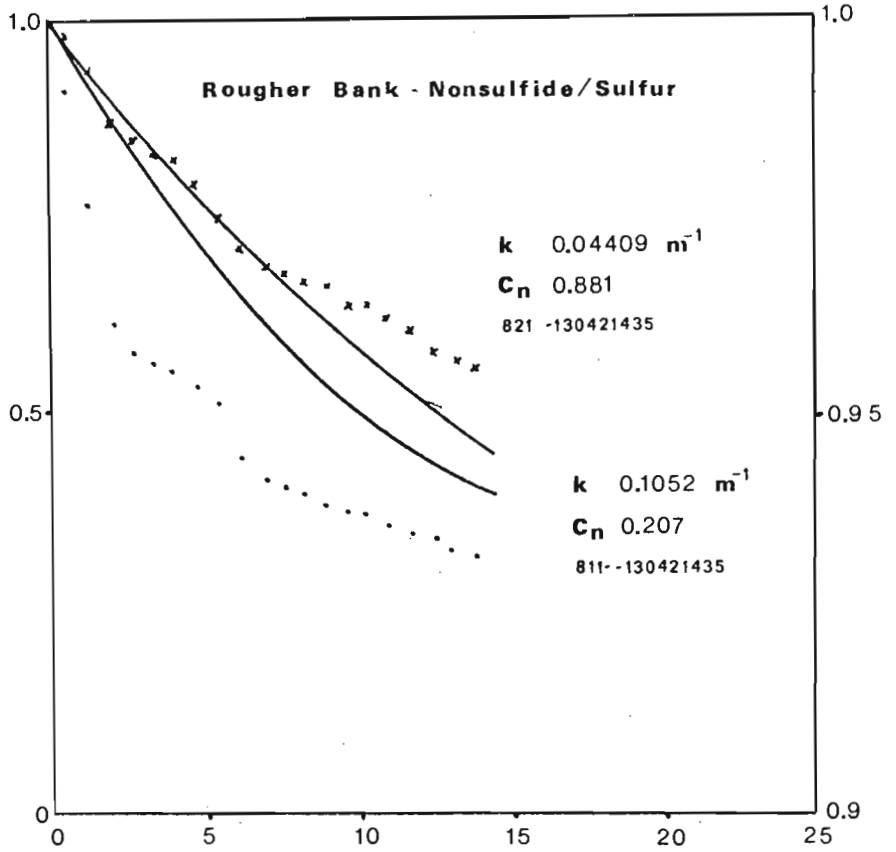
Fraction in Tailings vs Mean Time - mins

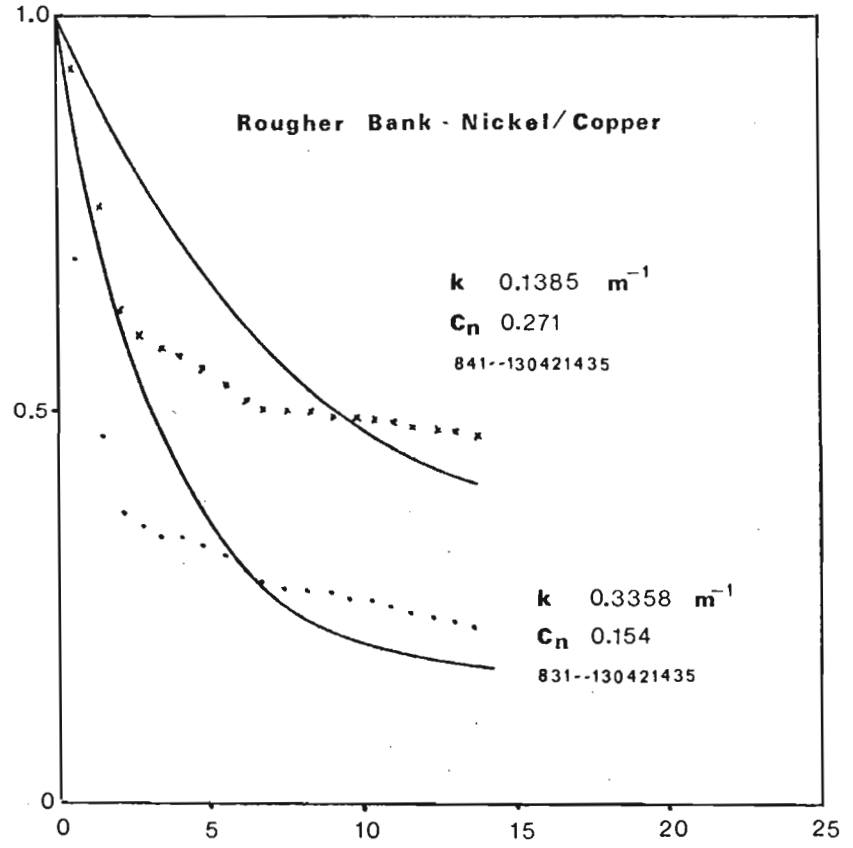




20. Continuous Tests - Discrete Model

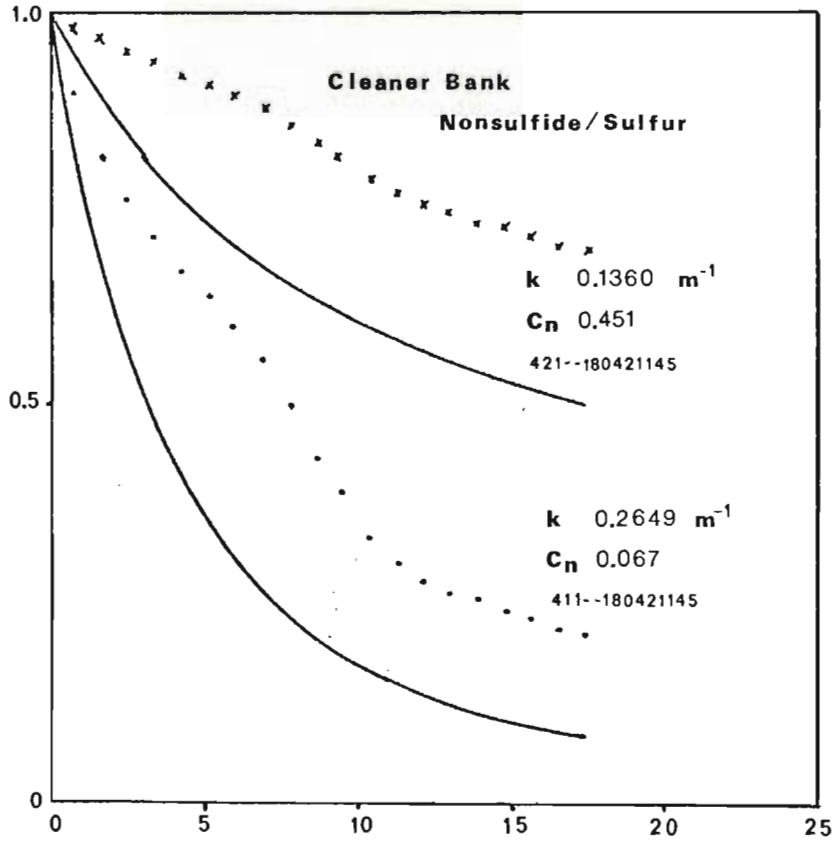
Fraction in Tailings vs Mean Time - mins

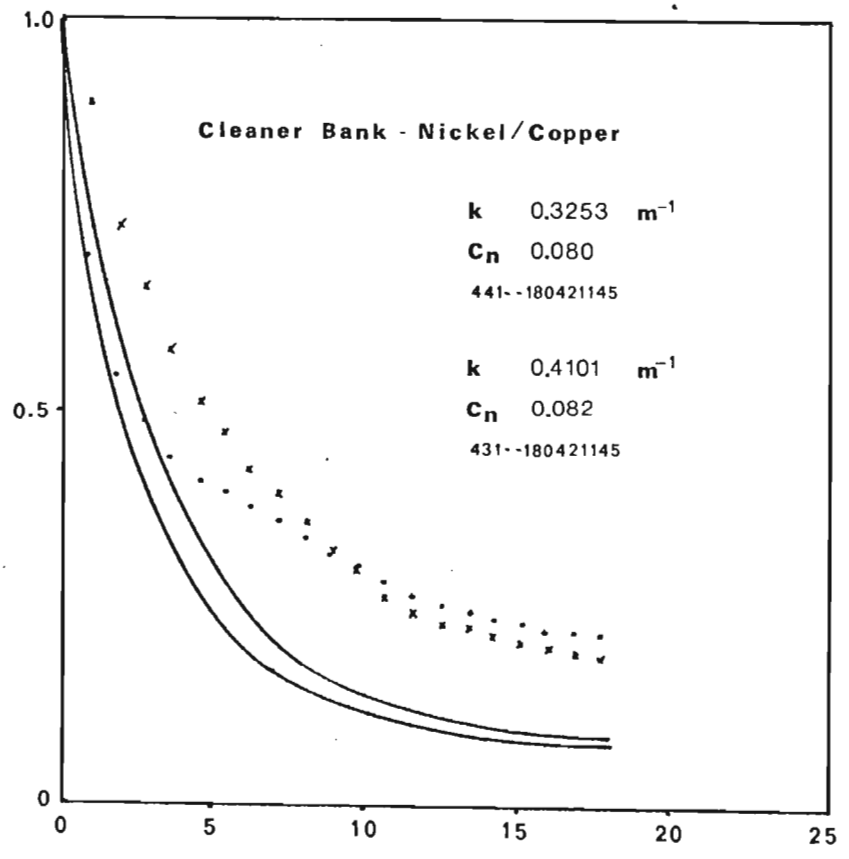




21. Predicted Continuous Operation - Discrete Model

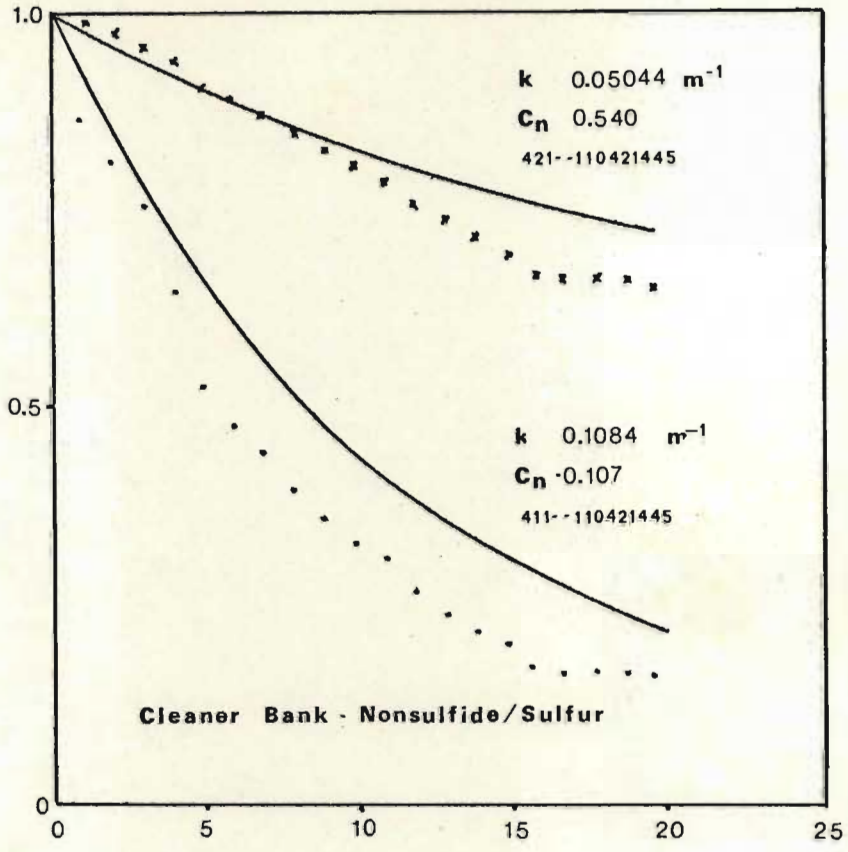
Fraction in Tailings vs Mean Time - mins

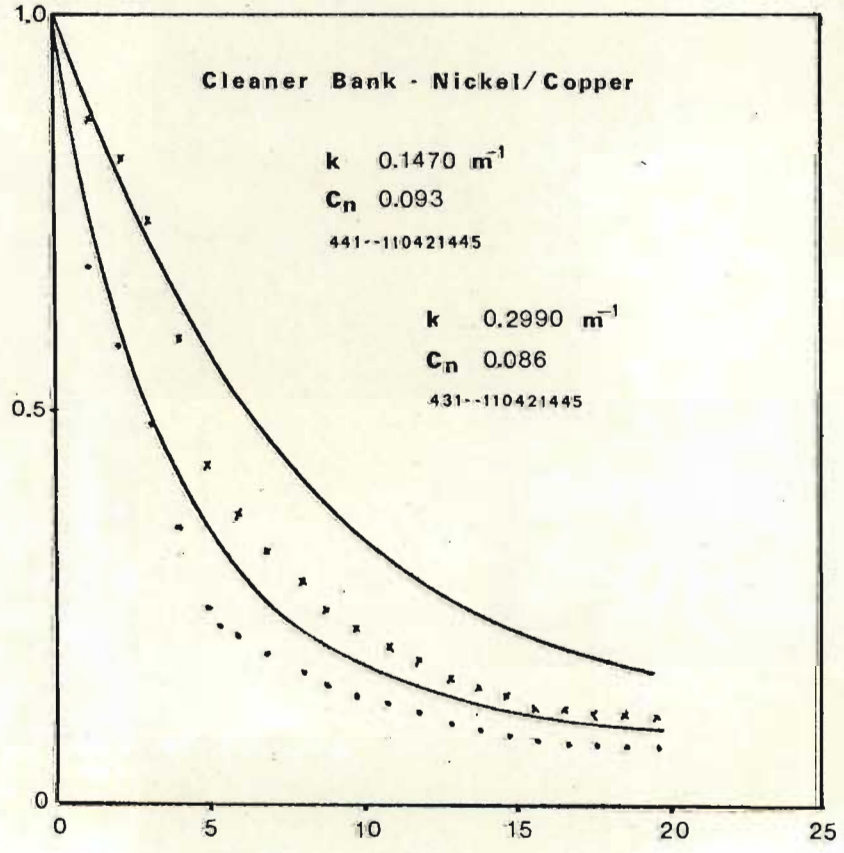




22. Predicted Continuous Operation - Discrete Model

Fraction in Tailings vs Mean Time - mins





23. Predicted Continuous Operation - Discrete Model

Fraction in Tailings vs Mean Time - mins

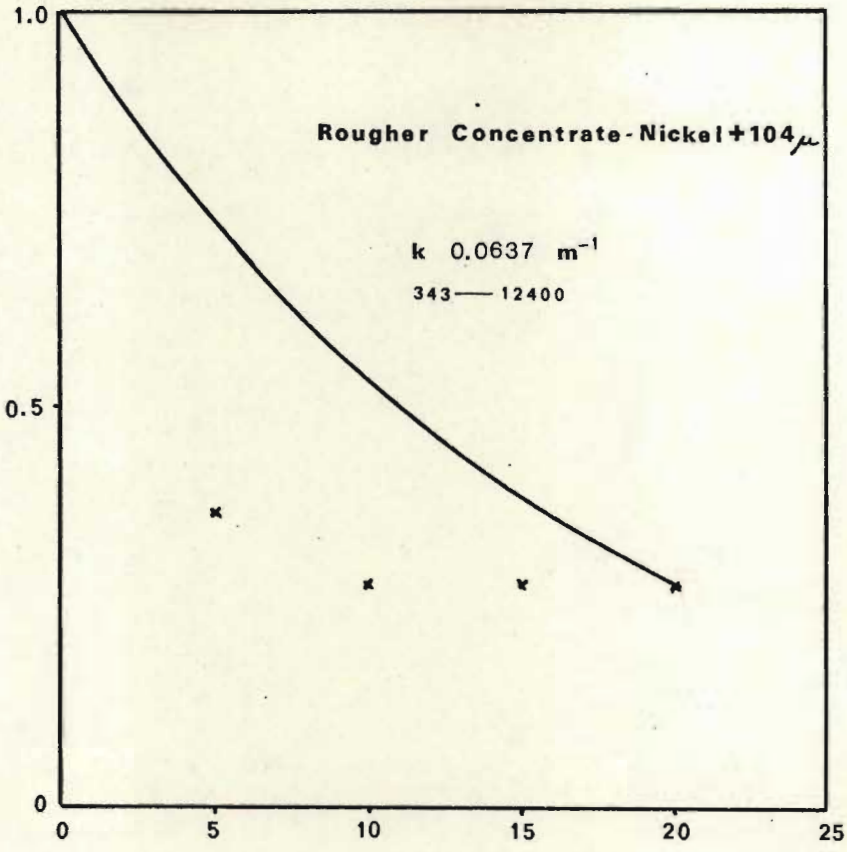
TABLE IV. BATCH TEST PREDICTION OF CONTINUOUS OPERATION - RATE PLUS NON-FLOATING MODEL

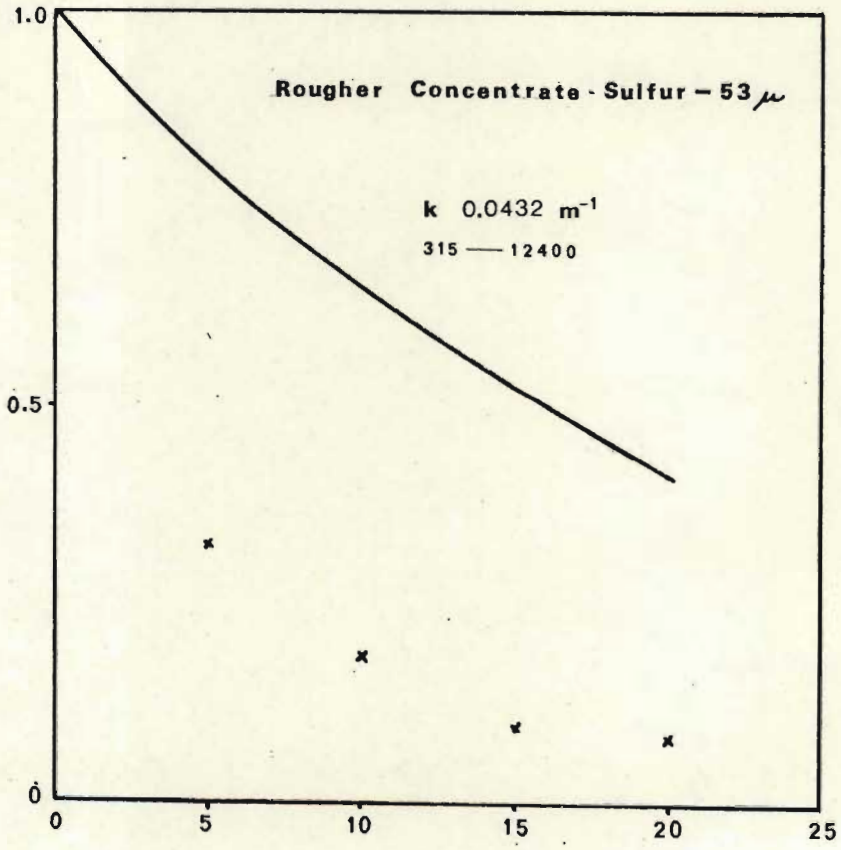
MATERIAL AND TEST	BATCH TEST CODE	CONTINUOUS TEST CODE	BATCH TEST PARAMETERS			TOTAL SQUARED ERROR IN CONCENTRATION PREDICTION
			Rate Constant min^{-1}	Non-floating Fraction	Sum of Squares	
Rougher Feed						
Sulphur	811--130411435	811--130421435	0,1052	0,207	0,01024	0,4635
Non-sulphide	821 "	821 "	0,04409	0,881	0,1763E-3	0,4173E-3
Copper	831 "	831 "	0,3358	0,154	0,5862E-2	0,2138
Nickel	841 "	841 "	0,1385	0,271	0,3432E-2	0,1773
Rougher Feed						
Sulphur	811--230411200	811--230421200	0,1341	0,236	0,01155	0,6742
Non-sulphide	821 "	821 "	0,05667	0,885	0,1274E-3	0,4503E-3
Copper	831 "	831 "	0,3452	0,179	0,6298E-2	0,5158
Nickel	841 "	841 "	0,2219	0,294	0,5120E-2	0,3615
Rougher Feed						
Sulphur	811--020511450	811--020521450	0,1147	0,292	0,9535E-2	1,188
Non-sulphide	821 "	821 "	0,05741	0,903	0,4222E-4	0,8002E-3

TABLE IV. (CONTINUED)

MATERIAL AND TEST	BATCH TEST CODE	CONTINUOUS TEST CODE	BATCH TEST PARAMETERS			TOTAL SQUARED ERROR IN CONCENTRATION PREDICTION
			Rate Constant min^{-1}	Non-floating Fraction	Sum of Squares	
Cleaner Feed						
Sulphur	411--110411445	411--110421445	0,1084	0,107	0,2666E-2	0,1863
Non-sulphide	421 "	421 "	0,05044	0,540	0,1059E-3	0,04503
Copper	431 "	431 "	0,2990	0,086	0,4631E-2	0,04354
Nickel	441 "	441 "	0,1470	0,093	0,1261E-2	0,1518
Cleaner Feed *						
Sulphur	411--180411150	411--180421145	0,2649	0,067	0,1529E-3	0,9115
Non-sulphide	421 "	421 "	0,1360	0,451	0,6999E-4	0,6161
Copper	431 "	431 "	0,4101	0,082	0,6536E-3	0,4653
Nickel	441 "	441 "	0,3253	0,080	0,6576E-3	0,4212
Cleaner Feed						
Sulphur	411--300411430	411--300421430	0,1884	0,181	0,3180E-4	0,01434
Non-sulphide	421 "	421-- "	0,1081	0,566	0,2706E-4	0,8882E-2

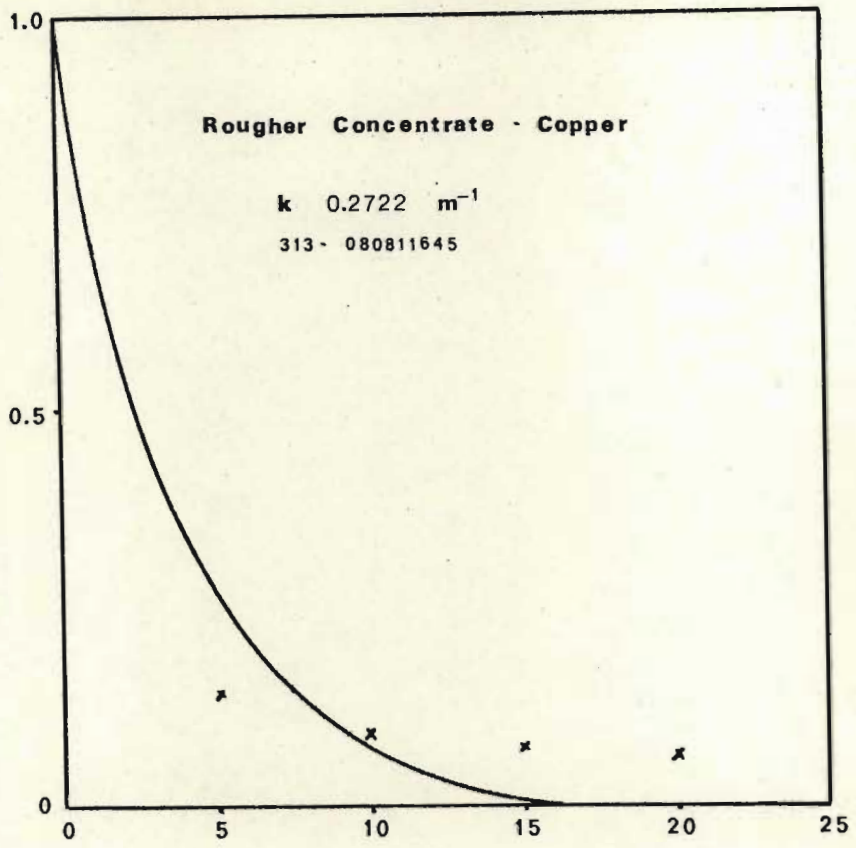
* This batch test was performed with a higher rate of froth removal than the others.

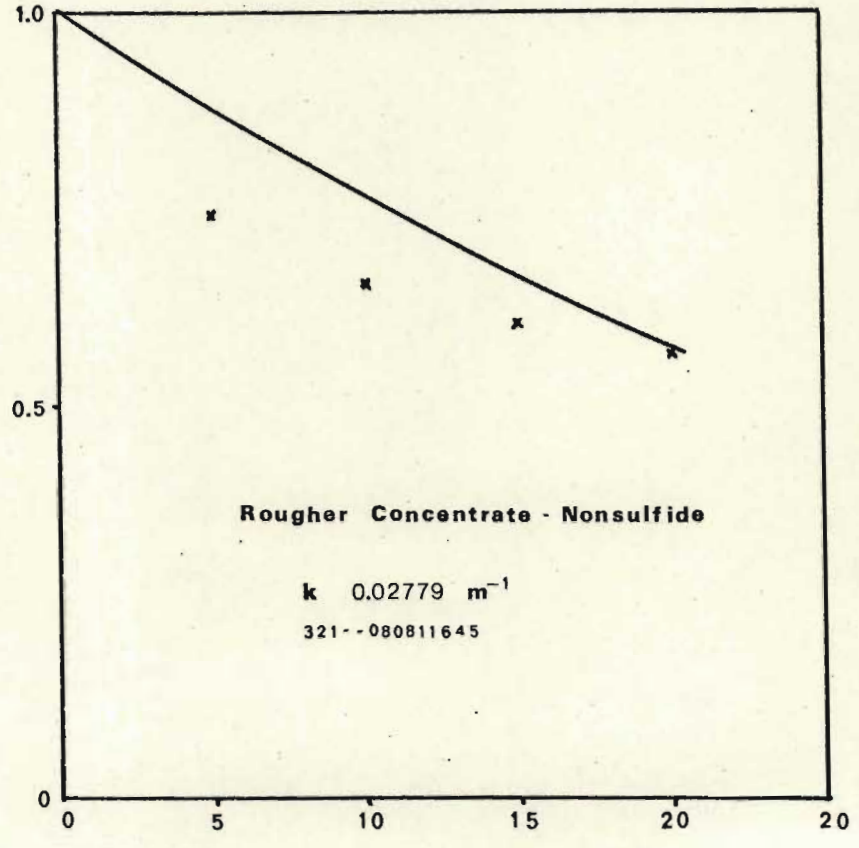




24. Predictions for Refloated Material - Discrete Model

Fraction in Tailings vs Time - mins





25. Predictions for Refloated Material - Discrete Model

Fraction in Tailings vs Time - mins

TABLE V. PREDICTIONS FOR REFLOATED MATERIAL - RATE PLUS NON-FLOATING MODEL

Material	Rougher Feed Test Code	Rougher Feed Regressed Parameters		Rougher Concentrate Test Code	Rougher Concentrate Regressed Parameters			Total Squared Error in Predicted Concentrati
		Rate Constant min^{-1}	Non-floating		Rate plus Non-Floating		Single Rate	
					Rate Constant (min^{-1})	Non-floating Fraction		
phur	811--080311935	0,1376	0,286	311--080312055	0,2353	0,084	0,1793	0,02660
	811--010311635	0,1968	0,362	311--010311800	0,4037	0,135	0,2211	0,03522
	811--220211920	0,1097	0,151	311--220212045	0,3628	0,097	0,2449	0,1644
	811--200211810	0,1524	0,208	311--200211945	0,3073	0,051	0,2554	0,06092
	811--140211805	0,1245	0,222	311--140211930	0,3668	0,074	0,2723	0,1396
phur (+208 microns)	812------12400	0,1322	0,861	312------12400	0,4759	0,398	0,0747	0,1967
(208-104 microns)	813 "	0,1238	0,597	313 "	0,2867	0,266	0,1058	0,05781
(104-53 microns)	814 "	0,1422	0,563	314 "	0,3215	0,145	0,1821	0,04375
(-53 microns)	815 "	0,0432	0,063	315 "	0,2543	0,083	0,1912	0,7461
per (208-104 microns)	833------12400	0,2245	0,438	333------12400	0,5969	0,118	0,3084	0,04532
(104-53 microns)	834 "	0,5051	0,213	334 "	0,6544	0,048	0,4723	0,6569E-2
(-53 microns)	835 "	0,05563	0,150	335 "	0,5938	0,029	0,4933	1,002
kel (208-104 microns)	843------12400	0,2424	0,815	343------12400	0,4108	0,274	0,1171	0,1772
(104-53 microns)	844 "	0,2664	0,515	344 "	0,3703	0,095	0,2517	0,01759
(-53 microns)	845 "	0,0637	0,232	345 "	0,3898	0,057	0,3073	0,6433
phur *	811--080811620	0,08672	0,123	311--080811645	0,2987	0,132	0,1798	0,1782
-sulphide *	821 "	0,02779	0,861	321 "	0,1610	0,557	0,03482	0,02925
er *	831 "	0,2722	0,241	331 "	0,5176	0,072	0,3563	0,2049
el *	841 "	0,1318	0,299	341 "	0,3625	0,106	0,2339	0,08949

* During these tests no dextrin was added to the rougher concentrate.

The Gamma Function Model

The gamma function model is inherently better equipped to describe the flotation of a mineralogical or metal species in that its basic structure goes a step further than that of the discrete model and recognizes the individual nature of the flotation feed particles. The gamma function model assigns a continuous distribution of first-order rate constants to any species; the distribution takes the form of a modified gamma function and is completely defined by the mean ($\bar{k}$) and variance (σ^2) of the distribution.

$$W(k) = \frac{(\bar{k}/\sigma^2)^{\bar{k}^2/\sigma^2} \; k^{(\bar{k}^2/\sigma^2 - 1)} \; e^{-k\bar{k}/\sigma^2}}{\Gamma(\bar{k}^2/\sigma^2)}$$

Any number of algebraic continuous distributions of the first-order rate constants will accurately fit flotation batch test data, a bi modal distribution has been suggested by Harris and Chakavarti.[33] The advantage of the modified gamma function lies in that it can describe a wide variety of batch test curves with only two parameters defining the distribution; thus it is flexible and simple.

The gamma function model because of its continuous distribution of first-order rate constants, may be expected to accurately describe batch tests of any material and should perform as well if not better in predicting continuous plant operation from batch data kinetic parameters. A continuous distribution of rate constants may be expected to perform better in describing the reflotation of a species as well; however, like the other single phase models, the gamma function model will lack

the ability to handle the froth phase.

From the large number of batch tests performed, several species of flotation curves and the corresponding fits of the gamma function model are presented in Figures 26, 27 and 28. The gamma function model accurately describes batch flotation of any species varying from non-sulphide (very slow floating material) to copper (very fast floating material). Even when a large fraction of non-floating material is evident the model fit is accurate. The rate constant distribution commonly takes one of two forms shown in Figure 29. When $\bar{k}^2/\sigma^2 - 1$ is negative, form A results and is indicative of a large fraction of non-floatable material. In the case of a species of this type it would be more theoretically correct to assign a non-floatable fraction to the species and describe the floatable fraction by the modified gamma function distribution. In example, for non-sulphide material it is obvious that some fraction is non-floatable and hence to describe the flotation of the whole non-sulphide species by a distribution of rate constants very near to zero is needlessly unrealistic.

Although the gamma function model describes continuous flotation by an integral without an analytical solution, i.e.

$$\alpha_j = \int_0^\infty \frac{W(k)\,dk}{(1+g)^j}$$

best fit parameters may be obtained by using Gauss-Laguerre quadrative methods and non-derivative search methods. Figures 30 and 31 illustrate the fit of the gamma function model to continuous data. The model form has some difficulty in fitting the combination of extremely fast and very slow floating material as in the case of copper material in the rougher feed;

other materials are handled accurately however.

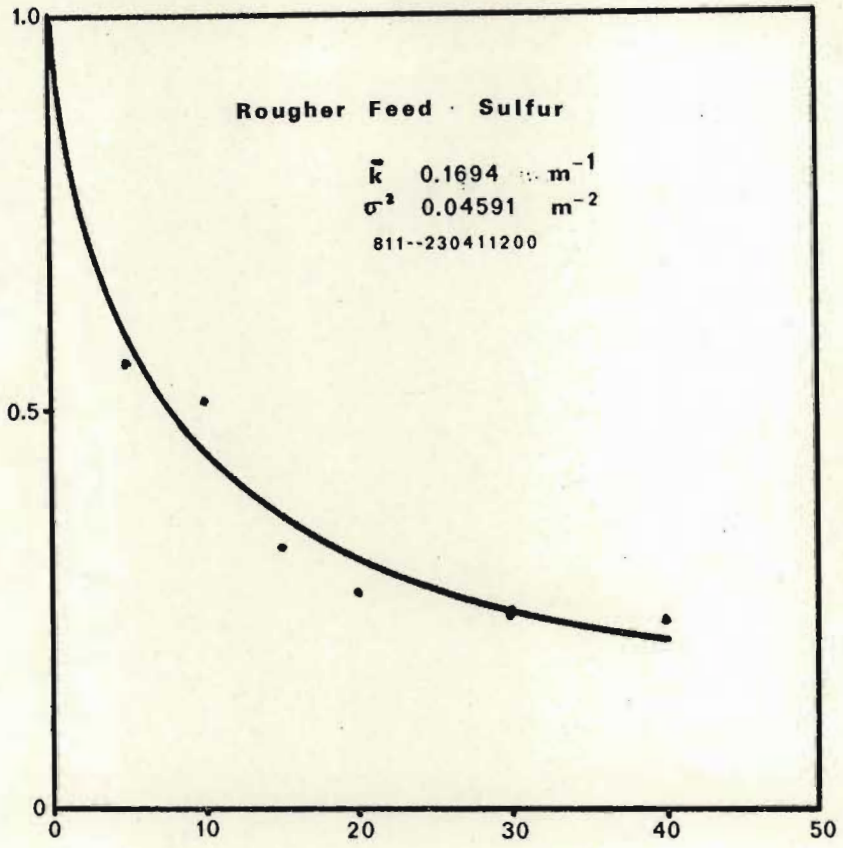
As with the discrete model, the gamma function model may be used to predict the operation of a continuous flotation bank from the kinetic parameters regressed from a batch test of the feed stream to that bank. Figures 32, 33 and 34 illustrate the predictive capacity of the model and Table VI summarizes all of the data of this nature. Again as with the discrete model, the problem of froth removal and the fact that the rate constants are equipment-specific, are both evident. In the majority of cases, however, the gamma function model more closely describes continuous flotation than does the rate plus non-floating model. It is more accurate simply because it assigns an entire distribution of rates to the flotation feed species.

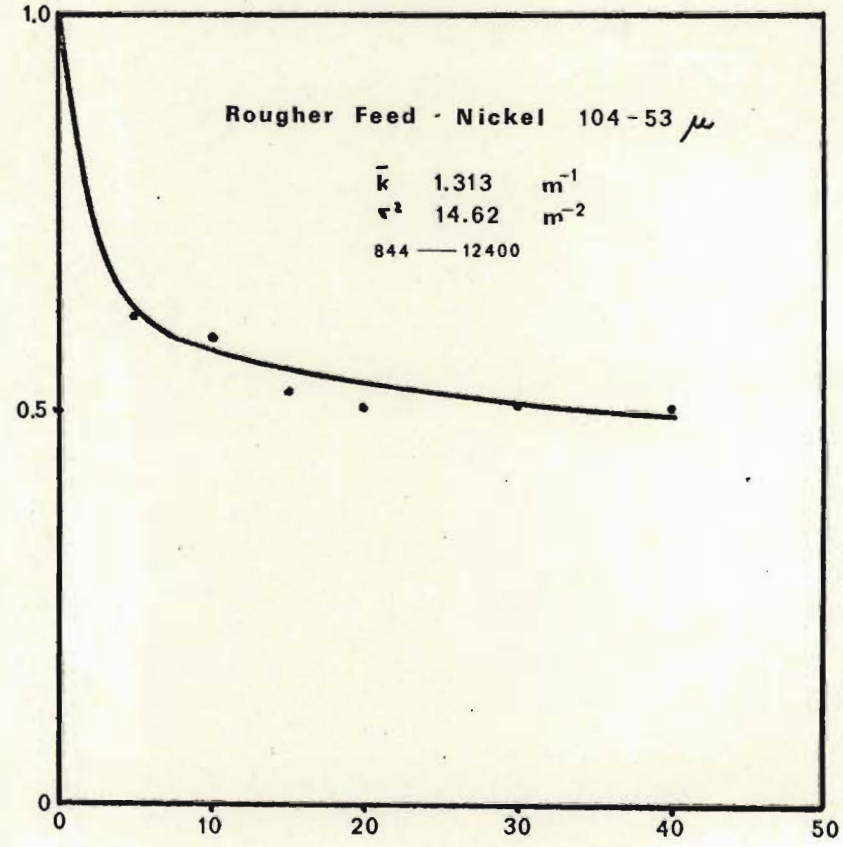
The applicability of the gamma function model must also be tested as with the rate plus non-floating model, with regard to the prediction of the reflotation of concentrate streams. The batch tests performed on the rougher feed stream and rougher concentrate stream may be analyzed. Given the kinetic parameters $\bar{k}$ and σ^2 regressed from the rougher feed batch test, the kinetic parameters for the plant rougher concentrate may be calculated.

$$\bar{K} = \frac{\int_0^\infty k\, \underline{r}^T \underline{\underline{g}}\, \underline{\underline{C}}^{-1} \underline{\underline{\varepsilon}}\, W(k)\, dk}{\int_0^\infty \underline{r}^T \underline{\underline{g}}\, \underline{\underline{C}}^{-1} \underline{\underline{\varepsilon}}\, W(k)\, dk}$$

$$\sigma^2 = \frac{\int_0^\infty (k-\bar{K})^2\, \underline{r}^T \underline{\underline{g}}\, \underline{\underline{C}}^{-1} \underline{\underline{\varepsilon}}\, W(k)\, dk}{\int_0^\infty \underline{r}^T \underline{\underline{g}}\, \underline{\underline{C}}^{-1} \underline{\underline{\varepsilon}}\, W(k)\, dk}$$

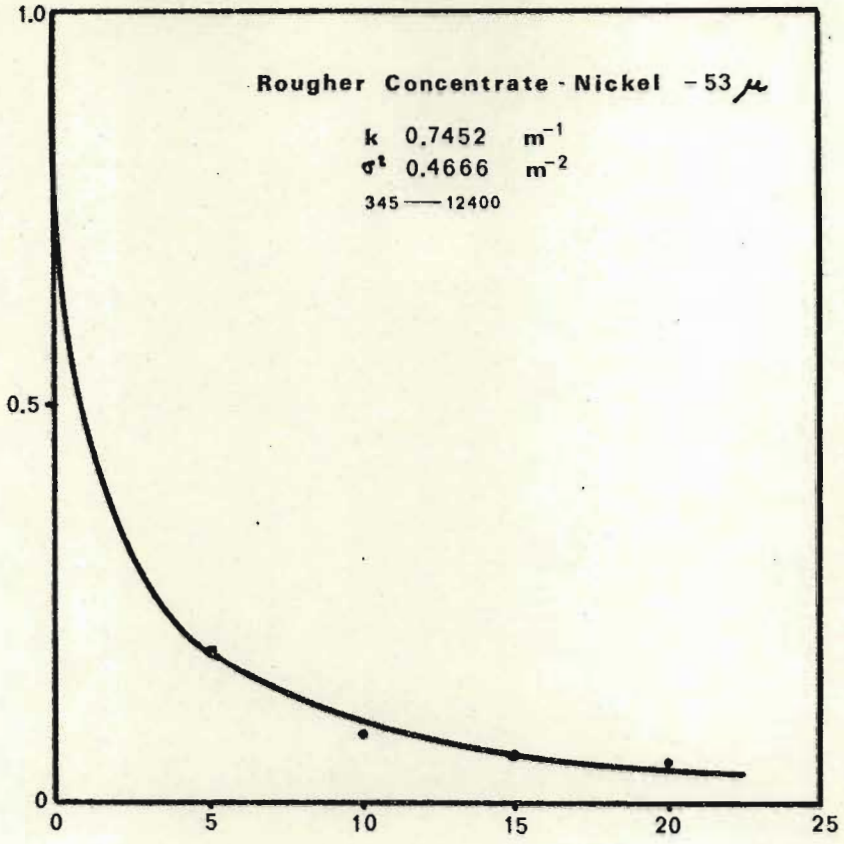
Once the kinetic parameters of the rougher concentrate stream have been calculated a predicted batch float of the rougher concentrate may be compared to the actual test results. Figures 35 and 36 illustrate the accuracy of the gamma function model in this capacity and Tables VII & VIII summarize all of the available data. In Table VIII, three parameters were used to describe the rougher feed test, two rate constant distribution parameters for the floatable material and a non-floating fraction. The addition of a third parameter here changes the calculated rate constant distribution in the rougher concentrate and improves the predictive abilities of the model, thus it may be of value to consider the gamma function rate constant distribution as descriptive of only the floatable material in a species. Again in the majority of cases, the gamma function model performs more accurately than the rate plus non-floating model. Due to the results of previous sections it might be expected that the accuracy for total sulphide material would be worse than that of copper and nickel, generally speaking this is the case. In total, predictions are still poor and especially so for material smaller than 53 microns.

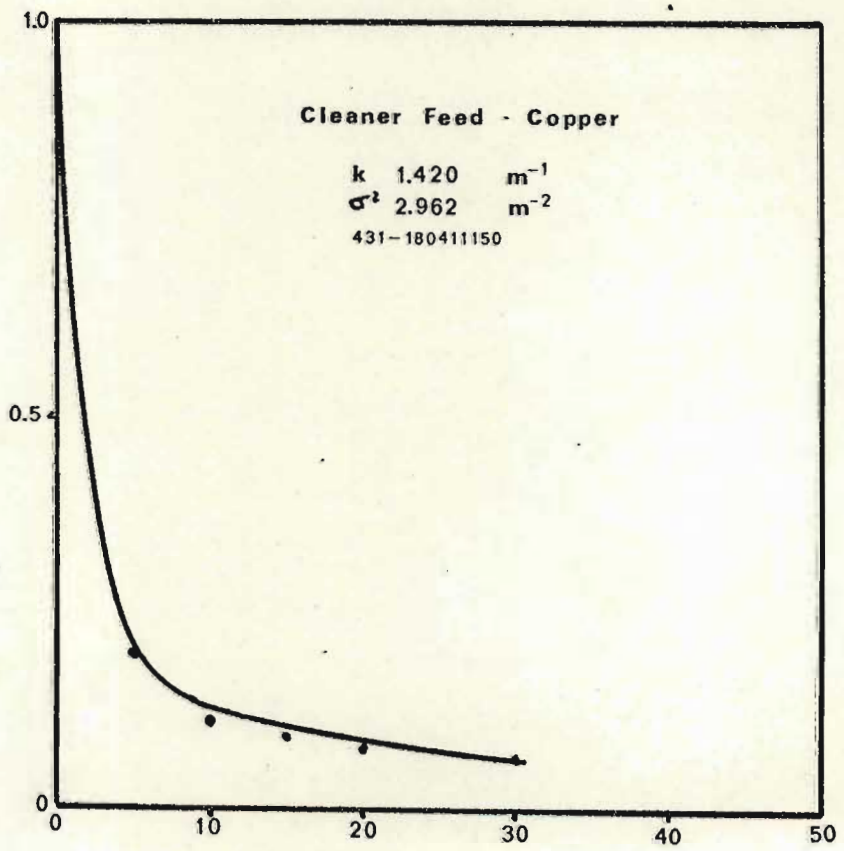




26. Batch Tests · Gamma Function Model

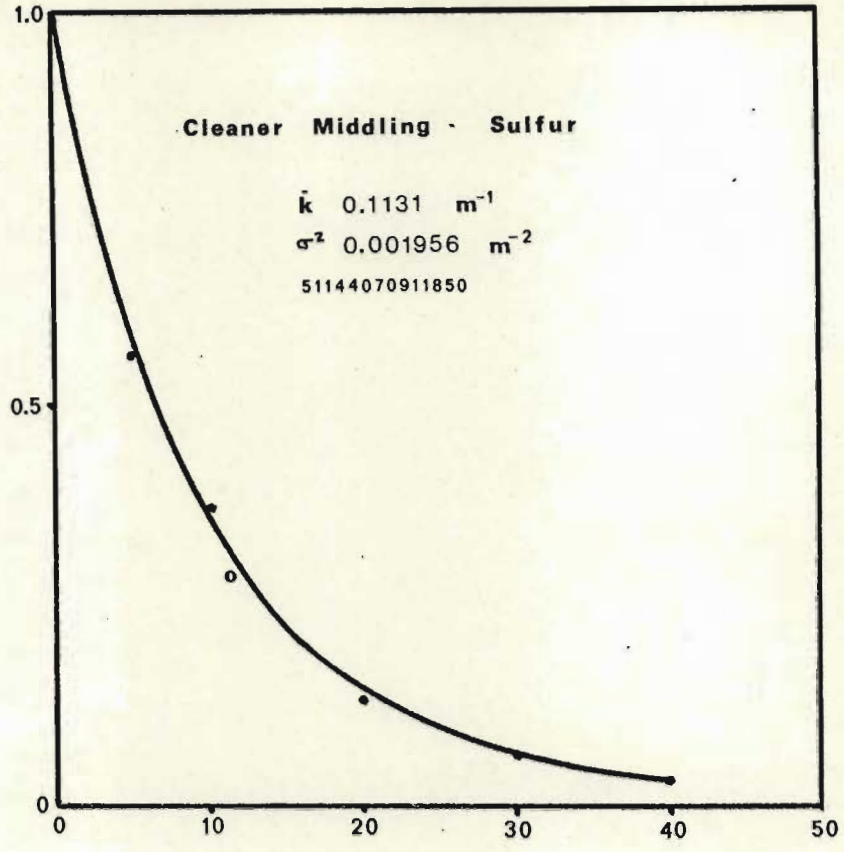
Fraction in Tailings vs. Time · mins

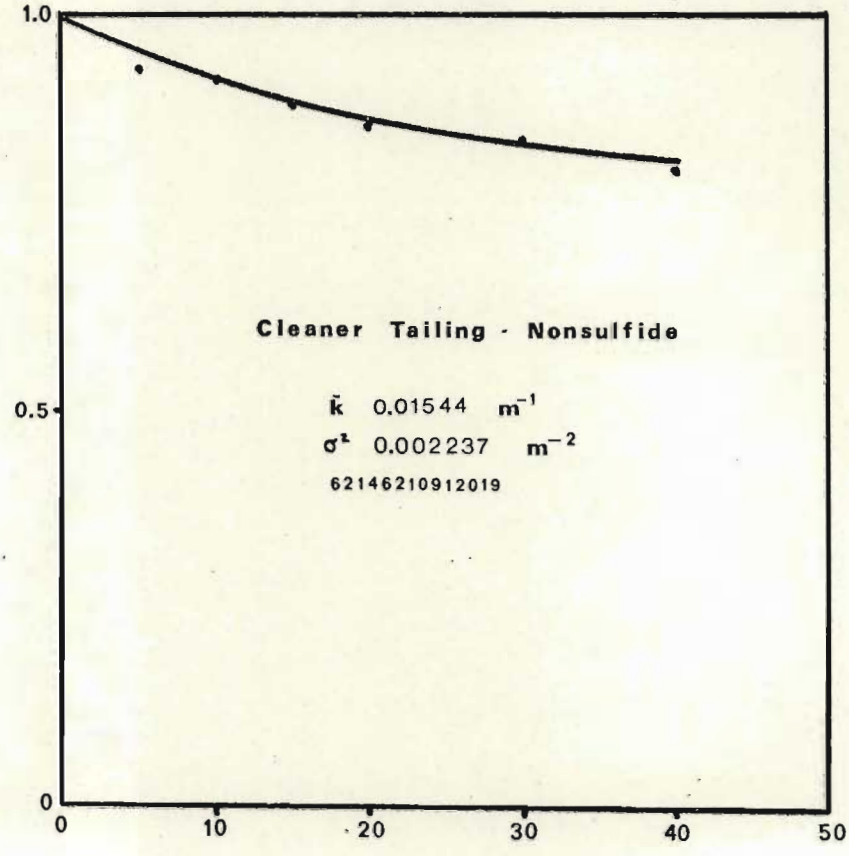




27. Batch Tests - Gamma Function Model

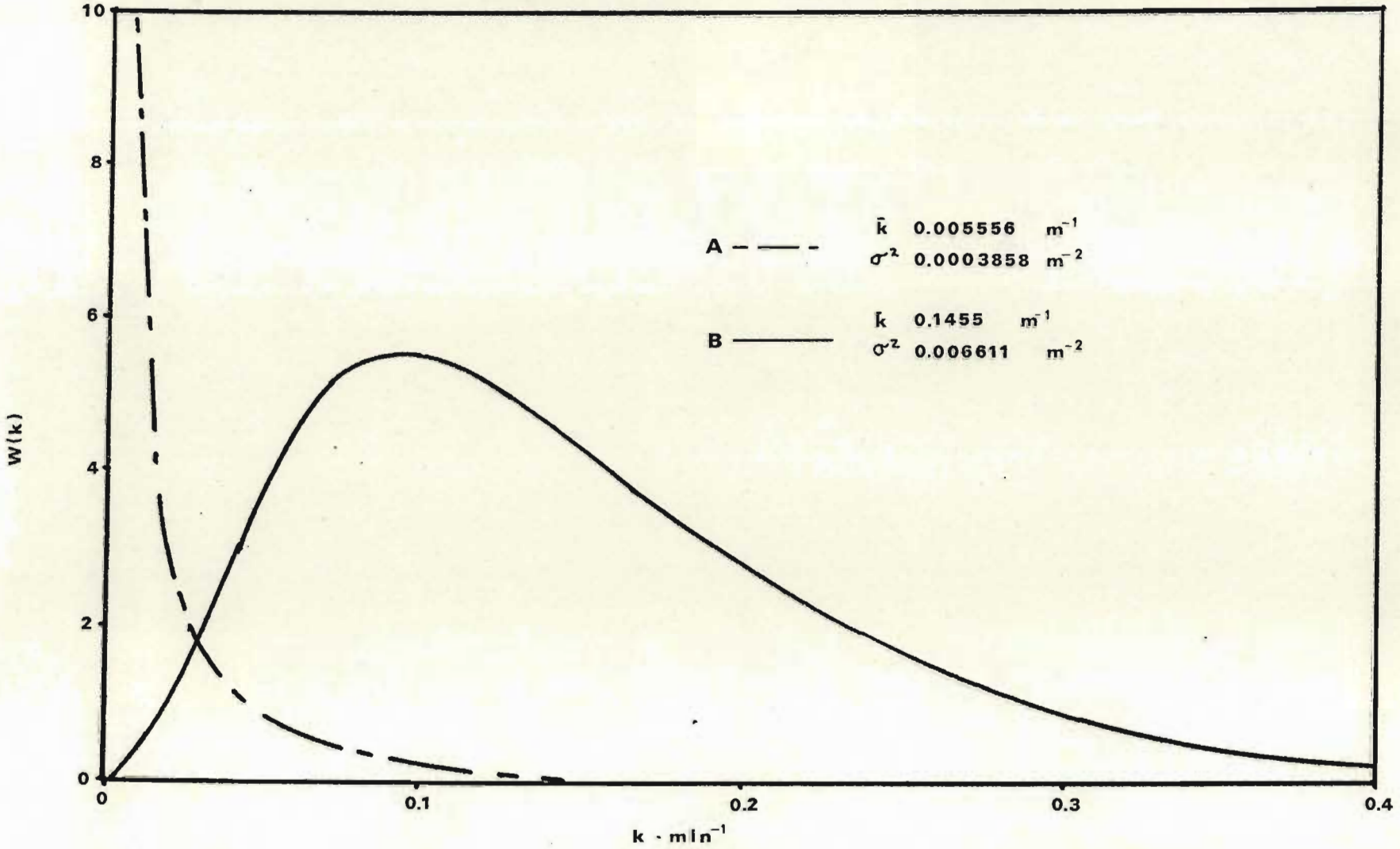
Fraction in Tailings vs Time - mins



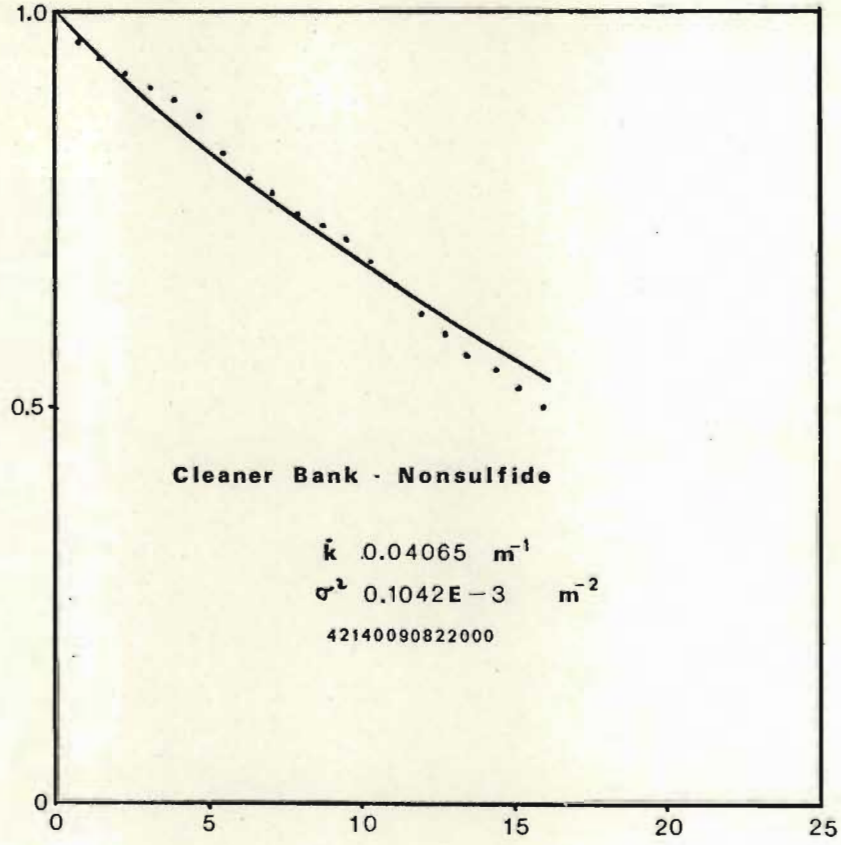


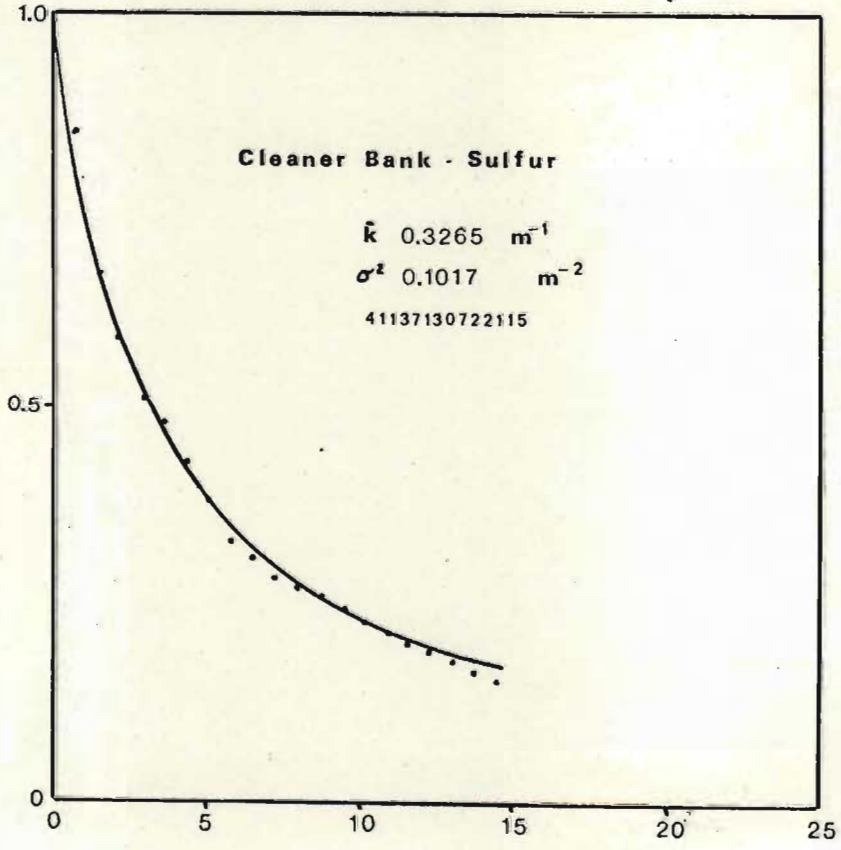
28. Batch Tests - Gamma Function Model

Fraction in Tailings vs Time - mins



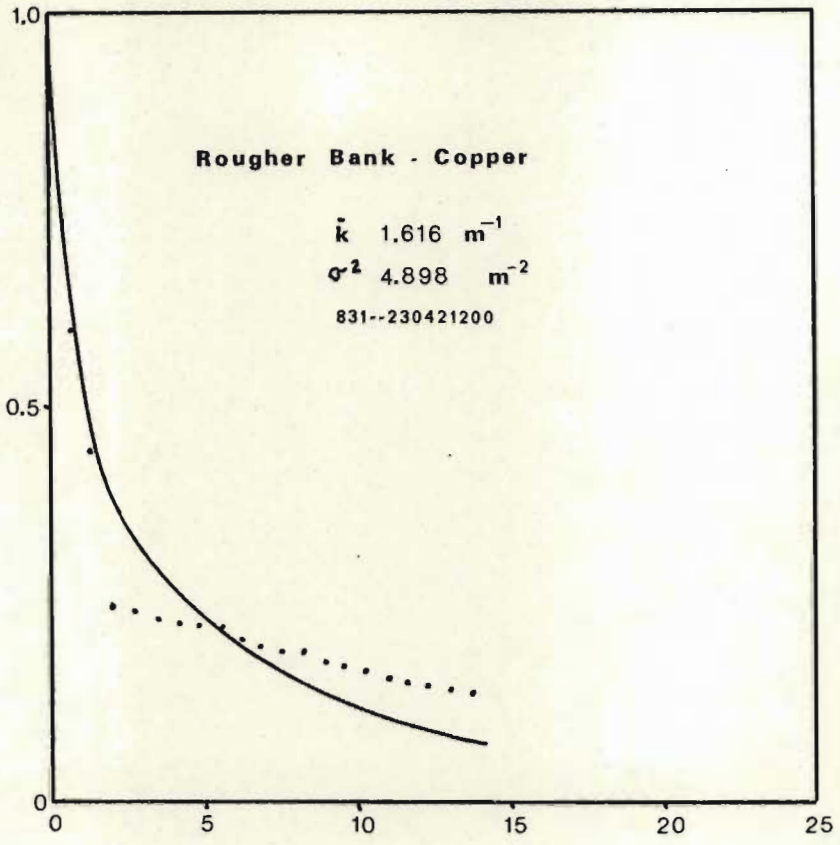
29. Typical Rate Distribution

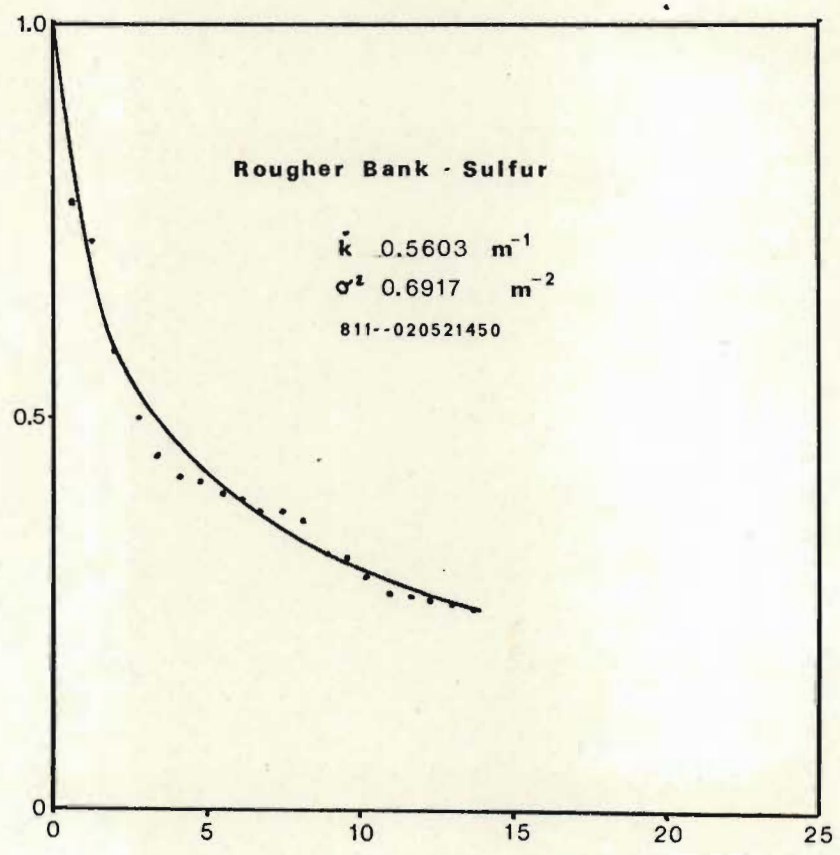




30. Continuous Tests - Gamma Function Model

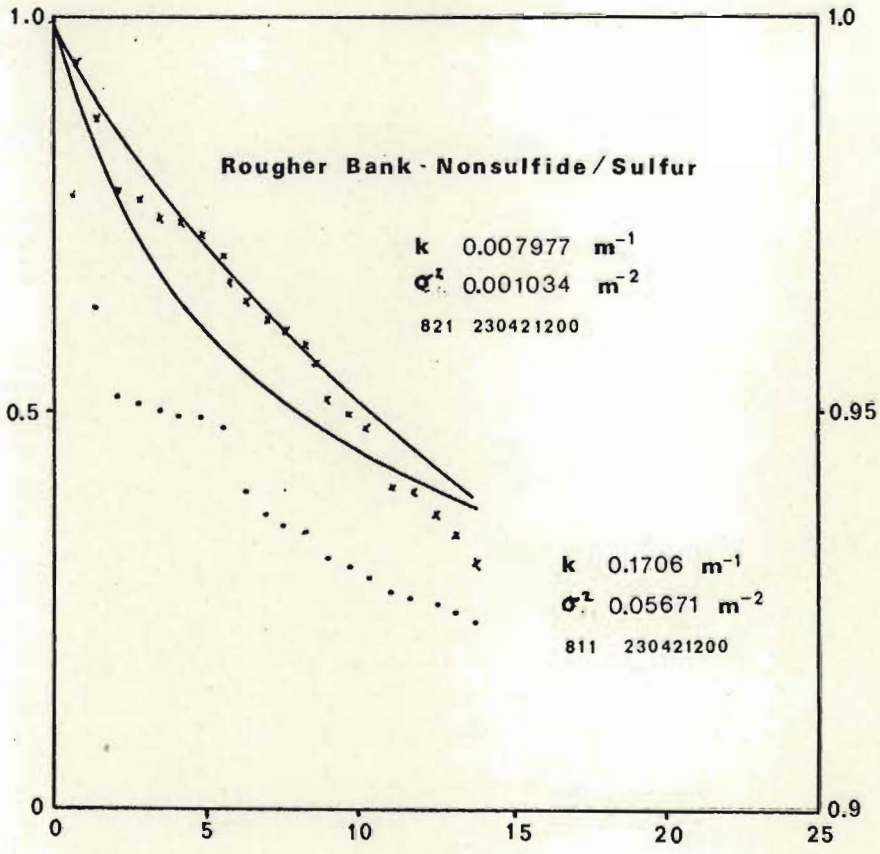
Fraction in Tailings vs Mean Time - mins

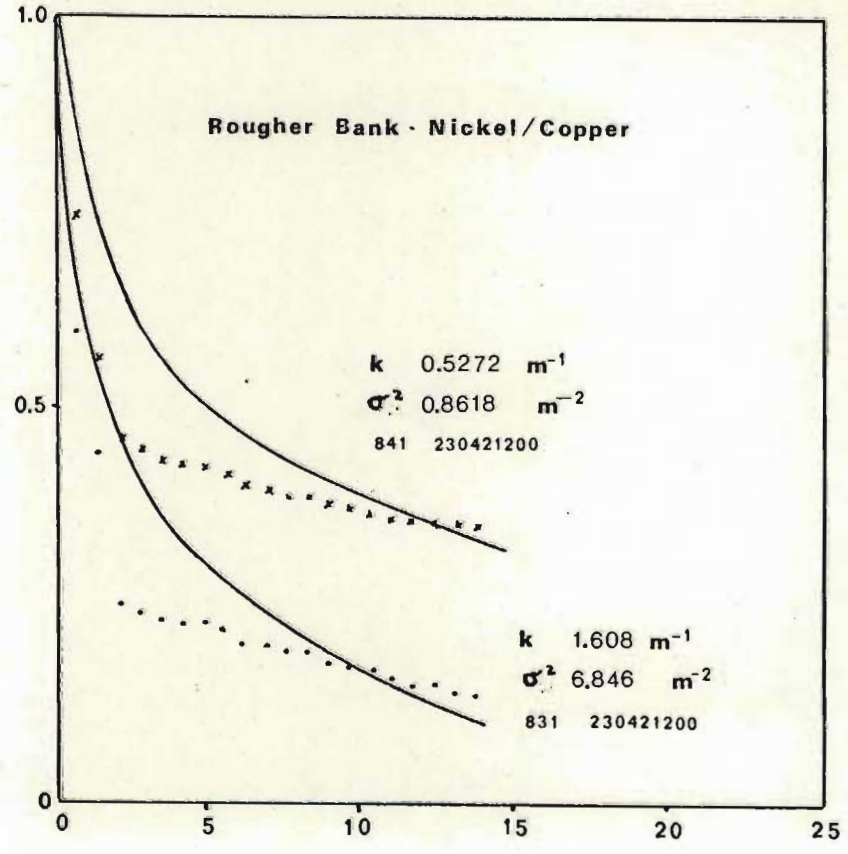




31. Continuous Tests - Gamma Function Model

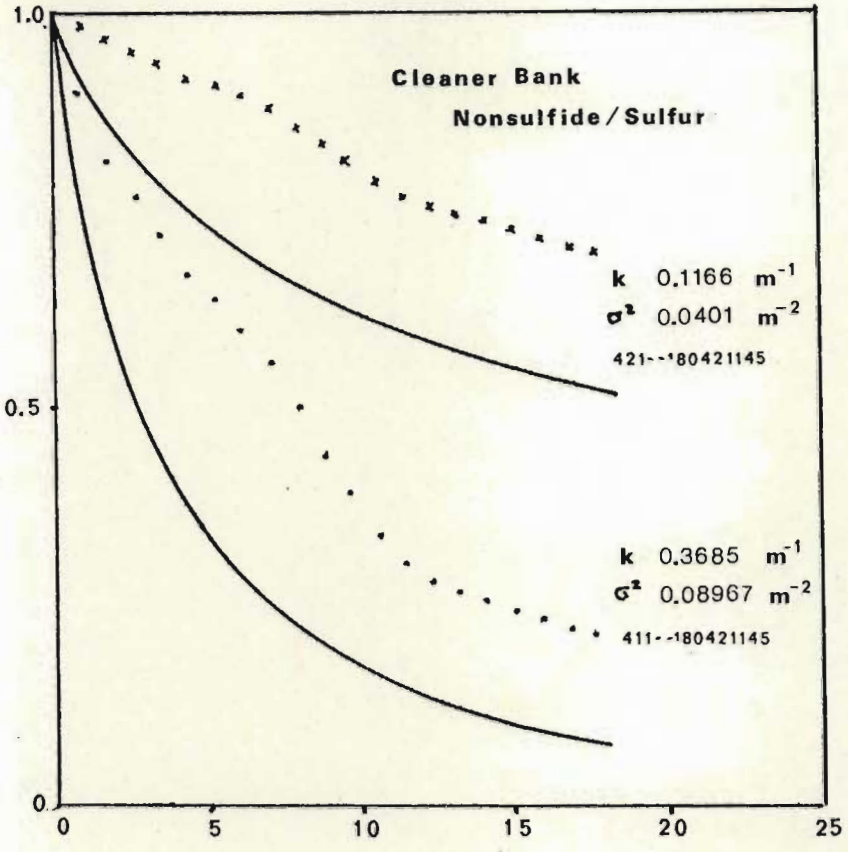
Fraction in Tailings vs Mean Time - mins

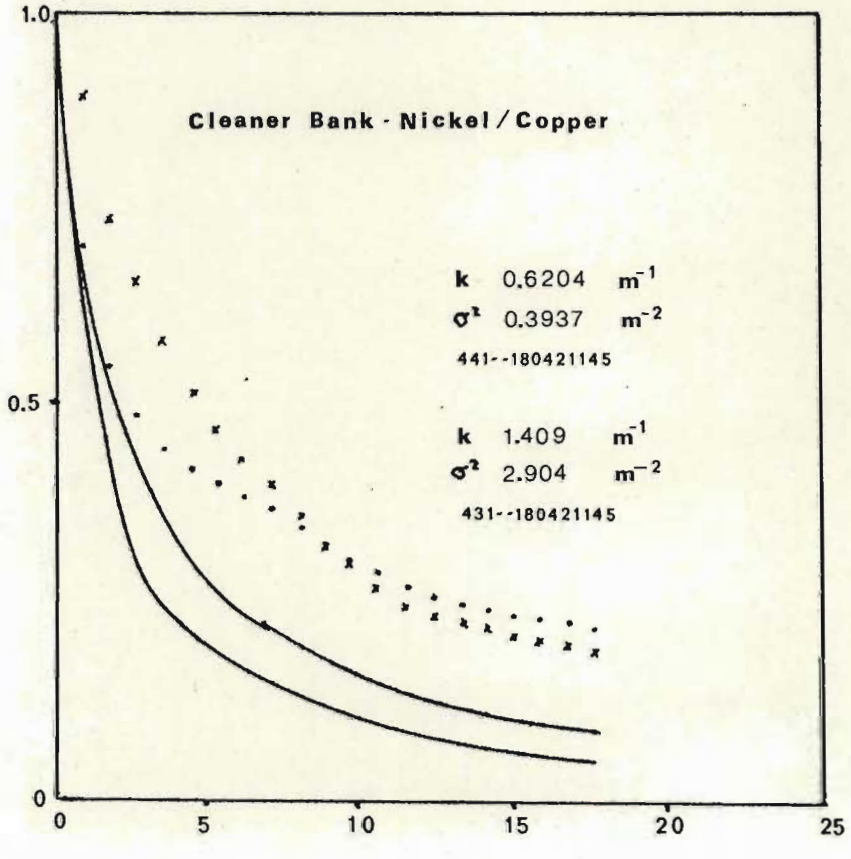




32. Predicted Continuous Operation - Gamma Function Model

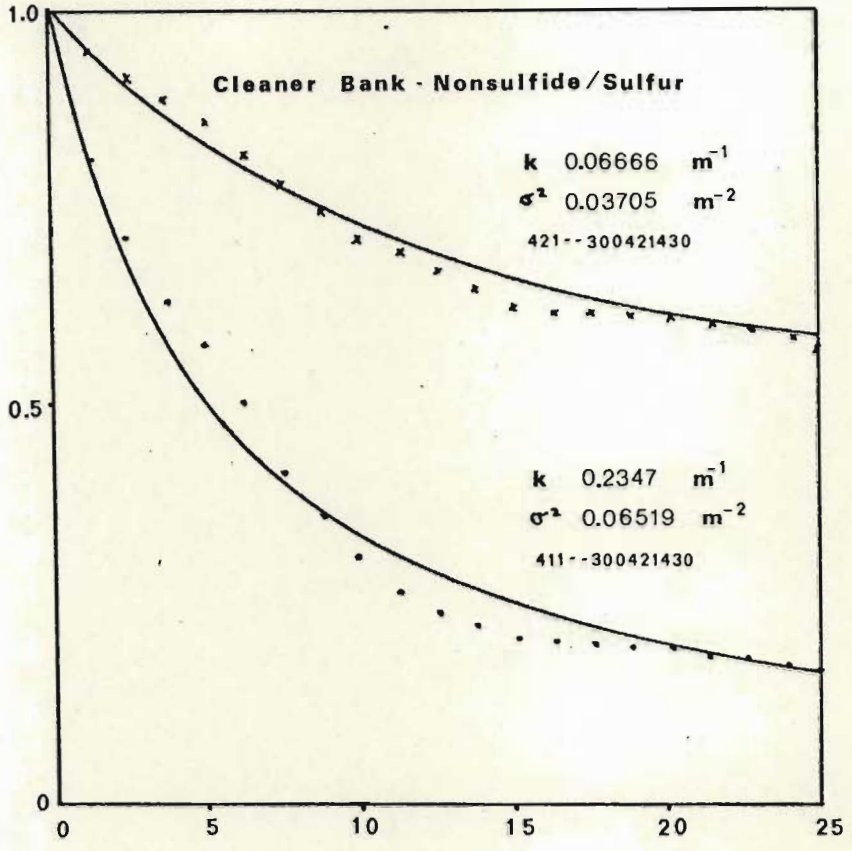
Fraction in Tailings vs Mean Time - mins

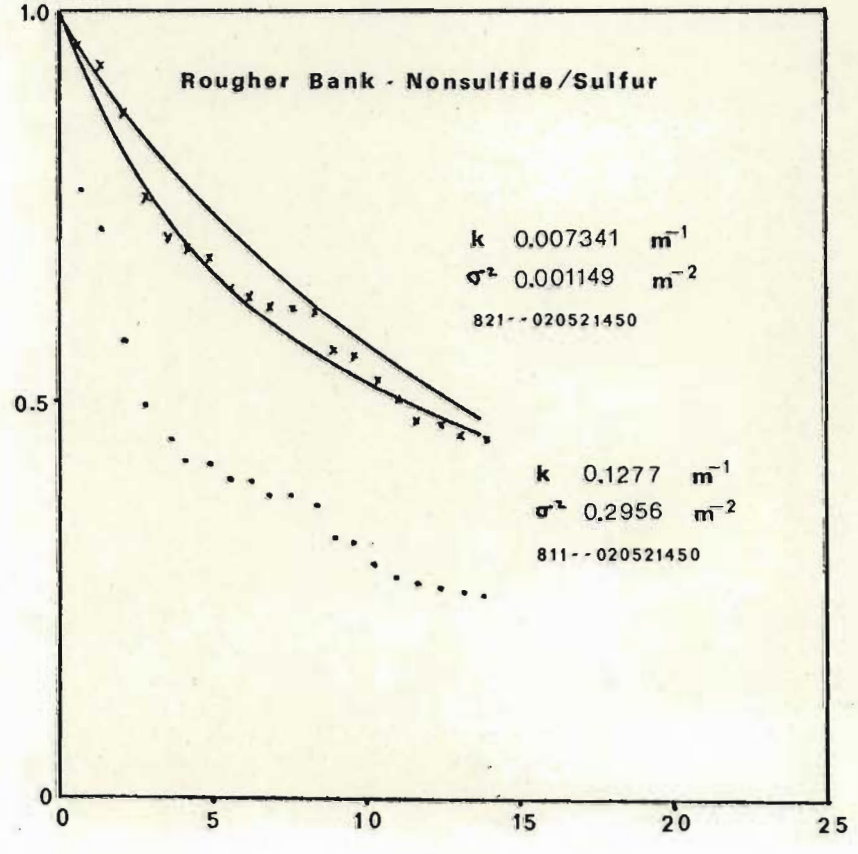




33. Predicted Continuous Operation - Gamma Function Model

Fraction in Tailings vs Mean Time - mins





34. Predicted Continuous Operation - Gamma Function Model

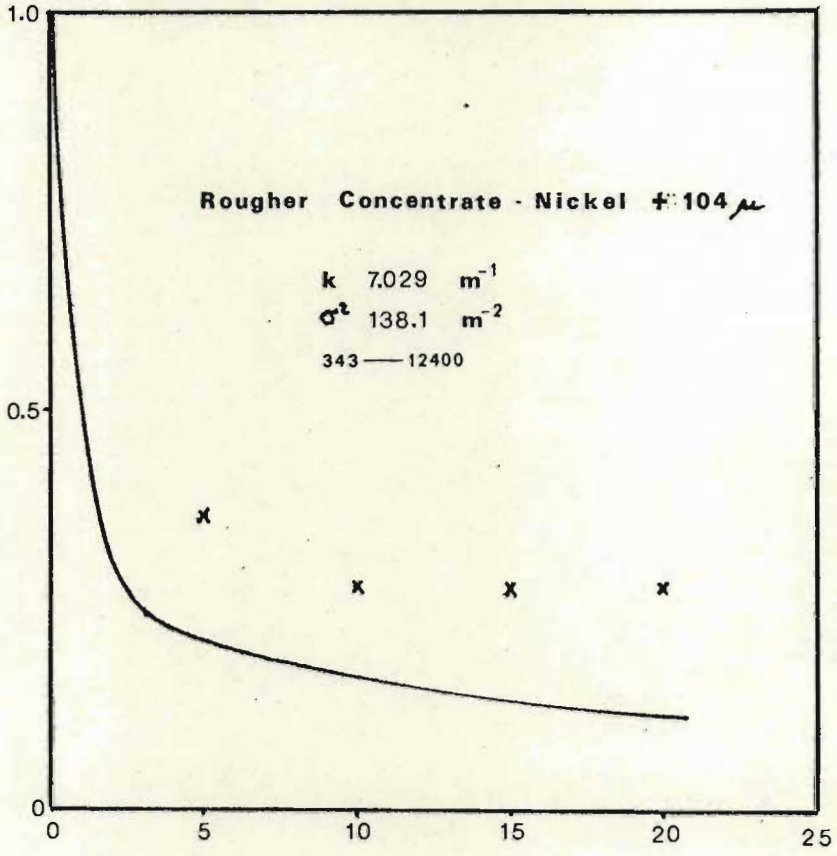
Fraction in Tailings vs Mean Tim

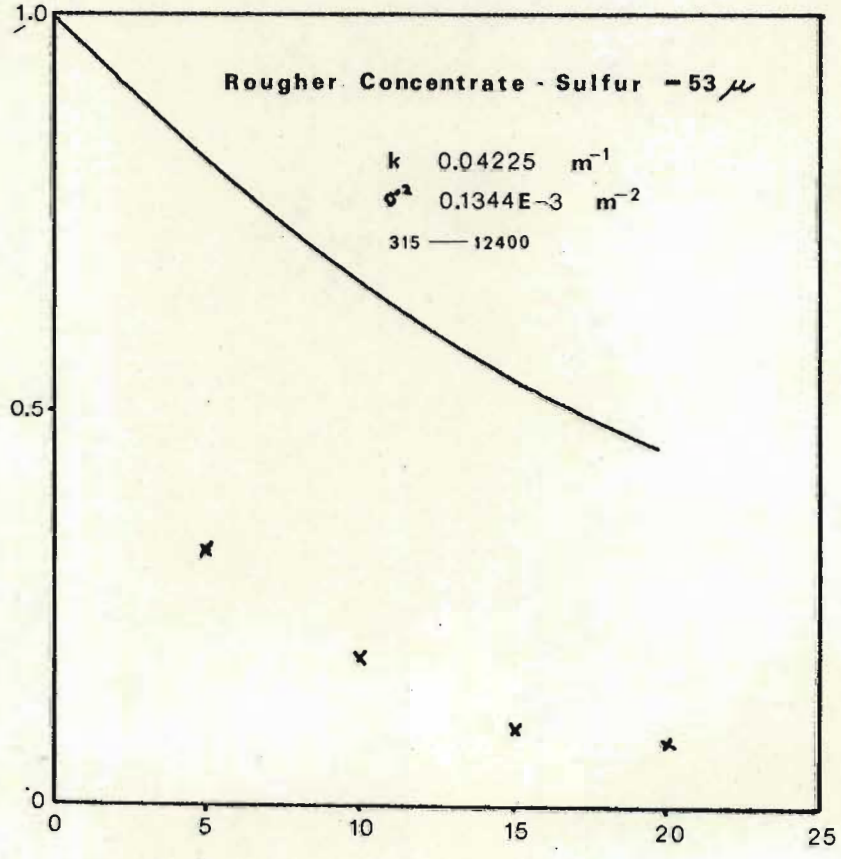
TABLE VI. BATCH TEST PREDICTION OF CONTINUOUS OPERATION - GAMMA FUNCTION MODEL

Material and Test	Batch Test Code	Continuous Test Code	Batch Test Parameters			Total Squared Error in Predicted Concentration.
			Mean Rate min^{-1}	Variance	Sum of Squares	
Rougher Feed						
Sulphur	811--130411435	811--130421435	0,1083	0,01231	0,01233	0,3933
Non-sulphide	821 "	821 "	0,5821E-2	0,4488E-3	0,2392E-3	0,3941E-3
Copper	831 "	831 "	1,313	3,904	0,9077E-3	0,08356
Nickel	841 "	841 "	0,1588	0,04377	0,8649E-2	0,09997
Rougher Feed						
Sulphur	811--230411200	811--230421200	0,1695	0,04593	0,9985E-2	0,5178
Non-sulphide	821 "	821 "	0,8028E-2	0,1054E-2	0,1982E-3	0,4134E-3
Copper	831 "	831 "	1,533	6,133	0,1055E-2	0,1207
Nickel	841 "	841 "	0,5041	0,7756	0,2933E-2	0,1361
Rougher Feed						
Sulphur	811--020511450	811--020521450	0,1274	0,02944	0,8488E-2	1,010
Non-sulphide	821 "	821 "	0,7143E-2	0,1061E-2	0,6556E-4	0,4977E-3
Cleaner Feed						
Sulphur	411--110411445	411--110421445	0,1256	0,01048	0,2538E-3	0,1765
Non-sulphide	421 "	421 "	0,02833	0,2300E-2	0,1041E-3	0,04736
Copper	431 "	431 "	0,7039	1,080	0,1234E-4	0,03491
Nickel	441 "	441 "	0,1936	0,02913	0,1454E-2	0,1698

TABLE VI. (Continued)

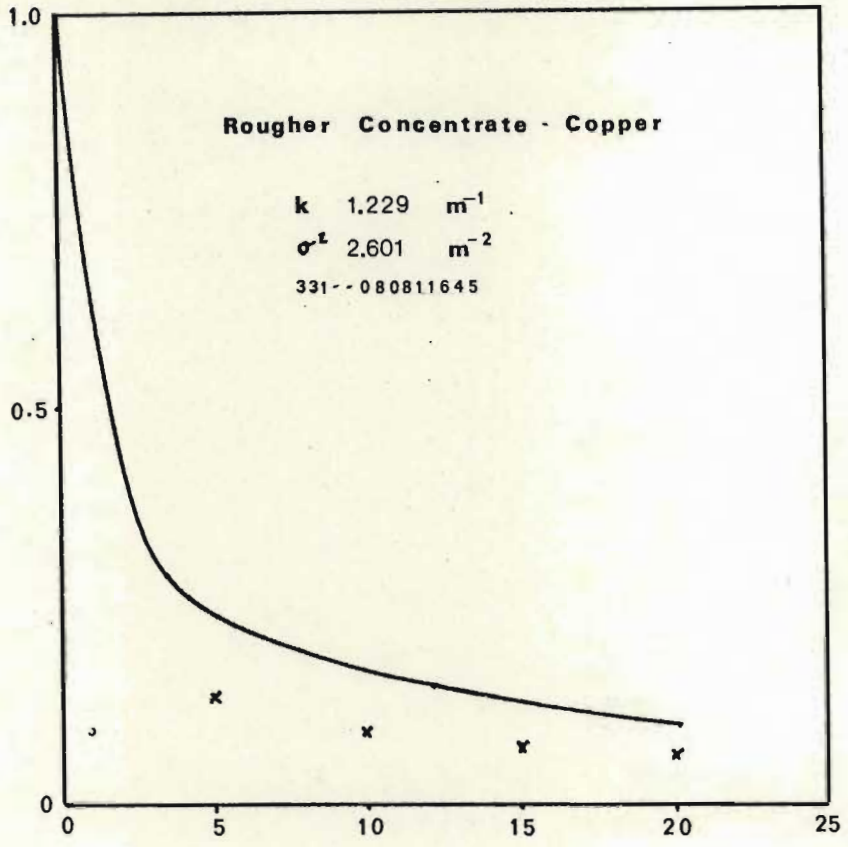
Material and Test	Batch Test Code	Continuous Test Code	Batch Test Parameters			Total Squared Error in Predicted Concentration.
			Mean Rate min^{-1}	Variance	Sum of Squares	
Cleaner feed*						
Sulphur	411--180411150	411--180421145	0,3956	0,1146	0,9913E-3	0,9686
Non-sulphide	421 "	421 "	0,1192	0,04261	0,1007E-2	0,6103
Copper	431 "	431 "	1,413	2,927	0,1377E-3	0,6491
Nickel	441 "	441 "	0,6531	0,4570	0,3100E-3	0,5205
Cleaner feed						
Sulphur	411--300411430	411--300421430	0,2462	0,07610	0,2126E-2	0,02963
Non-sulphide	421 "	421 "	0,06643	0,01690	0,7010E-3	0,01478

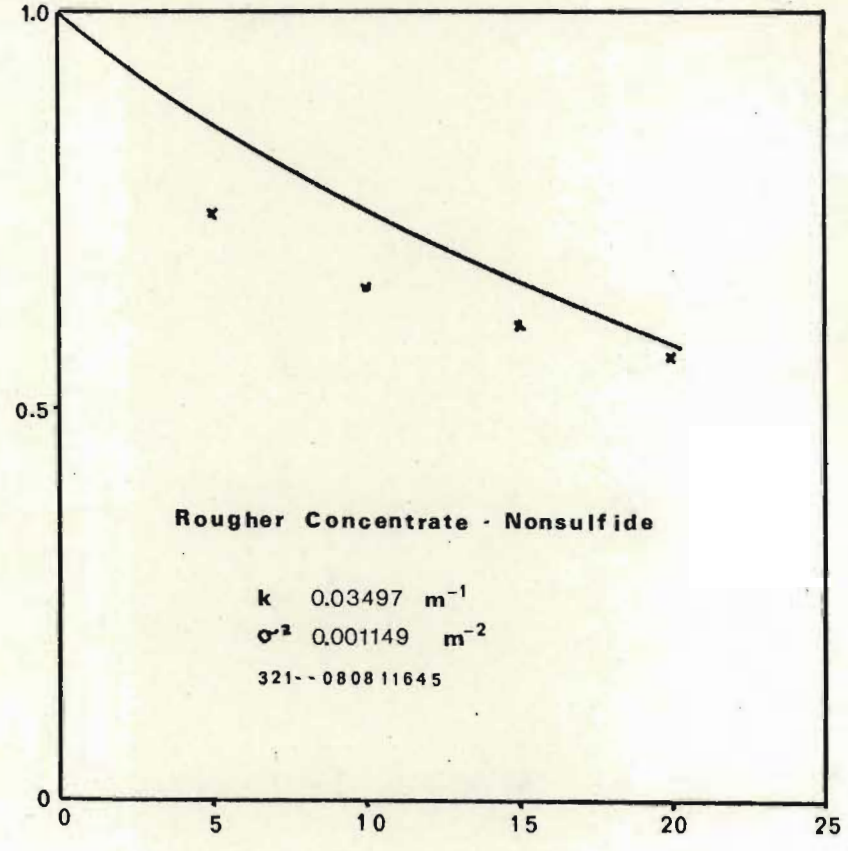




35. Predictions for Refloated Material - Gamma Function Model

Fraction in Tailings vs Time - mins





36. Predictions for Refloated Material - Gamma Function Model

Fraction in Tailings vs Time - mins

TABLE VII. PREDICTIONS FOR REFLOATED MATERIAL - GAMMA FUNCTION MODEL

Material	Rougher Feed Test			Rougher Concentrate Test			Rougher Concentrate Predicted Parameters		Total Squared Error in Concentration Prediction.
	Test Code	Regressed Parameters		Test Code	Regressed Parameters.				
		Mean min^{-1}	Variance		Mean min^{-1}	Variance	Mean min^{-1}	Variance	
Sulphur	811--080311935	0,1857	0,07282	311--080312055	0,2716	0,03898	0,2960	0,09353	0,02198
"	811--010311635	0,4024	0,5815	311--010311800	1,489	4,226	0,7046	0,8401	0,01056
"	811--220211920	0,1179	0,01095	311--220212045	0,7179	0,5837	0,1553	0,01226	0,1306
"	811--200211810	0,2252	0,08312	311--200211945	0,4007	0,07966	0,3208	0,09763	0,04851
"	811--140211805	0,1527	0,03358	311--140211930	0,6737	0,4133	0,2241	0,04017	0,1025
Sulphur (+208 microns)	812------12400	0,04558	0,05022	312------12400	3,308	66,11	0,4004	0,3270	0,04047
" (208-104 microns)	813 "	0,1115	0,07693	313 "	0,5243	0,6437	0,3329	0,1740	0,3749E-2
" (104-53 microns)	814 "	0,1657	0,1682	314 "	0,6182	0,5470	0,4486	0,3532	0,02947
" (-53 microns)	815 "	0,03987	0,1290E-3	315 "	0,3205	0,06371	0,04225	0,1344E-3	0,7879
Copper (+ 104 microns)	833 ------12400	0,7056	2,822	333------12400	7,067	129,6	1,508	5,021	0,02209
" (104-53 microns)	834 "	16,44	1281,	334 "	10,06	184,6	30,95	2037,	0,5805E-3
" (-53 microns)	835 "	0,04902	0,9251E-3	335 "	2,354	5,566	0,06132	0,1068E-2	0,9994
Nickel (+ 104 microns)	843------12400	0,7895	19,60	343------12400	4,122	74,87	7,029	138,1	0,09941
" (104-53 microns)	844 "	1,981	36,35	344 "	0,7994	0,7614	6,047	90,51	0,4381E-2
" (-53 microns)	845 "	0,05057	0,1606E-2	345 "	0,7016	0,3919	0,07030	0,1984E-2	0,6890
Sulphur *	811--080811620	0,08825	0,4221E-2	311--080811645	0,4828	0,2723	0,1108	0,4661E-2	0,1459
Non-sulphide *	821 "	0,4090E-2	0,1601E-3	321 "	0,1130	0,05209	0,03497	0,1149E-2	0,02093
Copper *	831 "	0,8467	2,065	331 "	2,600	10,56	1,229	2,601	0,02179
Nickel *	841 "	0,1486	0,04277	341 "	0,7684	0,7361	0,2411	0,05547	0,07503

* During these tests no dextrin was added to the rougher concentrate.

TABLE VIII. PREDICTIONS FOR REFLOATED MATERIAL - GAMMA FUNCTION MODEL WITH NON-FLOATING FRACTION

Material	Rougher Feed Test				Rougher Concentrate			Rougher Concentrate Predicted Parameters		Total Squared Error in Concentration Prediction.
	Test Code	Regressed Parameters			Test Code	Regressed Parameters				
		Mean min^{-1}	Variance	Cn		Mean min^{-1}	Variance	Mean min^{-1}	Variance	
Sulphur	811--080311935	0,1740	0,01316	0,260	311--080312055	0,2716	0,03898	0,1950	0,01325	0,8198E-2
"	811--010811635	0,2468	0,02787	0,333	311--010311800	1,489	4,226	0,2713	0,02772	0,01596
"	811--220211920	0,1154	0,6800E-3	0,156	311--220212045	0,7179	0,5837	0,1179	0,6821E-3	0,1441
"	811--200211810	0,1981	0,01792	0,179	311--200211945	0,4007	0,07966	0,2214	0,01799	0,03283
"	811--140211805	0,1392	0,4527E-2	0,204	311--140211930	0,6737	0,4133	0,1505	0,4555E-2	0,1099
Sulphur (+208 microns)	812------12400	0,1431	0,1008E-2	0,863	312------12400	3,308	66,11	0,1455	0,1006E-2	0,2108
" (208-104 microns)	813 "	0,1320	0,8306E-3	0,601	313 "	0,5243	0,6437	0,1343	0,8302E-3	0,05929
" (104-53 microns)	814 "	0,1492	0,3928E-2	0,548	314 "	0,6182	0,5470	0,1578	0,3925E-2	0,03507
" (-53 microns)	815 "	0,06813	0,2286E-3	0,263	315 "	0,3205	0,06371	0,07014	0,2318E-3	0,3458
Copper (+104 microns)	833------12400	0,2886	0,02886	0,434	333------12400	7,067	129,6	0,3048	0,02836	0,02657
" (104-53 microns)	834 "	1,205	0,9863	0,201	334 "	10,06	184,6	1,237	0,9794	0,2372E-2
" (- 53 microns)	835 "	0,07381	0,1989E-3	0,256	335 "	2,354	5,566	0,07536	0,2007E-3	0,6934
Nickel (+104 microns)	843------12400	0,2709	0,1392	0,813	343------12400	4,122	74,87	0,3922	0,1666	0,04192
" (104-53 microns)	844 "	0,2985	0,01571	0,507	344 "	0,7994	0,7614	0,3052	0,1545	0,01221
" (-53 microns)	845 "	0,08333	0,3306E-3	0,319	345 "	0,7016	0,3919	0,08544	0,3336E-3	0,4091
Sulphur *	811--080811620	0,09586	0,4376E-3	0,148	311--080811645	0,4828	0,2723	0,09801	0,4399E-3	0,1338
Non-sulphide *	821 "	0,05578	0,1850E-3	0,908	321 "	0,1130	0,05209	0,05792	0,1890E-3	0,09616
Copper *	831 "	0,4336	0,1347	0,203	331 "	2,600	10,56	0,4730	0,1342	0,8619E-2
Nickel *	841 "	0,1332	0,3526E-3	0,298	341 "	0,7684	0,7361			**

* During these tests no dextrin was added to the rougher concentrate.

** The computing facilities available cannot evaluate the necessary integrals for the parameters shown.

Performance of a Two-Phase Model

The froth phase in flotation is very complex, and in the flotation system it cannot be studied separately from the pulp phase, hence quantitative descriptions of the froth phase are difficult. Its importance in determining recovery and grade for various materials is illustrated in figures 37 and 38; the effect is indeed great. Several points about froth action have been well established by various researchers. Taggart[34] has shown the gradation of values through the froth layer, increased value at the top of the column and relatively poor grades just above the pulp level. The phenomena of upgrading a froth column through increasing its height is a natural function of the particles and system involved. When a froth bubble loaded with particles is disturbed or destroyed, a 'shower' of particles is released and falls back through the froth to the pulp phase. Bubbles may release particles through two primary means; 1) through bubble breakage, and 2) through bubble coalescence (this occurs very rapidly at the bottom of the froth layer). The released particles are carried back into the pulp by water draining from the froth column. The rate of return of particles to the pulp must be a complex function composed not only of the bubble breakage rate, the bubble coalescence rate, the concentration of solids in the froth and the degree of mineral-bubble adherence but also by the froth viscosity, fluid density and by the particle shape. A mathematical description of the sub-processes is too complex for investigation here and the study was limited to determining the importance of the froth phase and the role of frother concentration.

Frothers are chemically composed of heteropolar molecules, the polar portion of the molecule is hydrophilic and the other non-polar portion is hydrophobic. Thus the molecules tend to

concentrate at the air-water interface and lower the surface tension in a pure liquid. Air bubbles in the pulp are reduced in size and a froth is developed. The frothers used at RPM are Aerofroth 71 and cresylic acid, the phenol constituents of cresylic acid may be analyzed for in the various pulp streams by the method outlined in Colorimetric Chemical Analytical Methods.[35] Results of an analysis done on the cleaner bank at RPM are shown in Table X. It is normally assumed that the froth stability results from the concentration of frother, hence the normally voluminous froths in the cleaner sections of networks. This assumption is not supported by the cresylic acid analyses, however, as the majority of the frother remains in the tailings stream. Froth stability is also dependent upon the solids portion of the pulp, particularly the fine material. Gaudin[50] states that even in aqueous solutions of maximum frothing ability, the froth produced is very short lived. In a mineral suspension, however, the froth produced may last for several hours. It is not probable that the alcohol frother, AF71, acts in a different manner from the cresylic acid. Hence the main reason for stable and voluminous froths in the cleaner section would appear to be the nature of the solid material rather than the frother concentration. In example, in a typical plant situation at RPM, the rougher feed would contain approximately 50 grams/min (30 g/ton) of cresylic acid, the cleaner feed stream 6 grams/min (12,86 g/ton) and the final concentrate less than 0,2 grams/min. (4 g/ton). It is obvious from this data then that low froth production at the end of a rougher bank is not a result of lack of frother but more from a lack of floatable material which gives the froth its stability.

Several indications of the importance of the froth phase are found in the preceding sections. As implied by Arbiter

and Harris, the apparent rate of flotation reflects the froth removal rate in any instance where material appears in the froth phase more rapidly than the froth is removed. In general, flotation in the first few cells and in the cleaners is governed not by the kinetics of flotation but rather by the 'pull'; on the other hand, near the end of a bank of cells the process is kinetically determined.

An examination of the single-phase kinetic flotation models of previous chapters has proved them unsuitable mathematically for description of true flotation where the kinetic parameters are such that the froth removal rate has a marked effect on the flotation of a species, as in the cleaner section at RPM. This problem may be handled in two basic ways; one, the single phase kinetic parameters may be extended as not only functions of the species but also as functions of the froth removal rate, and two, a second phase (the froth phase) may be included in the kinetic formulation of the system and the resulting mathematical description.

Arbiter and Harris,[36] and Harris and Rimmer[37] have chosen the second of these methods. The two-phase model proposed by Arbiter and Harris in 1966, assumes both the pulp and the froth phases to be 'well mixed' and consistent with first-order kinetics. Thus for any chosen species a single rate constant will describe the transfer of the species from the pulp to the froth and a second first-order rate constant will describe the transfer of the species in the reverse direction. It has already been shown that for any mineralogical or metal species, a fraction of the species is non-floatable; addition of this fraction is expected to improve the accuracy of the models fit.

A large number of batch flotation tests, in which the removal rate of concentrate as well as the froth depth were recorded, were performed. A summary of the available data

which includes the information for a bank of continuously operating cells is presented in Tables IX and X. The flow-rate of froth was measured as the wet weight of concentrate per unit time and converted to volume flow per unit time. The volume of froth was calculated from cell dimensions and froth depth thus it includes the volume of air entrained in the froth.

Batch Test Results

Due to the close relationship between the froth phase and the pulp phase in flotation, it would be expected that the parameters involved would be very interdependent. Figure 39 shows the best fit of the two-phase model, a smooth curve is not fitted since the Qf/Vf value varies from point to point, and figure 40 shows the sum of squares surfaces respectively. Note the very poorly defined minimums where the sum of squares surface changes very little for changes in 'a' and 'b'.

Since the model incorporates the froth phase, parameters regressed from one test were used in predicting other tests. Figure 41 illustrates this. In figure 41.1 parameters were obtained from a test with high Qf/Vf values and used to predict a test with lower Qf/Vf values; in figure 41.2 the opposite was done. These figures indicated that the model attaches too great a significance to the Qf/Vf values.

The model's predictive capacity is improved by the addition of a non-floating fraction of the species. Figure 42 is an example of this. Parameter interdependence is still a serious problem however, as indicated by the sum of squares surface in figure 43.1. Predictions for different Qf/Vf

values still show the model's over-emphasis of the froth residence times as in figure 43.2, where 42.1 is predicted from 42.2.

Conclusions

The two-phase model as described by Harris and Rimmer is unsatisfactory in handling batch data. The addition of a non-floating fraction of the species improves the model but parameter significances are very poor. The value of Qf/Vf is given more importance in the model than what is suggested by the data and thus the model overreacts to changes in Qf/Vf.

TABLE IX. FROTH REMOVAL DATA - BATCH TESTS

Description and Test Code	Flotation Time (min)							Variable
	5	10	15	20	30	40	50	
Rougher Feed	0,0787	0,0772	0,139	0,314	0,478	0,480		Q/V min^{-1}
811--110212040	0,652	0,575	0,435	0,393	0,339	0,303		S
821 "	0,988	0,980	0,971	0,965	0,956	0,948		NS
Rougher Feed	0,104	0,0801	0,0697	0,0714	0,0637	0,0645		Q/V
811--130211915	0,762	0,632	0,576	0,538	0,477	0,431		S
821 "	0,993	0,988	0,982	0,978	0,970	0,963		NS
Rougher Feed	0,279	0,261	0,320	0,233	0,334	0,280		Q/V
811--140211805	0,571	0,494	0,354	0,304	0,236	0,204		S
821 "	0,970	0,950	0,927	0,914	0,895	0,882		NS
Rougher Feed	0,752	0,626	1,40	1,18	2,20	3,40		Q/V
811--200211810	0,484	0,464	0,311	0,254	0,202	0,179		S
821 "	0,955	0,944	0,914	0,894	0,873	0,859		NS
Rougher Feed	1,495	1,065	1,079	0,730	0,634	0,531		Q/V
811--220211920	0,606	0,470	0,310	0,245	0,183	0,156		S
821 "	0,968	0,942	0,919	0,905	0,884	0,873		NS
Rougher Feed	0,0733	0,0593	0,0607	0,0624	0,0597	0,0698		Q/V
811--260212030	0,732	0,657	0,551	0,504	0,436	0,407		S
821 "	0,992	0,986	0,981	0,976	0,969	0,964		NS
Rougher Feed	0,667	0,461	2,220	1,52	0,645	0,546		Q/V
811--010311625	0,550	0,512	0,401	0,351	0,287	0,260		S
821 "	0,967	0,954	0,932	0,921	0,905	0,893		NS
Rougher Feed	1,92	1,45	1,42	1,00	0,746	0,618		Q/V
811--080311935	0,574	0,512	0,401	0,351	0,287	0,260		S
821 "	0,966	0,952	0,934	0,923	0,907	0,898		NS
Rougher Feed	0,336	0,354	0,700	0,837	0,547	0,341		Q/V
811--130411435	0,633	0,565	0,336	0,281	0,244	0,230		S
821 "	0,980	0,967	0,938	0,924	0,911	0,905		NS
831 "	0,288	0,245	0,174	0,153	0,134	0,124		CU
841 "	0,626	0,495	0,326	0,305	0,290	0,284		NI
Rougher Feed	0,727	0,592	0,545	0,359	0,237	0,185		Q/V
811--230411200	0,560	0,518	0,328	0,273	0,248	0,237		S

TABLE IX. (Continued)

Description and Test Code	Flotation Time (min)							Variable
	5	10	15	20	30	40	50	
821--230411200	0,971	0,959	0,931	0,916	0,906	0,899		NS
831 "	0,301	0,268	0,200	0,174	0,157	0,148		CU
841 "	0,489	0,429	0,321	0,298	0,286	0,280		NI
Rougher Conc.	0,0162	0,0185	0,0151	0,0175	0,0217	0,0245		Q/V
311--110212145	0,775	0,576	0,419	0,319	0,196	0,111		S
321 "	0,947	0,893	0,831	0,776	0,686	0,617		NS
Rougher Conc.	0,0235	0,0230	0,0240	0,0231	0,0231	0,0223		Q/V
311--130212035	0,612	0,404	0,294	0,215	0,177	0,100		S
321 "	0,960	0,924	0,896	0,868	0,839	0,799		NS
Rougher Conc.	0,534	0,443	1,010	1,13				Q/V
311--140211930	0,220	0,104	0,079	0,069				S
321 "	0,805	0,703	0,650	0,617				NS
Rougher Conc.	0,598	0,375	0,439	0,390				Q/V
311--200211945	0,251	0,105	0,061	0,046				S
321 "	0,751	0,635	0,557	0,520				NS
Rougher Conc.	0,516	0,429	0,687	0,905				Q/V
311--220212045	0,242	0,126	0,103	0,091				S
321 "	0,752	0,647	0,598	0,566				NS
Rougher Conc.	0,0540	0,0615	0,0618	0,0584	0,0572	0,0659		Q/V
311--260212155	0,756	0,580	0,496	0,394	0,237	0,162		S
321 "	0,962	0,929	0,899	0,872	0,811	0,770		NS
Rougher Conc.	0,954	0,869	1,18	1,025				Q/V
311--010311800	0,247	0,161	0,138	0,126				S
321 "	0,839	0,773	0,741	0,718				NS
Rougher Conc.	1,00	1,13	3,29	0,961				Q/V
311--080312055	0,369	0,166	0,110	0,095				S
321 "	0,811	0,699	0,631	0,600				NS
Cleaner Feed	0,0394	0,0528	0,0693	0,0813	0,0889	0,134	0,252	Q/V
411--110411445	0,590	0,407	0,305	0,232	0,144	0,111	0,095	S
421 "	0,894	0,815	0,754	0,711	0,645	0,606	0,571	NS
431 "	0,270	0,168	0,127	0,106	0,078	0,064	0,053	CU
441 "	0,516	0,294	0,205	0,162	0,110	0,088	0,074	NI

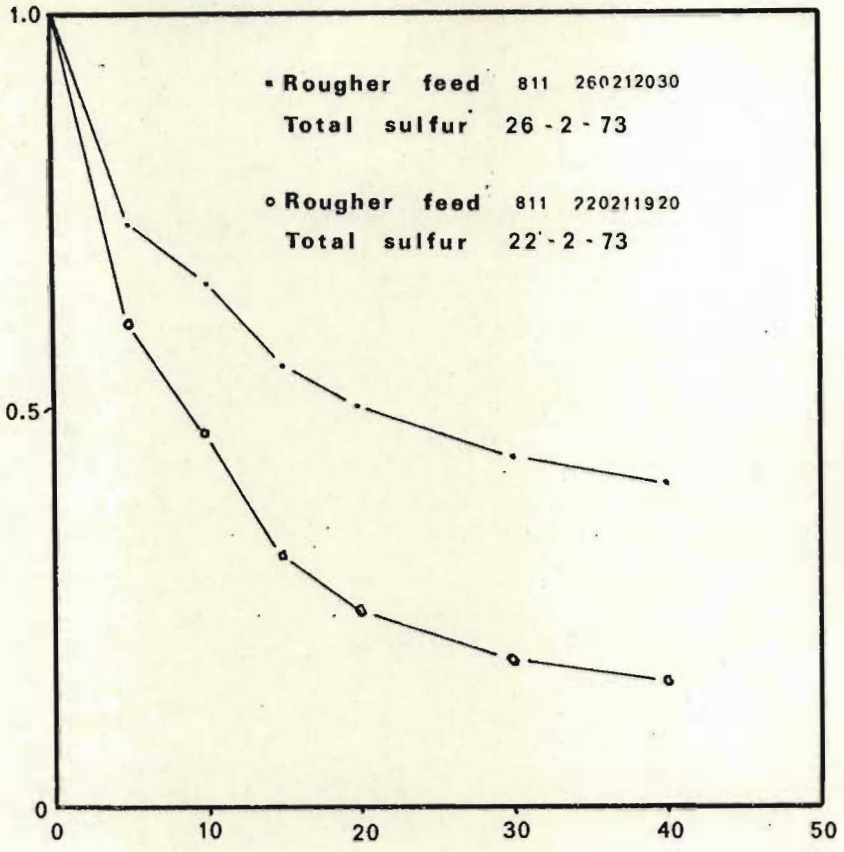
TABLE IX (Continued)

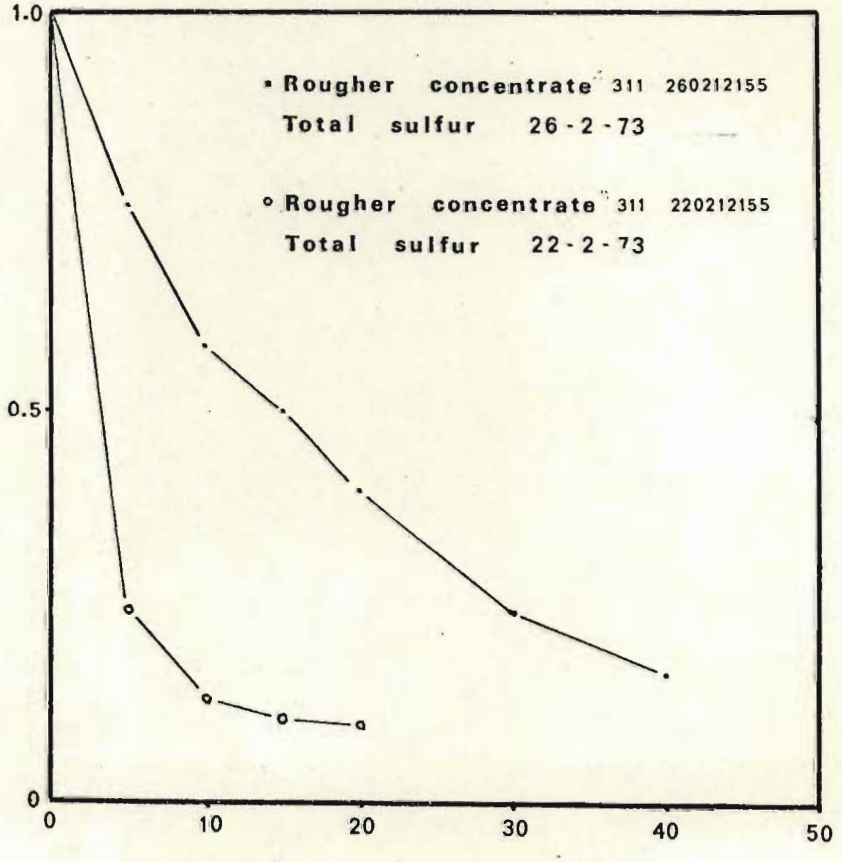
Description and Test Code	Flotation Time (min)							Variable
	,5	10	15	20	30	40	50	
Cleaner Feed	0,270	0,512	0,630	0,532	0,407			Q/V
411--180411150	0,312	0,136	0,091	0,074	0,058			S
421 "	0,729	0,588	0,526	0,492	0,456			NS
431 "	0,195	0,115	0,090	0,078	0,066			CU
441 "	0,254	0,132	0,096	0,080	0,064			NI

* Q/V is the flowrate of froth divided by the froth volume.
S, NS, CU, NI symbolize the fraction of sulphur, non-sulphide, copper and nickel in the tailings.

TABLE X. FROTH REMOVAL DATA - CONTINUOUS TESTS

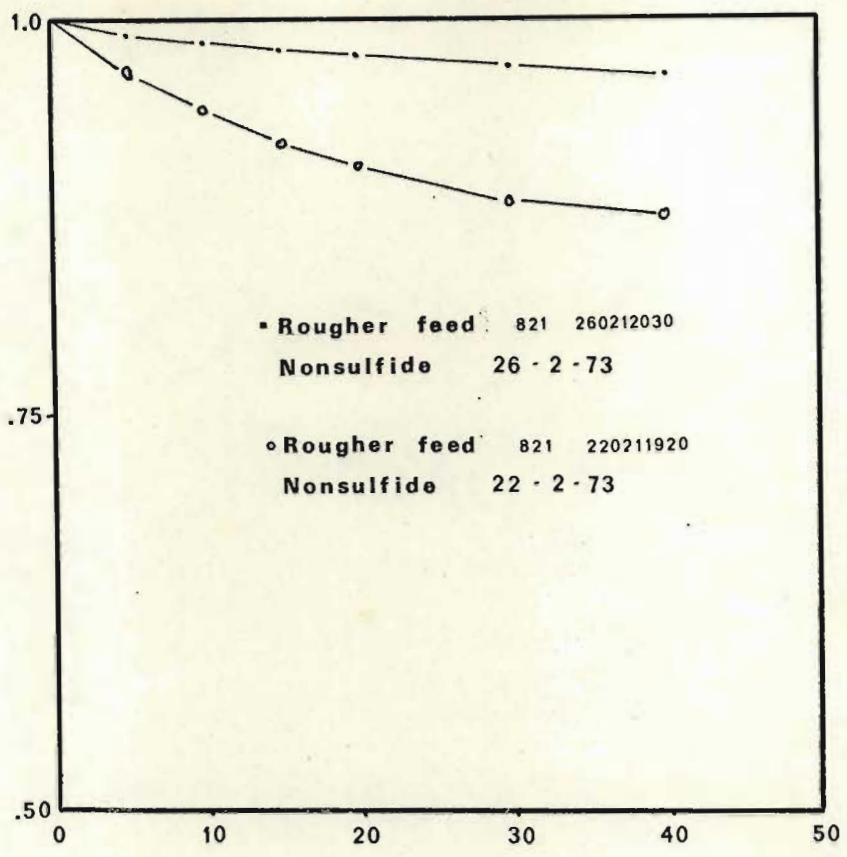
	Cleaner Bank 411--110421445								Cleaner Bank 411--300421430					
	Mean Time (min)	Sulphur Fraction	Nonsulphide Fraction	Copper Fraction	Nickel Fraction	Froth Flow Kg/min	Froth % Solids	Cresylic Acid Flowrate g/min	Mean Time	Sulphur Fraction	Nonsulphide Fraction	Froth Flow Kg/min	Froth % Solids	Froth Depth (cms)
	0,0	1,000	1,000	1,000	1,000	-		5,92	0,0	1,000	1,000	-	-	-
1 1	0,97	0,862	0,978	0,681	0,869	10,16	27,9	0,028	1,25	0,829	0,957	15,3	39,5	36
2	1,95	0,808	0,970	0,578	0,814	3,95	28,9	0,012	2,51	0,719	0,923	12,2	38,0	"
3	2,93	0,752	0,959	0,479	0,742	5,52	33,0	0,006	3,76	0,637	0,889	14,8	29,9	"
4	3,90	0,642	0,937	0,349	0,590	11,90	31,0	0,005	5,02	0,582	0,862	10,2	33,7	"
5	4,88	0,524	0,904	0,252	0,434	19,89	25,3	0,004	6,29	0,507	0,823	21,6	23,2	34
6	5,87	0,474	0,884	0,213	0,371	13,85	22,3	0,009	7,54	0,420	0,780	21,9	25,3	"
7	6,84	0,438	0,868	0,189	0,326	9,35	25,6	0,003	8,80	0,366	0,747	13,7	30,1	"
8	7,82	0,392	0,846	0,168	0,281	14,79	22,0	0,010	10,06	0,316	0,713	19,8	21,6	"
9	8,80	0,357	0,822	0,150	0,246	14,92	22,5	-	11,31	0,269	0,695	12,0	19,4	22
10	9,78	0,329	0,801	0,138	0,222	14,75	20,5	-	12,58	0,242	0,671	13,1	22,5	"
11	10,76	0,307	0,783	0,128	0,204	13,13	20,4	-	13,83	0,225	0,653	8,6	26,1	"
12	11,74	0,271	0,756	0,116	0,181	19,51	19,6	-	15,09	0,204	0,626	14,1	22,9	"
13	12,72	0,239	0,733	0,104	0,163	21,12	16,0	-	16,35	0,203	0,623	1,1	25,8	26
14	13,69	0,217	0,712	0,095	0,149	22,57	12,9	-	17,60	0,201	0,621	0,9	34,0	"
15	14,67	0,199	0,693	0,087	0,137	19,55	14,0	-	18,87	0,200	0,619	0,9	19,1	"
16	15,66	0,173	0,668	0,078	0,123	25,42	13,9	-	20,13	0,195	0,610	3,2	34,8	"
17	16,63	0,169	0,663	0,076	0,120	4,08	18,1	-	21,38	0,189	0,600	4,9	23,7	24
18	17,61	0,168	0,661	0,075	0,118	1,53	21,6	-	22,64	0,184	0,593	3,5	23,9	24
19	18,59	0,167	0,659	0,074	0,117	1,70	15,3	-	23,90	0,176	0,583	5,1	23,6	"
20	19,57	0,162	0,650	0,070	0,112	6,37	19,6	-	25,16	0,164	0,567	8,7	22,5	"

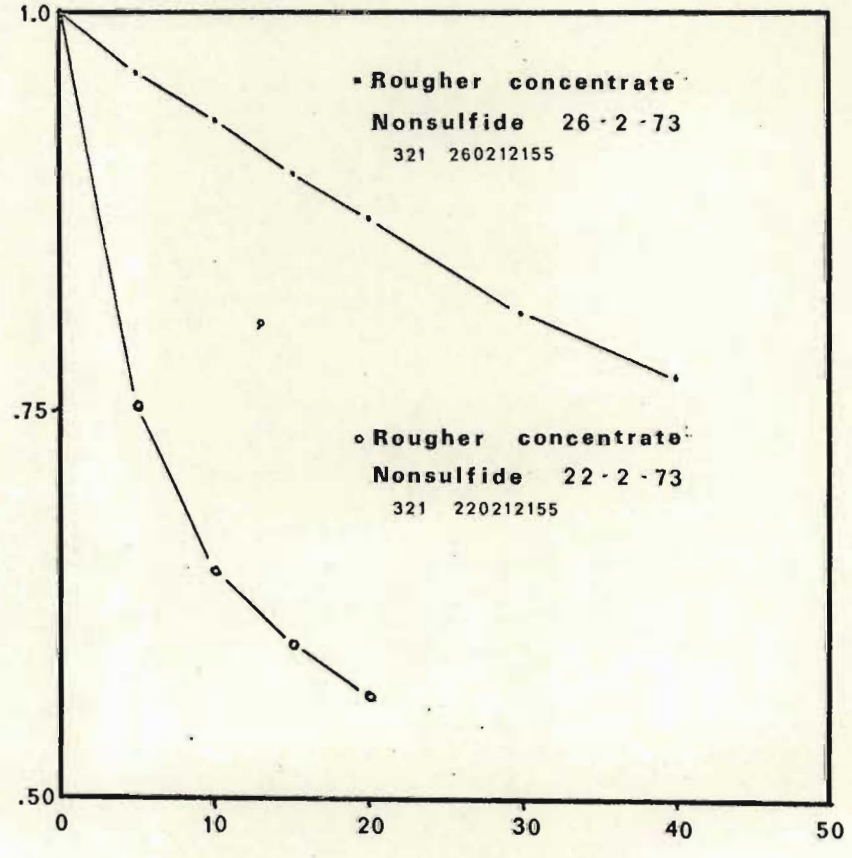




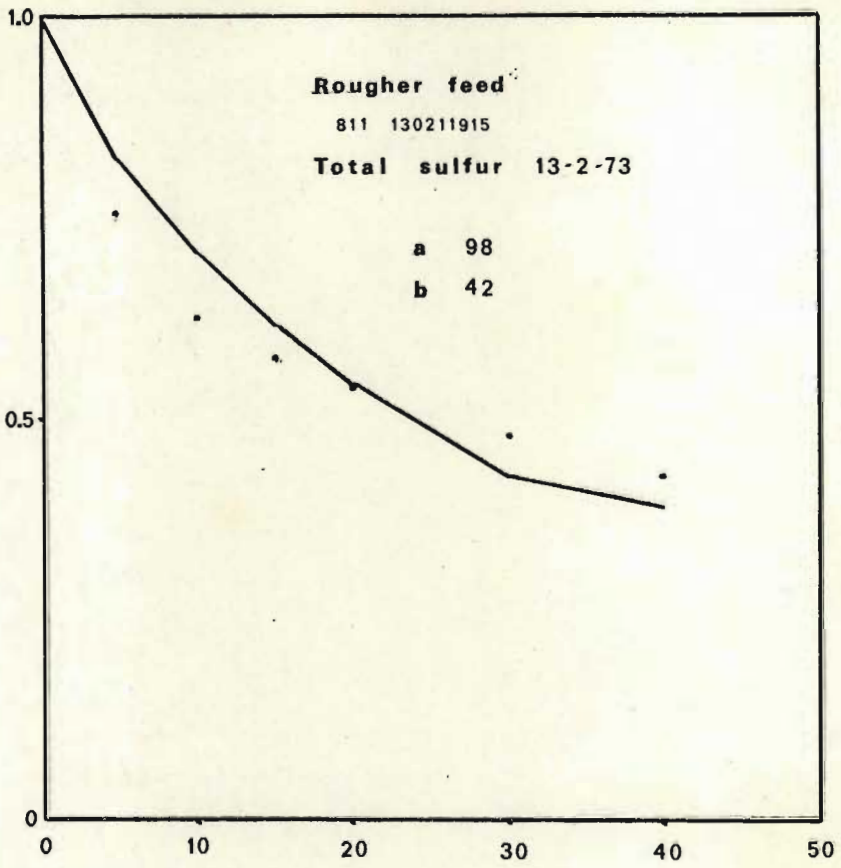
37. Importance of the Froth Phase

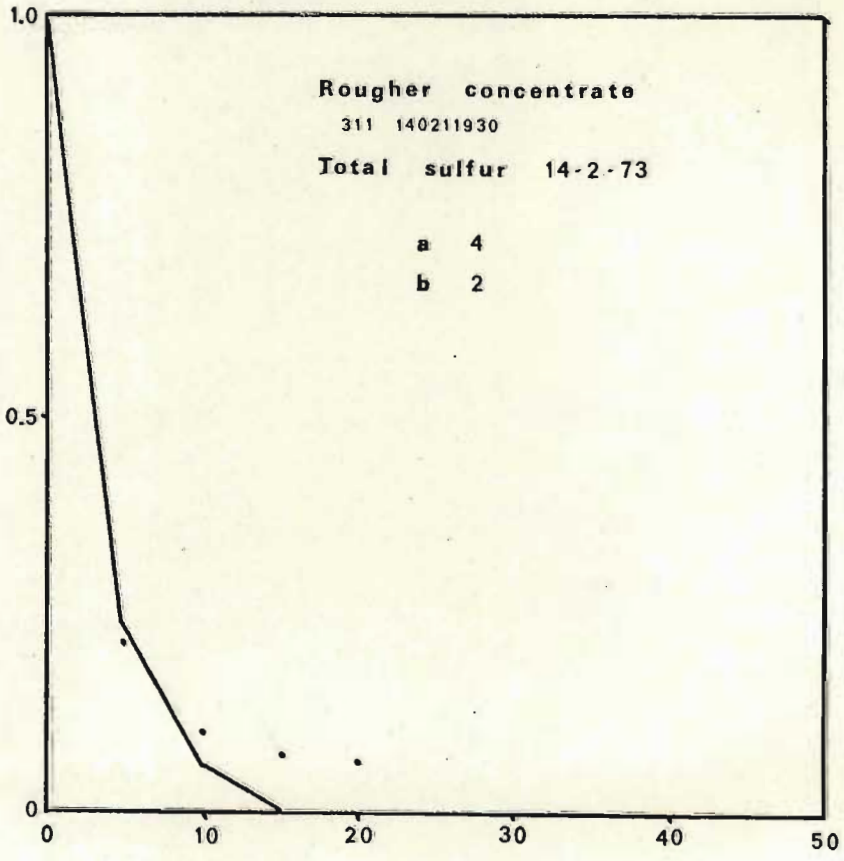
Fraction in Tailings vs. Time - mins





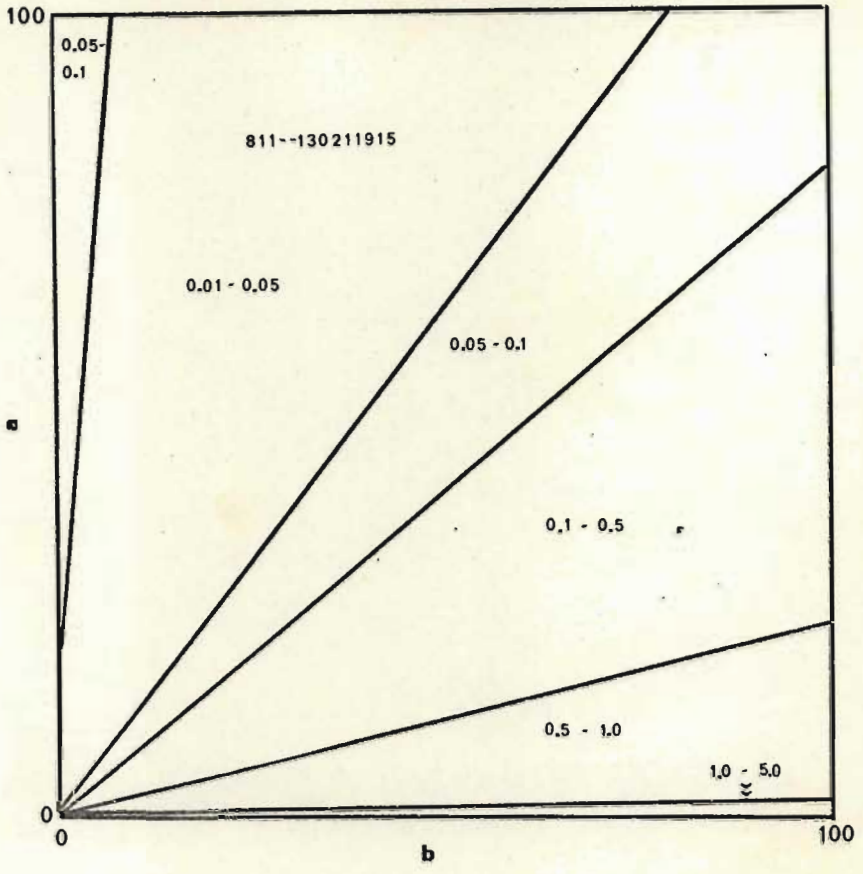
38. Importance of the Froth Phase Fraction in Tailings vs. Time - mins

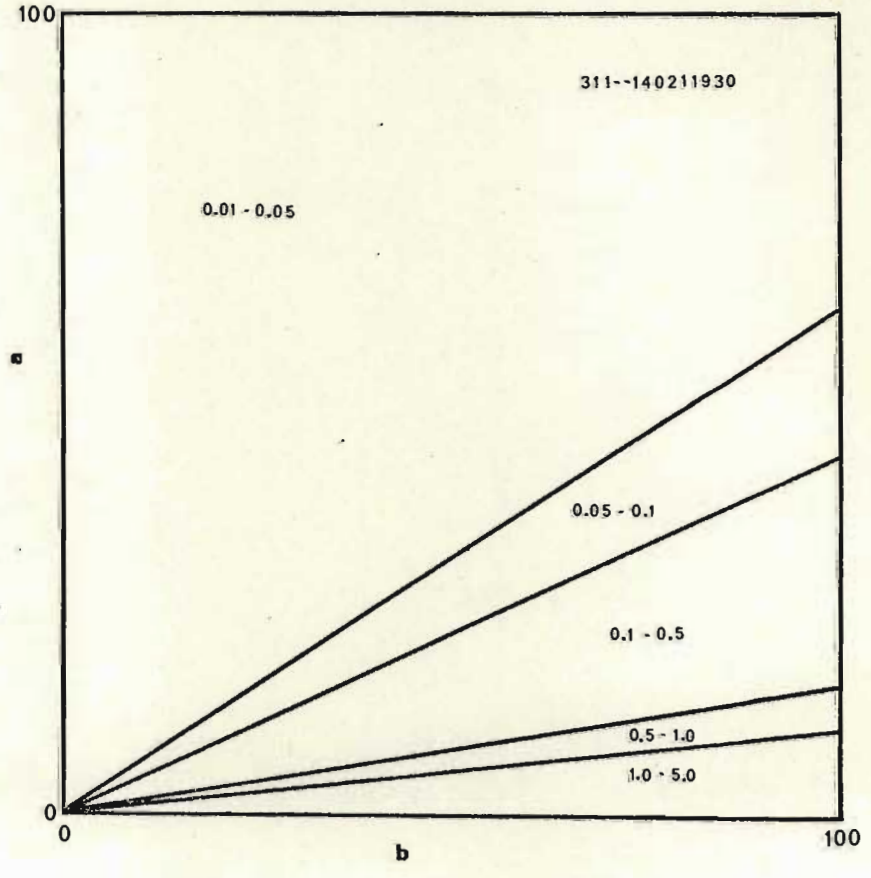




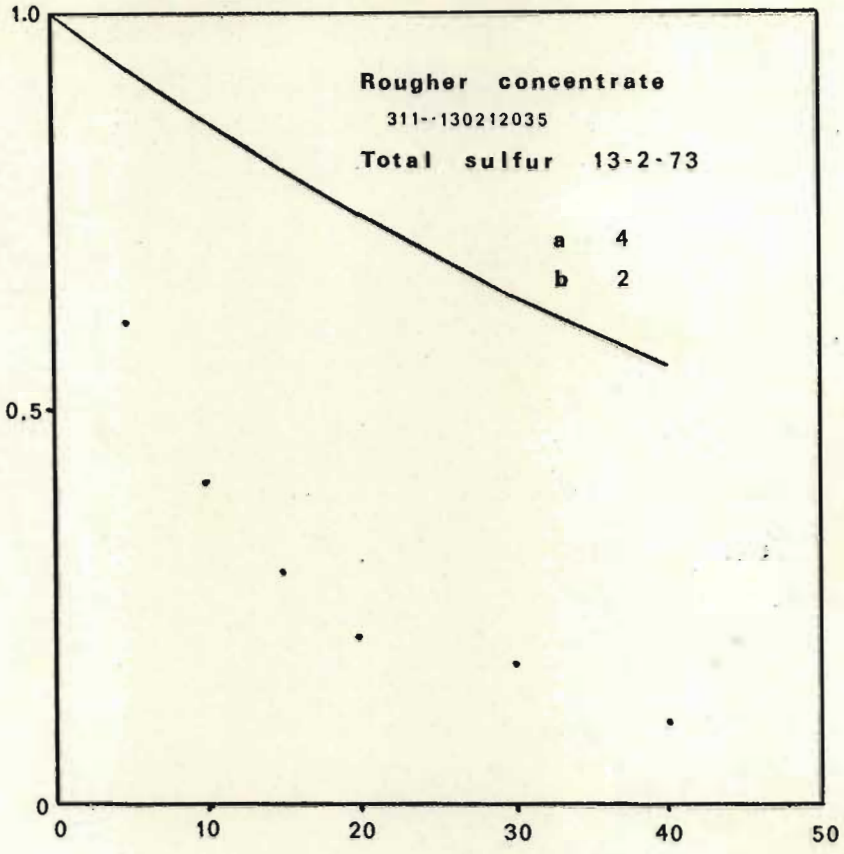
39. Batch Tests - Two Phase Model

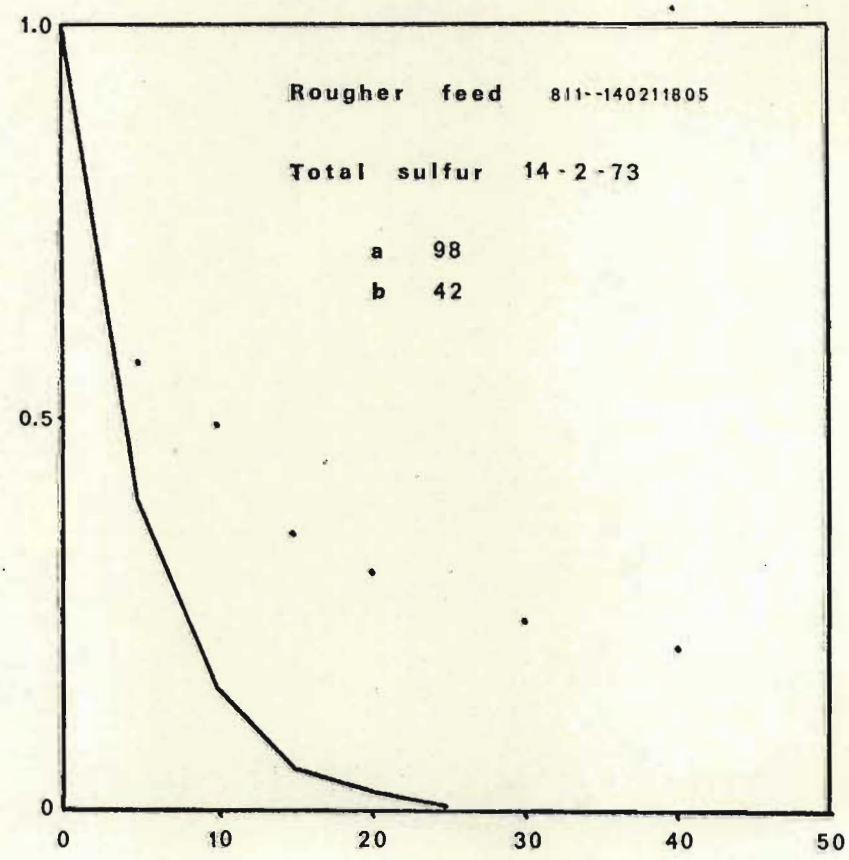
Fraction in Tailings vs. Time - mins





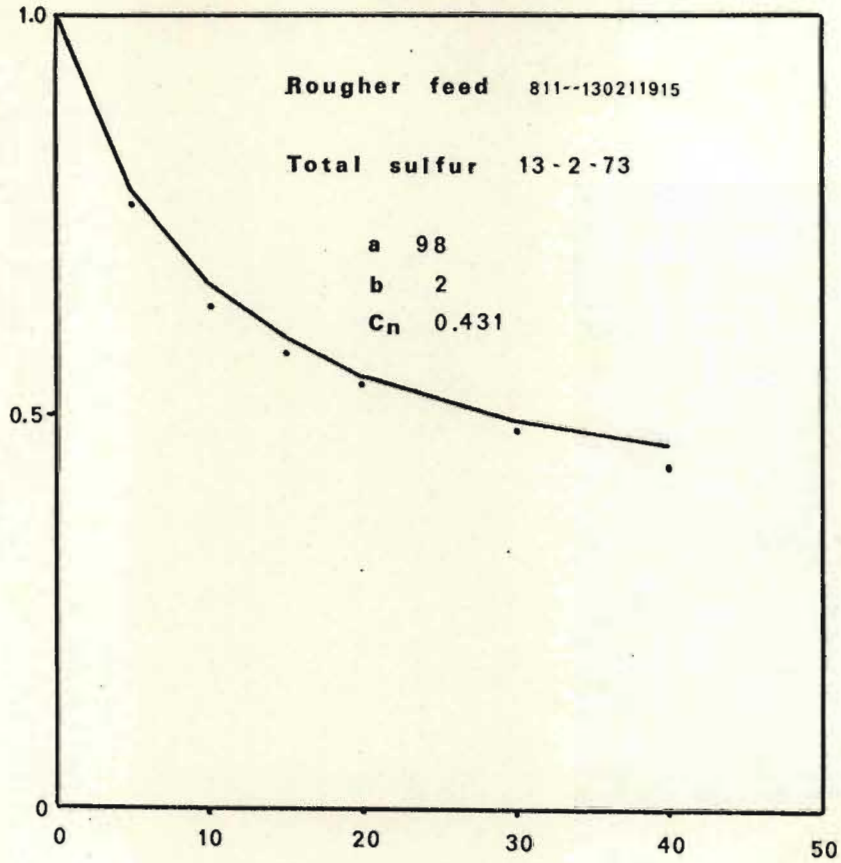
40. Sum of Squares Surfaces - Two Phase Model

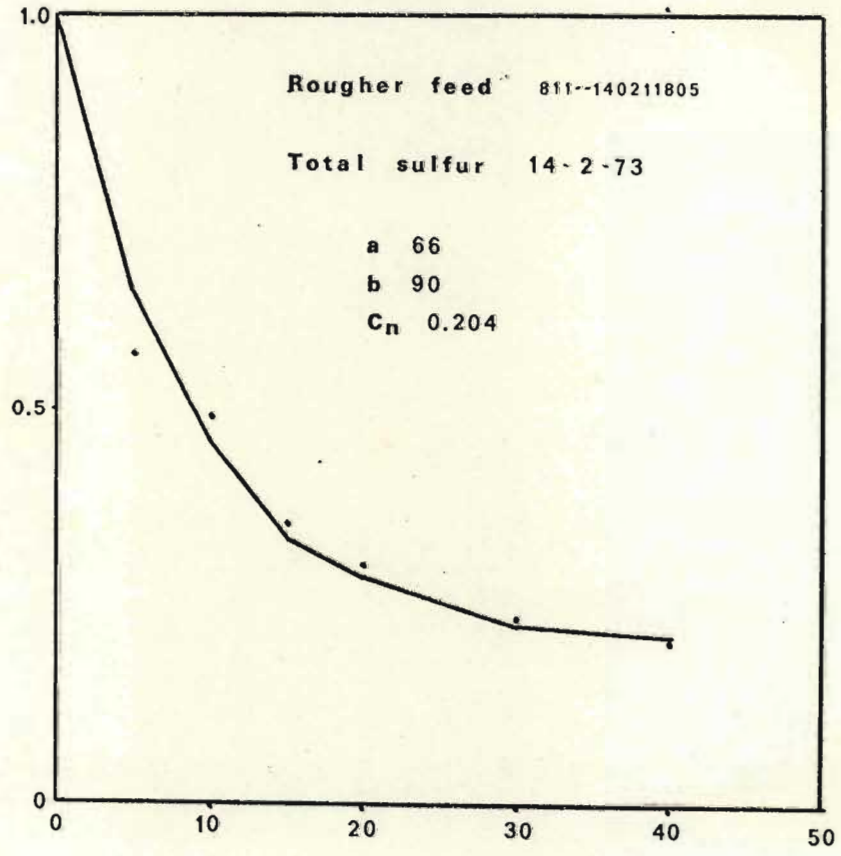




41. Two Phase Model Predictions

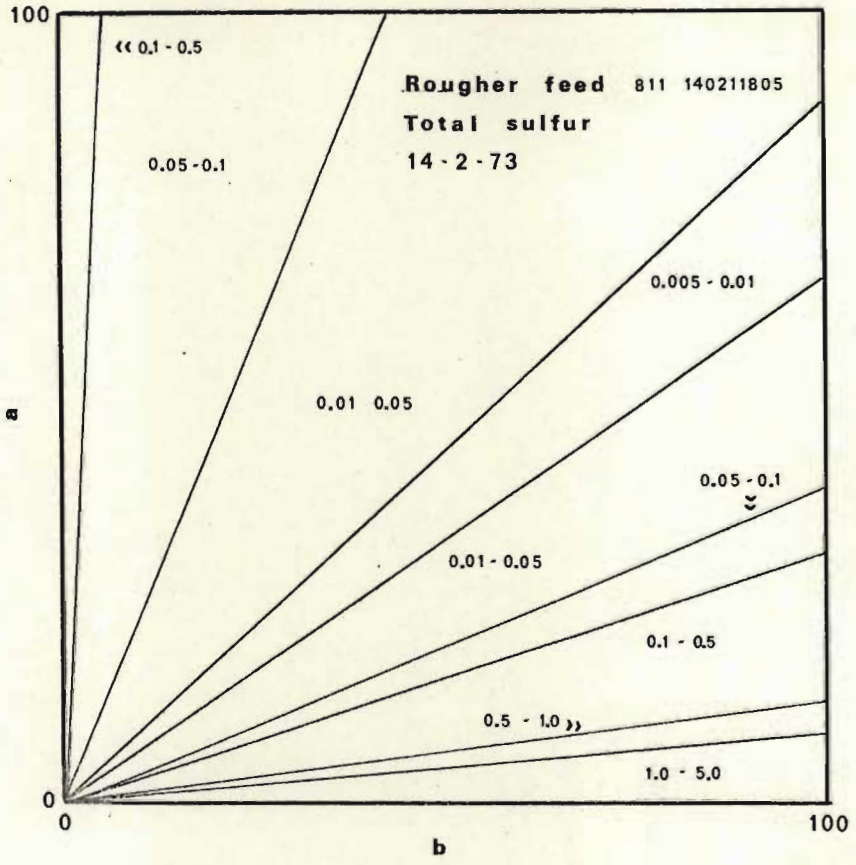
Fraction in Tailings vs. Time - mins

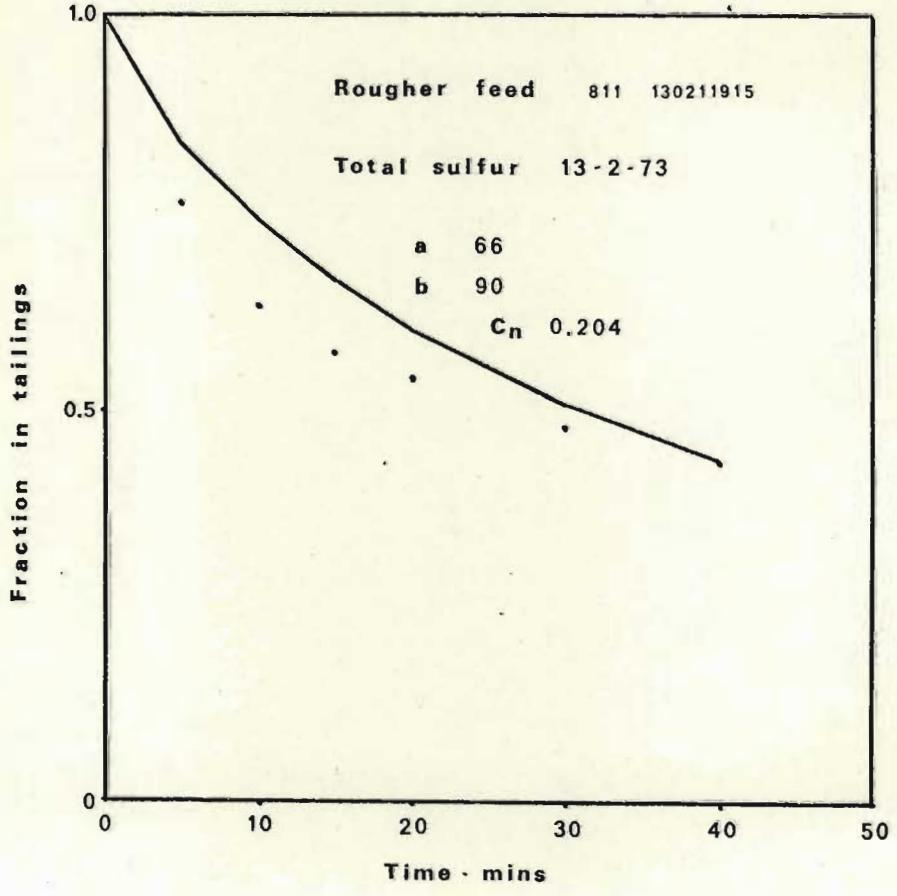




42. Batch Tests - Two Phase Model

Fraction in Tailings vs. Time - mins





43. Two Phase Model - Sum of Squares Surface and Batch Prediction

CONCLUSIONS ABOUT THE FOUR FLOTATION MODELS

Up to this point a considerable amount of data regarding the performance of four chemical analogy models in batch operation, continuous operation, batch prediction of continuous operation and prediction of the reflotation of materials has been presented. Several generalisations may be made about the applicability of these models before proceeding toward the difficult stage of circuit design analysis.

Clearly the single rate first-order model is unsuited for any application, whether it be the simple description of batch flotation of the more complicated predictive requirements. A single rate constant cannot adequately describe the flotation behaviour of the very individual particles in a mineralogical or metal species. The factors of shape and surface condition vary for any mineralogical species to a degree which leads to gross inadequacies for the single rate model.

The two-phase first order model is also severely limited. Although the model has the attraction of including the froth phase mathematically, the parameter interaction and general poor parameter definition as well as an over-reaction to changes in the froth flowrate make the model presently unsuitable. The description of species flotation by a single rate parameter (transfer from pulp to froth) has been proved lacking in the case of the single rate model; the addition of a non-floating fraction improves the model's batch description but does not alleviate the other shortcomings. It is likely that any flotation model which attempts to directly incorporate the actions of the froth phase will suffer from the problem of parameter interdependence. In flotation, the pulp and froth phase are experimentally difficult to separate and their examination together will always lead to poor parameter estima-

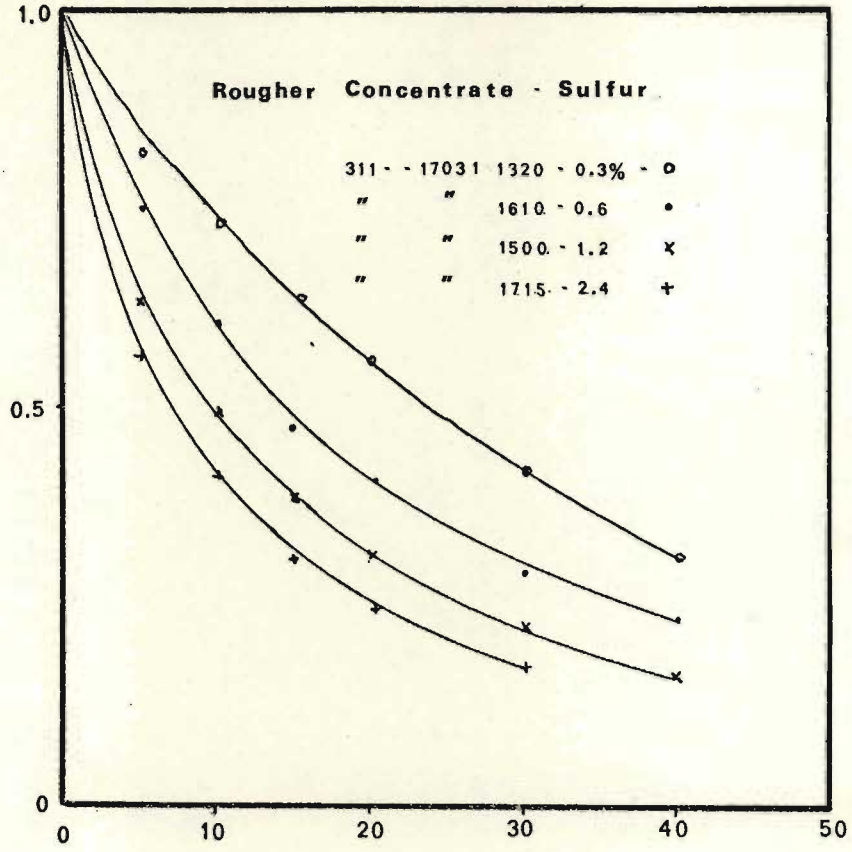
tions. The overall complexity of the froth phase and its close dependence upon the pulp phase make froth modelling a difficult problem to attack realistically. However the froth phase, particularly the froth removal rate, is of such importance that any realistic flotation model cannot fail to incorporate froth behaviour in its application.

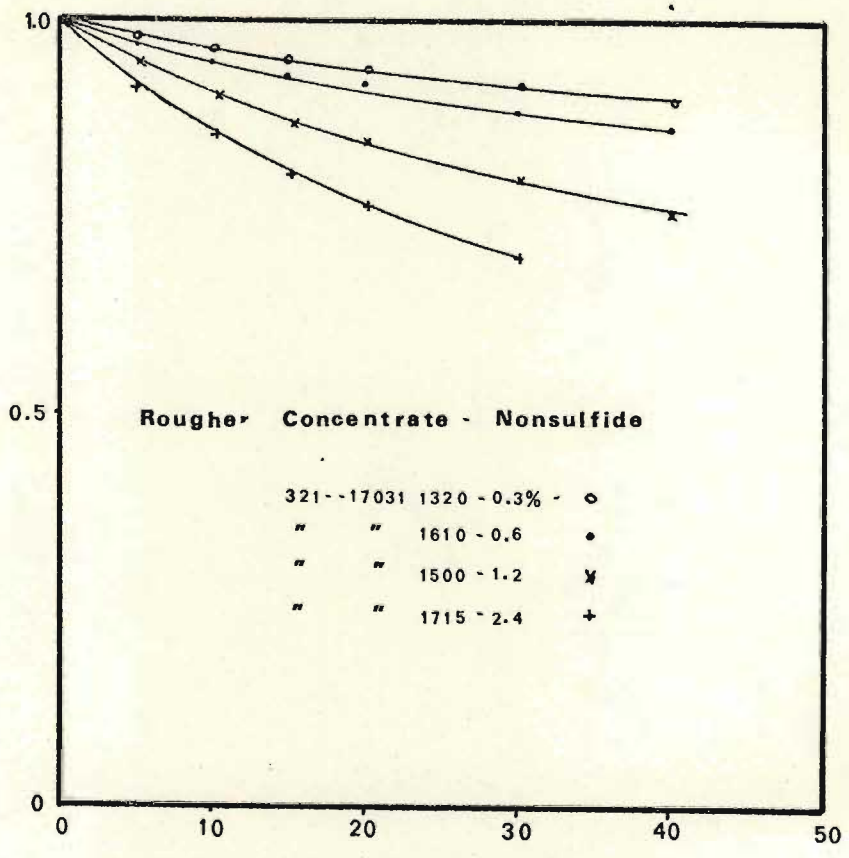
The other two single phase models, the rate plus non-floating model and the gamma function model, may be made to react to frothing conditions if their kinetic parameters prove to be reasonable functions of the froth removal rate. The data from figures 44 and 45 is used to present the kinetic parameters $\bar{k}$ (mean) and σ^2 (variance) as functions of the froth removal rate in figures 46 and 47. The rate distribution parameters appear to change smoothly with the change in froth removal rate, except for nickel material. Movement of the parameters for the rate plus non-floating model is not so consistent, partially because the non-floating fraction is not always distinct. Theoretically, however, the non-floatable fraction in this case should not alter with the froth removal rate. This abnormality clearly places the rate plus non-floating model at a disadvantage as can be seen by the figures in Table XI.

In addition to the complications of the froth phase, two other problems were evident in the past data anlysis. Firstly, the regressed parameters for any model are equipment-specific. This is illustrated by the predictions of continuous operation with the batch test parameters; while the sulphide predictions are poor, predictions of copper and nickel are within the limits of froth behaviour. Either model, the rate plus non-floating or the gamma function model, is adequate but the gamma function model appears to predict the bulk of the continuous operations more accurately.

Secondly, the assumed time invariancy of a particle's floatability is questionable. Certainly both models fall short of accurate prediction of the rougher concentrate flotation from the kinetic parameters of the rougher feed; the species normally floats faster than predicted. The formation of the rate plus non-floating model leads to large discrepancies where once floated material exhibits a non-floating fraction. The addition of a non-floating fraction and the description of the floatable material by the gamma function rate constant distribution is more plausibly the true situation, and does provide slightly superior results. Still, the majority of industrial works add reagents in several stages for the express purpose of altering the particles' floatability, it can only be hoped that such an effect is minimal within the broader scope of the modelling objectives.

The gamma function model clearly possesses several advantages and provides the most plausible framework for the development of a production-design tool. It is inherently more descriptive of the individual particles within a mineralogical species and has shown itself to be slightly more accurate in the description of batch data, and continuous data as well as being more accurate in a predictive capacity. Refer to Tables XII, XIII and XIV. As the gamma function model is mathematically capable of describing the operation of various flotation networks and has shown promise, its applicability to the analysis of an industrial flotation circuit will be examined in the following chapter.

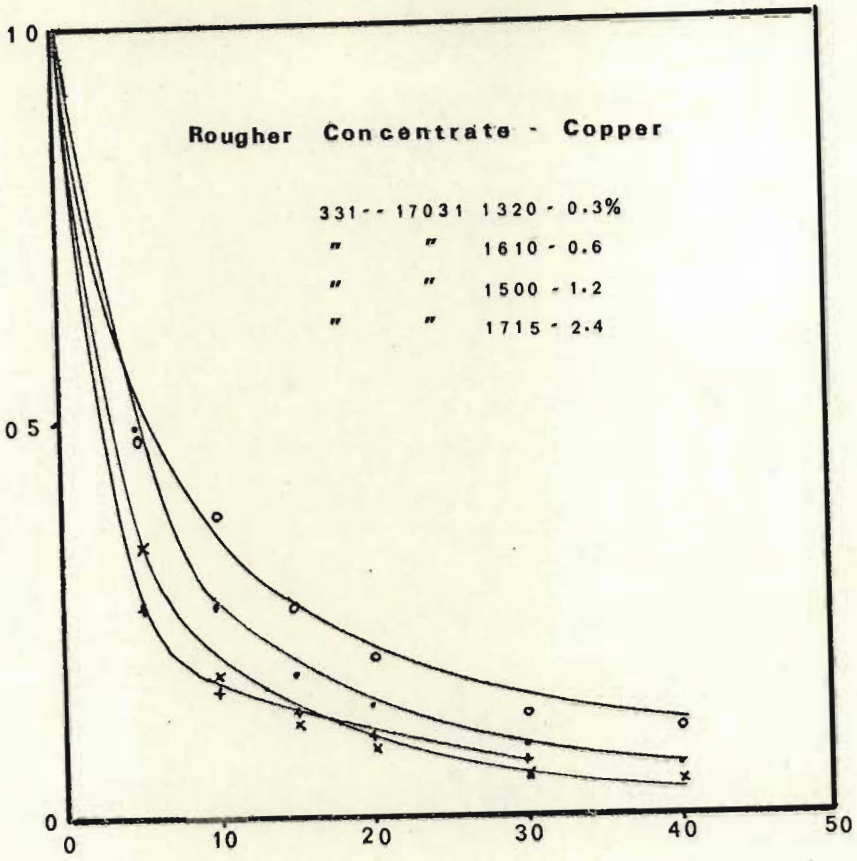


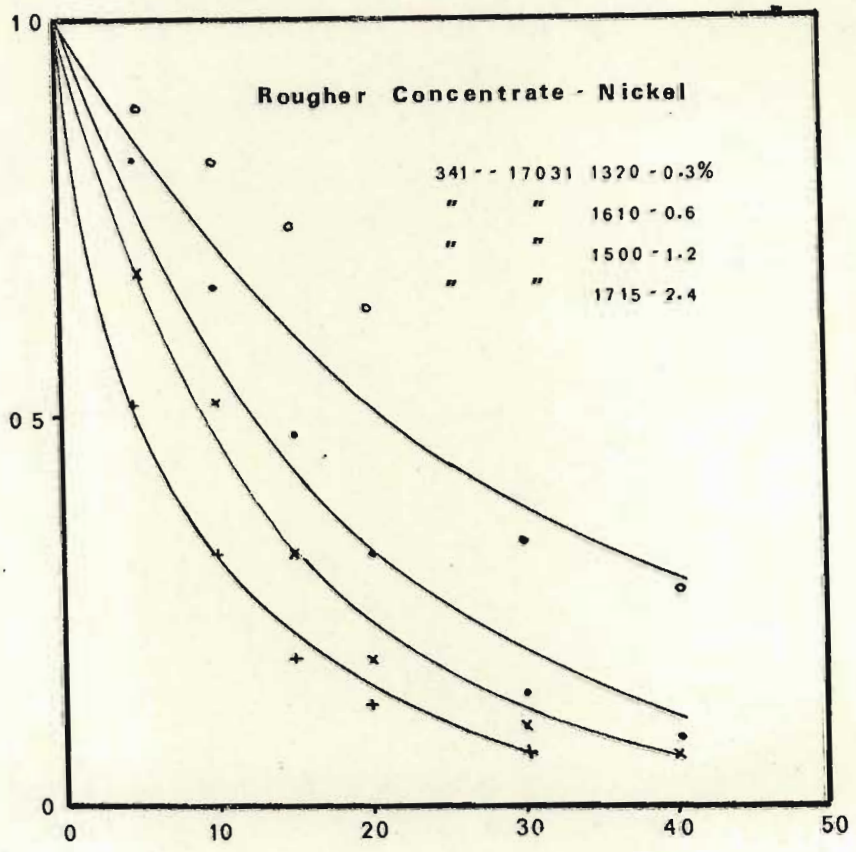


44. Batch Tests - Froth Removal Rate

Gamma Function Model

Fraction in Tailings vs Time - mins

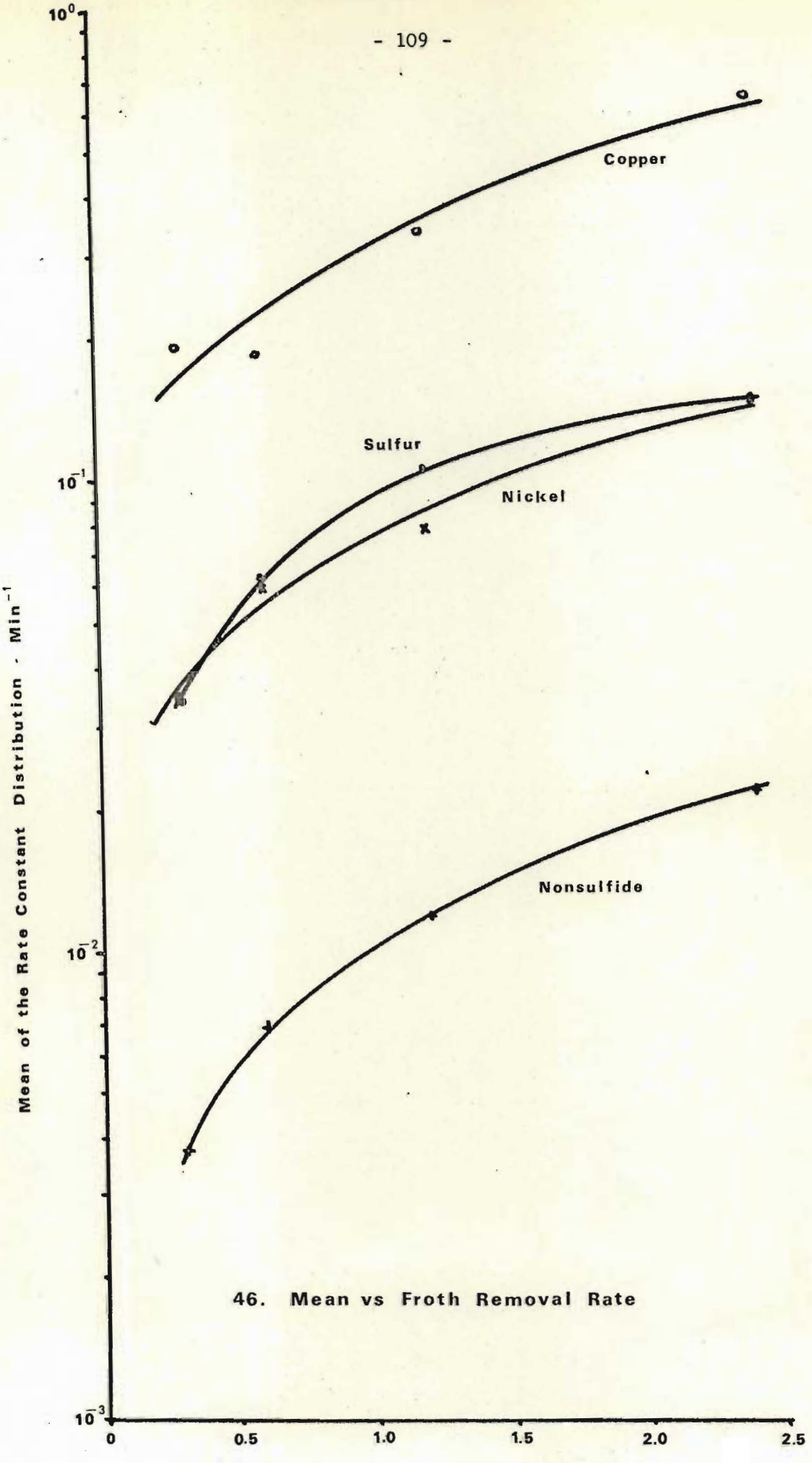




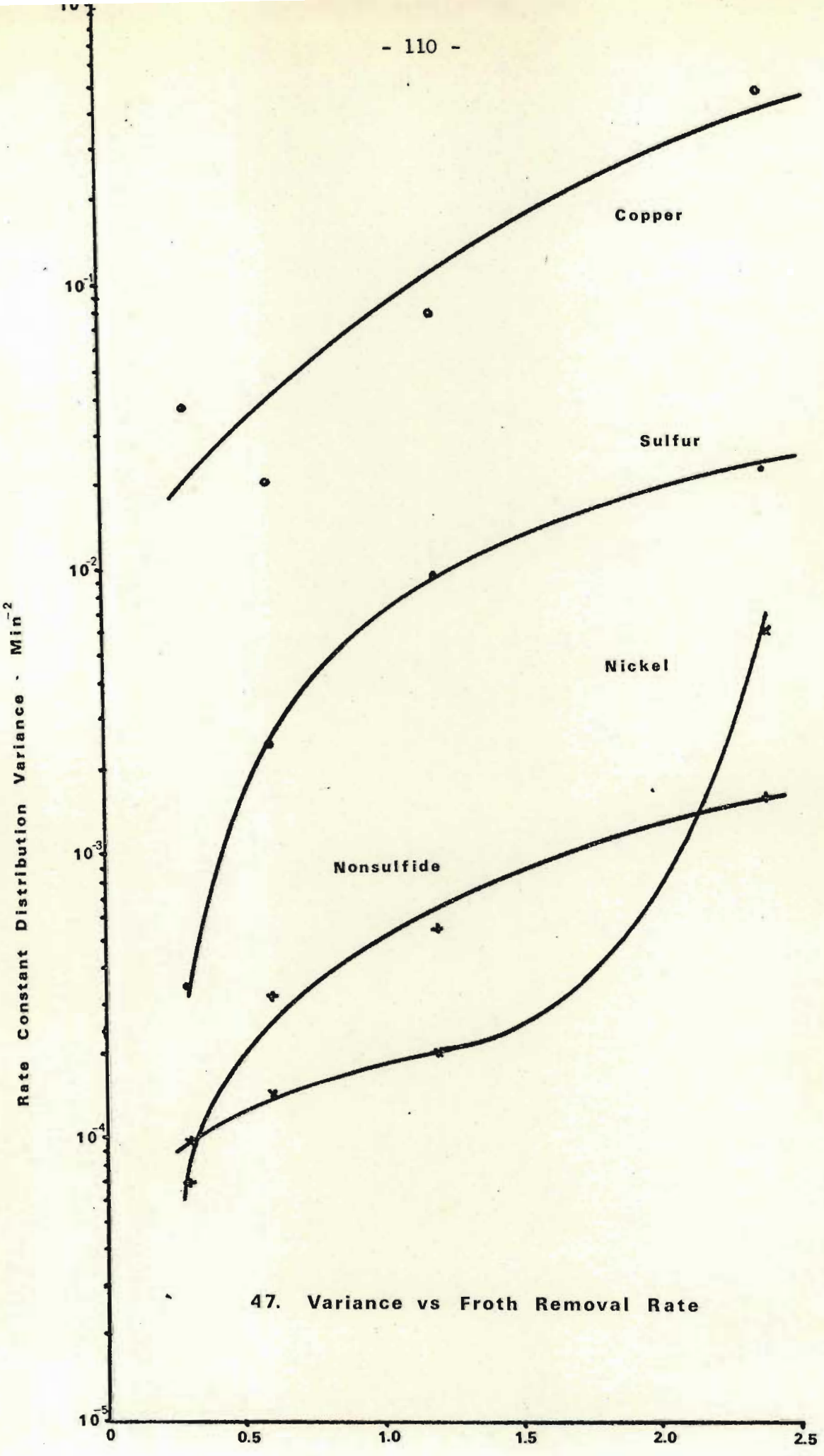
45. Batch Tests - Froth Removal Rate

Gamma Function Model

Fraction in Tailings vs Time - mins



46. Mean vs Froth Removal Rate



47. Variance vs Froth Removal Rate

TABLE XI. FROTH REMOVAL RATE DATA - ROUGHER CONCENTRATE BATCH TESTS

Species and Test Code	% of Cell Volume removed as Concentrate/min	Rate Plus Nonfloating Model			Gamma Function Model		
		k	C_n	SSQS	k	σ^2	SSQS
Sulfur							
311--170311320	0,3	0,06727	0,370	0,01445	0,03371	0,3403E-3	0,1425E-2
" " 1610	0,6	0,06637	0,181	0,7216E-3	0,06120	0,2426E-2	0,2280E-3
" " 1500	1,2	0,09720	0,169	0,4244E-2	0,1040	0,9413E-2	0,9667E-3
" " 1715	2,4	0,1359	0,183	0,2524E-2	0,1476	0,02100	0,2499E-3
Nonsulfide							
321--170311320	0,3	0,01859	0,799	0,2304E-4	0,3744E-2	0,6806E-4	0,1700E-4
" " 1610	0,6	0,1507	0,894	0,3692E-2	0,6910E-2	0,3134E-3	0,3816E-5
" " 1500	1,2	0,03702	0,704	0,1162E-3	0,01164	0,5330E-3	0,4031E-4
" " 1715	2,4	0,05050	0,622	0,1469E-3	0,02154	0,1480E-2	0,4734E-4
Copper							
331--170311320	0,3	0,1100	0,089	0,01827	0,1919	0,03684	0,3144E-2
" " 1610	0,6	0,1146	0,022	0,01414	0,1855	0,02026	0,6800E-3
" " 1500	1,2	0,1309	0,000	0,04408	0,3320	0,07644	0,1244E-3
" " 1715	2,4	0,3177	0,095	0,3719E-2	0,6253	0,4515	0,3660E-3
Nickel							
341--170311320	0,3	0,06210	0,305	0,1200	0,03461	0,8854E-4	0,06455
" " 1610	0,6	0,07922	0,139	0,05169	0,05878	0,1503E-3	0,02395
" " 1500	1,2	0,09344	0,064	0,9415E-2	0,07765	0,1986E-3	0,3225E-2
" " 1715	2,4	0,1327	0,056	0,1074E-2	0,1401	0,5779E-2	0,4249E-3

TABLE XII - MODEL COMPARISON

PREDICTION OF CONTINUOUS OPERATION

Material	Mean Error in Predicted Concentration for Model Form	
	Rate plus Non-floating	Gamma Function
Sulphur - Rougher	,19	,17
" Cleaner	,135	,136
Copper - Rougher	,13	,07
Cleaner	,11	,13
Nickel - Rougher	,11	,07
" Cleaner	,12	,13
Non-sulphide - Rougher	,005	,0045
" Cleaner	,10	,10
Total - Rougher	,169	,137
Cleaner	,151	,160

TABLE XIII - MODEL COMPARISON

PREDICTIONS FOR REFLOATED MATERIAL

Species	Mean Error in Predicted Concentration for Model Form		
	Rate plus Non-floating	Gamma Function	
		$(\bar{k}, \sigma^2)$	$(\bar{k}, \sigma^2, Cn)$
Sulphur	0,18	0,16	0,148
Copper	0,23	0,23	0,191
Nickel	0,22	0,21	0,175
Total	0,20	0,19	0,16

TABLE XIV - MODEL COMPARISON

BATCH TEST DESCRIPTION

Species	Mean Error in Fitted Concentration for Model Form	
	Rate plus Non-floating	Gamma Function
Sulphur - Rougher	,04	,03
" - Cleaner	,009	,01
Copper - Rougher	,03	,02
" Cleaner	,01	,005
Nickel - Rougher	,03	,04
" Cleaner	,009	,01
Non-sulphide - Rougher	,004	,004
" Cleaner	,005	,009
Total -	,0074	,0067

All batch tests from continuous prediction and prediction of refloated material are used.

V. USE OF THE GAMMA FUNCTION MODEL AS A CIRCUIT DESIGN AID.

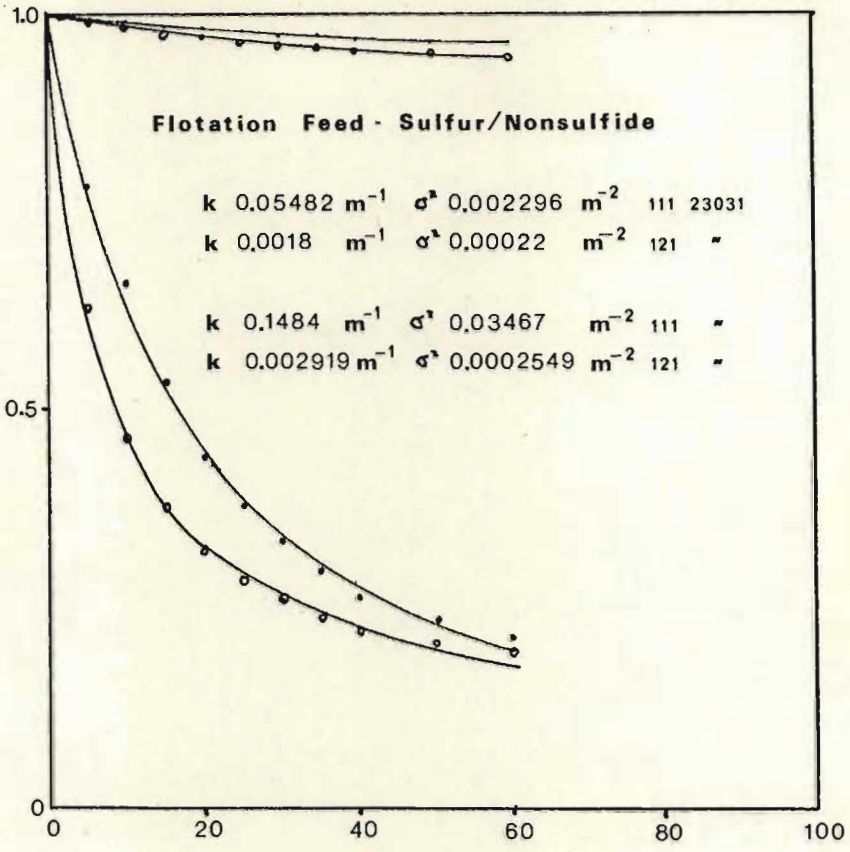
The mathematical technique developed by Woodburn (refer to section VIII) allows a rapid method of flotation circuit design analysis based on what is the most descriptive of the first-order irreversible models investigated; the continuous distribution gamma function model. In the previous chapters, the gamma function model has been proved accurate in the description of batch flotation and its predictive capabilities have been proved reasonable considering the difficulties of the froth removal rate and of the machine dependability of parameters. In continuation, the gamma function model's assets and liabilities can be further illustrated by a circuit design analysis of the Rustenburg ores.

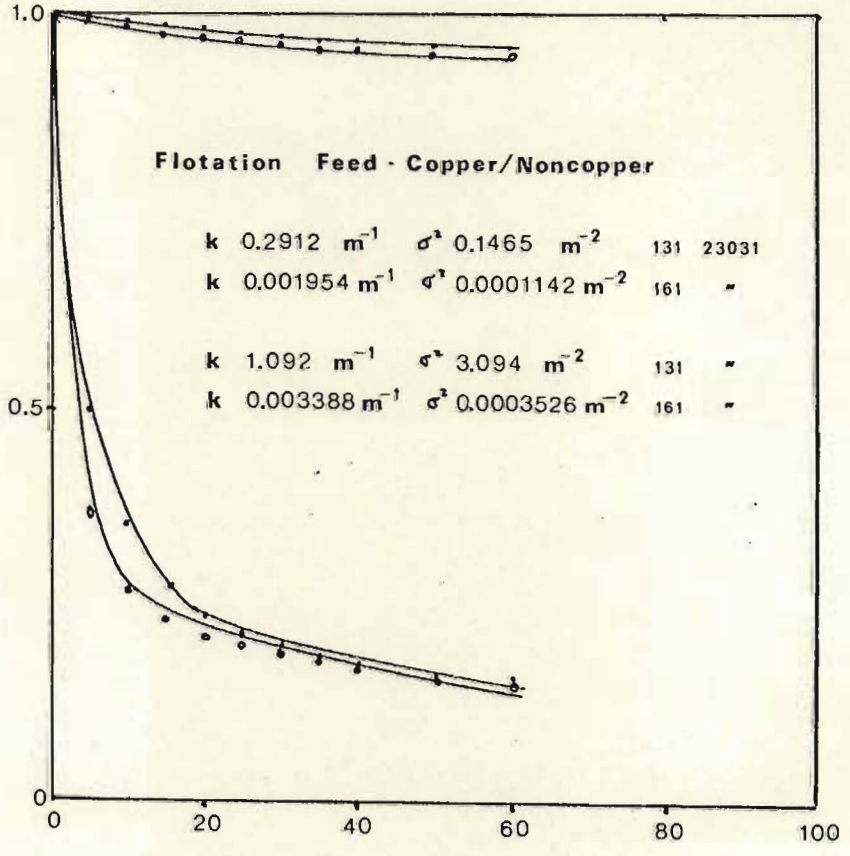
The true test of a model's predictive capability is the analysis of plant scale flotation in an industrial circuit by means of batch tests on the plant feed stream. Two flotation tests on the plant feed stream were performed, their concentration versus time curves are shown in figures 48 and 49. The froth removal rates were not measured, however, it is assumed that the faster apparent flotation rate is more representative of the plant performance and simulations of the various plant circuits are based on the regressed parameters of test 2. The batch tests and the sampling campaigns on the various circuits described later were not performed at the same time. In general operation of the plant, results are fairly steady showing that the feed material does not alter radically over long periods thus the batch test was performed on a material which is reasonably representative of the feed to the various circuits.

Simulation of Various Hypothetical Circuits.

In the simulation of the following circuits it is necessary to provide some basis of comparison, this is best done from the point of view of plant utilization, i.e. maximum metallurgical results from a limited capital investment. It is convenient to assume a plant availability of twenty-five equal volumed units which may be arranged in any combination to produce the desired circuit. Four basic circuits are of interest: the rougher-scavenger circuit, the rougher-cleaner-recleaner circuit, the rougher-cleaner circuit and the rougher-cleaner-cleaner middlings circuit. Flow diagrams are provided in Figure 50. In the generation of the grade-recovery curves of Figure 51, an increasing number of cells were used to produce the final concentrate stream. In the case of the rougher-cleaner-recleaner circuit and the rougher-cleaner-cleaner middlings circuit, the number of cleaner and cleaner middlings cells, respectively, were kept constant at four units.

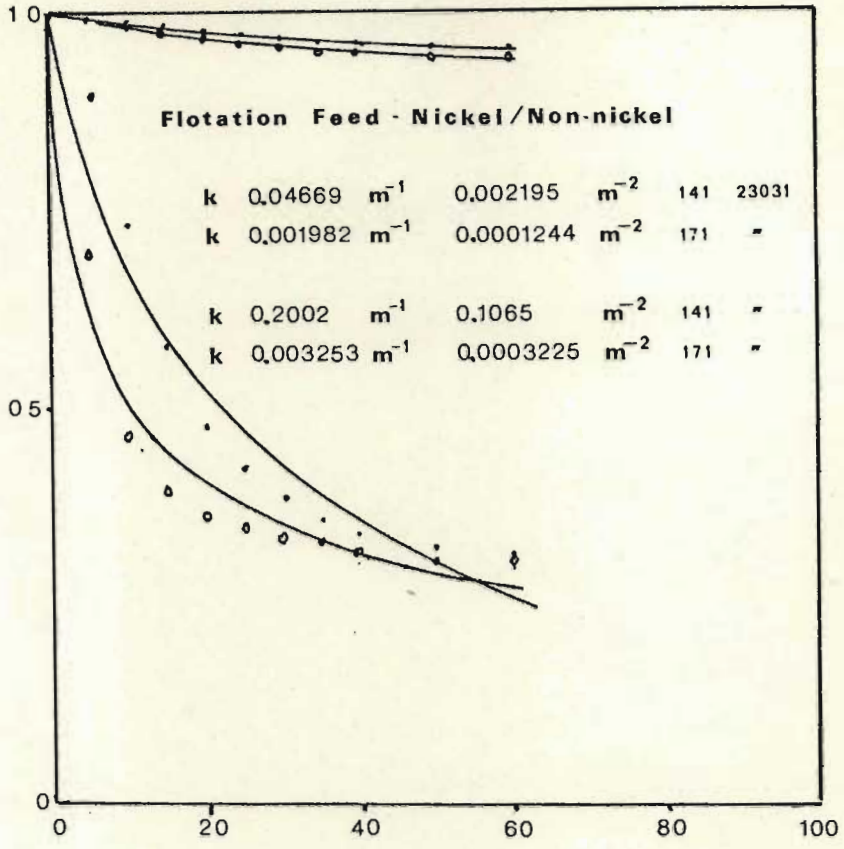
The simulations shown have many of the trends that experience would dictate. Firstly, recovery increases as grade is dropped in the simple rougher-scavenger circuit. Secondly, in all those circuits which utilize cleaner capacity a sharp drop in the recovery-grade curve is experienced when losses due to the shortening of the rougher bank override increases in recovery due to the production of a lower grade concentrate. Thirdly, the faster floating copper material is suited to a cleaner operation while the slower floating nickel material is not. Slow floating materials are both difficult to recover and difficult to produce at good grades; the nearer the flotation rate of the valuable species gets to the rate of flotation of the gangue species, the less advantageous a cleaning operation becomes.





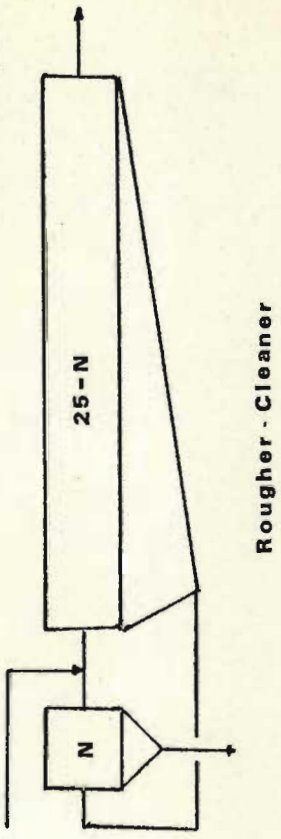
48. Batch Tests - Gamma Function Model

Fraction in Tailings vs Time - mins

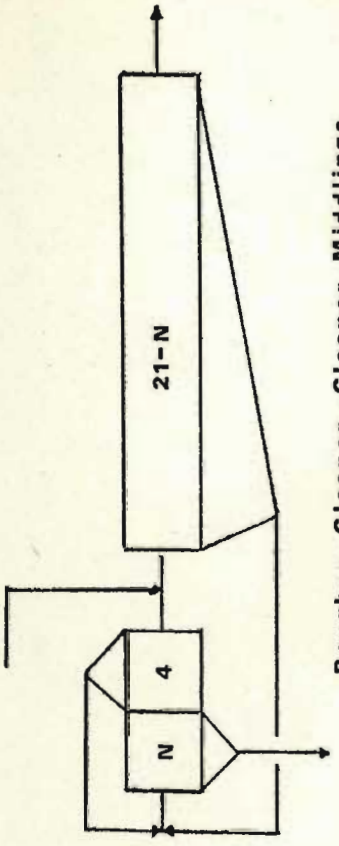


49. Batch Tests - Gamma Function Model

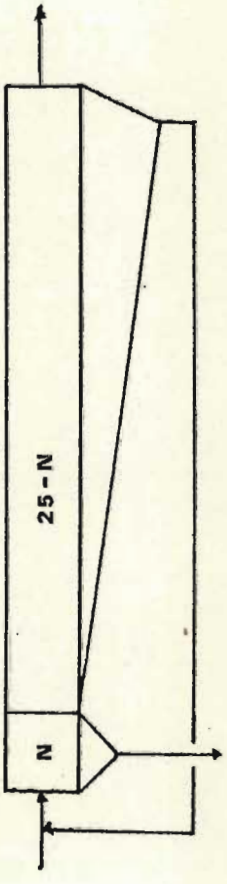
Fraction in Tailings vs Time - mins



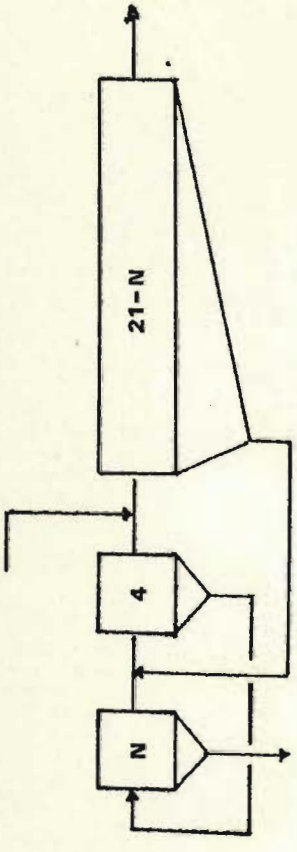
Rougher - Cleaner



Rougher - Cleaner - Cleaner Middlings

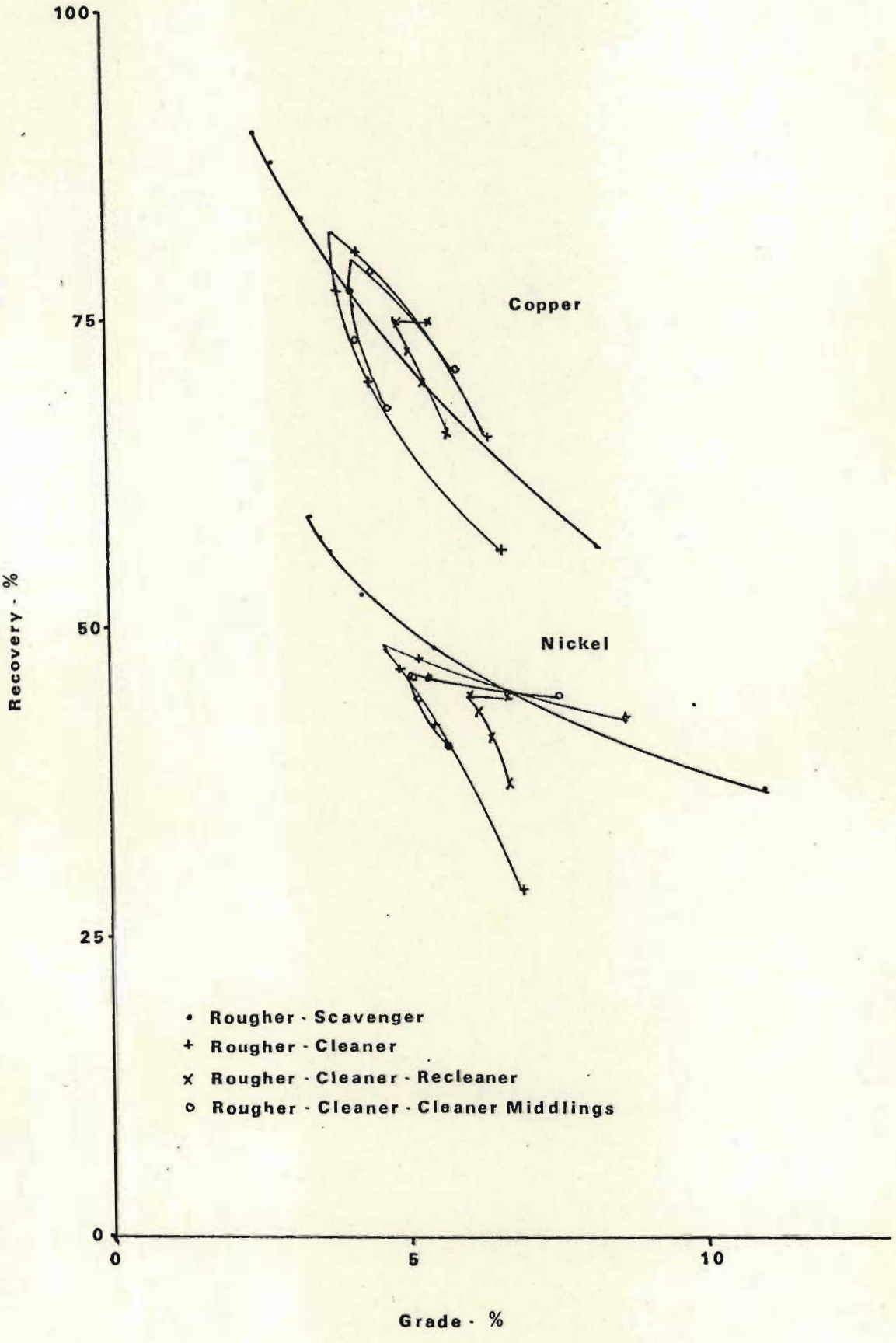


Rougher - Scavenger



Rougher - Cleaner - Recleaner

50. Hypothetical Circuits



51. Simulated Grade-Recovery Curves

Comparison of Actual Circuit Data with Simulations

Although the gamma function model may be capable of predicting operational trends for various circuit configurations as it has done above, it is also of value to determine the range of error in a circuit simulation. Several flotation cell configurations were tested at Rustenburg over a period of several months. The flotation section tested at Rustenburg may be simplified into a forty unit network; twenty of these units have four times the effective volume of the remaining twenty. The smaller volumed units serve as cleaners. Use of Woodburn's technique (refer to Section VIII) then involves the simulation of flotation in these forty units when the units are arranged so as to produce a rougher-scavenger, a rougher-cleaner, a rougher-cleaner-recleaner, a rougher-cleaner-cleaner middlings, and a rougher-scavenger-cleaner-cleaner middlings circuit. For the simulation, a ratio of concentrate volume to tailings volume was assumed for each cell. The assumed value for rougher and scavenger cells is 0,011 and for cleaner, recleaner or cleaner middling cells is 0,025. Figure 52 illustrates the circuits as operated, smaller cells are indicated as such. Data for the rougher-scavenger circuit is available on a periodic basis throughout the test period and sulphur, copper and nickel analyses are available. All other data represents test averages; copper and nickel analyses are available only for one case of the rougher-cleaner-cleaner middlings circuit. Simulated results and actual assays are presented in Tables XV and XVI. Table XVII contains general operational data for the test circuits.

Generally the model performs poorly; the batch test kinetic parameters predict higher tailings losses and higher final concentrate grades in the majority of cases. The only exception being the rougher-cleaner circuit sulphide simulation

where the model predictions compare very well with the operational results. It is worthwhile to note that the model's simulated values in the rougher-scavenger circuit lie outside the range of one and one-half standard deviations from the average value as assayed; an exception is the percentage of nickel in the final concentrate. The two probable sources of error have already been discussed, namely, the dependence of the batch kinetic parameters upon froth removal rate and machine design. As is illustrated in the section on the applicability of the four flotation models, an increase in froth removal rate will increase the apparent rate of flotation of both the gangue and the valued species. It is possible that the froth removal rate in the batch test used for simulation was lower than that which corresponds to plant operation since an increased froth removal rate would lower both tailings grades and concentrate grades.

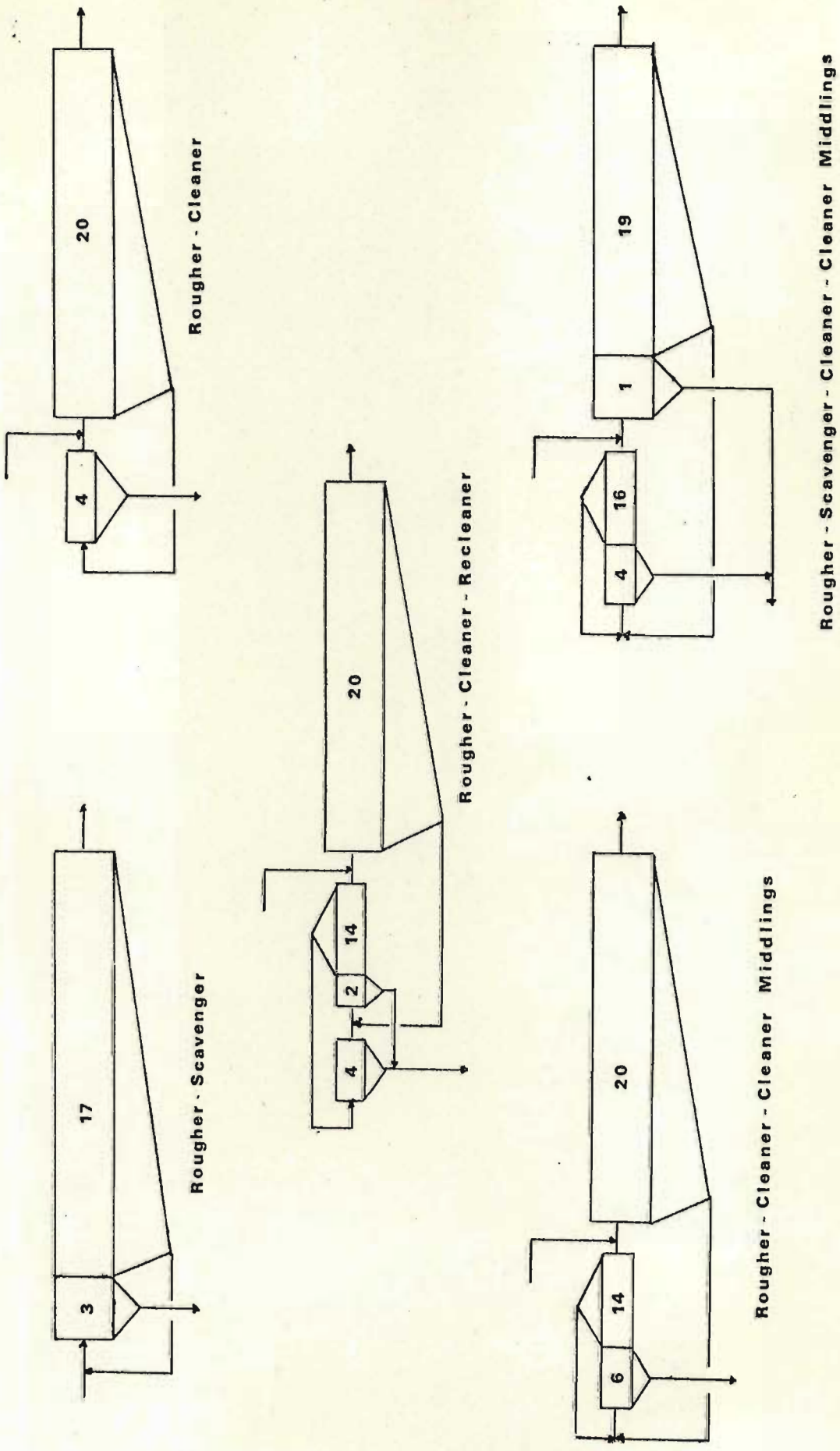
Conclusions

The requirement of a reasonably accurate production and design tool has not been completely met. Of the four models investigated, the gamma function first-order irreversible model has proved most acceptable in the conditions tested. Use of the gamma function model as a production and design aid is of some value, however, considerable caution should be exercised in the use of simulated results.

The major problems associated with the use of the gamma function model in its present form are

1) some species may be particularly susceptible to the flotation action in different machines, thus the measured rate parameters are machine dependent ,

2) the froth phase is an important and integral part of flotation, its removal rate does in fact determine the apparent rate of species flotation.



52. Tested Circuits

TABLE XV. DATA SUMMARY FOR ROUGHER-SCAVENGER CIRCUIT

Time	Measured Feed Assay			Tailings Assays						Concentrate Assays					
				Measured Value			Model Value			Measured Value			Model Value		
	% Cu	% Ni	% S	% Cu	% Ni	% S	% Cu	% Ni	% S	% Cu	% Ni	% S	% Cu	% Ni	% S
'30	0,065	0,164	0,38	0,016	0,064	0,14	0,024	0,097	0,23	3,05	5,73	15,0	3,77	6,27	14,8
300			1,08	0,016	0,059	0,14			0,65	3,33	4,85	13,3			33,3
315	0,077	0,264	0,61			0,14	0,028	0,158	0,37			12,1	4,48	9,81	22,0
330			0,47	0,018	0,080	0,13			0,28	3,63	6,81	17,5			17,9
345	0,071	0,182	0,45			0,13	0,026	0,108	0,27			13,3	4,13	6,94	17,1
900			0,46	0,015	0,078	0,15			0,28	4,34	6,48	15,1			17,8
915	0,068	0,180	0,42			0,15	0,025	0,108	0,26			18,6	4,07	7,02	16,4
930			0,48	0,015	0,072	0,17			0,30			12,2			18,6
945	0,075	0,195	0,44			0,15	0,027	0,116	0,26	3,34	7,05	13,8	4,35	7,40	16,8
000			0,42	0,021	0,090	0,18			0,25			14,5			15,9
015	0,063	0,185	0,40			0,19	0,022	0,108	0,23	3,89	5,04	13,1	3,48	6,73	15,0
030			0,46	0,016	0,073	0,13			0,27			12,5			17,1
045	0,073	0,166	0,43			0,14	0,026	0,098	0,25	3,75	6,26	16,4	4,11	6,21	16,2
100			0,43	0,018	0,077	0,15			0,26			14,2			16,4
115	0,069	0,168	0,42			0,13	0,025	0,099	0,25	3,68	4,62	13,9	3,95	6,35	16,0
130			0,51	0,018	0,074	0,15			0,30			18,4			18,6
145	0,078	0,187	0,48			0,13	0,028	0,110	0,28	3,72	7,19	17,2	4,36	6,91	17,7
200			0,50	0,014	0,074	0,14			0,30			13,8			18,6
215	0,096	0,174	0,41			0,15	0,034	0,102	0,24	3,51	6,26	15,7	5,35	6,50	15,6
245			0,46	0,013	0,094	0,19			0,27	3,10	5,85	14,9			17,1
300	0,074	0,214	0,46			0,14	0,027	0,127	0,27			14,9	4,25	8,00	17,4

TABLE XV. (Continued)

ime	Measured Feed Assay			Tailings Assays						Concentrate Assays					
				Measured Value			Model Value			Measured Value			Model Value		
	% Cu	% Ni	% S	% Cu	% Ni	% S	% Cu	% Ni	% S	% Cu	% Ni	% S	% Cu	% Ni	% S
315			0,47	0,010	0,114	0,18			0,28	3,38	7,51	18,6			17,6
345	0,076	0,187	0,56	0,008	0,099	0,16	0,026	0,107	0,32	3,16	10,95	16,9	4,06	6,62	19,5
400	0,074	0,135	0,43			0,19	0,027	0,079	0,25			13,5	4,18	5,12	16,2
415			0,55	0,008	0,099	0,20			0,33	3,35	4,50	11,1			20,0
430	0,084	0,198	0,46			0,18	0,030	0,116	0,27			11,7	4,68	7,29	17,1
445			0,44			0,15			0,26	3,36	4,47	12,2			16,2
500	0,093	0,219	0,45	0,006	0,090	0,16	0,032	0,127	0,26			12,3	5,06	7,86	16,6
515			0,56			0,16			0,33	3,47	6,24	15,8			20,2
530	0,098	0,228	0,52	0,024	0,099	0,17	0,035	0,133	0,30			12,0	5,37	8,24	18,8
545			0,52			0,19			0,31	2,59	4,97	13,0			19,0
AGE	0,077	0,190	0,475	0,015	0,084	0,16	0,028	0,112	0,29	3,45	6,16	14,4	4,35	7,08	18,0
DEV.	0,011	0,030	0,146	0,005	0,015	0,02	0,004	0,018	0,08	0,39	1,58	2,2	0,53	1,06	3,3

TABLE XVI. DATA SUMMARY FOR CIRCUIT ANALYSIS

Circuit Description		Feed Assay (measured)	Tails Assay (measured)	Tails Assay (Model)	Concentrate Assay (measured)	Concentrate Assay (model)	Test Length (hours)
Rougher-Scavenger-Cleaner Cleaner-Middling	(% S)	0,42	0,14	0,16	7,4	10,6	12
Rougher-Cleaner	(% S)	0,43	0,10	0,10	13,4	12,3	8,3
Rougher-Cleaner-Recleaner	(% S)	0,47	0,18	0,25	12,8	18,6	8
Rougher-Cleaner-Cleaner-Middling							
TEST 1	(% S)	0,41	0,15	0,20	8,6	14,5	12
TEST 2	(% S)	0,43	0,15	0,25	7,0	18,5	9,8
TEST 3	(% S)	0,43	0,16	0,20	7,0	14,3	12
	(% Cu)	0,077	0,0134	0,0143	2,51	3,76	12
	(% Ni)	0,191	0,092	0,103	3,42	5,45	12
TEST 4	(% S)	0,44	0,14	0,18	7,1	12,3	11,5

TABLE XVII.

Test Circuit	General Remarks
Rougher-Scavenger-Cleaner-Cleaner Middlings	Average particle size slightly finer than normal. Residence time in rougher-scavenger nearly double of normal operation.
Rougher-Cleaner	Average particle size normal, residence time in rougher also normal.
Rougher-Cleaner-Recleaner	Particle size and residence time in rougher normal.
Rougher-Cleaner-Cleaner Middling	Test 1, Very fine average particle size; residence time in rougher slightly longer than normal. Test 2, Normal operation. Test 3, Normal operation. Test 4, Particle size slightly coarser than normal; residence time in rougher bank nearly double normal.
Rougher-Scavenger	Normal operation.

Mathematical Formulation of the Models

The mathematical description of each model is discussed and the equations necessary to handle various predictions are outlined in detail in the following sections in order to clarify the results of the investigation. Each of the investigated models is discussed separately.

The Single Rate Model

The single rate model may be developed for the batch case as follows

$$dc/dt = -kC$$

hence through integration and initial conditions

$$C = C_o e^{-kt}$$

The addition of the 'perfect' mixing residence time distribution below

$$E(t) = \frac{1}{T} e^{-t/T}$$

leads to the equation for continuous operation of a single cell

$$C = C_o \int_o^\infty e^{-kt} \frac{1}{T} e^{-t/T} dt$$

$$C = \frac{C_o}{(1 + kT)}$$

By repeated application of the above equation, the equation for a series of 'n' equal volumed units may be developed.

$$C = \frac{C_o}{(1 + kT)^n}$$

As the single rate model describes the flotation of any mineral species by a single first-order rate constant, the same equations and rate constant apply to the species at any point within a flotation network.

The Rate plus Non-floating Model

Mathematically this model is a simple extension of the single rate model into a general discrete distribution model as follows :

$$dC_i/dt = -k_i t$$

thus for batch operation

$$C = \sum_{i=1}^{n} C_i e^{-k_i t} \quad ; \quad \sum_{i=1}^{n} C_i = C_o$$

and for continuous operation

$$C = \sum_{i=1}^{N} \frac{C_i}{(1 + k_i T)^n}$$

For the forms used in fitting data at Rustenburg where only two fractions were assumed, Cn, the fraction which is unfloatable and the remaining material which floats according

to a single rate constant, the equation for a batch test becomes

$$C = (C_o - C_n)\, e^{-kt} + C_n$$

and for continuous operation

$$C = \left[(C_o - C_n) / (1 + kT)^n\right] + C_n$$

In the application to various circuits with the rate plus non-floating model, the concentrate from any cell contains no material with a zero rate constant, in fact all the concentrate material must behave as floatable material with a rate constant of 'k'. Thus in the particular application of predicting rougher concentrate flotation from the rougher feed batch test, the rougher feed batch test is described by the equation

$$C = (C_o - C_n)\, e^{-kt} + C_n$$

and the rougher concentrate batch test is described by

$$C = C_o\, e^{-kt}$$

The Gamma Function Model

Woodburn and Loveday[38] have assumed a form of the gamma function to describe a distribution of first-order rate constants for any mineral species; the assumed distribution is

as follows

$$W(k) = \frac{(\bar{k}/\sigma^2)^{(\bar{k}^2/\sigma^2)} \; k^{(\bar{k}^2/\sigma^2 - 1)} \; e^{-k\bar{k}/\sigma^2}}{\Gamma(\bar{k}^2/\sigma^2)}$$

since

$$\int_0^\infty k^{(\bar{k}^2/\sigma^2 - 1)} e^{-k\bar{k}/\sigma^2} dk = \frac{\Gamma(\bar{k}^2/\sigma^2)}{(\bar{k}/\sigma^2)^{(\bar{k}^2/\sigma^2)}}$$

$$\int_0^\infty W(k)\, dk = 1$$

for all $\bar{k} > 0$ and $\sigma^2 > 0$.

By simply applying the batch equation for the discrete distribution model, the equation for the gamma function model may be developed

$$C = \sum_{i=1}^{n} C_i e^{-k_i t}$$

$$C = \sum_{i=1}^{n} W\left(\frac{k_i + k_{i+1}}{2}\right) \exp\left[\left(\frac{k_i + k_{i+1}}{-2}\right) t\right]$$

and letting $n \longrightarrow \infty$

$$C = \int_0^\infty W(k)\, e^{-kt} dk$$

or

$$C = \int_0^\infty \frac{(\bar{k}/\sigma^2)^{(\bar{k}^2/\sigma^2)}\, k^{(\bar{k}^2/\sigma^2 - 1)}\, e^{-k\bar{k}/\sigma^2 - kt}}{\Gamma(\bar{k}^2/\sigma^2)}\, dk$$

and upon collection of terms and integration

$$C = \frac{(\bar{k}/\sigma^2)^{(\bar{k}^2/\sigma^2)}}{\Gamma(\bar{k}^2/\sigma^2)} \int_0^\infty k^{(\bar{k}^2/\sigma^2 - 1)}\, e^{-k(\bar{k}/\sigma^2 + t)}\, dk$$

$$C = \left(\frac{\bar{k}/\sigma^2}{\bar{k}/\sigma^2 + t}\right)^{(\bar{k}^2/\sigma^2)}$$

In the continuous operation of a single cell,

$$C = \int_0^\infty \left(\frac{\bar{k}/\sigma^2}{\bar{k}/\sigma^2 + t}\right)^{(\bar{k}^2/\sigma^2)} \frac{1}{\tau}\, e^{-t/\tau}\, dt$$

however, since $\bar{k}$ and σ^2 define only the distribution of rate constants in the feed and this distribution will alter throughout the circuit, repeated application of the above equation is incorrect. Loveday has developed a form of the above equation which may be used to predict operation of a series of cells. Using the 'perfectly mixed' residence time distribution for a single unit and Laplace transforms, the residence time distribution for a series of cells is

$$E(t) = \frac{1}{\tau(n-1)!} \left(\frac{t}{\tau}\right)^{n-1} e^{-t/\tau}$$

where n is the number of equal residence time units with a mean holding time of T.
Thus for continuous operation of a series of cells

$$C = \int_0^\infty \int_0^\infty W(k)\, e^{-kt}\, E(t)\, dk\, dt$$

$$C = \frac{1}{T^n (n-1)!} \int_0^\infty \left(\frac{\bar{k}/\sigma^2}{\bar{k}/\sigma^2 + t} \right)^{(\bar{k}^2/\sigma^2)} t^{n-1}\, e^{-t/T}\, dt$$

A general mathematical formulation of flotation networks has been put forward by Dieft[39]. Woodburn has expanded Dieft's work and developed the general network equations for the gamma function or any distributed parameter model.

With Professor E.T. Woodburn's permission, I am publishing his work here.

Consider a tank 'j' as shown in figure 53. Let $d\alpha(k)dk$ be the mass flow of tailings which has an apparent first-order rate constant in the region k to k + dk. Using the notation that the fraction of the tailings stream from cell j which is feed to cell i equals a_{ij} ; and similarly the fraction of the concentrate stream from cell j which feeds cell i equal to b_{ij}. The feed to any cell j

becomes

$$\sum_{\substack{k=1 \\ k \neq i,j}}^{k=N} (a_{jk} + q_k b_{jk})\, d\alpha_k + dF_j = (1 + q_j)\, d\alpha_j$$

Defining $a_{jj} = -1$ and $b_{jj} = -1$ then

$$\sum_{k=1}^{k=N} (a_{jk} + b_{jk} q_k)\, d\alpha_k = -dF_j$$

and in matrix notation

$$\underline{\underline{C}}\, d\underline{\alpha} = -d\underline{F}$$

or

$$d\underline{\alpha} = -\underline{\underline{C}}^{-1}$$

If all mass flows are normalized and the rate constant distribution of the feed stream is W(k), then

$$d\underline{F} = \underline{\varepsilon}\, W(k)\, dk$$

where ε is a column matrix containing the fraction of fresh feed to each cell, and

$$\underline{\alpha} = -\int_0^{\infty} \underline{\underline{C}}^{-1}\, \underline{\varepsilon}\, W(k)\, dk$$

The same form of the above formula may be used to generate a series of residence times for each of the flotation

units in a circuit. It is convenient and reasonably practical to work in terms of the volume split within a cell. Thus following the same argument as for the solids but with a constant value for gi; now the ratio of concentrate volume to tailings volume, the mean residence time in each unit may be calculated directly.

$$\underline{\alpha}_v = -\underline{C}^{-1}\,\underline{\varepsilon}\,F_v$$

The fractional recovery of any species is defined as the total mass flowrate of that species in all concentrate streams which are not recycled, for a fresh feed of unity for that species. Thus the partial fractional recovery associated with the production of concentrate from cell j is

$$r_j = -\sum_{i=1}^{i=N} b_{ij}$$

one minus the sum of all fractions recycled (remembering that $b_{jj} = -1$).

The concentrate flowrate of a species is

$$\underline{g}\,d\underline{\alpha} = -\underline{g}\,\underline{C}^{-1}\,d\underline{F}$$

where $\underline{g}$ is a matrix such that

$$g_{jj} = kT_j$$

(where Tj is mean residence time in cell j)

$$g_{ij} = 0 \qquad (i \neq j)$$

Therefore the fractional recovery of any given mineral species is defined by

$$R = -\int_0^\infty \underline{r}^T \underline{\underline{g}}\, \underline{\underline{C}}^{-1}\, d\underline{F}$$

or

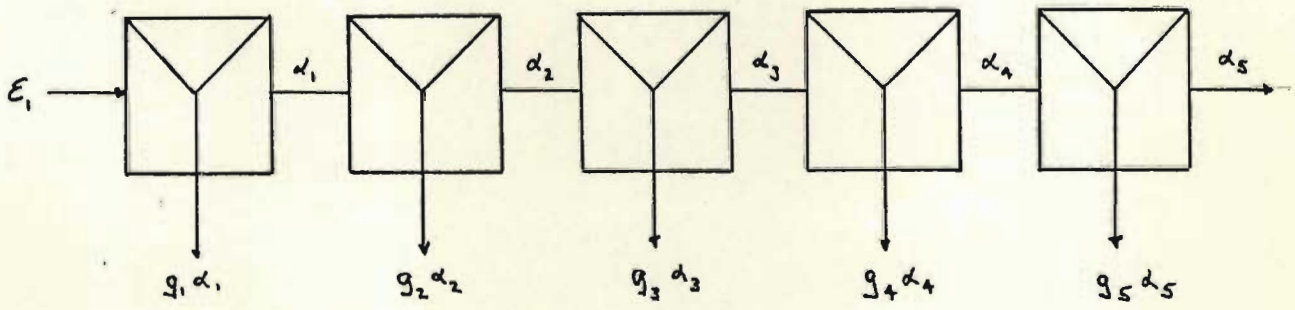
$$R = -\int_0^\infty \underline{r}^T \underline{\underline{g}}\, \underline{\underline{C}}^{-1}\, \underline{\underline{\varepsilon}}\, W(k)\, dk$$

The rate constant distribution in any concentrate stream is defined by

$$\bar{k} = \frac{\int_0^\infty k\, \underline{r}^T \underline{\underline{g}}\, \underline{\underline{C}}^{-1}\, \underline{\underline{\varepsilon}}\, W(k)\, dk}{\int_0^\infty \underline{r}^T \underline{\underline{g}}\, \underline{\underline{C}}^{-1}\, \underline{\underline{\varepsilon}}\, W(k)\, dk}$$

$$\sigma^2 = \frac{\int_0^\infty (k-\bar{k})^2\, \underline{r}^T \underline{\underline{g}}\, \underline{\underline{C}}^{-1}\, \underline{\underline{\varepsilon}}\, W(k)\, dk}{\int_0^\infty \underline{r}^T \underline{\underline{g}}\, \underline{\underline{C}}^{-1}\, \underline{\underline{\varepsilon}}\, W(k)\, dk}$$

In example of the above formulation, consider a simple bank of five continuously operating flotation cells with the total feed entering the first cell and with no concentrate streams being recycled.



For this case then $\varepsilon_1 = 1$, $\varepsilon_i = 0$ for $1 < i \leq 5$. By definition $a_{jj} = b_{jj} = -1$ and $g_i = k\tau_i$ if a perfectly mixed unit is assumed, thus $C_{jj} = -[1 + g_j]$.
Due to the cell configuration $a_{j+1,j} = 1$ for $1 \leq j < 5$; $a_{jk} = 0$ for $1 \leq k \leq 5$, $1 \leq j \leq 5$ but $j \neq k, k+1$; and $b_{jk} = 0$ $1 \leq k \leq 5$, $1 \leq j \leq 5$ but $j \neq k$.

In this case then the configuration matrix will be

$$\underline{\underline{C}} = \begin{bmatrix} C_{11} & 0 & 0 & 0 & 0 \\ 1 & C_{11} & 0 & 0 & 0 \\ 0 & 1 & C_{11} & 0 & 0 \\ 0 & 0 & 1 & C_{11} & 0 \\ 0 & 0 & 0 & 1 & C_{11} \end{bmatrix}$$

Its inverse will be

$$\underline{\underline{C}}^{-1} = \begin{bmatrix} C_{11}^{-1} & 0 & 0 & 0 & 0 \\ -C_{11}^{-2} & C_{11}^{-1} & 0 & 0 & 0 \\ C_{11}^{-3} & -C_{11}^{-2} & C_{11}^{-1} & 0 & 0 \\ -C_{11}^{-4} & C_{11}^{-3} & -C_{11}^{-2} & C_{11}^{-1} & 0 \\ C_{11}^{-5} & -C_{11}^{-4} & C_{11}^{-3} & -C_{11}^{-2} & C_{11}^{-1} \end{bmatrix}$$

The general expression for $\underline{\alpha}$ the tailings flow is,

$$\underline{\alpha} = \int_0^\infty \underline{\underline{C}}^{-1} \underline{\varepsilon}\, W(k)\, dk$$

thus we have the following matrix expression,

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \end{bmatrix} = \int_0^\infty \begin{bmatrix} \frac{1}{1+q_1} & 0 & 0 & 0 & 0 \\ \frac{1}{(1+q_1)^2} & \frac{1}{1+q_1} & 0 & 0 & 0 \\ \frac{1}{(1+q_1)^3} & \frac{1}{(1+q_1)^2} & \frac{1}{1+q_1} & 0 & 0 \\ \frac{1}{(1+q_1)^4} & \frac{1}{(1+q_1)^3} & \frac{1}{(1+q_1)^2} & \frac{1}{1+q_1} & 0 \\ \frac{1}{(1+q_1)^5} & \frac{1}{(1+q_1)^4} & \frac{1}{(1+q_1)^3} & \frac{1}{(1+q_1)^2} & \frac{1}{(1+q_1)} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} W(k)\,dk$$

or

$$\alpha_j = \int_0^\infty \frac{W(k)\,dk}{(1+q_1)^j}$$

The above integrals for the calculation of species recovery, concentrate rate constant distribution and fractional tailings flow, when the modified gamma function distribution of rate constants is used, do not have an analytical solution. However, use of generalized Laguerre polynomials and Gauss quadrature techniques simplifies

digital integration to a realizable form. The general integral of the form,

$$\int_0^\infty F(x)\, x^\beta e^{-x}\, dx$$

may be digitally solved by generation of the correct roots and weights for the following expression.

$$\int_0^\infty F(x)\, x^\beta e^{-x}\, dx = \sum_{i=1}^{i=NN} f(x_i)\, W_i$$

The author is indebted to Dr. I. Kropholler of Loughborough University for the contribution of a Fortran program for calculation of the necessary roots and weights which are both a function of β.

The Two-phase Model.

This model, the only froth incorporating model to be discussed, was examined in 1966 by Harris and Rimmer[40]. The equations are merely presented here and their application discussed.

For the batch or semi-batch case,

$$C = \frac{C_o}{E-F}\left[E e^{Ft} - F e^{Et}\right]$$

where E and F are the roots for 'Z' to the following equation

$$(Z + a)(Z + b + Qf/Vf) = ab$$

'a' and 'b' being the rate constant for mass transfer from pulp to froth and froth to pulp respectively. The term Qf/Vf is assumed constant with time, and represents the froth flowrate divided by the froth volume.

Since Qf/Vf was not to be function of time, calculation of concentration with time was done in the following manner for batch tests,

$$C(t_i) = \frac{C(t_{i-1})}{E - F}\left[Ee^{Ft} - Fe^{Et}\right]$$

where 'i' corresponds to the time interval, i.e. 0 - 5 mins., 5 - 10 mins. etc. Qf/Vf is measured over this interval and assumed constant.

The equations developed by Harris and Rimmer for continuous operation are not presented here because the model showed very poor results in batch testing analysis and no further investigation was thought necessary at this time. Data for a single bank of continuously operated Fagergren 40 cu.ft. cells in which Qf/Vf were measured is available however.

Parameter Estimation Techniques

Many methods are available for the best estimation of parameters for any model; broadly speaking, they may be divided into three general categories; derivative, non-derivative and search techniques. The methods of Fletcher

and Powell[41,42] Rosenbrock[43] Hooke and Jeeves[44] Nelder and Mead[45], and Law and Bailey[46] are all well known.

In the estimation of parameters for all of the four models, the single rate model, the rate plus non-floating model, the gamma function model and the two-phase model, the whole spectrum of estimation techniques was required.

Parameter Estimation for the Single Rate and Rate plus Non-floating Models.

Estimation of the single rate constant of the single rate model and of the non-floating fraction and positive rate constant for the rate plus non-floating model was performed by a derivative method programmed by J.A. Head[47]; the following is a description of the programs mathematical base.

The problem is basically the minimization of the sum of squares as defined by

$$\Delta = \sum_{i=1}^{N} \left[F(\bar{P}, x_i) - Y_i \right]^2$$

for Δ to be minimized

$$\frac{d\Delta}{dP_j} = 0 \qquad j = 1,2,3 \ldots m$$

for all 'j' parameters.

Given the initial estimate of Pj^o ; j = 1,2,3 ... m , the sum of squares as a function of $\bar{P}$ may be expanded in an m-dimensional Taylor series about the point $\bar{P}^o$

$$\Delta(\bar{P}) = \Delta(\bar{P}^{\circ}) + \sum_{j=1}^{m} \left(\frac{\partial \Delta}{\partial P_j}\right)(P_j - P_j^{\circ}) + \tfrac{1}{2} \sum_{j=1}^{m} \sum_{k=1}^{m} \left(\frac{\partial^2 \Delta}{\partial P_j \partial}\right.$$

$$(P_j - P_j^{\circ})(P_k - P_k^{\circ}) + \ldots\ldots$$

Assuming that the initial P_j° are near to $\bar{P}$ which minimize Δ, higher order terms may be neglected. For increased accuracy, the equation may be extended but requires increased computation of analytical derivatives.

If we define the following matrices

$$C_j = \frac{\partial \Delta(\bar{P})}{\partial P_j}$$

and

$$A_{jk} = \frac{\partial^2 \Delta(\bar{P})}{\partial P_j \partial P_k}$$

then approximately

$$\Delta(\bar{P}) \approx \Delta(\bar{P}^{\circ}) + \sum_{j=1}^{m} C_j (P_j - P_j^{\circ}) + \tfrac{1}{2} \sum_{j=1}^{m} \sum_{k=1}^{m} A_{jk} (P_j - P_j^{\circ})(P_k - P_k^{\circ})$$

The matrices Cj and Ajk may be computed from the following

$$C_j = \sum_{i=1}^{N} \left[F(\bar{P}^\circ, X_i) - Y_i\right] \left[\frac{\partial F(\bar{P}^\circ, X_i)}{\partial P_j}\right]$$

$$A_{jk} = \sum_{i=1}^{N} \left[F(\bar{P}^\circ, X_i) - Y_i\right] \left[\frac{\partial^2 F(\bar{P}^\circ, X_i)}{\partial P_j \partial P_k}\right] + \left[\frac{\partial F(\bar{P}^\circ, X_i)}{\partial P_j}\right] \left[\frac{\partial F(\bar{P}^\circ, X_i)}{\partial P_k}\right]$$

from the derivative of Δ and assuming $\frac{\partial \Delta(\bar{P}^\circ)}{\partial P_m} \approx 0$

we have

$$\frac{\partial \Delta(\bar{P})}{\partial P_m} = 0 \approx 0 + C_m + \sum_{k=1}^{m} A_{mk} (P_k - P_k^\circ)$$

multiplying by the inverse of matrix A_{mk}

$$0 \approx \sum_{k=1}^{m} A_{km}^{-1} C_m + P_k - P_k^\circ$$

Through repeated application or iterative procedures, the best estimates of the parameter array may be found.

Parameter Estimation for the Gamma Function Model.

The derivative dependent method of Head and the Law and Bailey technique proved unstable in estimation of the batch test kinetic parameters for the gamma function model. An alternative method, that of Powell[48], proved very successful provided reasonable initial estimates of the parameters were

computed. The initial estimates were computed as follows :

let

$$a = \bar{x}^2/\sigma^2 - 1$$

and

$$b = \bar{x}/\sigma^2$$

then the gamma function batch test equation becomes for normalized flows :

$$C = \left(\frac{b}{b+t}\right)^{a+1}$$

taking logarithms of both sides, we have

$$Y = \ln C = (a+1)(\ln b - \ln(b+t))$$

let the function to be minimized be

$$SS = \sum_{i=1}^{n} (Y_i - (a+1)(\ln b - \ln(b+t_i)))^2$$

at the minimum

$$\frac{dSS}{da} = 0$$

hence

$$\frac{dSS}{da} = 2\sum_{i=1}^{n} (Y_i - (a+1)(\ln b - \ln(b+t_i)))(-(\ln b - \ln(b+t_i))) = 0$$

therefore letting

$$X_i = \ln b - \ln(b+t_i)$$

$$\sum_{i=1}^{n} -Y_i X_i + (a+1) X_i^2 = 0$$

$$a+1 = \frac{\sum_{i=1}^{n} Y_i X_i}{\sum_{i=1}^{n} X_i^2}$$

thus a one-parameter search along the 'b' axis will provide a minimum very close to the true minimum; the Powell routine need only start from this near minimum. This initial search weights the smaller of the 'C' or concentration points hence with material which reach nearly 100 percent recovery, the initial minimum is less accurate.

The concentration versus mean residence time curve for a continuously operating bank is described as follows :

$$\underline{\alpha} = \int_0^{\infty} \underline{\underline{C}}^{-1} \underline{\varepsilon}\, W(k)\, dk$$

which may be simplified to

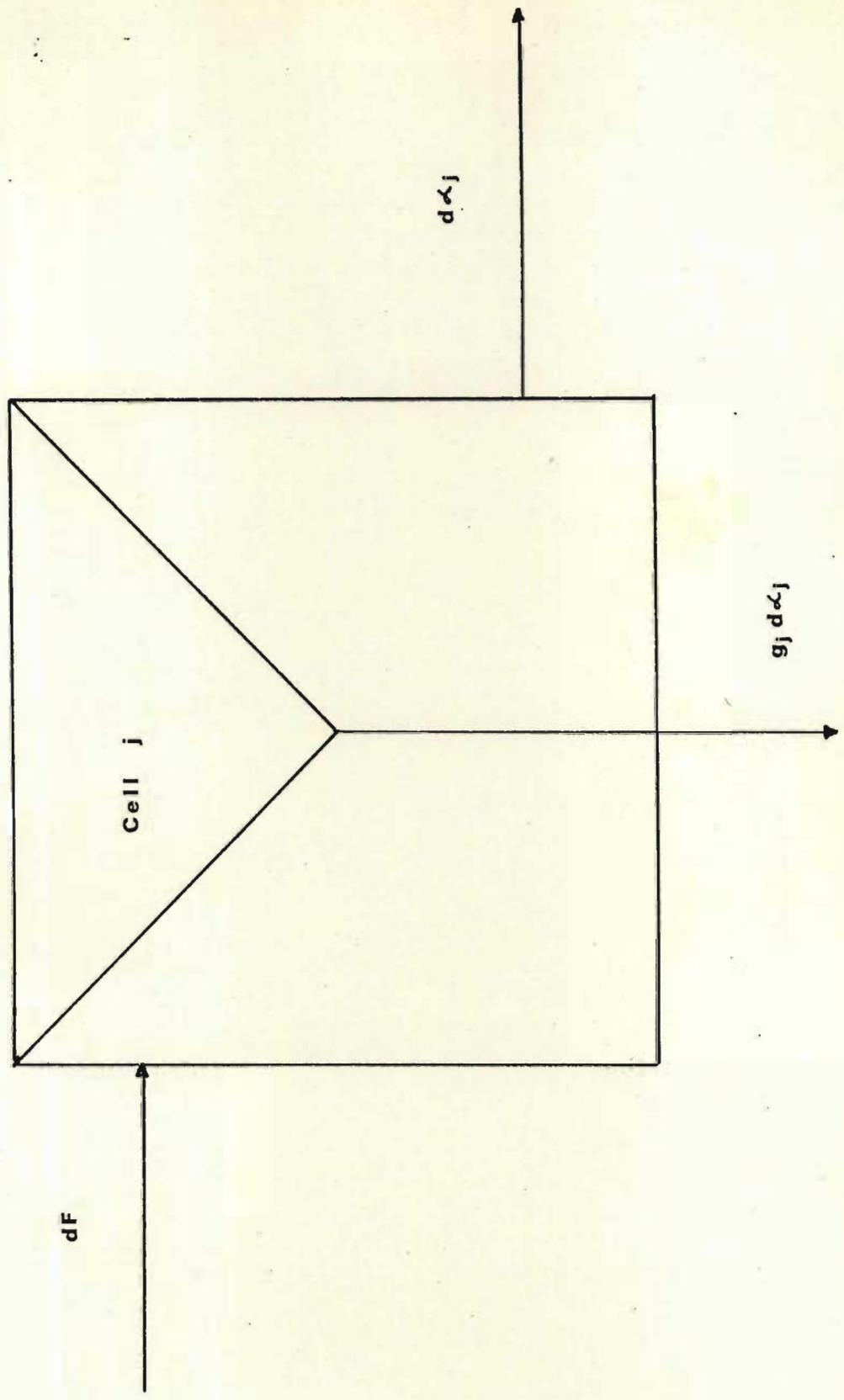
$$\alpha_j = \int_0^{\infty} \frac{W(k)\, dk}{(1+q_1)^j}$$

Parameter estimation from continuous test data is lengthy even though the above equation may be rapidly solved by the previously discussed Gauss-Laguerre quadrature technique. The regression was however performed on data from a bank of twenty cells. Initial parameter estimates were obtained by

assuming that the continuous data fit the batch equation, thereafter the non-derivative Powell search method was used to find the minimum sum of squares using the actual continuous operation equation.

Parameter Estimation for the Two-Phase Model.

The two-phase model showed poor predictive powers and a large interdependence between parameters. The very flat minimum of the sum of squares surface for batch data required the use of a direct grid search for stable estimation of the two rate parameters. Other slow direct search methods such as that of Hooke and Jeeves[49] would probably prove to be applicable.



53. Flotation Cell

IX. TEST METHODS AND DATA COLLECTION

In the course of this investigation over fifty batch tests, thirty continuous tests and numerous plant circuit tests were performed. The raw data for batch and continuous tests is available in part two of the thesis. Unless otherwise specifically stated, reagent additions were not changed from normal plant operation. Batch tests may be sub-divided primarily on the basis of their froth removal rates into five groups. In the discussion below the means of data collection and consequent data reduction are described.

All plant batch tests, that is, batch tests performed on plant pulp streams were performed in a five litre Denver cell with a Fagergren impeller. Impeller speed was 2000 rpm. Laboratory tests were performed in a two litre Denver cell with a Denver impeller at 1600 rpm.

All batch and continuous tests are assigned code numbers which aid in the identification and correlation of data.

Group One - Batch Tests

Included in this group are batch tests on the flotation feed stream in the plant and laboratory tests on crushed and batch ground mill feed.

The general method of weighing and assaying the final products of a batch test was used to generate a recovery versus time curve built back from the final tailings.

Froth removal in these tests was not measured but froth was removed at a rapid and consistent rate.

Group Two - Batch Tests

Tests in this group represent a sub-division of batch tests performed on various plant streams at Rustenburg Platinum Mines. The basic problem in using batch test data is how to handle the froth removal. Initially, plant froth removal rates were approximated visually but this method proved to be too indefinite and was abandoned after only a few tests. Thus the froth removal rate was not recorded and was rather arbitrary.

Group Three - Batch Tests

Again these batch tests were performed on various plant streams at Rustenburg. These tests represent a slight improvement over the method in group two, in this case the froth removal rate was held constant through the measurement of the wet weight of concentrate removed in a specific time interval and by regulation of the make-up water addition rate. As in group two, the froth was allowed to flow freely over the froth weir and was not mechanically removed by scraping, neither was the froth depth measured.

Group Four - Batch Tests

Another improvement was made to the batch test procedure of group three, in this group of plant batch tests, the froth was removed at a measured rate through regulation of the pulp level and use of a froth paddle. The pulp level was measured before and after each concentrate interval. Thus after an approximation of the change in level due to air in the pulp is accounted for, figures for the volume of froth and froth flowrate may be computed for each stage of concen-

trate removal.

Data processing followed the normal batch test procedure with the incorporation of the calculation of froth flowrate/ froth volume for application of the two-phase model.

Group Five - Batch Tests

In doing the batch tests of the previous groups, it was well understood that the rate or distribution of rates which was assumed to govern flotation of a species, was in fact not being measured; the issue was complicated by the froth phase. In order to closely approximate the rate parameters, the effects of the froth phase must be minimized by very rapid froth removal. Batch tests of this group were performed with this in mind. The froth was removed by a hand froth paddle as rapidly as was physically possible. In most instances pulp height and the wet weight of concentrate were recorded for each concentrate interval.

Data processing followed the normal procedure for batch testing with the calculation of froth flowrate/froth volume needed for application of the two-phase model.

Group Six - Continuous Tests

A number of tests were conducted on a continuous flotation bank, in order to test the various models' abilities in the continuous case.

The production of residence time recovery curves from a bank of continuously operating cells is statistically difficult, particularly with a low-grade ore. Several methods may be used to obtain data for a recovery curve for a bank of cells; two methods were attempted at Rustenburg. The

simplest method of obtaining a recovery curve is to take a pulp sample from each cell composed of small samples from various points below the froth layer and assay it for the particular species. By ignoring concentrate tonnages (they are usually small compared to tailings tonnages), the recovery curve may be produced directly. Unfortunately, samples taken in this manner are extremely scattered even when considerable care is taken in the sampling procedure. (Figure 54)

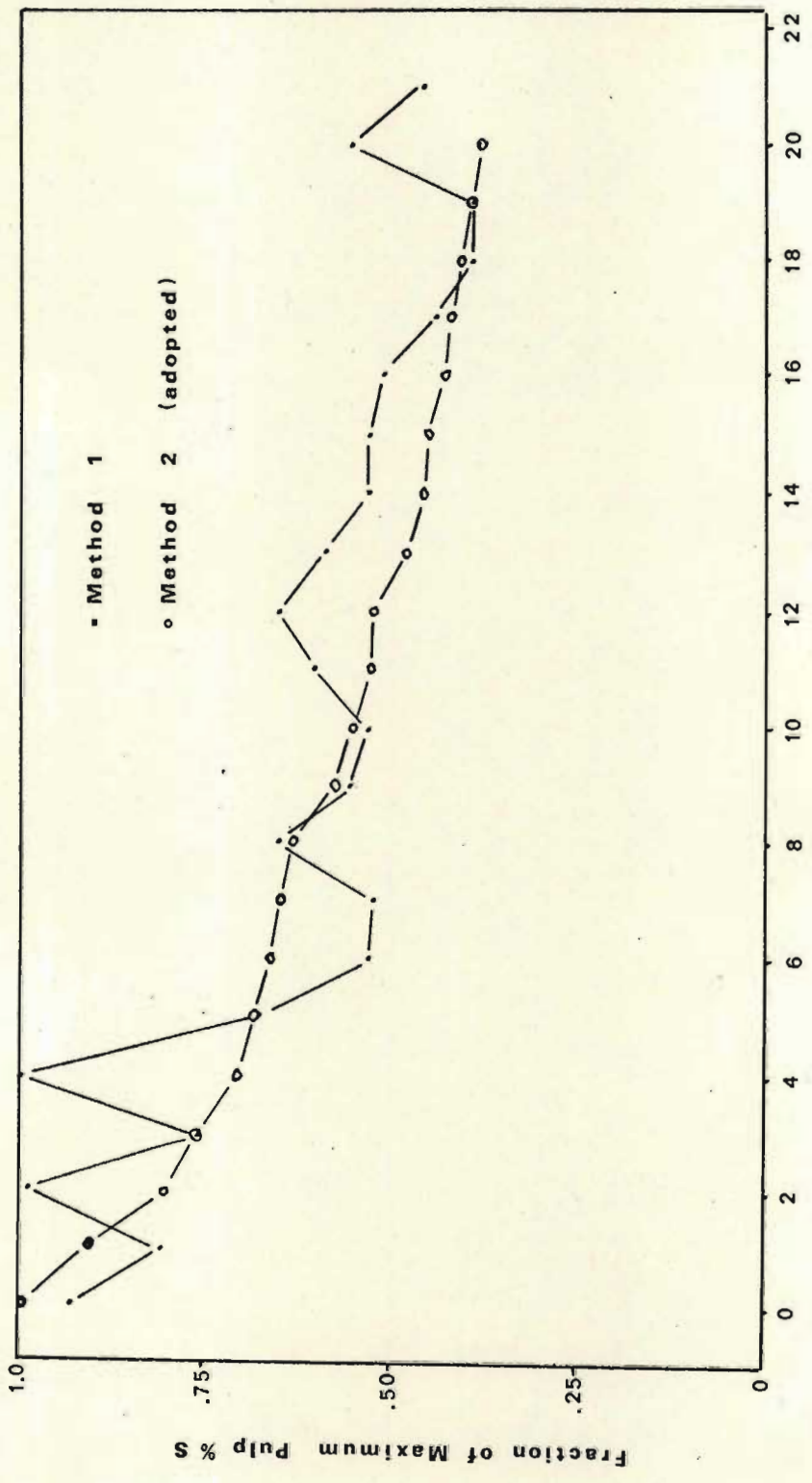
The above method was abandoned in favour of a slightly better method. This second method consists of timing samples of concentrate from each cell, then weighing and assaying them. Measurements from a magnetic flowmeter and a gamma density gauge provide a feed tonnage and several samples of the tailings stream are taken for assay. The recovery-residence time curve is calculated directly from the feed tonnage, concentrate tonnages, concentrate assays and the mean tailings assay. Due to the cell and launder design, it is impossible to collect the entire concentrate flow for a specified time thus a correction factor must be incorporated.

Group Seven - Special Tests

This group contains batch tests at various froth removal rates and are used determining the effect of froth removal upon rate parameters of the single phase models. Froth depth was not measured but a set wet weight of concentrate was removed in each time interval.

Data for Various Flotation Circuits

This eighth data group is not included in part two of the thesis, the data is used as a check on the accuracy of circuit simulations by the gamma function model.
Data consists of assays and measured flows (density and volume) of all main flotation streams for any specified flotation circuit. Samples for assay from each stream were hand-cut every fifteen or twenty minutes for an eight or sixteen hour test. Every effort was made to ensure steady operation of the entire plant and consistently accurate sampling. No attempts were made at determining the response of the flotation circuit to any variables as steady-state operation was the primary objective. Thus the flotation data represents an accurate analysis of the flotation circuit kinetics for a long steady-state period.



54. Continuous Operation Curves

LIST OF SYMBOLS

a_{ij} - fraction of the tailings material from cell j which is fed to cell i.

b_{ij} - fraction of the concentrate material from cell j which is fed to cell i.

C - concentration of a species.

Co - initial concentration.

Cn - non-floatable fraction of a species.

$\underline{\underline{C}}$ - configuration matrix.

E(t) - residence time distribution.

$\underline{F}$ - column matrix of species distribution in a flotation feed stream.

gj - ratio of concentrate material to tailings material for cell j.

k - first-order rate constant. (min^{-1})

$\bar{k}$ - mean of the modified gamma function distribution of first-order rate constants. (min^{-1})

n - number of flotation cells or perfectly stirred units.

$\underline{r}$ - a column matrix which contains the partial fractional recovery from each flotation unit.

R - total fractional recovery of a species.

LIST OF SYMBOLS (Continued)

t	-	time
T	-	mean retention time in a continuous cell.
v	-	volumetric flowrate to a set of cells.
V	-	effective volume of a single stirred unit.
$W(k)$	-	the distribution of first-order rate constants in the gamma function model.
W_i	-	weights in the Gauss-Laguerre integration formula.
j	-	tailings material flow from a cell j.
ε	-	column matrix of fractional feed flows to various flotation cells.
Γ	-	gamma function.
σ^2	-	variance of the gamma function model distribution of rate constants. (min^{-2})
μ	-	microns.
β	-	power of X in the general integration form.
X_i	-	roots in the integration solution.

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INDUSTRIAL APPLICATION OF A FLOTATION MODEL

PART TWO

TABLE OF CONTENTS

ALL OF THE FOLLOWING DATA IS IN THE FORM OF NORMALIZED CONCENTRATION
CURVES VERSUS TIME (MEAN TIME FOR CONTINUOUS OPERATION)
FOR EACH TEST THERE EXISTS A HEADER CARD WHICH ROUGHLY IDENTIFIES THE TEST
FOLLOWED BY A DATA CARD (2) - THE THIRD CARD (AND FOURTH WHERE NDATA
IS GREATER THAN 20) CONTAINS THE NORMALIZED CONCENTRATIONS IN A FIELD OF
FOUR, I.E. .526 IS WRITTEN AS 0526 - THE FOURTH CARD (OR FIFTH AND SIXTH)
CONTAINS THE CORRESPONDING TIMES (0500 IS 5.0 MINS)

THE IDENTIFIER CARD (ONE) CONTAINS A SERIES OF IDENTIFYING NUMBERS AS
HEADER CARD - ALPHA INFORMATION ABCDDEEEEFGGGG
A IDENTIFYS THE PLANT STREAM TESTED OR ITS EQUIVALENT
1 EQUALS FLOTATION FEED *** 2 EQUALS FLOTATION FINAL TAILING
3 EQUALS ROUGHER CONCENTRATE *** 4 EQUALS CLEANER FEED
5 EQUALS CLEANER MIDDLINGS RETURN *** 6 EQUALS CLEANER TAILINGS
7 IS NON EXISTENT *** 8 EQUALS ROUGHER FEED (1 PLUS 6)

B INDICATES THE SPECIES ASSAYED FOR
1 EQUALS TOTAL SULFUR VALUE *** 2 EQUALS NONSULFIDE MATERIAL
3 EQUALS CHALCOPYRITE (CU) *** 4 EQUALS PENTLANDITE (NI)
5 EQUALS THE CALCULATED PYRRHOTITE (FE7S8)

C INDICATES THE SIZE FRACTION FOR THAT SPECIES
1 EQUALS ALL SIZES TOGETHER *** 2 EQUALS THE RANGE GREATER 208 MICRONS
3 EQUALS THE RANGE 208 TO 104 MICRONS *** 4 EQUALS 104 TO 53 MICRONS
5 EQUALS MATERIAL LESS THAN 53 MICRONS

DD IS THE TEST RUN NUMBER IF ANY

EEEE IS THE DATE OF THE TEST - DAY/MONTH

F SIGNIFYS THE TEST MODE *** 1= PLANT BATCH 2=CONTINUOUS BANK
3= LABORATORY BATCH
GGGG IS THE TEST TIME IN HOURS AND MINUTES

********** GROUP ONE **********

THE FOLLOWING TEST IS USED IN CIRCUIT ANALYSIS * SEE PAGES 115-127

FLOT FEED BATCH 2303 NO 1 SULF 111 23031

011
1000078206600537043803810334029402640237021 5
0000050010001500200025003000350040005000 6000

FLOT FEED BATCH 2303 NO 1 NON SULF 121 23031

011
1000099209860982097609730970096609620959095 6
0000050010001500200025003000350040005000 6000

FLOT FEED BATCH 2303 NO 1 COPPER 131 23031

011
1000050503490276023502130197018301710161015 3
0000050010001500200025003000350040005000 6000

FLOT FEED BATCH 2303 NO 1 NON CU 161 23031

011
1000099109850980097409710967096309590955095 2
0000050010001500200025003000350040005000 6000

FLOT FEED BATCH 2303 NO 1 NICKEL 141 23031

011
1000089207310577047404240387035803410327031 7
0000050010001500200025003000350040005000 6000

FLOT FEED BATCH 2303 NO 1 NON NI 171 23031

011
1000099109850980097509710967096309600956095 3
0000050010001500200025003000350040005000 6000

FLOT FEED BATCH NO 1 FES2 151 23031

011
1000083407360606049104200361030902690231020 2
0000050010001500200025003000350040005000 6000

FLOT FEED BATCH 2303 NO 2 SULFUR 111 23031

011
1000062904640378032202870263023802200208019 9
0000050010001500200025003000350040005000 6000

FLOT FEED BATCH 2303 NO 2 NON SULF 121 23031

011
1000098809800973096709620958095309490946094 2
0000050010001500200025003000350040005000 6000

```
    FLOT FEED BATCH 2303 NO 2 COPPER                                131  23031
  011
1000035802650229020601910180016901600152014 5
0000050010001500200025003000350040005000600 0
    FLOT FEED BATCH 2303 NO 2 NON CU                                161  23031
  011
1000098709780970096409590955095009460943093 9
0000050010001500200025003000350040005000600 0
    FLOT FEED BATCH 2303 NO 2 NICKEL                                141  23031
  011
1000069204610392036003450336032803210316031 1
0000050010001500200025003000350040005000600 0
    FLOT FEED BATCH 2303 NO 2 NON NI                                171  23031
  011
1000098709780971096509600956095109470943094 0
0000050010001500200025003000350040005000600 0
    FLOT FEED BATCH NO 2 FES2                                       151  23031
  011
1000071405460432035403040270023202070191017 9
0000050010001500200025003000350040005000600 0

    THE FOLLOWING TESTS ARE USED IN A COMPARISON OF LABORATORY BATCH FLOTATION
    AND PLANT BATCH FLOTATION * SEE PAGES 25-36

    MINERAL LAB TEST SULFUR  3409/1                                 111       33409
04013
1000025701750145012901160109010400980093009000880084
0000050010001500200025003000350040004500500055006
    MINERAL LAB TEST SULFUR  3409/2                                 111       33409
04013
1000030601960157013401190112010600920085008100780076
0000050010001500200025003000350040004500500055006
    MINERAL LAB TEST COPPER 3409/1                                  131 1     33409
04013
1000020201690155014601380134013001270124012201200116
0000050010001500200025003000350040004500500055006
    MINERAL LAB TEST COPPER 3409/2                                  131 2     33409
04013
1000020401640148013701300126012201150111010801060104
0000050010001500200025003000350040004500500055006
    MINERAL LAB TEST NICKEL  3409/1                                 141 1     33409
```

```
04013
10000385033803200311030503010298029502920291028902860
00000500100015002000250030003500400045005000550060
    MINERAL LAB TEST NICKEL  3409/2                    141 2    33409
04013
10000427036003390326031803130309030102970294029202900
00000500100015002000250030003500400045005000550060
    MINERAL LAB TEST FE7S8 3409/1                      151 1    33409
04013
10000233012000810059004200320025001800110008000400
00000500100015002000250030003500400045005000550060
    MINERAL LAB TEST FE7S8 3409/2                      151 2    33409
04013
10000305015701050074005500450038002000110006000300
00000500100015002000250030003500400045005000550060
    MINERAL BATCH LAB 100/NORITE 3733                  111 1    33733
04013
10000282024202250215020501980193018701830179017601720
00000500100015002000250030003500400045005000550060000
    MINERAL BATCH LAB 100/NORITE 3733                  111 2    33733
04013
10000321027502470232022102130206019901940190018501810
00000500100015002000250030003500400045005000550060
    MINERAL BATCH LAB 100/REEF  3737                   111 1    33737
04013
10000475039003450321030602930284027602680261025302460
00000500100015002000250030003500400045005000550060
    MINERAL BATCH LAB 100/REEF  3737                   111 2    33737
04013
10000563046304070367034603260313030302930286027702690
00000500100015002000250030003500400045005000550060
    MINERAL BATCH LAB 100/MERENSKY  3742               111 1    33742
04013
10000418030202490213018901720160014701380129012301150
00000500100015002000250030003500400045005000550060
    MINERAL BATCH LAB 100/MERENSKY  3742               111 2    33742
04013
10000428029602200182016101450133012301140106010000940
00000500100015002000250030003500400045005000550060
```

********** GROUP TWO **********

THE MAJORITY OF THESE TESTS ARE USED TO ILLUSTRATE THE RATES OF FLOTATION
IN VARIOUS PLANT FLOTATION STREAMS * SEE PAGES 44-46

MINERAL BATCH RO CONC 46 31146210911711
0400710080050090010
1000052303840299025302130151
0000050010001500200025003500
GANGUE BATCH RO CONC 46 32146210911711
0400710200110300 1010
1000086707920735069806600597
0000050010001500200025003500
MINERAL BATCH CL FEED 46 41146210911515
0400710080050090010
1000048903660303024401890133
0000050010001500200030004000
GANGUE BATCH CL FEED 46 42146210911515
0400710200110300 1010
1000081207340682063305760521
0000050010001500200030004000
MINERAL BATCH CL MID 44 51144070911850
0400610200110300 0010
1000056603740132006300 34
0000050010002000300040005000
MINERAL BATCH CL MID 46 51146210911625
0400710200110300 1010
1000074305720478038503450233
0000050010001500200025003500
GANGUE BATCH CL MID 44 52144070911850
0400610200110300 0010
1000075205790339022601 67
0000050010002000300040005000
GANGUE BATCH CL MID 46 52146210911625
0400710200110300 1010
1000087807760710063205970507
0000050010001500200025003500
MINERAL BATCH CL TAILS 44 61144070911610

```
0400610200110300010
10000771067105580443033 2
0000050013002000300040 00
    MINERAL BATCH CL TAILS 46                    61146210912019
0400710080050090001 0
10000840080307240645060404 87
0000050010001500200030004000
    GANGUE BATCH CL TAILS 44                     62144070911610
0400610200110300010
10000888081307770730069 2
0000050013002000300040 00
    GANGUE BATCH CL TAILS 46                     62146210912019
0400710200110300101 0
10000931091808870861084608 07
0000050010001500200030004000
    MINERAL BATCH RO FEED 44                     81144070911700
0400710080050090001 0
10000715062805380500045904 22
0000050010002000300040005000
    MINERAL BATCH RO FEED 46                     81146210911912
0400710080050090001 0
10000697056005300489045604 15
0000050010002000300040005000
    GANGUE BATCH RO FEED 44                      82144070911700
0400710200110300101 0
10000982097109530945093809 29
0000050010002000300040005000
    GANGUE BATCH RO FEED 46                      82146210911912
0400710200110300101 0
10000985097609730967096209 56
0000050010002000300040005000
    MINERAL BATCH RO FEED WET -208               81 49131011102
04007
10000802064905910467030102 51
0000050010001500200030004000500060 00
    MINERAL BATCH RO FEED WET +104               81 49131011453
04007
10000806071906530556049504 58
0000050010001500200030004000500060 00
    GANGUE BATCH RO FEED WET -208                82 49131011102
```

```
04007
1000098309700960094709290912
0000050010001500200030004000500060000
    GANGUE  BATCH RO FEED WET +104                                  82 49131011453
04007
1000098309700953094009220908
0000050010001500200030004000500060000
```

********** GROUP THREE **********

FROTH WET WEIGHTS WERE CONTROLLED THROUGH PULP LEVEL AND MEASURED
I.E. 'PULL 100' MEANS 100 GRAMS OF WET CONCENTRATE COLLECTED IN 5 MINUTES
A FROTH PADDLE WAS NOT USED

```
    GANGUE BATCH RO FEED +208  PULL 200/5                    82200041111600
040071
10000998099209900987097209 51
00000500100015002000300040 00
    GANGUE BATCH RO FEED +104      PULL 200/5                82300041111600
040071
10000997099409930991098809 85
00000500100015002000300040 00
    GANGUE BATCH RO FEED +53       PULL 200/5                82400041111600
040071
10000995099309920990098809 85
00000500100015002000300040 00
    GANGUE BATCH RO FEED -53 PULL 200/5                      82500041111600
040071
10000976096309500939092009 07
00000500100015002000300040 00
    MINERAL BATCH RO FEED +208   PULL 200/5                  81200041111600
040071
10000951087208340795060104 25
00000500100015002000300040 00
    MINERAL BATCH RO FEED +104    PULL 200/5                 81300041111600
040071
10000795075207310697065006 17
00000500100015002000300040 00
    MINERAL BATCH RO FEED +53      PULL 200/5                81400041111600
040071
10000761071706930670063206 07
00000500100015002000300040 00
    MINERAL BATCH RO FEED -53      PULL 200/5                81500041111600
040071
10000706062205390473039603 50
00000500100015002000300040 00
    MINERAL BATCH RO CONC PULL 150                           31100061110300
040071
```

```
1000064104760366028502400194
0000050010001500200025003500
    MINERAL BATCH RO CONC  PULL 100                    31100061110525
040071
1000067704690345024701950154
0000050010001500200025003500
    GANGUE BATCH RO CONC PULL 150                      32100061110300
040071
1000096809480926090008790853
0000050010001500200025003500
    GANGUE BATCH RO CONC PULL 100                      32100061110525
040071
1000096709420924090408890869
0000050010001500200025003500
    MINERAL BATCH RO FEED 47 PULL 180                  81147280911210
0400710100050140101O
1000075406440589048103600293
0000050010001500200030004000
    MINERAL BATCH RO FEED 47 PULL 350                  81147280911415
040071009005014010
1000069605990545044403420292
0000050010001500200030004000
    MINERAL BATCH RO FEED 47 PULL 100                  81147280911602
0400710100050140101O
1000077006960643053804890423
0000050010001500200030004000
    MINERAL BATCH RO FEED 47 PULL 220                  81147280911650
0400710100050140101O
1000066406330557042503470275
0000050010001500200030004000
    MINERAL BATCH RO FEED 48 PULL 100                  81148051011206
0401110100050140101O
1000081007540719063305770541052305000483046 7
0000050010001500200030004000500060007000 8000
    MINERAL BATCH RO FEED 48 PULL 300                  81148051011340
0401010100050140101O
1000063305660538046304250404037003460324
0000050010001500200030004000500060007000
    MINERAL BATCH +208 WET RO FEED 48 PULL 180         81248051011544
0400710100050140101O
```

```
10000746069906770604056205 36
0000050010001500200030004000
    GANGUE BATCH RO FEED 47 PULL 180                    82147280911210
0400710100050140101 0
10000991098209760967095609 42
0000050010001500200030004000
    GANGUE BATCH RO FEED 47 PULL 350                    82147280911415
0400710090050140101 0
10000981097109630954093909 29
0000050010001500200030004000
    GANGUE BATCH RO FEED 47 PULL 100                    82147280911602
0400710100050140101 0
10000993098909840977097209 65
0000050010001500200030004000
    GANGE BATCH RO FEED 47 PULL 220                     82147280911650
0400710100050140101 0
10000986098309750959094709 37
0000050010001500200030004000
    GANGUE BATCH RO FEED 48 PULL 100                    82148051011206
0401110100050140101 0
10000995099209900986098109780975097309700968
0000050010001500200030004000500060007000 8000
    GANGUE BATCH RO FEED 48 PULL 300                    82148051011340
0401010100050140101 0
10000982097309640955094409330920091209 08
0000050010001500200030004000500060007000
    GANGUE BATCH +208 WET RO FEED 48 PULL 180           82248051011544
0400710100050140101 0
10000990098609840977096809 61
0000050010001500200030004000
```

********** GROUP FOUR **********

THESE TESTS ARE USED PRIMARILY IN CONNECTION WITH THE TWO PHASE FLOTATION
MODEL * SEE PAGES 88-103
FROTH HEIGHT AND FROTH FLOW WERE MEASURED AND CONTROLLED

MINERAL BATCH RO CONC 100/5 NUTST 31100020212130
04007
1000078205790382023801280079
0000050010001500200030004000
MINERAL BATCH RO CONC 100/5 NWTST 31100070212110
04007
1000071705300375025001450096
0000050010001500200030004000
MINERAL BATCH RO CONC 100/5 NWTST 31100110212145
04007
1000077505760419031901960111
0000050010001500200030004000
MINERAL BATCH RO CONC 100/5 NWTST 311 120312110
04005
1000074206270478036802440177
00000500100015002000300040005000
MINERAL BATCH RO CONC 100/5 NWTST 31100130212035
04007
1000061204040294021501770100
0000050010001500200030004000
MINERAL RO CONC 100/5 NWTST 311 220212155
04005
1000075605800496039402370162
00000500100015002000300040005000
RO CONC MINERAL 100/5 311 260212155
007
1000075605800496039402370162
0000050010001500200030004000
GANGUE BATCH RO FEED 100/5 NWTST 32100020212130
04007
1000093708790819077307140674
0000050010001500200030004000
GANGUE BATCH RO CONC 100/5 NWTST 32100070212110

```
04007
10000960091608720814077507 38
00000500100015002000300040005000 6000
    GANGUE  BATCH RO CONC 100/5 NWTST                    321001102121 45
04007
10000947089308310776068606 17
00000500100015002000300040 00
    GANGUE  BATCH RO CONC 100/5 NWTST                    321  120312110
04005
10000957092908910852078807 43
00000500100015002000300040005000
    GANGUE  BATCH RO CONC 100/5 NWTST                    321001302120 35
04007
10000960092408960868083907 99
00000500100015002000300040 00
    GANGUE  RO CONC 100/5 NWTST                          321  220212155
04005
10000962092908990872081107 70
00000500100015002000300040005000
     RO CONC GANGUE 100/5                                321  260212155
  007
10000962092908990872081107 70
00000500100015002000300040 00
    MINERAL BATCH RO FEED 200/5                          811000102120 25
04007
10000820071406550575052904 47
00000500100015002000300040 00
    MINERAL BATCH RO FEED 100/5 NUTST                    811000202120 30
04007
10000744066206140535047604 36
00000500100015002000300040 00
    MINERAL BATCH RO FEED 100/5 NWTST                    811000702120 00
04007
10000685059605370504044303 90
00000500100015002000300040 00
    MINERAL BATCH RO FEED 100/5 NWTST                    811001102120 40
04007
10000652057504350393033903 03
00000500100015002000300040 00
    MINERAL BATCH RO FEED 100/5 NWTST                    811  120311905
04007
```

```
10000815070706460615055705O3
0000050010001500200030004000S000
    MINERAL BATCH RO FEED 100/5 NWTST                    81100130211915
04007
10000762063205760538047704 31
0000050010001500200030004000
    MINERAL BATCH RO FFED 100/5 NWTST                    811  260212030
04007
10000732065705510504043604 7
0000050010001500200030004000S000
    GANGUE BATCH RO FEED 200/5                           82100010212025
04007
10000988097909710963095309 2
0000050010001500200030004000
    GANGUE BATCH RO FEED 100/5  NUTST                    82100020212030
04007
10000993098709820976096609 9
0000050010001500200030004000
    GANGUE BATCH RO FEED 100/5 NWTST                     82100070212000
04007
10000990098409770972096309 6
0000050010001500200030004000
    GANGUE  BATCH RO FEED 100/5 NWTST                    82100110212040
04007
10000988098009710965095609 8
0000050010001500200030004000
    GANGUE  BATCH RO FEED 100/5 NWTST                    821  120311905
04007
10000995099109870984097909 4
0000050010001500200030004000S000
    GANGUE  BATCH RO FEED 100/5 NWTST                    82100130211915
04007
10000993098809820978097009 3
0000050010001500200030004000
    GANGUE  BATCH RO FFED 100/5 NWTST                    821  260212030
04007
10000992098609810976096909 4
0000050010001500200030004000S000
    GANGUE  BATCH RO CONC  INFINITE NWTST                321  010311800
04005
```

```
100008390773074107l8
0000050010001500200030004000500O
    GANGUE  BATCH RO CONC INFINITE NWTST                    321  080312055
04005
1000081106990631060O
0000050010001500200030004000500O
    GANGUE  BATCH RO CONC INFINITE NWTST                    32100140211930
04005
1000080507030650061 7
00000500100015002000
    GANGUE  BATCH RO CONC INFINITE NWTST                    321  200211945
04005
1000075106350557052O
0000050010001500200030004000500O
    GANGUE  BATCH RO CONC INFINITE NWTST                    321  220212045
04005
1000075206470598056 6
0000050010001500200030004000500O
    GANGUE  BATCH RO FEED INFINITE NWTST                    821  010311625
04007
1000096709540932092109050893
0000050010001500200030004000500O
    GANGUE  BATCH RO FEED INFINITE NWTST                    821  080311935
04007
1000096609520934092309070898
0000050010001500200030004000500O
    GANGUE  BATCH RO FEED INFINITE NWTST                    82100140211805
04007
1000097009500927091408950882
000005001000150020003000400O
    GANGUE  BATCH RO FEED  INFINITE NWTST                   821  200211810
04007
1000095509440914089408730859085l
0000050010001500200030004000500O
    GANGUE  BATCH RO FEED INFINITE NWTST                    821  220211920
04007
1000096809420919090508840873
0000050010001500200030004000500O
```

```
C    THE FOLLOWING SIZE CURVES WERE BUILT FROM COMPOSITES OF THE SAMPLES FROM
C    TESTS 321--010311800 321--080312055 321--140211930 321--200211945 AND
C    321--220212045
C
     GANGUE  BATCH RC INF +208                                  322      12400
04005
10000962095709560955
00000500100015002000300040005000
     GANGUE  BATCH RC INF +104                                  323      12400
04005
10000911089008790879
00000500100015002000300040005000
     GANGUE  BATCH RC INF +53                                   324      12400
04005
10000892085108350829
00000500100015002000300040005000
     GANGUE  BATCH RC INF -53                                   325      12400
04005
10000775065905940556
00000500100015002000300040005000
C
C    THE FOLLOWING SIZE CURVES WERE BUILT FROM COMPOSITES OF THE SAMPLES FROM
C    TESTS 821--010311935 821--080311935 821--140211805 821--200211810 AND
C    821--220211920
C
     GANGUE  BATCH INFINITE +208                                822      12400
04007
10000998099809960996099609960996
00000500100015002000300040005000
     GANGUE  BATCH INFINITE +104                                823      12400
04007
10000996099509920991099109910991
00000500100015002000300040005000
     GANGUE  BATCH INFINITE +53 RF                              824      12400
04007
10000993099209890987098509850985
00000500100015002000300040005000
     GANGUE  BATCH RF INF -53                                   825      12400
04007
10000921088108310800075807310731
00000500100015002000300040005000
```

********** GROUP FIVE **********

THESE TESTS THAT FOLLOW ARE USED IN ASSESSING THE VALIDITY OF ' K
CONSISTENCY ' * SEE PAGES 54 AND 71

BATCH RO FEED COPPER 8/8/74 NO DEXTRIN TO RC 831 080811620
04007
1000040603360283025402200203
0000050010001500200030004000
BATCH RO FEED NICKEL 8/8/74 NO DEXTRIN TO RC 841 080811620
04007
1000066304850390035803140298
0000050010001500200030004000
BATCH RO FEED SULF 8/8/74 NO DEXTRIN TO RC 811 080811620
04007
1000064305270363028401790148
0000050010001500200030004000
BATCH RO FEED GANGUE 8/8/74 NO DEX TO RC 821 080811620
04007
1000098209680953094009200908
0000050010001500200030004000
MINERAL BATCH RO FEED INFINITE NWTST 811 010311625
04007
1000055005120407037403600333
0000050010001500200030004000 5000
MINERAL BATCH RO FEED INFINITE NWTST 811 080311935
04007
1000057405120401035102870260
0000050010001500200030004000 5000
MINERAL BATCH RO FEED INFINITE NWTST 811 00140211805
04007
1000057104940354030402360204
0000050010001500200030004000
MINERAL BATCH RO FEED INFINITE NWTST 811 200211810
04007
1000048404640311025402020179 0104
0000050010001500200030004000 5000
MINERAL BATCH RO FEED INFINITE NWTST 811 220211920

```
04007
10000606047003100245018301560
00000500100015002000300040005000
     ROUGHER FEED BATCH SULF INF I   22/2  WITH ACID                811  220211920
  008
100006060470031002450183015600970
00000500100015002000300040004500

    THESE SIZE FLOTATION CURVES ARE BUILT FROM THE COMPOSITE SAMPLES OF TESTS
    811--010311625 811--080311935 811--140211805 811--220211920

    MINERAL BATCH INFINITE +208                                      812      12400
04007
10000920091808730866086408630
00000500100015002000300040005000
    MINERAL BATCH INFINITE +104                                      813      12400
04007
10000776076306480623060406010
00000500100015002000300040005000
    MINERAL BATCH INFINITE +53 RF                                    814      12400
04007
10000723070606400592055805480
00000500100015002000300040005000
    MINERAL BATCH RF INF -53                                         815      12400
04007
10000867074505130404030402630
00000500100015002000300040005000
    COPPER BATCH RO FEED INF COMB +104                               833      12400
04007
10000578057804340434043043404340
00000500100015002000300040004000
    COPPER BATCH RO FEED INF COMB +53                                834      12400
04007
10000270025702180201020102010
00000500100015002000300040004000
    COPPER BATCH RO FEED INF COMB -53                                835      12400
04007
10000828065304970413030602560
00000500100015002000300040004000
    NICKEL BATCH RO FEED INF COMB +104                               843      12400
```

```
04007
1000085708570813081308130813
0000050010001500200030004000
    NICKEL BATCH RO FEED INF COMB +53                    844      12400
04007
1000062005930526050705070507
0000050010001500200030004000
    NICKEL BATCH RO FEED INF COMB -53                    845      12400
04007
1000089206570466040503520319
0000050010001500200030004000
    CALC FE7S8 RO FEED +104 INF COMB                     853      12400
04007
1000081107900662061905880584
0000050010001500200030004000
   CALC FE7S8 RO FEED +53 INF COMB                       854      12400
04007
1000082808130742068206340620
0000050010001500200030004000
   CALC FE7S8 RO FEED -53 INF COMB                       855      12400
04007
1000086607730525040302950253
0000050010001500200030004000
     BATCH RO CONC NO DEX 8/8/74 SULF                    311  080811645
04005
10000317019801480115
0000050010001500200030004000
     BATCH RO CONC NO DEX 8/8/74 GANGUE                  321  080811645
04005
1000074606530606O564
0000050010001500200030004000
     BATCH RO CONC NO DEX 8/8/74 COPPER                  331  080811620
04005
1000014000920O730060
0000050010001500200030004000
     BATCH RO CONC NO DEX 8/8/74 NICKEL                  341  080811620
04005
1000024701440113O093
0000050010001500200030004000
    MINERAL BATCH RO CONC  INFINITE NWTST                311  010311800
```

```
04005
10000247016101380126
00000500100015002000300040005000
    MINERAL BATCH RO CONC INFINITE NWTST                          311  080312055
04005
10000369016601100095
00000500100015002000300040005000
    MINERAL BATCH RO CONC INFINITE NWTST                          31100140211930
04005
10000220010400790069
00000500100015002000
    MINERAL BATCH RO CONC INFINITE NWTST                          311  200211945
04005
10000251010500610046
00000500100015002000300040005000
    MINERAL BATCH RO CONC INFINITE NWTST                          311  220212045
04005
10000242012601030091
00000500100015002000300040005000

    THESE SIZE FLOTATION CURVES ARE BUILT UP FROM COMPOSITES OF SAMPLES FROM
    TESTS 311--010311800 311--080312055 311--140211930 311--200211945  AND
    311--220212045

    MINERAL BATCH RC INF +208                                     312      12400
04005
10000453041003970395
00000500100015002000300040005000
    MINERAL BATCH RC INF +104                                     313      12400
04005
10000439031302710269
00000500100015002000300040005000
    MINERAL BATCH RC INF +53                                      314      12400
04005
10000318017301560146
00000500100015002000300040005000
    MINERAL BATCH RC INF -53                                      315      12400
04005
10000326018500950079
00000500100015002000300040005000
```

```
    COPPER BATCH RO CONC INF COMB +104                    333      12400
04005
10000163011901190119
00000500100015002000
    COPPER BATCH RO CONC +53 INF COMB                     334      12400
04005
10000084005600470044
00000500100015002000
    COPPER BATCH RO CONC INF COMB -53                     335      12400
04005
10000078004400280020
00000500100015002000
    NICKEL BATCH RO CONC INF COMB +104                    343      12400
04005
10000369027802780278
00000500100015002000
    NICKEL BATCH RO CONC INF COMB +53                     344      12400
04005
10000237011700990095
00000500100015002000
    NICKEL BATCH RO CONC INF COMB -53                     345      12400
04005
10000189008600610050
00000500100015002000
   CALC FE7S8 RO CONC +104 INF COMB                       353      12400
04005
10000621043703610356
00000500100015002000300040000
   CALC FE7S8 RO CONC +53 INF COMB                        354      12400
04005
10000415022502060192
00000500100015002000
   CALC FE7S8 RO CONC -53 INF COMB                        355      12400
04005
10000433025201240104
00000500100015002000
```

THESE TESTS ARE USED TO TEST THE PREDICTIVE CAPACITIES OF THE MODELS IN
THAT THESE BATCH TESTS WERE PERFORMED ON THE FEED TO A CONTINUOUS BANK AND
MAY BE USED TO PREDICT THE OPERATION OF THAT BANK * SEE PAGES 53 AND 71

BATCH RO FEED 13/4/73 1435 INF SULF 811 130411435
04007
1000063305650336028102440230
0000050010001500200030004000
GANGUE BATCH RO FEED INF 13/4/73 1435 821 130411435
04007
1000098009670938092409110905
0000050010001500200030004000
ROUGHER FEED INF 13/4/73 1435 COPPER BATCH 831 130411435
04007
1000028802450174015301340124
0000050010001500200030004000
ROUGHER FEED INF 13/4/73 1435 NICKEL BATCH 841 130411435
04007
1000062604950326030502900284
0000050010001500200030004000
RO FEED BATCH INF 23/4/73 1200 811 230411200
04007
1000056005180328027302480237
0000050010001500200030004000
GANGUE BATCH RO FEED 23/4/73 INF 1200 821 230411200
04007
1000097109590931091609060899
0000050010001500200030004000
RO FEED BATCH INF 23/4/73 1200 COPPER 831 230411200
04007
1000030102680200017401570148
0000050010001500200030004000
RO FEED BATCH INF 23/4/73 1200 NICKEL 841 230411200
04007
1000048904290321029802860280
0000050010001500200030004000
RO FEED INF BATCH 2/5/73 1450 SULF 811 020511450
04007
1000062805900407035503120298

```
00000500100015002000300040005000
  RO FEED INF BATCH 2/5/73 1450 GANGUE                    821  020511450
04007
10000973096309430932092009l4
00000500100015002000300040005000
    MINERAL CL FEED BATCH 200/5 1445 11/4/73              411  110411445
04008
10000590040703050232014401110095
00000500100015002000300040005000
    GANGUE BATCH CL FEED 11/4/73 1445 200/5               421  110411445
04008
10000894081507540711064506060571
00000500100015002000300040005000
  CLEANER FEED BATCH 11/4/73 COPPER  1445                 431  110411445
04008
10000270016801270106007800640053
00000500100015002000300040005000
  CLEANER FEED BATCH 11/4/73 NICKEL  1445                 441  110411445
04008
10000516029402050162011000880074
00000500100015002000300040005000
    CL FEED BATCH 18/4/73 MINERAL 1150 INF                411  180411150
04006
10000312013600910074005l
00000500100015002000300040005000
    GANGUE CL FEED 18/4/73 1150 INF                       421  180411150
04006
10000729058805260492045l
00000500100015002000300040005000
  CLEANER FEED BATCH INF 18/4073 1150 COPPER              431  180411150
04006
10000195011500900078006l
00000500100015002000300040005000
  CLEANER FEED BATCH INF 18/4073 1150 NICKEL              441  180411150
04006
10000254013200960080006l
00000500100015002000300040005000
  CLEANER FEED BATCH SULF INF 30/4/73 1430                411  300411430
04006
10000498030902270202018l
```

```
000005001000150020003000
  CLEANER FEED BATCH GANG INF 30/4/73 1430                    421  300411430
04006
1000082207110649061805 83
000005001000150020003000
```

********** GROUP SIX **********

THESE TESTS WERE PERFORMED IN ORDER TO TEST THE VARIOUS MODELS CAPABILITIES
IN DESCRIBING CONTINUOUS FLOTATION * SEE PAGES 52 AND 70

GANGUE PLANT RO FEED 47 +104 82347280921645

04011
1000099909980997099709960995099509950994099 4
0000020404090614081910231228143316381842204 7

GANGUE PLANT RO FEED +53 82447280921645

04011
1000099809960996099509950994099409940994099 4
0000020404090614081910231228143316381842204 7

GANGUE PLANT RO FEED 47 -53 82547280921645

04011
1000099209820978097509690961095509400936093 3
0000020404090614081910231228143316381842204 7

MINERAL PLANT RO FEED 47 +208 81247280921645

04011
1000091207800741071406780660065106330619060 3
0000020404090614081910231228143316381842204 7

MINERAL PLANT RO FEED 47 +104 81347280921645

04011
1000092108390821081107790761075207430736073 2
0000020404090614081910231228143316381842204 7

MINERAL PLANT RO FEED 47 +53 81447280921645

04011
1000073806480614060905650547053505230519051 7
0000020404090614081910231228143316381842204 7

MINERAL PLANT RO FEED 47 -53 81547280921645

04011
1000071706180589057105020437041003590350033 5
0000020404090614081910231228143316381842204 7

GANGUE PLANT RO FEED 47 +208 82247280921645

04011
1000099809960994099209890986098609830981097 7
0000020404090614081910231228143316381842204 7

MINERAL PLANT RUN 34 CLEANER 22/6/72 41134220621215

```
04021
10000889075606860641061305980577055005390519050904960486047104590449043604260414
0403
   0  64 129 193 258 323 387 452 517 581 646 711 775 840 905 9691034109911631228
1293
    MINERAL PLANT CL FEED 37                                          41137130722115
0402110100040070001O
10000850067105930514048204330382033003100283027302600247022902140200019001780167
0155
   0  72 144 217 290 363 435 508 581 654 726 799 872 945101710901163123613081381
1454
    PLANT RUN 40 9/8/72 MINERAL                                       41140090822000
04021
10000781069806240588055005180417036903470314030302860251022101930171015301400126
0114
   0  79 159 239 319 399 479 558 638 719 799 878 958103811181198127813571438151 8
1598
    GANGUE PLANT TEST CL FEED 34                                      42134220621215
0402110200110300001O
10000990097409620954094509410933092309160907090008920884087508670860084908380825
0813
   0  64 129 193 258 323 387 452 517 581 646 711 775 840 905 9691034109911631228
1293
    GANGUE PLANT CL FEED 37                                           42137130722115
0402110200110300101O
10000978094809320916090608900874085008370822081308040793077807660754074007280714
0699
   0  72 144 217 290 363 435 508 581 654 726 799 872 945101710901163123613081381
1454
    PLANT RUN 40 CLE GANGUE                                           42140090822000
04021
10000962094409210906088808700818078807710742073107150683065206170591056605460522
0499
   0  79 159 239 319 399 479 558 638 719 799 878 958103811181198127813571438151 8
1598
    PLANT RUN NO 32 MINERAL 25/5/72                                    8113225052
04021
10000878071706900660063006260610060105720566056105530541053705230517051305080502
0499
   0  55 111 167 223 279 335 391 447 503 559 615 671 727 783 839 895 95110071063
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1119
    MINERAL TEST RUN 35 RO FEED PLANT                                    81135290621135
0402110200110300010
1000088808450826080107760756074406760650063206040583055805390527050805000494048604
0479
   0  45  90 136 181 227 272 318 364 409 454 500 545 591 637 682 728 773 819 864
 910
    MINERAL PLANT RO FEED 35                                             81135290621635
0402110200100501101O
1000081807800677064306180599059105850564055905440536052205100490045604450436042904
0425
   0  46  93 140 186 233 280 326 373 420 467 513 560 607 653 700 747 794 840 887
 934
    MINERAL PLANT RO FEED 36                                             81136060721610
0402110200110300101O
1000083607740761075007380728070506970604055705450533052105110495047604650463044804
0443
   0  46  93 141 188 235 282 330 377 424 471 521 565 612 660 707 754 801 848 896
 943
    MINERAL PLANT RO FEED 36                                             81136060722010
0402110200110300101O
1000091207960789078007680756072006840628057705610533051404920466044404350431042704
0415
   0  45  90 135 180 226 271 316 361 407 452 498 543 588 633 678 724 769 814 859
 905
    MINERAL PLANT RO FEED 37                                             81137130721720
0402110100040070001O
1000091807820744070306800658063506160573056105430531051305010478047104550448044104
0428
   0  62 124 187 249 311 374 436 498 561 623 685 748 810 872 935 997106011221184
1247
    MINERAL PLANT RO FEED 37                                             81137130722050
0402110100040070001O
1000090308000764070006840664064606300576055105350522047704600455043504220408039404
0376
   0  87 175 263 351 439 527 615 702 791 879 966105411421230131814061494158216701
1758
    MINERAL PLANT RO FEED 38                                             81138190721615
0402110200100501101O
1000083007610685066306330599058705610546049304660454044604450442044104240420041 5
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0402
   0  81 163 244 326 408 490 571 653 735 816 898 980106211431225130713881470155216
33
    MINERAL PLANT RO FEED 38                                          811381907220000402110200100501101010000809079406760655063806010581055605350501048304730468046604620455044104300424041
6
   0  87 175 263 351 438 526 614 702 789 877 965105311401228131614041491157916671755
    MINERAL PLANT RUN 42 24/8/72                                      811422408215100402110000839078807290687067006480635061006020601058705850581057905660557055305480543053
8
   0  48  97 146 195 244 293 342 391 440 489 538 587 636 685 734 783 832 881 930 979
    MINERAL PLANT RO FEED 44                                          811440709218000402110100040070001010000914088708560827081508060796077707540735072107030690067706680631062006000591057
2
   0  58 117 177 235 294 354 412 471 531 590 648 708 767 825 885 944100310621121118
0
    MINERAL PLANT RO FEED 47                                          811472809212100402110100040070001010000741072006850669065406440639062905290506049204870469046104540447044204390435043
0
   0 102 204 307 409 511 614 716 819 921102311261228133014331535163817401842194520
47
    MINERAL PLANT RO FEED 47                                          811472809214100402110100040070001010000791076807040675063606250612060405950569053205180509046704580449044204380433042
5
   0 102 204 307 410 512 615 718 820 922102511281230133314361538164117431846194920
51
    MINERAL PLANT RO FEED 47                                          811472809216030402110100040070001010000738068006550615059605880562055205400521050504910481047004640451043404290418040
9
   0 112 224 336 448 561 673 785 898101011231235134714601572168417961909202121332246
    MINERAL PLANT RO FEED 47                                          81147280921645
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0402110100040070001O
1000080407730721069806870674066606590646060705730559054905370529049104810478047 3
0470
   0 109 219 329 439 548 658 768 878 9881098120713171427153716471757186619762086
2196
    PLANT RUN 32 GANGUE 25/5/72                                          8213225052
04021
1000099709910989098709800979097809760974097309730972096809670965096309630961096 0
0959
   0  55 111 167 223 279 335 391 447 503 559 615 671 727 783 839 895 95110071063
1119
    GANGUE PLANT RO FEED 35                                              821352906211 35
0402110200110300101O
1000099509900988098309780974097309670965096409610957094609430940093209310930092 8
0926
   0  45  90 136 181 227 272 318 364 409 454 500 545 591 637 682 728 773 819 864
 910
    GANGUE PLANT RO FEED 35                                              821352906216 35
0402110200110300101O
1000099509930987098209780975097309720970097009670965096209580954094209390938093 6
0935
   0  46  93 140 186 233 280 326 373 420 467 513 560 607 653 700 747 794 840 887
 934
    GANGUE PLANT RO FEED 36                                              821360607216 10
0402110200110300101O
1000099309890987098609820981097809760946094009380937093509330931092709250925092 2
0921
   0  46  93 141 188 235 282 330 377 424 471 521 565 612 660 707 754 801 848 896
 943
    GANGUE PLANT RO FEED 36                                              821360607220 10
0402110200110300101O
1000099709930992099109880987098109660953094609450942093709290923091909170916091 5
0914
   0  45  90 135 180 226 271 316 361 407 452 498 543 588 633 678 724 769 814 859
 905
    GANGUE PLANT RO FEED 37                                              821371307217 20
0402110200110300101O
1000099709930990098609810978097609700966096409630961095909560951095009480945094 4
0942
   0  62 124 187 249 311 374 436 498 561 623 685 748 810 872 935 997106011221184
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1247
    GANGUE PLANT RO FEED 37                                                821371307220 50
0402110200110300 1010
1000099609910987097909760974097209690962095909570955094509410940093409320927092 4
0920
   0  87 175 263 351 439 527 615 702 791 879 966105411421230131814061494158216701
1758
    GANGUE PLANT RO FEED 38                                                821381907216 15
0402110200110300 1010
1000099109890982097909680950094809440944094109390938093709370936093609330933093 1
0929
   0  81 163 244 326 408 490 571 653 735 816 898 980106211431225130713881470155 2
1633
    GANGUE PLANT RO FEED 38                                                821381907220 00
0402110200110300 1010
1000099609950989098709800968096309590957095609530952095109510949094909460944094 2
0940
   0  87 175 263 351 438 526 614 702 789 877 965105311401228131614041491157916671
1755
    GANGUE  PLANT RUN 42 24/8/72                                           821422408215 10
04021
1000099409910987098109790975097109650965096509640964096309630962096109610960095 9
0959
   0  48  97 146 195 244 293 342 391 440 489 538 587 636 685 734 783 832 881 930
 979
    GANGUE PLANT RO FEED 44                                                821440709218 00
0402110200110300 1010
1000099809970995099109900990098809860985098309820980097909780977097009690966096 5
0962
   0  58 117 177 235 294 354 412 471 531 590 648 708 767 825 885 944100310621121
1180
    GANGUE PLANT RO FEED 47                                                821472809212 10
0402110100050140 1010
1000099309920990098709860984098309810978097609740974097109700968096709650964096 3
0962
   0 102 204 307 409 511 614 716 819 921102311261228133014331535163817401842194 5
2047
    GANGUE PLANT RO FEED 47                                                821472809214 10
0402110100050140 1010
1000099609950992098709840983098009790979097709740972097109590958095609540954095 3
```

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0951
   0 102 204 307 410 512 615 718 820 922102511281230133314361538164117431846194 9
2051
    GANGUE PLANT RO FEED 47                                          82147280921603
0402110100050140101 0
1000099409910989098509830982097409730972097109690968096709650964096209590959095 7
0955
   0 112 224 336 448 561 673 785 898101011231235134714601572168417961909202121332
2246
    GANGUE PLANT RO FEED 47                                          82147280921645
0402110100050140101 0
1000099609950993099009890987098609850984098209790978097609750973096709650965096 4
0964
   0 109 219 329 439 548 658 768 878 988109812071317142715371647175718661976208 6
2196
    COPPER PLANT RO FEED 47 1410                                     83147280921410
04021
1000053105020433040803910381037803620359035203430339032903040300029602940292028 9
0287
   0 102 204 307 410 512 615 718 820 922102511281230133314361538164117431846194 9
2051
    NICKEL PLANT RO FEED 47 1410                                     84147280921410
04021
1000079507690711067406460633063006140610060005870580057205410535053005260524052 1
0519
   0 102 204 307 410 512 615 718 820 922102511281230133314361538164117431846194 9
2051
```

THE FOLLOWING TESTS ON A CONTINUOUS BANK OF CELLS HAVE COMPLEMENTARY BATCH TEST DATA ON THEIR FEED STREAMS * SEE PAGES 53 AND 71

ROUGHER A3 13/4/73 1435 MINERAL 811 130421435
04021
1000091207650616057805670558053805130443041704090401038803820376036003510340033103210
0 68 137 207 276 345 414 483 552 621 690 759 829 898 967103611051174124313121381
ROUGHER A3 13/4/73 1435 GANGUE 821 130421435
04021
1000099809930987098409830982097909750971096909680967096609640964096209610958095709560
0 68 137 207 276 345 414 483 552 621 690 759 829 898 967103611051174124313121381
ROUGHER BANK COPPER 13/4/72 1430 831 130421430
04021
1000069504790373035403460342033103170294028202770274026802620259025002450236023202260
0 68 137 207 276 345 414 483 552 621 690 759 829 898 967103611051174124313121381
ROUGHER BANK NICKEL 13/4/72 1430 841 130421430
04021
1000093407630628059705820573055705370516050605030500049504920491048604820478047604720
0 68 137 207 276 345 414 483 552 621 690 759 829 898 967103611051174124313121381
ROUGHER A3 23/4/73 1200 MINERAL 811 230421200
04021
1000076706330521051004980494049104800401037203580351031703070295027402690259025002370
0 68 137 205 274 343 411 480 549 618 686 755 824 892 961102910981167123613041373
ROUGHER A3 23/4/73 1200 GANGUE 821 230421200
04021
1000099409870978097709750974097309700964096209600959095209500948094109400937093509310
0 68 137 205 274 343 411 480 549 618 686 755 824 892 961102910981167123613041373

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 ROUGHER BANK 2304/73 1200 COPPER                                    831  230421200
04021
1000060004460249024102320228022602200204019801920190017701720168015601520148014 4
0139
   0  68 137 205 274 343 411 480 549 618 686 755 824 892 961102910981167123613041
1373
 ROUGHER BANK 2304/73 1200 NICKEL                                    841  230421200
04021
1000075005650460044904360431042804180401039503900388037903750372036303610358035 5
0351
   0  68 137 205 274 343 411 480 549 618 686 755 824 892 961102910981167123613041
1373
  ROUGHER TEST 2/5/73 1450 SULFUR                                    811  020521450
04021
1000077707240585049804530428042404040398038403810369032703200298027702720266026 1
0256
   0  68 137 205 274 342 411 479 548 616 685 753 822 890 959102710961164123313011
1370
  ROUGHER TEST 2/5/73 1450 GANGUE                                    821  020521450
04021
1000099509930987097609710970096909650964096309630962095709560953095009480947094 6
0945
   0  68 137 205 274 342 411 479 548 616 685 753 822 890 959102710961164123313011
1370
    CLEANER 11/4/73 A SECT 1445 SULF                                 411  110421445
04021
1000086208080752064205240474043803920357032903070271023902170199017301690168016 7
0162
   0  97 195 293 390 488 587 684 782 880 978107611741272136914671566166317611859
1957
    CLEANER A SECTION 11/4/73 1445 GANGUE                            421  110421445
04021
1000097809700959093709040884086808460822080107830756073307120693066806630661065 9
0650
   0  97 195 293 390 488 587 684 782 880 978107611741272136914671566166317611859
1957
  CLEANER BANK 11/4/73 1445 COPPER                                   431  110421445
04021
1000068105780479034902520213018901680150013801280116010400950087007800760075007 4
0070
```

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   0  97 195 293 390 488 587 684 782 880 97810761174127213691467156616631761185
1957
  CLEANER BANK 11/4/73 1445 NICKEL                                    441  110421445
04021
10000869081407420590043403710326028102460222020401810163014901370123012001180117
0112
   0  97 195 293 390 488 587 684 782 880 97810761174127213691467156616631761185
1957
    CLEANER A SECTION 18/4/73 MINERAL                                 411  180421145
04021
10000902081207660720066906370602055804990435039503380306028102710259024602320225
0214
   0  87 175 263 350 438 526 614 702 790 877 96510531141122813161405149215801668
1755
    CLEANER A SECT 18/4/73 GANGUE 1145                                421  180421145
04021
10000984096309490935091809060894087808550830081307870768075307450737072607140708
0696
   0  87 175 263 350 438 526 614 702 790 877 96510531141122813161405149215801668
1755
 CLEANER BANK 18/4/73 1145 COPPER                                     431  180421145
04021
10000704054804860444041603980384036703450320030502840266025302470241023402260222
0217
   0  87 175 263 350 438 526 614 702 790 877 96510531141122813161405149215801668
1755
 CLEANER BANK 18/4/73 1145 NICKEL                                     441  180421145
04021
10000894073706600584051704760436039903590320029602670247023302260217020901990194
0186
   0  87 175 263 350 438 526 614 702 790 877 96510531141122813161405149215801668
1755
  CLEANER BANK 30/4/73 1430 SULFS                                     411  300421430
04021
10000829071906370582050704200366031602690242022502040203020102000195018901840176
0164
   0 125 251 376 502 629 754 88010061131125813831509163517601887201321382264239O
2516
  CLEANER BANK 30/4/73 1430 GANGUE                                    421  300421430
04021
```

100009570923088908620823078007470713069506710653062606230621061906100600059305830567

0 125 251 376 502 629 754 880100611311258138315091635176018872013213822642390 2516

********** GROUP SEVEN **********

THESE SPECIAL TESTS ARE USED IN ANALYZING THE FROTH PHASE OF FLOTATION *
SEE PAGE 105

```
 MINERAL BATCH RO CONC 17-3-74 FROTH TEST 15 G/MIN          311  170311320
  007
1000081607260636055104180311
0000050010001500200030004000
   GANGUE   BATCH RO CONC 17-3-74 FROTH TEST 15 G/MIN       321  170311320
  007
1000097909660952093909160892
0000050010001500200030004000
   MINERAL BATCH RO CONC 17-3-74 FROTH TEST 60G/MIN         311  170311500
  007
1000063004900386030702200159
0000050010001500200030004000
   GANGUE  BATCH RO CONC 17-3-74 FROTH TEST 60G/MIN         321  170311500
  007
1000094409080877085008040765
0000050010001500200030004000
   MINERAL BATCH RO CONC 17-3-74 FROTH TEST 30G/MIN         311  170311610
  007
1000074706000496040502920232
0000050010001500200030004000
   GANGUE  BATCH RO CONC 17-3-74 FROTH TEST 30G/MIN         321  170311610
  007
1000096809440925090708770854
0000050010001500200030004000
   MINERAL BATCH RO CONC 17-3-74 FROTH TEST 120G/MIN        311  170311715
  006
1000056404100308024801 71
0000050010001500200030004000
   GANGUE  BATCH RO CONC 17-3-74 FROTH TEST 120G/MIN        321  170311715
  006
1000090708490803076607 01
0000050010001500200030004000
    RO CONC 15G/MIN BATCH 1320     COPPER                   331  170311320
```

```
  007
1000048103850270020401310109
0000050010001500200030004000
     BATCH RO CONC NICKEL 15 G/MIN                    341  170311320
  007
1000089108190742063403360150
0000050010001500200030004000
     BATCH RO CONC  COPPER 60 G/MIN                   331  170311500
  007
1000033601730114008300530043
0000050010001500200030004000
     BATCH RO CONC NICKEL 60 G/MIN                    341  170311500
  007
1000068705070317018500960059
0000050010001500200030004000
     BATCH RO CONC COPPER 30 G/MIN                    331  170311610
  007
1000049102670187014100900067
0000050010001500200030004000
     BATCH RO CONC NICKEL 30 G/MIN                    341  170311610
  007
1000082806650485032401430084
0000050010001500200030004000
      BATCH RO CONC COPPER 120 G/MIN                  331  170311715
  006
1000026101770113009600059
0000050010001500200030004000
     BATCH RO CONC NICKEL 120 G/MIN                   341  170311715
  006
1000051903260188012800063
0000050010001500200030004000
```

```
// FOR FTSRT                STARTS ANYFT
*IOCS(CARD)
C
C** PROGRAM FOR INITIALIZATION OF REGRESSION OF BATCH OR CONTINUOUS TEST DATA
C   BY THE DERIVATIVE METHOD DESCRIBED ON PAGES 142-144
C** ICL - AN ARRAY OF SEVEN WHICH SPECIFIES WHICH TEST CODES ARE TO BE REGRESSED
C   IF ICL IS ZERO ALL TESTS INPUT WILL BE REGRESSED.
C** IWT - AN ARRAY OF FIVE USED TO SPECIFY WHICH MODEL FORM IS TO BE REGRESSED.
C   HOLDS A VALUE OF EITHER ZERO OR UNITY.  THE GENERAL FORM OF THE BATCH TEST
C   EQUATION IS C=(IWT1*A*EXP(-100.*T)+(1.-IWT1*A-IWT2*B)*EXP(-IWT3*C*T**D-IWT4*
C   C*T-IWT5*T**D)+IWT2*B   WHERE A,B,C,D ARE POSSIBLE PARAMETERS.  THIS MAY BE
C   SIMPLIFIED IF ONLY THE SINGLE RATE AND RATE PLUS NONFLOATING MODELS ARE OF
C   INTEREST.  SEE ALSO SUBROUTINES THEO AND DERIV.
C** A SIMILAR CONTINUOUS EQUATION IS USED. C=(1.-IWT1*A-IWT2*B)/(1.+C*IWT3*T)**N
C   +IWT2*B   WHERE A,B,C ARE POSSIBLE PARAMETERS.
C
      EXTERNAL SETUP
      COMMON Y(21),THEOR(21),SIQSG(21),T(21),RES(21),PF(4),PPF(4,4),P(4)
     1,X(4),C(4),A(4,4),ITIT(30),NPAR,NDATA,ITMAX,NCSTP,NCTCT,CHMIN,IWT(
     25),CHISQ,K,ID(7),ILC(7),LC
      K=0
C
C**  FREE FORMAT CARD INPUT TO ARRAY Y
C
      CALL RDCRD (13,MN,Y)
      DO 1 L=1,7
 1    ILC(L)=Y(L)
      DO 2 L=1,5
 2    IWT(L)=Y(L+7)
      LC=Y(13)
      LC=1
      CALL LINK (SETUP)
      END
```

```
// FOR SETUP
*IOCS(CARD)
C
C** READS IN BATCH/CONTINUOUS TEST DATA AND DESCRIPTION FOR ANYFT DERIVATIVE
C   REGRESSION PROGRAM.
C** CALCULATES INITIAL PARAMETER ESTIMATES AND CHECKS TEST CODE TO SEE IF TEST
C   IS TO BE REGRESSED.
C** LINKED TO BY FTSRT OR ANYFT. LINKS TO BLOK1.
C
      EXTERNAL BLOK1
      COMMON Y(21),THEOR(21),SIQSG(21),T(21),RES(21),PF(4),PPF(4,4),P(4)
     1,X(4),C(4),A(4,4),ITIT(30),NPAR,NDATA,ITMAX,NCSTP,NCTCT,CHMIN,IWT(
     25),CHISQ,JL,ID(7),ILC(7),LC
      DO 7 M=1,400
      READ (2,500) ITIT,ID,NDATA
500   FORMAT (30A2,6X,3I1,I2,I4,I1,I4/2X,I3)
      READ (2,501) (Y(I),I=1,NDATA)
501   FORMAT (20F4.3)
      READ (2,502) (T(I),I=1,NDATA)
502   FORMAT (20F4.2)
      NIT=0
      NITC=0
      DO 8 K=1,7
      IF(ILC(K)) 8,8,9
9     NIT=NIT+1
      IF(ILC(K)-ID(K)) 8,10,8
10    NITC=NITC+1
8     CONTINUE
      IF(NIT-NITC) 7,14,7
7     CONTINUE
      CALL EXIT
14    GO TO (40,41),LC
41    DO 42 I=1,NDATA
42    T(I)=SQRT(T(I))
40    ITMAX=9
      NCSTP=3
      NCTCT=8
      NPAR=4
      P(1)=(1.-Y(2)+.05)*IWT(1)
      P(2)=(Y(NDATA)-.05)*IWT(2)
```

```
      TTT=0.
      SSS=0.
      XX1=Y(1)-IWT(1)*P(1)-IWT(2)*P(2)
      DO 73 I=1,NDATA
      XX0=Y(I)              -IWT(2)*P(2)
      SSS=ALOG(XX0/XX1)+SSS
73    TTT=TTT-T(I)
      P(3)=SSS/TTT
      CALL LINK (BLOK1)
      END
```

```
// FOR BLOK1
*IOCS(1443 PRINTER)
C
C** DATA OUTPUT FOR BATCH/CONTINUOUS TEST REGRESSIONS FOR SINGLE RATE OR RATE
C   PLUS NONFLOATING FIRST ORDER MODELS.  LINKED TO BY SETUP.  LINKS TO ANYFT.
C
      COMMON DATA(21),THEOR(21),SIGSQ(21),ABCIS(21),RES(21),PF(4),PPF(4,
     14),PAR(4),OLD(4),C(4),A(4,4),ITIT(30),NPAR,NDATA,ITMAX,NCSTP,NCTCT
     2,CHMIN,IWT(5),CHISQ,KH,ID(7)
      WRITE(3,3) ITIT,ID
      WRITE(3,4) NPAR
      DO 5 I=1,NPAR
      WRITE (3,6) I,PAR(I)
 5    CONTINUE
      WRITE (3,7) ITMAX,NCSTP,NCTCT
      WRITE (3,8) NDATA,(DATA(I),I=1,NDATA)
      WRITE(3,9) (ABCIS(I),I=1,NDATA)
C
C**  MAY BE ALTERED TO WEIGHT INPUT DATA FOR REGRESSION
 10   DO 11 I=1,NDATA
 11   SIGSQ(I)=1.
C
      WRITE(3,12)
 45   CHISQ=10.**10
      WRITE(3,25) ITIT
      NIT=0
      NCUT=0
      NCUTT=0
      CALL LINK (ANYFT)
 3    FORMAT (1H1,30A2,7I5)
 4    FORMAT ('  THE FOLLOWING SUPPLIED BY USER   '/ 20X,' NUMBER OF PAR
     1AMETERS =  ',I3)
 6    FORMAT (20X,'INITIAL VALUES FOR PARAMETER NO. ',I4,' IS ',E16.8)
 7    FORMAT (20X,'MAX NO. OF ITERATIONS ALLOWED',I5/20X,'MAX NO. OF CON
     1SECUTIIVE CUT STEPS  ',I5/20X,'MAX TOTAL NO. OF CUT STEPS ',I5)
 8    FORMAT ( /20X,'THE',I4,'  DATA POINTS ARE'/20(5E20.8/))
 9    FORMAT ( /20X,'THE CORRESPONDING ABCISSA ARE'/20(5E20.8/))
 12   FORMAT (20X,' STANDARD DEVIATION = 1. FOR EACH')
 25   FORMAT (/20X,'     ITERATION SEQUENCE FOR ',30A2/,'     ITERATION
     1CUT   CHI SQUARED        P A R A M E T E R S     ',/)
      END
```

```
// FOR ANYFT
*IOCS(1443 PRINTER)
C
C** MAIN REGRESSION PROGRAM FOR BATCH/CONTINUOUS TEST DATA FOR USE WITH SINGLE
C   RATE AND RATE PLUS NONFLOATING MODELS.  LINKS TO SETUP.
C** DATA - CONTAINS OBSERVED SPECIES CONCENTRATIONS
C** THOER - CONTAINS CALCULATED SPECIES CONCENTRATIONS
C** SIGSQ - NOT USED
C** ABCIS - CONTAINS FLOTATION TIMES CORRESPONDING TO SPECIES CONCENTRATIONS
C** RES - NOT USED
C** PF - CONTAINS FIRST DERIVATIVE OF THOER WITH RESPECT TO EACH PARAMETER.
C** PPF - CONTAINS SECOND ORDER DERIVATIVES
C** PAR - CONTAINS PRESENT PARAMETER VALUES  OLD - CONTAINS PREVIOUS PARAMETER
C   VALUES.
C** C,A - MATRICES USED IN THE REGRESSION
C** ITIT - A 60 CHARACTER TEST DESCRIPTION
C** NPAR - NUMBER OF PARAMETERS TO BE REGRESSED
C** NDATA - NUMBER OF OBSERVED CONCENTRATIONS
C** ITMAX - MAXIMUM NUMBER OF ITERATIONS TO EXIT
C** NCSTP - MAXIMUM NUMBER OF CUT STEPS PER ITERATION
C** NCTCT - MAXIMUM NUMBER OF CUT STEPS TO EXIT
C** CHMIN - PROGRAM EXITS ON A SUM OF SQUARES LESS THAN CHMIN
C** IWT - MODEL CODE
C** CHISQ - PRESENT SUM OF SQUARES
C** ID - TEST IDENTITY CODE
C
      EXTERNAL SETUP
      COMMON DATA(21),THOER(21),SIGSQ(21),ABCIS(21),RES(21),PF(4),PPF(4,
     14),PAR(4),OLD(4),C(4),A(4,4),ITIT(30),NPAR,NDATA,ITMAX,NCSTP,NCTCT
     2,CHMIN,IWT(5),CHISQ,KH,ID(7)
      CHMIN=0.
 54   PREV=CHISQ
C
C** CALL TO SUBROUTINE TO GENERATE THEORETICAL MODEL VALUES
C
 55   CALL THEO
      CHISQ=0
      DO 60 I=1,NDATA
 60   CHISQ=CHISQ+(THOER(I)-DATA(I))**2
      WRITE (3,70) NIT,NCUT,CHISQ,(PAR(I),I=1,NPAR)
```

```
 70   FORMAT (6X,I5,I8,6E16.7/35X,5E16.7/)
      IF(CHISQ-PREV) 100,1003,1003
 1003 IF(NIT) 57,100,57
 57   NCUT=NCUT+1
      IF(NCUT-NCSTP) 1004,1004,100
 1004 NCUTT=NCUTT+1
      IF(NCUTT-NCTCT) 1005,1005,900
 1005 DO 81 I=1,NPAR
 81   PAR(I)=OLD(I)+(PAR(I)-OLD(I))*.5
      GO TO 55
 100  IF(CHISQ-CHMIN) 900,1006,1006
 1006 NIT=NIT+1
      NCUT=0
      DO 101 I=1,NPAR
      C(I)=0.
      DO 101 J=1,NPAR
      A(I,J)=0.
 101  CONTINUE
      DO 110 I=1,NDATA
C
C** CALL TO SUBROUTINE TO GENERATE FIRST AND SECOND PARTIAL DERIVATIVES
C
      CALL DERIV (I)
      DO 110 M=1,NPAR
      C(M)=C(M)+2.*(THOER(I)-DATA(I))*PF(M)
      DO 110 J=1,NPAR
      A(J,M)=A(J,M)+(2.          *((THOER(I)-DATA(I))*PPF(J,M)+PF(J)*PF(M
     1)))
 110  CONTINUE
C
C** CALL TO SUBROUTINE FOR DIRECT MATRIX INVERSION
C
      CALL MINV5 (A,NPAR)
 1007 IF(NIT-ITMAX) 1008,1008,900
 1008 DO 120 K=1,NPAR
      X=0.
      DO 121 KK=1,NPAR
 121  X=X+A(K,KK)*C(KK)
      OLD(K)=PAR(K)
 120  PAR(K)=PAR(K)-X
```

```
      GO TO 54
900   CALL LINK (SETUP)
      END
```

```
// FOR THEO                    VAR FOR PLANT
      SUBROUTINE THEO
C
C** CALCULATES THE THEORETICAL CONTINUOUS TEST CONCENTRATIONS FROM REGRESSED
C   PARAMETERS.  USED BY ANYFT.
C
      COMMON Y(21),THEOR(21),SIQSG(21),T(21),RES(21),PF(4),PPF(4,4),P(4)
     1,X(4),C(4),A(4,4),ITIT(30),NPAR,NDATA,ITMAX,NCSTP,NCTCT,CHMIN,IWT(
     25),CHISQ,JL,ID(7),ILC(7),LC
      P(1)=P(1)*IWT(1)
      P(2)=P(2)*IWT(2)
      P(3)=P(3)*IWT(4)
      THEOR(1)=1.
      DO 1 I=2,NDATA
 1    THEOR(I)=(1.-P(1)*IWT(1)-P(2)*IWT(2))/(1.+P(3)*T(2))**(I-1)+P(2)*I
      WT(2)
      RETURN
      END
```

```
// FOR DERIV
      SUBROUTINE DERIV(K)
C
C** CALCULATES THE FIRST AND SECOND ORDER ANALYTICAL PARTIAL DERIVATIVES OF THE
C   THEORETICAL CONTINUOUS TEST CONCENTRATIONS WITH RESPECT TO DESIGNATED
C   PARAMETERS.  USED BY ANYFT.
C
      COMMON Y(21),THEOR(21),SIQSG(21),T(21),RES(21),PF(4),PPF(4,4),P(4)
     1,X(4),C(4),A(4,4),ITIT(30),NPAR,NDATA,ITMAX,NCSTP,NCTCT,CHMIN,IWT(
     25),CHISQ,JL,ID(7),ILC(7),LC
      P(1)=P(1)*IWT(1)
      P(2)=P(2)*IWT(2)
      P(3)=P(3)*IWT(4)
      DO 38 J=1,4
      PF(J)=0.
      DO 38 L=1,4
38    PPF(J,L)=0.
      I=K-1
      PF(1)=-1./(1.+P(3)*T(2))**I*IWT(1)
      PF(2)=(PF(1)+1.  )*IWT(2)
      PF(3)=-I*T(2)*(1.-P(1)-P(2))/(1.+P(3)*T(2))**(I+1)
      PPF(1,3)=I*T(2)/(1.+P(3)*T(2))**K
      PPF(2,3)=PPF(1,3)
      PPF(3,1)=PPF(1,3)
      PPF(3,2)=PPF(1,3)
      PPF(3,3)=I*T(2)**2.*K*(1.-P(1)-P(2))/(1.+P(3)*T(2))**(I+2)
      PPF(1,3)=PPF(1,3)*IWT(1)
      PPF(2,3)=PPF(2,3)*IWT(2)
      PPF(3,1)=PPF(3,1)*IWT(1)
      PPF(3,2)=PPF(3,2)*IWT(2)
      RETURN
      END
```

```
// FOR DERIV
C
C** CALCULATES THE FIRST AND SECOND ORDER ANALYTICAL PARTIAL DERIVATIVES OF THE
C   THEORETICAL BATCH TEST CONCENTRATIONS WITH RESPECT TO THE DESIGNATED MODEL
C   PARAMETERS. USED BY ANYFT.
C
      SUBROUTINE DERIV(I)
      COMMON Y(21),THEOR(21),SIQSG(21),T(21),RES(21),PF(4),PPF(4,4),P(4)
     1,X(4),C(4),A(4,4),ITIT(30),NPAR,NDATA,ITMAX,NCSTP,NCTCT,CHMIN,IWT(
     25),CHISQ
      Z=-IWT(3)*P(3)*T(I)**P(4)-IWT(4)*P(3)*T(I)-IWT(5)*T(I)**P(4)
      R=(1.-IWT(1)*P(1)-IWT(2)*P(2))
      W=(-IWT(3)*T(I)**P(4)-IWT(4)*T(I))
      V=(-IWT(3)*P(3)*T(I)**P(4)*ALOG(T(I))-IWT(5)*T(I)**P(4)*ALOG(T(I))
     1)
      PF(1)=IWT(1)*EXP(-100.*T(I))-IWT(1)*EXP(Z)
      PF(2)=-IWT(2)*EXP(Z)+IWT(2)
      PF(3)=R*W*EXP(Z)
      PF(4)=R*V*EXP(Z)
      PPF(1,1)=0.
      PPF(2,2)=0.
      PPF(3,3)=R*W**2*EXP(Z)
      PPF(4,4)=R*V**2*EXP(Z)+R*(-IWT(3)*P(3)*T(I)**P(4)*(ALOG(T(I)))**2-
     1IWT(5)*T(I)**P(4)*(ALOG(T(I)))**2)*EXP(Z)
      PPF(1,2)=0.
      PPF(1,3)=-IWT(1)*W*EXP(Z)
      PPF(1,4)=-IWT(1)*V*EXP(Z)
      PPF(2,1)=0.
      PPF(2,3)=-IWT(2)*W*EXP(Z)
      PPF(2,4)=-IWT(2)*V*EXP(Z)
      PPF(3,1)=PPF(1,3)
      PPF(3,2)=PPF(2,3)
      PPF(3,4)=R*(-IWT(3)*T(I)**P(4)*ALOG(T(I)))*EXP(Z)+R*W*V*EXP(Z)
      PPF(4,1)=PPF(1,4)
      PPF(4,2)=PPF(2,4)
      PPF(4,3)=PPF(3,4)
      RETURN
      END
```

```
// FOR THEO
C
C** CALCULATES THEORETICAL MODEL VALUES FOR THE BATCH CASE OF THE SINGLE RATE
C    AND RATE PLUS NONFLOATING MODEL. USED BY PROGRAM ANYFT.
C** ARRAY THEOR CONTAINS THE THEORETICAL VALUES AS NORMALIZED CONCENTRATIONS.
C
      SUBROUTINE THEO
      COMMON Y(21),THEOR(21),SIGSG(21),T(21),RES(21),PF(4),PPF(4,4),P(4)
     1,X(4),C(4),A(4,4),ITIT(30),NPAR,NDATA,ITMAX,NCSTP,NCTCT,CHMIN,IWT(
     25),CHISQ
      DO 1 I=1,NDATA
 1    THEOR(I)=IWT(1)*P(1)*EXP(-100.*T(I))+(1.-IWT(1)*P(1)-IWT(2)*P(2))*
     1EXP(-IWT(3)*P(3)*T(I)**P(4)-IWT(4)*P(3)*T(I)-IWT(5)*T(I)**P(4))+IW
     2T(2)*P(2)
      RETURN
      END
```

```
// XEQ FSTRT   FX1
 0 0 0 0 0 0 0 0 1 0 1 0
    GANGUE PLANT RO FEED 47 +104                                  823472809216 45
04011
1000099909980997099709960995099509950995099409 94
0000020404090614081910231228143316381842204 7
    GANGUE PLANT RO FEED +53                                      82447280921645
04011
1000099809960996099509950994099409940994099409 94
0000020404090614081910231228143316381842204 7
    GANGUE PLANT RO FEED 47 -53                                   82547280921645
04011
1000099209820978097509690961095509400936093 3
0000020404090614081910231228143316381842204 7
    MINERAL PLANT RO FEED 47 +208                                 81247280921645
04011
1000091207800741071406780660065106330619060 3
0000020404090614081910231228143316381842204 7
    MINERAL PLANT RO FEED 47 +104                                 81347280921645
04011
1000092108390821081107790761075207430736073 2
0000020404090614081910231228143316381842204 7
    MINERAL PLANT RO FEED 47 +53                                  81447280921645
04011
1000073806480614060905650547053505230519051 7
0000020404090614081910231228143316381842204 7
    MINERAL PLANT RO FEED 47 -53                                  81547280921645
04011
1000071706180589057105020437041003590350033 5
0000020404090614081910231228143316381842204 7
     GANGUE PLANT RO FEED 47 +208                                 82247280921645
04011
1000099809960994099209890986098609830981097 7
0000020404090614081910231228143316381842204 7
```

```
// FOR REGAL                    ALF,CN BATCH REG ** VA02A
*IOCS(CARD)
*IOCS(TYPEWRITER)
C
C** ALTERNATE PROGRAM FOR THE REGRESSION OF BATCH DATA FOR THE RATE PLUS
C   NONFLOATING MODEL USING THE POWELL SEARCH METHOD.
C** IS,ID - TEST IDENTIFICATION
C** E - PARAMETER TOLERANCES
C** ZZZ - WORKING ARRAY FOR VA02A
C** Y - OBSERVED SPECIES CONCENTRATIONS
C** T - FLOTATION RETENTION TIMES
C** P - MODEL PARAMETERS
C** PRED - BEST FIT SPECIES CONCENTRATIONS
C
      DIMENSION IS(30),ID(7)
      DIMENSION E(3),ZZZ(100),Y(13),T(13),P(3),PRED(13)
      DATA E /.001,.005,.1/
 2    CALL BREAD (T,Y,IS,ID,NDATA)
      P(2)=Y(NDATA)-.02
      XX=0.
      X=0.
      DO 1 I=1,NDATA
      X=X+ALOG(Y(I)-P(2))
 1    XX=XX+T(I)
      P(1)=-X/XX
      NNM=2+3*(NDATA+2*3/2)
      ES=5.
      CALL VA02A (NDATA,2,PRED,P,E,ES,-20,200,ZZZ,NNM,Y,T)
      W=0.
      DO 302 I=1,NDATA
      PRED(I)=(1.-P(2))*EXP(-P(1)*T(I))+P(2)
302   W=W+(Y(I)-PRED(I))**2.0
      WRITE (3,100) IS,ID,NDATA
100   FORMAT (30A2,8I5)
      WRITE (3,103) P,W,(Y(I),T(I),PRED(I),I=1,NDATA)
103   FORMAT (4E12.4/13(3F10.4/)/)
      GO TO 2
99    CALL EXIT
      END
```

```
// FOR MODSQ                    ALF,CN BATCH REG
      SUBROUTINE MODSQ (FF,M,N,P,F,TT)
C
C** SUBROUTINE FOR USE WITH POWELL SEARCH (VA02A) REGRESSION OF BATCH TEST
C   PARAMETERS OF THE RATE PLUS NONFLOATING MODEL
C
      DIMENSION FF(1),P(3),F(1),TT(1)
      DO 1 I=1,M
      XXX=(1.-P(2))*EXP(-P(1)*TT(I))+P(2)
 1    F(I)=FF(I)-XXX
      RETURN
      END
```

```
// XEQ REGAL   FX
   MINERAL VOLLY FLOT FEED 23/3/73 4/3                    111  23031
04011
1000070605600458038003340298026602420222020 7
0000050010001500200025003000350040005000 6000
   GANGUE  VOLLY FLOT FEED 23/3/73 4/3                    121  23031
04011
1000099009830978097209680966096009560952094 9
0000050010001500200025003000350040005000 6000
      COPPER BATCH VOLLY FLOT FEED 4/3 23-3-73            131  23031
  011
1000043203070252022002020188017601660156014 9
0000050010001500200025003000350040005000 6000
     NICKELR BATCH VOLLY FLOT FEED 4/3 23-3-73           141  23031
  011
1000079205960484041703840362034303310322031 4
0000050010001500200025003000350040005000 6000
 FLOT FEED BATCH NON CU MAT 23/3/73                      161  23071
  011
1000098909820975096909650961095609520949094 6
0000050010001500200025003000350040005000 6000
 FLOT FEED BATCH NON NI MAT 23/3/73                      171  23071
  011
1000098909820974097009660962095709540950094 6
0000050010001500200025003000350040005000 6000
```

```
// FOR REGDL
*IOCS(CARD)
*IOCS(TYPEWRITER)
      EXTERNAL DIST
      DIMENSION E(3)
      DIMENSION ZZZ(100)
      DIMENSION Y(13),T(13),P(3),FLIM(4),TT(20),PRED(13)
      DATA FLIM /-.9,20.,0.,300./
      DATA E/.01,.05,.001/
C** MAINLINE PROGRAM FOR THE REGRESSION OF BATCH FLOTATION DATA TO FIT THE GAMMA
C   FUNCTION MODEL.
C** THE POWELL SEARCH METHOD IS USED TO GENERATE MEAN AND VARIANCE OF THE BEST
C   FITTING RATE CONSTANT DISTRIBUTION FOR BOTH CASES 1) ZERO MATERIAL IN THE
C   SPECIES IS NONFLOATABLE 2) Y(NDATA) OR LAST OBSERVED FRACTION OF THE SPECIES
C   IS CONSIDERED TO BE NONFLOATABLE.
C** E - PARAMETER TOLERANCES FOR EXIT FROM POWELL SEARCH
C** ZZZ - WORKING ARRAY FOR VA02A
C** Y - OBSERVED NORMALIZED SPECIES CONCENTRATION FROM BATCH TEST
C** T - BATCH TEST RETENTION TIMES
C** P - PARAMETER DISTRIBUTION VALUES
C** FLIM - PARAMETER SEARCH LIMITS FOR CASE TWO GRID SEARCH
C** TT - TEST IDENTIFICATION AND CODE
C** PRED - CALCULATED SPECIES CONCENTRATIONS FROM BEST FIT PARAMETERS
C** A - ON OUTPUT IS MEAN OF RATE DISTRIBUTION
C** B - ON OUTPUT IS VARIANCE OF RATE DISTRIBUTION
C** W - SUM OF SQUARES AT BEST FIT
C** NDATA - NUMBER OF OBSERVED CONCENTRATIONS
 107  FORMAT (' STAGE ONE ',3E12.4)
 9    READ (2,100) TT,NDATA
 100  FORMAT (20A4/2X,I3)
      READ (2,101) (Y(I),I=1,NDATA)
 101  FORMAT (20F4.3)
      READ (2,102) (T(I),I=1,NDATA)
 102  FORMAT (20F4.2)
C
C** GET FIRST GUESS PARAMETER VALUES AS DESCRIBED ON PAGES 144-145
C
      W=10.**10.
      P(3)=0.
      DEL=10.
```

```
      P(2)=-9.99
      DO 300 L=1,4
      DO 400 K=1,10
      P(2)=P(2)+DEL
      X=0.
      XX=0.
      DO 2 J=2,NDATA
      X=X+ALOG(Y(J))*ALOG(P(2)/(P(2)+T(J)))                                  C
 2    XX=XX+(ALOG(P(2)/(P(2)+T(J))))**2.
      P(1)=X/XX-1.0
      CALL DIST1(PRED,NDATA,T,P)
      S=0.
      DO 3 J=1,NDATA
 3    S=S+(Y(J)-PRED(J))**2.
      IF(S-W) 401,402,402
401   X1=W
      W=S
      A=P(1)
      B=P(2)
      GO TO 400
402   P(2)=P(2)-2.*DEL
      IF(P(2)) 404,405,405
404   P(2)=.01
405   DEL=DEL/5.
      W=X1
      GO TO 300
400   CONTINUE
300   CONTINUE
      P(1)=A
      P(2)=B
      ES=10.
      NNM=2+3*(NDATA+2*3/2)
C
C** INITIAL GUESSES AVAILABLE, BEGIN POWELL SEARCH FOR BEST FIT
C
      CALL VA02A (NDATA,2,PRED,P,E,ES,-5,100,ZZZ,NNM,Y,T)
      CALL DIST1 (PRED,NDATA,T,P)
      W=0.
      DO 301 I=1,NDATA
301   W=W+(Y(I)-PRED(I))**2.0
```

```
      A=(P(1)+1.)/P(2)
      B=(P(1)+1.)/P(2)**2.
C
C** OUTPUT RESULTS FOR CASE ONE
C
      WRITE (3,107) A,B
      WRITE (3,100) TT,NDATA
      WRITE (3,103) P,W,(Y(I),T(I),PRED(I),I=1,NDATA)
      P(3)=Y(NDATA)
C
C** PERFORM A GRID SEARCH TO OBTAIN INITIAL PARAMETER ESTIMATES FOR CASE TWO
C
      CALL GRIDS (30,30,FLIM,P,Y,NDATA,DIST,T,W,PRED)
      WRITE (3,107) P
      ES=20.
C                        )T,Y,MNN,ZZZ,001,5-,SE,E,P,DERP,3,ATADN( A20AV LLAC
C                                                          )2/3*3+ATADN(*4+3=MNN
      NNM=2+3*(NDATA+2*3/2)
C
C** PERFORM POWELL SEARCH FOR CASE TWO
C
      CALL VA02A (NDATA,2,PRED,P,E,ES,-20,200,ZZZ,NNM,Y,T)
      CALL DIST1 (PRED,NDATA,T,P)
      W=0.
      DO 302 I=1,NDATA
  302 W=W+(Y(I)-PRED(I))**2.0
C
C** OUTPUT RESULTS FOR CASE TWO
C
      WRITE (3,100) TT,NDATA
      WRITE (3,103) P,W,(Y(I),T(I),PRED(I),I=1,NDATA)
      A=(P(1)+1.)/P(2)
      B=(P(1)+1.)/P(2)**2.
  103 FORMAT (4E12.4/13(3F10.4/)/)
      WRITE (3,107) A,B
      GO TO 9
   99 CALL EXIT
      END
```

```
// FOR MODSQ                          DIST PARA BATCH WITH VA02A
      SUBROUTINE MODSQ (FF,M,N,P,F,TT)
C
C** SUBROUTINE FOR POWELL REGRESSION OF BATCH DATA FOR THE GAMMA FUNCTION MODEL
C
      DIMENSION FF(1),P(3),F(1),TT(1)
      DO 1 I=1,M
      XXX=(P(2)/(P(2)+TT(I)))**(P(1)+1.)*(1.-P(3))+P(3)
 1    F(I)=(FF(I)-XXX)
      RETURN
      END
```

```
// XEQ REGDL   FX
     BATCH RO FEED COPPER 8/8/74 NO DEXTRIN TO RC                    831  080811620
04007
10000406033602830254022002O3
00000500100015002000300040O0
     BATCH RO FEED NICKEL 8/8/74 NO DEXTRIN TO RC                    841  080811620
04007
10000663048503900358031402 98
00000500100015002000300040O0
     BATCH RO FEED SULF 8/8/74 NO DEXTRIN TO RC                      811  080811620
04007
10000643052703630284017901 48
00000500100015002000300040O0
     BATCH RO FEED GANGUE 8/8/74 NO DEX TO RC                        821  080811620
04007
10000982096809530940092009 8
00000500100015002000300040O0
    MINERAL BATCH RO FEED INFINITE NWTST                             811  010311625
04007
10000550051204070374036003 3
00000500100015002000300040005000
    MINERAL BATCH RO FEED INFINITE NWTST                             811  080311935
04007
10000574051204010351028702 60
00000500100015002000300040005000
    MINERAL BATCH RO FEED INFINITE NWTST                             81100140211805
04007
10000571049403540304023602 04
00000500100015002000300040O0
```

```
// FOR INT
*IOCS(CARD)
C
C** USED TO INITIATE THE TWO PARAMETER GRID SEARCH PROGRAM FOR THE REGRESSION OF
C   BATCH DATA FOR THE TWO PHASE MODEL
C** ILC - TEST CODE
C** AA - LIMITS OF THE SEARCH AREA ON PULP TO FROTH RATE PARAMETER
C** BB - LIMITS OF THE SEARCH AREA ON THE FROTH TO PULP RATE PARAMETER
C
      EXTERNAL SEAR2
      COMMON DATA(13),TIME(13),PRED(13),ISA(50),ID(7),IS(30),ILC(7),AA(2
     1),BB(2),F(13),NDATA,DB,DA,IWT(2),SG(13)
C**      ASK FOR DATA SPECIFICATIONS
C**      FITS TWO PARAMETER MODELS BY SEARCHING
C**      ALSO PLOTS SUM OF SQUARES SURFACE FOR THE SEARCH AREA
      READ (2,100) ILC,AA,BB,IWT
  100 FORMAT (7I5,4F8.2,2I5)
      READ (2,101)  (F(I),I=2,13)
  101 FORMAT (13F6.4)
      F(1)=0.
      DA=(AA(2)-AA(1))/50.
      DB=(BB(2)-BB(1))/50.
      CALL LINK (SEAR2)
      END
```

```
// FOR SEAR2
*IOCS(1443 PRINTER)
*IOCS(CARD)
C
C** MAIN PROGRAM FOR TWO PARAMETER GRID SEARCH
C** TIME - BATCH FLOTATION TIME
C** DATA - OBSERVED SPECIES TAILINGS CONCENTRATION
C** PRED - FLOTATION MODELS BEST FIT OF THE TAILINGS CONCENTRATION
C** F - CONTAINS FROTH FLOW/FROTH VOLUME (RECIPROCAL MINS)
C** IS,ID - TEST DESCRIPTION
C** SAVA,SAVB - BEST FIT PARAMETERS
C** S - SUM OF SQUARED DEVIATIONS
C
      EXTERNAL INT
      DIMENSION P(11)
      COMMON DATA(13),TIME(13),PRED(13),ISA(50),ID(7),IS(30),ILC(7),AA(2
     1),BB(2),F(13),NDATA,DB,DA,IWT(2),SG(13)
      DATA P/0.,.0005,.001,.005,.01,.05,.1,.5,1.,5.,1000./
      READ (2,110) IS,ID,NDATA
 110  FORMAT (30A2,6X,3I1,I2,I4,I1,I4/2X,I3)
      READ (2,111) (DATA(I),I=1,NDATA)
 111  FORMAT (20F4.3)
      READ (2,112) (TIME(I),I=1,NDATA)
 112  FORMAT (20F4.2)
 12   WRITE (3,101)
 101  FORMAT (1H1)
      W=10.**10.
      DO 1 I=1,50
      A=AA(2)-DA*I
      DO 11 J=1,50
      B=BB(1)+DB*J
      ISA(J)=9
C
C** CALL TO TWO PHASE MODEL SUBROUTINE TO GENERATE PREDICTED SPECIES
C   CONCENTRATIONS
C
      CALL MODSQ (A,B,S)
      DO 3 K=1,10
      MK=K+1
      XX=(P(MK)+P(K))/2.
```

```
      IF(ABS(S-XX)-(P(MK)-P(K))/2.) 4,4,3
 4    ISA(J)=K-1
 3    CONTINUE
      IF(W-S) 11,11,2
 2    W=S
      SAVA=A
      SAVB=B
 11   CONTINUE
 1    WRITE (3,102) ISA
 102  FORMAT (8X,50I2)
      CALL MODSQ (SAVA,SAVB,S)
C
C** OUTPUT SECTION
C
      WRITE (3,103) IS,ID,SAVA,SAVB,S,AA,BB
 103  FORMAT (//1X,30A2,7I5/' PARAMETERS =  ',2E12.4/' SUM OF SQUARES =    '
     1   ',E12.4/' SEARCH AREA =  ',2E12.4,5X,2E12.4)
      WRITE (3,1000) (TIME(I),DATA(I),PRED(I),F(I),SG(I),I=1,NDATA)
 1000 FORMAT (5E12.4)
 7    CONTINUE
      CALL LINK (INT)
      END
```

```
// XEQ INT     FX
    8    1    1    0 1302    1 1915 0.     100.    0.      100.       1
.104  .0801 .0697 .0714 .0637 .0645
   MINERAL BATCH RO FEED 100/5 NWTST                              81100130211915
04007
10000762063205760538047704 31
00000500100015002000300040 00
    8    1    1    0 1402    1 1805 0.     100.       0.   100.       1    0
 .279  .261  .320  .233  .334  .280
   MINERAL BATCH RO FEED INFINITE NWTST                           81100140211805
04007
10000571049403540304023602 04
00000500100015002000300040 00
    3    1    1    0 1402    1 1930  0.    100.    0.      100.       1    0
 .534  .443  1.01  1.13
   MINERAL BATCH RO CONC INFINITE NWTST                           31100140211930
04005
10000220010400790069
00000500100015002000
    3    1    1    0 1102    1 2145 0.     100.     0.      100.       1
.0162 .0185 .0151 .0175 .0217 .0245
   MINERAL BATCH RO CONC 100/5 NWTST                              31100110212145
04007
10000775057604190319019601 11
00000500100015002000300040 00
```

```
// FOR GCONR                          GAMMA CONT REGRESSION
*IOCS(TYPEWRITER)
*IOCS(CARD)
C
C** PROGRAM TO ESTIMATE THE GAMMA FUNCTION MODEL RATE PARAMETERS FROM
C   CONTINUOUS TEST DATA
C** T - MEAN CELL RETENTION TIMES * CUMULATIVE
C** Y - OBSERVED SPECIES CONCENTRATIONS
C** PRED - PREDICTED SPECIES CONCENTRATIONS
C** IS,ID - TEST IDENTIFICATION ARRAYS
C** P - BEST ESTIMATED RATE DISTRIBUTION PARAMETERS
C
      DIMENSION T(21),Y(21),PRED(21),ID(7),IS(30),P(4),E(2),ZZZ(100)
      DATA E/.01,.05/
 98   CALL BREAD (T,Y,IS,ID,NDATA)
C
C** ESTIMATE FIRST PARAMETER VALUES FROM BATCH REGRESSION OF CONTINUOUS DATA
C
      W=10.**10.
      DEL=10.
      P(2)=-9.99
      DO 300 L=1,4
      DO 400 K=1,10
      P(2)=P(2)+DEL
      X=0.
      XX=0.
      DO 2 J=2,NDATA
      X=X+ALOG(Y(J))*ALOG(P(2)/(P(2)+T(J)))
 2    XX=XX+(ALOG(P(2)/(P(2)+T(J))))**2.
      P(1)=X/XX-1.0
      CALL DIST1(PRED,NDATA,T,P)
      S=0.
      DO 3 J=1,NDATA
 3    S=S+(Y(J)-PRED(J))**2.
      IF(S-W) 401,402,402
401   X1=W
      W=S
      A=P(1)
      B=P(2)
      GO TO 400
```

```
 402  P(2)=P(2)-2.*DEL
      IF(P(2)) 404,405,405
 404  P(2)=.01
 405  DEL=DEL/5.
      W=X1
      GO TO 300
 400  CONTINUE
 300  CONTINUE
      P(1)=A
      P(2)=B
C
C** INITIAL PARAMETER ESTIMATE FOR POWELL SEARCH COMPLETED
C
      ES=10.
      P(3)=(P(1)+1.)/P(2)
      P(4)=P(3)/P(2)
      NNM=2+3*(NDATA+2*3/2)
C
C** START POWELL SEARCH
C
      CALL VA02A (NDATA,2,PRED,P,E,ES,-5,200,ZZZ,NNM,Y,T)
C
C** OUTPUT DATA AND MODEL FIT
C
      WRITE (1,100) IS,ID,P
 100  FORMAT (//30A2,2X,7I5/4E16.6/)
      CALL MODSQ (Y,NDATA,100,P,PRED,T)
      XX=0.
      DO 1000 I=1,NDATA
      XX=XX+(Y(I)-PRED(I))**2.
      WRITE (1,101) Y(I),T(I),PRED(I),XX
 1000 CONTINUE
 101  FORMAT (4E16.6)
      GO TO 98
 99   CALL EXIT
      END
```

```
// FOR MODSQ                          GAMMA CONT REGRESSION
      SUBROUTINE MODSQ (Y,NDATA,N,P,PRED,T)
C
C** SUBROUTINE VARIATION FOR CONTINUOUS FLOTATION DATA * GENERATES THEORETICAL
C   VALUES FOR USE BY VA02A
C** USES SUBROUTINE KROPH TO GENERATE ROOTS AND WEIGHTS FOR A GENERALIZED GAUSS
C   LAGUERRE QUADRATURE INTEGRATION OF THE FORM DESCRIBED IN PAGES 139-140
C
C** Y - OBSERVED NORMALIZED SPECIES CONCENTRATIONS
C** NDATA - NUMBER OF OBSERVATIONS
C** N - NUMBER OF VARIABLES I. E. 2 EXCEPT IN A SPECIAL CASE WHERE PRED ARRAY
C   CONTAINS PREDICTED SPECIES CONCENTRATIONS (N=100)
C** P - PARAMETER VALUES OF THE RATE CONSTANT DISTRIBUTION, P(1) IS MEAN**2/
C   VARIANCE -1, P(2) IS MEAN/VARIANCE
C** PRED - CONTAINS THE DIFFERENCE BETWEEN OBSERVED AND PREDICTED SPECIES
C   CONCENTRATIONS EXCEPT AS ABOVE UNDER N
C** T - CONTAINS MEAN FLOTATION TIMES CORRESPONDING TO Y
C
      DIMENSION Y(1),PRED(1),T(2),P(4),RTZ(15),W(15)
      X=P(1)
      D1=RCGAM(X+1.000)
      CALL KROPH(X,D1,RTZ,W,15)
      DO 300 I=1,NDATA
      ZED=I-1
      PRED(I)=0.
      DO 301 K=1,15
  301 PRED(I)=PRED(I)+W(K)/D1/(1.+RTZ(K)*T(2)/P(2))**ZED
      IF(N-100) 290,300,290
  290 PRED(I)=Y(I)-PRED(I)
  300 CONTINUE
      RETURN
      END
```

```
// XEQ GCONR   FX
    MINERAL PLANT RUN 34 CLEANER 22/6/72                                    41134220621215
04021
10000889075606860641061305980577055005390519050904960486047104590449043604260414
0403
   0  64 129 193 258 323 387 452 517 581 646 711 775 840 905 9691034109911631228
1293
    MINERAL PLANT CL FEED 37                                                41137130722115
040211010004007000l0
10000850067105930514048204330382033003100283027302600247022902140200019001780167
0155
   0  72 144 217 290 363 435 508 581 654 726 799 872 945101710901163123613081381
1454
    PLANT RUN 40 9/8/72 MINERAL                                             41140090822000
04021
10000781069806240588055005180417036903470314030302860251022101930171015301400126
0114
   0  79 159 239 319 399 479 558 638 719 799 878 958103811181198127813571438151 8
1598
    GANGUE PLANT TEST CL FEED 34                                            42134220621215
0402110200110300001 0
10000990097409620954094509410933092309160907090008920884087508670860084908380825
0813
   0  64 129 193 258 323 387 452 517 581 646 711 775 840 905 9691034109911631228
1293
```

```
// FOR GRIDS                    (N,M,FLIM,P,DATA,NPT,SRTH,ABCIS)
      SUBROUTINE GRIDS (N,M,FLIM,P,DATA,NPT,SRTH,ABCIS,W,PRED)
C
C** SUBROUTINE TO PERFORM A TWO-PARAMETER GRID SEARCH
C** N - NO. OF EQUAL SPACED SEARCH POINTS FOR P(1)
C** M -     '                '          '       '  P(2)
C** FLIM - CONTAINS PARAMETER SEARCH LIMITS
C** P - ON RETURN CONTAINS BEST PARAMETER ESTIMATES
C** DATA - AN ARRAY OF OBSERVED ORDINATES
C** NPT - NUMBER OF OBSERVED DATA POINTS
C** SRTH - NAME OF SUBROUTINE TO GENERATE PREDICTED ORDINATES FROM ABCISSA AND
C    PARAMETERS
C** ABCIS - OBSERVED ABCISSA VALUES
C** W - MINIMUM SUM OF SQUARES FOUND
C** PRED - AN ARRAY OF PREDICTED ORDINATES
C
      DIMENSION FLIM(4),P(3),DATA(13),PRED(13),ABCIS(13)
      DA=(FLIM(2)-FLIM(1))/FLOAT(N)
      DB=(FLIM(4)-FLIM(3))/FLOAT(M)
      W=10.**10.
      NN=N+1
      MM=M+1
      DO 1 I=1,NN
      P(1)=DA*(I-1)+FLIM(1)
      DO 1 J=1,MM
      P(2)=DB*(J-1)+FLIM(3)
      CALL SRTH (PRED,NPT,ABCIS,P)
      S=0.
      DO 2 K=1,NPT
2     S=S+(PRED(K)-DATA(K))**2.
      IF(S-W) 3,1,1
3     A=P(1)
      B=P(2)
      W=S
1     CONTINUE
      P(1)=A
      P(2)=B
      RETURN
      END
```

```
// FOR VA02A                                POWELL SEARCH TECHNIQUES
      SUBROUTINE  VA02A(M,N,F,X,E,STEP,LIST,MXITN,W,NNM,F9,TT)
C
C** SUBROUTINE TO IMPLEMENT POWELL SEARCH TECHNIQUES.  USES VA01A AND USER
C   SUPPLIED MODSQ.
C** M IS THE NUMBER OF DATA POINTS
C** N - IS THE NUMBER OF PARAMETERS
C** F - IS AN ARRAY OF M LENGTH CONTAINING THE SQUARE OF THE ERROR AT EACH POINT
C** X - IS AN ARRAY OF N LENGTH CONTAINING PARAMETER VALUES
C** E - IS AN ARRAY OF PARAMETER TOLERANCES AS EXIT CRITERION
C** STEP - IS A FACTOR WHICH WHEN MULTIPLIED BY E GIVES THE MAXIMUM SINGLE MOVE
C   MADE IN THE X PARAMETER VALUE
C** LIST - IF ZERO NO PRINTING IS DONE.  IF LIST GREATER THAN ZERO, DETAILS
C   OF THE SEARCH ARE PRINTED EVERY LIST ITERATIONS.  IF LIST IS LESS THAN ZERO
C   PRINTING IS THE SAME AS CASE TWO EXCEPT FUNCTIONAL VALUES ARE NOT PRINTED.
C** MXITN - MAXIMUM NUMBER OF ITERATIONS BEFORE EXIT
C** W - AN ARRAY OF AT LEAST NNM REAL WORDS
C** NNM - 2+3*(M+2*3/2)
C** F9 - AN ARRAY OF M LENGHT CONTAINING OBSERVED DATA POINTS * Y OR ORDINATES
C** TT - AN ARRAY OF M LENGTH HOLDING X OR ABCISSA VALUES FOR F9
C
      DIMENSION X(1),E(1),W(1),F(1),F9(1),TT(1)
      LUNO=LUNO
      MANDN=M+N
      KST=N+MANDN
      NPLUS=N+1
      KINV=NPLUS*(MANDN+1)
      KSTO=KINV-MANDN-1
C
C** USER SUPPLIED
C
      CALL MODSQ (F9,M,N,X,F,TT)
      NN=N+N
      K=NN
      DO1 I=1,M
      K=K+1
      W(K)=F(I)
    1 CONTINUE
      IINV=2
      K=KST
```

```
      I=1
    2 X(I)=X(I)+E(I)
C
C** USER SUPPLIED
C
      CALL MODSQ (F9,M,N,X,F,TT)
      X(I)=X(I)-E(I)
      DO 3 J=1,N
      K=K+1
      W(K)=0.
      W(J)=0.
    3 CONTINUE
      SUM=0.
      KK=NN
      DO 4 J=1,M
      KK=KK+1
      F(J)=F(J)-W(KK)
      SUM=SUM+F(J)*F(J)
    4 CONTINUE
      IF (SUM) 5,5,6
5     CONTINUE
C                                                          I )7,3( ETIRW
                                                                      C
      WRITE (1,7) I
    7 FORMAT (5X,8HVA02A E(,I3,20H) UNREASONABLY SMALL)
      DO 8 J=1,M
      NN=NN+1
      F(J)=W(NN)
    8 CONTINUE
      GO TO 10
    6 SUM=1./SQRT(SUM)
      J=K-N+I
      W(J)=E(I)*SUM
      DO 9 J=1,M
      K=K+1
      W(K)=F(J)*SUM
      KK=NN+J
      DO 11 II=1,I
      KK=KK+MANDN
      W(II)=W(II)+W(KK)*W(K)
   11 CONTINUE
```

```
 9 CONTINUE
   ILESS=I-1
   IGAMA=N+I-1
   ICINV=N-ILESS
   ICINP=ICINV+1
   IF (ILESS) 13,13,14
13 W(KINV)=1.
   GO TO 15
14 B=1.
   DO 16 J=NPLUS,IGAMA
   W(J)=0.
16 CONTINUE
   KK=KINV
   DO 17 II=1,ILESS
   IIP=II+N
   W(IIP)=W(IIP)+W(KK)*W(II)
   JL=II+1
   IF (JL-ILESS) 18,18,19
18 DO 20 JJ=JL,ILESS
   KK=KK+1
   JJP=JJ+N
   W(IIP)=W(IIP)+W(KK)*W(JJ)
   W(JJP)=W(JJP)+W(KK)*W(II)
20 CONTINUE
19 B=B-W(II)*W(IIP)
   KK=KK+ICINP
17 CONTINUE
   B=1./B
   DO 21 II=NPLUS,IGAMA
   BB=-B*W(II)
   DO 22 JJ=II,IGAMA
   W(KK)=W(KK)-BB*W(JJ)
   KK=KK+1
22 CONTINUE
   W(KK)=BB
   KK=KK+ICINV
21 CONTINUE
   W(KK)=B
15 GO TO (27,24),IINV
24 I=I+1
```

```
      IF (I-N) 2,2,25
   25 IINV=1
      FF=0.
      KL=NN
      DO 26 I=1,M
      KL=KL+1
      F(I)=W(KL)
      FF=FF+F(I)*F(I)
   26 CONTINUE
      ICONT=1
      ISS=1
      MC=N+1
      IPP=IABS(LIST)*(IABS(LIST)-1)
      ITC=0
      IPS=1
      IPC=0
27    IPC=IPC-IABS(LIST)
      IF (IPC) 28,29,29
28    CONTINUE
C                                                          FF,CM,CTI )03,3( ETIRW
      WRITE (1,30) ITC,MC,FF                                                    C
   30 FORMAT (//5X,9HITERATION,I4,I9,16H CALLS OF CALCUN,5X,2HF=,E24.14)
C                                                         )N,1=I,)I(X( )13,3( ETIRW
      WRITE (1,31) (X(I),I=1,N)                                                 C
   31 FORMAT (5X,9HVARIABLES,/(5E24.14))
      IF(LIST)1000,1001,1001
1001  CONTINUE
C                                                         )M,1=I,)I(F( )23,3( ETIRW
      WRITE (1,32) (F(I),I=1,M)                                                 C
   32 FORMAT (5X,9HFUNCTIONS,/(5E24.14))
 1000 IPC=IPP
      GO TO (29,33),IPS
   29 GO TO (34,35),ICONT
35    IF(CHANJ-1.)10,10,36
10    IF(LIST)1003,33,37
1003  CONTINUE
C                                                                 )2001,3( ETIRW
      WRITE (1,1002)                                                            C
 1002 FORMAT(//5X,31HVA02A FINAL VALUES OF VARIABLES)
      GO TO 1004
```

```
37    CONTINUE
C                                                                    )83,3( ETIRW
      WRITE (1,38)                                                                C
   38 FORMAT (//5X,45HVA02A FINAL VALUES OF FUNCTIONS AND VARIABLES)
 1004 IPS=2
      GO TO 28
33    RETURN
   36 ICONT=1
   34 ITC=ITC+1
      K=N
      KK=KST
      DO 39 I=1,N
      K=K+1
      W(K)=0.
      KK=KK+N
      W(I)=0.
      DO 40 J=1,M
      KK=KK+1
      W(I)=W(I)+W(KK)*F(J)
   40 CONTINUE
   39 CONTINUE
      DM=0.
      K=KINV
      DO 41 II=1,N
      IIP=II+N
      W(IIP)=W(IIP)+W(K)*W(II)
      JL=II+1
      IF (JL-N) 42,42,43
   42 DO 44 JJ=JL,N
      JJP=JJ+N
      K=K+1
      W(IIP)=W(IIP)+W(K)*W(JJ)
      W(JJP)=W(JJP)+W(K)*W(II)
   44 CONTINUE
      K=K+1
   43 IF (DM-ABS(W(II)*W(IIP))) 45,41,41
   45 DM=ABS(W(II)*W(IIP))
      KL=II
   41 CONTINUE
      II=N+MANDN*KL
```

```
      CHANJ=0.
      DO 46 I=1,N
      JL=N+I
      W(I)=0.
      DO 47 J=NPLUS,NN
      JL=JL+MANDN
      W(I)=W(I)+W(J)*W(JL)
   47 CONTINUE
      II=II+1
      W(II)=W(JL)
      W(JL)=X(I)
      IF(ABS(E(I)*CHANJ)-ABS(W(I)))48,48,46
48    CHANJ=ABS(W(I)/E(I))
   46 CONTINUE
      DO 49 I=1,M
      II=II+1
      JL=JL+1
      W(II)=W(JL)
      W(JL)=F(I)
   49 CONTINUE
      FC=FF
      ACC=0.1/CHANJ
      IT=3
      XC=0.
      XL=0.
      IS=3
      XSTEP=-AMIN1(0.5,STEP/CHANJ)                                    ATTACHED
      IF(CHANJ-1.)50,50,51
   50 ICONT=2
51    CALL VD01A(IT,XC,FC,6,ACC,0.1,XSTEP)                            ATTACHED
      GO TO (52,53,53,53),IT
   52 MC=MC+1
      IF(MC-MXITN)54,54,55
55    CONTINUE
C                                                    NTIXM )65,3( ETIRW
                                                                      C
      WRITE (1,56) MXITN
   56 FORMAT(5X,5HVA02A,I6,17H  CALLS OF CALFUN)
      ISS=2
      GO TO 53
   54 XL=XC-XL
```

```
      DO 57 J=1,N
      X(J)=X(J)+XL*W(J)
   57 CONTINUE
      XL=XC
C
C** USER SUPPLIED
C
      CALL MODSQ (F9,M,N,X,F,TT)
      FC=0.
      DO 58 J=1,M
      FC=FC+F(J)*F(J)
   58 CONTINUE
      GO TO (59,59,60),IS
   60 K=N
      IF (FC-FF) 61,51,62
   61 IS=2
      FMIN=FC
      FSEC=FF
      GO TO 63
   62 IS=1
      FMIN=FF
      FSEC=FC
      GO TO 63
   59 IF (FC-FSEC) 64,51,51
64    K=KSTO
      GO TO (75,74),IS
   75 K=N
   74 IF (FC-FMIN) 65,51,66
   66 FSEC=FC
      GO TO 63
   65 IS=3-IS
      FSEC=FMIN
      FMIN=FC
   63 DO 67 J=1,N
      K=K+1
      W(K)=X(J)
   67 CONTINUE
      DO 68 J=1,M
      K=K+1
      W(K)=F(J)
```

```
   68 CONTINUE
      GO TO 51
53    K=KSTO
      KK=N
      GO TO (69,70,69),IS
   70 K=N
      KK=KSTO
   69 SUM=0.
      DM=0.
      JJ=KSTO
      DO 71 J=1,N
      K=K+1
      KK=KK+1
      JJ=JJ+1
      X(J)=W(K)
      W(JJ)=W(K)-W(KK)
   71 CONTINUE
      DO 72 J=1,M
      K=K+1
      KK=KK+1
      JJ=JJ+1
      F(J)=W(K)
      W(JJ)=W(K)-W(KK)
      SUM=SUM+W(JJ)*W(JJ)
      DM=DM+F(J)*W(JJ)
   72 CONTINUE
      GOTO(73,10),ISS
73    J=KINV
      KK=NPLUS-KL
      DO 76 I=1,KL
      K=J+KL-I
      J=K+KK
      W(I)=W(K)
      W(K)=W(J-1)
   76 CONTINUE
      IF (KL-N) 77,78,78
   77 KL=KL+1
      JJ=K
      DO 79 I=KL,N
      K=K+1
```

```
      J=J+NPLUS-I
      W(I)=W(K)
      W(K)=W(J-1)
   79 CONTINUE
      W(JJ)=W(K)
      B=1./W(KL-1)
      W(KL-1)=W(N)
      GO TO 88
   78 B=1./W(N)
   88 K=KINV
      DO 80 I=1,ILESS
      BB=B*W(I)
      DO 81 J=I,ILESS
      W(K)=W(K)-BB*W(J)
      K=K+1
   81 CONTINUE
      K=K+1
   80 CONTINUE
      IF (FMIN-FF) 82,83,83
83    CHANJ=0.
      GO TO 84
   82 FF=FMIN
      CHANJ=ABS(XC)*CHANJ
   84 XL=-DM/FMIN
      SUM=1./SQRT(SUM+DM*XL)
      K=KSTO
      DO 85 I=1,N
      K=K+1
      W(K)=SUM*W(K)
      W(I)=0.
   85 CONTINUE
      DO 86 I=1,M
      K=K+1
      W(K)=SUM*(W(K)+XL*F(I))
      KK=NN+I
      DO 87 J=1,N
      KK=KK+MANDN
      W(J)=W(J)+W(KK)*W(K)
   87 CONTINUE
   86 CONTINUE
```

```
GO TO 14
END
```

```
// FOR VD01A      (ITEST,X,F,MXITN,ABSAC,RELAC,XSTEP)
      SUBROUTINE VD01A(ITEST,X,F,MXITN,ABSAC,RELAC,XSTEP)
C**
C** USED BY VA02A * NOT USER SPECIFIC

      GO TO (1,2,2),ITEST
    2 IS=6-ITEST
      ITEST=1
      IINC=1
      XINC=XSTEP+XSTEP
      MC=IS-3
      IF(MC)4,4,15
    3 MC=MC+1
      IF(MXITN-MC)12,15,15
   12 ITEST=4
   43 X=DB
      F=FB
      IF(FB-FC)15,15,44
   44 X=DC
      F=FC
   15 RETURN
    1 GO TO (5,6,7,8),IS
    8 IS=3
    4 DC=X
      FC=F
      X=X+XSTEP
      GO TO 3
    7 IF(FC-F)9,10,11
   10 X=X+XINC
      XINC=XINC+XINC
      GO TO 3
    9 DB=X
      FB=F
      XINC=-XINC
      GO TO 13
   11 DB=DC
      FB=FC
      DC=X
      FC=F
   13 X=DC+DC-DB
```

```
      IS=2
      GO TO 3
    6 DA=DB
      DB=DC
      FA=FB
      FB=FC
   32 DC=X
      FC=F
      GO TO 14
    5 IF(FB-FC)16,17,17
   17 IF(F-FB)18,32,32
   18 FA=FB
      DA=DB
   19 FB=F
      DB=X
      GO TO 14
   16 IF(FA-FC)21,21,20
   20 XINC=FA
      FA=FC
      FC=XINC
      XINC=DA
      DA=DC
      DC=XINC
   21 XINC=DC
      IF((D-DB)*(D-DC))32,22,22
   22 IF(F-FA)23,24,24
   23 FC=FB
      DC=DB
      GO TO 19
   24 FA=F
      DA=X
   14 IF(FB-FC)25,25,29
   25 IINC=2
      XINC=DC
      IF(FB-FC)29,45,29
   29 D=(FA-FB)/(DA-DB)-(FA-FC)/(DA-DC)
      IF(D*(DB-DC))33,33,37
   37 D=0.5*(DB+DC-(FB-FC)/D)
      IF(ABS(D-X) - ABS(ABSAC))34,34,35
35    IF(ABS(D-X)- ABS(D*RELAC)) 34,34,36
```

```
34 ITEST=2
   GO TO 43
36 IS=1
   X=D
   IF((DA-DC)*(DC-D))3,26,38
38 IS=2
   GO TO (39,40),IINC
39 IF(ABS(XINC)-ABS(DC-D))41,3,3
33 IS=2
   GO TO (41,42),IINC
41 X=DC
   GO TO 10
40 IF(ABS(XINC-X)-ABS(X-DC))42,42,3
42 X=0.5*(XINC+DC)
   IF((XINC-X)*(X-DC))26,26,3
45 X=0.5*(DB+DC)
   IF((DB-X)*(X-DC))26,26,3
26 ITEST=3
   GO TO 43
   END
```

```
// FOR AMIN1      F(A,B)
      FUNCTION AMIN1(A,B)
C
C** COMPARES NUMBERS A AND B, MINIMUM NUMBER IS PASSED TO LEFT HAND SIDE
C
      IF(A-B)20,20,10
10    AMIN1=B
      GOTO30
20    AMIN1=A
30    RETURN
      END
```

```
// FOR DIST
      SUBROUTINE DIST (PRED,N,T,P)
C
C** SUBROUTINE TO GIVE GAMMA FUNCTION MODEL BATCH CONCENTRATIONS FROM PARAMETERS
C    AND TIME ARRAY * USED BY REGDL
C
      DIMENSION PRED(1),T(1),P(3)
      DO 1 I=1,N
      PRED(I)=(P(2)/(P(2)+T(I)))**(P(1)+1.)
      PRED(I)=PRED(I)*(1.-P(3))+P(3)                                    NF
    1 CONTINUE
      RETURN
      END
```

```
// FOR DIST1
      SUBROUTINE DIST1(PRED,N,T,P)
C
C** SUBROUTINE TO CALCULATE THE GAMMA FUNCTION MODEL BATCH CURVE GIVEN FLOAT
C   TIMES AND PARAMETERS
C** PRED - AN ARRAY OF LENGTH N CONTAINING CALCULATED NORMALIZED CONCENTRATIONS
C** N - NUMBER OF FLOAT TIMES TO BE EVALUATED
C** T - AN ARRAY OF LENGTH N CONTAINING THE BATCH FLOTATION TIMES
C** P - AN ARRAY OF RATE DISTRIBUTION PARAMETERS
C   P(1) IS MEAN**2/VARIANCE-1
C   P(2) IS MEAN/VARIANCE
C   P(3) IS THE NORMALIZED FRACTION OF NONFLOATING MATERIAL
C
      DIMENSION PRED(1),T(1),P(3)
      DO 1 I=1,N
      PRED(I)=(P(2)/(P(2)+T(I)))**(P(1)+1.)
      PRED(I)=PRED(I)*(1.-P(3))+P(3)                                    NFL    C
 1    CONTINUE
      RETURN
      END
```

```
// FOR BCONT                    ALPHA CN CONTINUIOS PREDICTS
*IOCS(CARD)
*IOCS(1443 PRINTER)
C
C** PROGRAM TO PREDICT CONTINUOUS BANK FLOTATION GIVEN BATCH TEST PARAMETERS FOR
C   THE RATE PLUS NONFLOATING MODEL AND THE SINGLE RATE MODEL
C** T - CONTAINS OBSERVED MEAN RETENTION TIMES FOR THE FLOTATION BANK
C** Y - CONTAINS ACTUAL OBSERVED SPECIES NORMALIZED CONCENTRATIONS FOR THE
C   FLOTATION BANK
C** P - CONTAINS PREDICTED SPECIES CONCENTRATIONS FOR THE FLOTATION BANK
C** A - CONTAINS FIRST ORDER RATE CONSTANT AND THE NONFLOATING FRACTION OF THE
C   SPECIES
C** IS,ID - TEST IDENTIFICATION (CONTINUOUS TEST)
C** S - PROGRESSIVE SUM OF SQUARES OF THE DEVIATIONS BETWEEN OBSERVED (Y) AND
C   PREDICTED (P)
C
      DIMENSION IS(30),ID(7)
      DIMENSION T(21),Y(21),P(21),A(2)
 2    CONTINUE
      CALL RDCRD (1,NNV,T)                                        DUMMY
      CALL RDCRD (2,NV,A)
      CALL BREAD (T,Y,IS,ID,NDATA)
 100  FORMAT (30A2/7I5)
      WRITE (3,100) IS,ID
      WRITE (3,101) A
      S=0.
      DO 1 I=1,NDATA
      ZED=I-1
      P(I)=(1.-A(2))/(1.+A(1)*T(2))**ZED+A(2)
      S=S+(Y(I)-P(I))**2.0
 1    WRITE (3,101) T(I),Y(I),P(I),S
 101  FORMAT (4E12.4)
      GO TO 2
 99   CALL EXIT
      END
```

```
// XEQ BCONT   FX
 -.04173    8.868
 .1052   .207
    ROUGHER A3 13/4/73 1435 MINERAL                                  811  130421435
04021
10000912076506160578056705580538051304430417040904010388038203760360035103400331
0321
   0  68 137 207 276 345 414 483 552 621 690 759 829 898 967103611051174124313121
1381
 -.9256     12.65
 .04409 .881
    ROUGHER A3 13/4/73 1435 GANGUE                                   821  130421435
04021
10000998099309870984098309820979097509710969096809670966096409640962096109580957
0956
   0  68 137 207 276 345 414 483 552 621 690 759 829 898 967103611051174124313121
1381
 -.5519     .3572
 .3358 .154
  ROUGHER BANK COPPER 13/4/72 1430                                   831  130421430
04021
10000695047903730354034603420331031702940282027702740268026202590250024502360232
0226
   0  68 137 207 276 345 414 483 552 621 690 759 829 898 967103611051174124313121
1381
 -.4263     3.593
 .1385 .271
  ROUGHER BANK NICKEL 13/4/72 1430                                   841  130421430
04021
10000934076306280597058205730557053705160506050305000495049204910486048204780476
0472
   0  68 137 207 276 345 414 483 552 621 690 759 829 898 967103611051174124313121
1381
 -.3763     3.654
 .1341 .236
    ROUGHER A3 23/4/73 1200 MINERAL                                  811  230421200
04021
```

```
// FOR GCONT                    GAMMA CONTINUOUS PREDICTS
*IOCS(TYPEWRITER)
*IOCS(CARD)
C
C** PROGRAM TO READ GAMMA FUNCTION MODEL BATCH PARAMETERS AND CONTINUOUS TEST
C   DATA.  BATCH PARAMETERS ARE USED TO PREDICT CONTINUOUS OPERATION.
C   SEE PAGES 71 AND 138
C** T - CONTAINS MEAN RETENTION TIMES FOR CONTINUOUS TEST BANK OF CELLS
C** Y - CONTAINS OBSERVED SPECIES CONCENTRATIONS AT TIME T.
C** IS,ID - CONTAINS CONTINUOUS TEST DESCRIPTION AND CODE
C** P - CONTAINS PREDICTED SPECIES CONCENTRATIONS
C** A - CONTAINS BATCH TEST REGRESSED PARAMETERS
C** RTZ,W - CONTAINS ROOTS AND WEIGHTS FOR GAUSS-LAGUERRE QUADRATURE INTEGRATION
C** SS - CONTAINS PROGRESSIVE SUM OF SQUARES OF THE DEVIATION BETWEEN Y AND P
C
      DIMENSION T(21),Y(21),ID(7),P(21),IS(30)
      DIMENSION RTZ(15),W(15)
      DIMENSION A(4)
      IOUT=3
 98   READ (2,200) A
      READ (2,200) A
      A1=A(1)
      D1=RCGAM(A(1)+1.)
      CALL KROPH (A1,D1,RTZ,W,15)                                    QUADD
 200  FORMAT (4F12.5)
      CALL BREAD (T,Y,IS,ID,NDATA)
      DO 300 I=1,21
      ZED=I-1
      P(I)=0.
      DO 301 K=1,15
 301  P(I)=P(I)+W(K)/D1/(1.+RTZ(K)*T(2)/A(2))**ZED
C                 DEZ**))2(A/)2(T*)K(ZTR+.1(/ID/)1(A**)K(ZTR*)K((W+)I(P=)I(P  103
 300  CONTINUE
      A(3)=(A(1)+1.)/A(2)
      A(4)=A(3)/A(2)
 99   WRITE (IOUT,100) IS,ID,A
      SS=0.
      DO 1 I=1,NDATA
      SS=SS+(Y(I)-P(I))**2.0
 1    WRITE (IOUT,102) T(I),Y(I),P(I),SS
```

```
      WRITE (IOUT,103) (RTZ(I),W(I),I=1,15)
103   FORMAT (/' ROOTS AND WEIGHTS USED'/15(2E20.9/))
      GO TO 98
100   FORMAT (/30A2/7I5/4E12.4//)
102   FORMAT (4E12.4)
101   CALL EXIT
      END
```

```
// XEQ GCONT   FX
 .1052   .207
 -.04173    8.868
    ROUGHER A3 13/4/73 1435 MINERAL                                    811  130421435
04021
10000912076506160578056705580538051304430417040904010388038203760360035103400331
0321
   0  68 137 207 276 345 414 483 552 621 690 759 829 898 967103611051174124313121
1381
 .04409 .881
 -.9256     12.65
    ROUGHER A3 13/4/73 1435 GANGUE                                     821  130421435
04021
10000998099309870984098309820979097509710969096809670966096409640962096109580957
0956
   0  68 137 207 276 345 414 483 552 621 690 759 829 898 967103611051174124313121
1381
 .3358 .154
 -.5519     .3572
  ROUGHER BANK COPPER 13/4/72 1430                                     831  130421430
04021
10000695047903730354034603420331031702940282027702740268026202590250024502360232
0226
   0  68 137 207 276 345 414 483 552 621 690 759 829 898 967103611051174124313121
1381
 .1385 .271
 -.4263     3.593
  ROUGHER BANK NICKEL 13/4/72 1430                                     841  130421430
04021
10000934076306280597058205730557053705160506050305000495049204910486048204780476
0472
   0  68 137 207 276 345 414 483 552 621 690 759 829 898 967103611051174124313121
1381
 .1341 .236
 -.3763     3.654
```

```
// FOR CLEAN                    ROUGHER BATCH TO CLEANER BATCH PREDICTIONS
*IOCS(TYPEWRITER)
*IOCS(CARD)
C
C** CALCULATION OF ROUGHER CONCENTRATE RATE CONSTANT DISTRIBUTION (GAMMA MODEL)
C   FROM THE RATE CONSTANT DISTRIBUTION OF THE FEED TO THE ROUGHER BANK AS
C   DEFINED BY A BATCH FLOTATION TEST.  SEE PAGE  136
C** IS,ID - TEST IDENTIFICATION (ROUGHER CONCENTRATE FLOAT)
C** A - CONTAINS ROUGHER BATCH PARAMETERS AS REGRESSED
C** AA,BB,BARK,SIGMA - CONTAIN CALCULATED ROUGHER CONCENTRATE PARAMETER
C** Y - CONTAINS ACTUAL BATCH TEST DATA ON ROUGHER CONCENTRATE - NORMALIZED
C   SPECIES CONCENTRATIONS
C** T - CONTAINS ROUGHER CONCENTRATE BATCH FLOTATION TIMES
C** P - CONTAINS SPECIES CONCENTRATIONS FOR THE ROUGHER CONCENTRATE BATCH FLOAT
C   WHICH ARE PREDICTED THROUGH THE REGRESSED ROUGHER FEED BATCH PARAMETERS
C** SUMSQ - SUM OF THE SQUARES OF THE DEVIATIONS BETWEEN Y AND P
C
      DIMENSION RTZ(25),W(25),Y(7),T(7),A(5),P(7),IS(30),ID(7),RA(3)          C
      DIMENSION F(25)                                                        C
 200  FORMAT (F12.5,F13.7,F11.5,F12.5,F12.2)
 201  FORMAT (30A2,6X,3I1,I2,I4,I1,I4/2X,I3)
 202  FORMAT (20F4.3)
 203  FORMAT (20F4.2)
C
C** READ IN ROUGHER FEED BATCH PARAMETERS
C
 98   READ (2,200) A
      NX=20
      READ (2,200) A
      READ (2,200) A
      A(3)=A(1)
      A(4)=A(2)
      A(1)=A(3)**2./A(4)-1.000
      A(2)=A(3)/A(4)
      IF(A(1)+1.) 99,99,97
 97   READ (2,201) IS,ID,NDATA
      READ (2,202) (Y(I),I=1,NDATA)
      READ (2,203) (T(I),I=1,NDATA)
C
C** CALCULATE MEAN RETENTION TIME IN THE CELLS OF THE CONTINUOUS ROUGHER BANK
```

```
C
      TT=4.*622./A(5)
      ALF=A(1)
      ALF=A(1)+1.0000                                                NEW KRPC
      DO 500 I=1,3
      RA(I)=0.
                                                                     NEW KRPC
      GO TO 1
C                                                          2,1,1 ).8-FLA(FI
 2    DO 3 K=1,15
      F(K)=0.
      DO 3 L=1,20
      ZED=L
 3    F(K)=F(K)+Z(K)**ALF*TT/(1.0+Z(K)*TT/A(2))**ZED
      DO 4 K=1,15
 4    RA(I)=RA(I)+WZ(K)*F(K)
      GO TO 6
 1    D1=RCGAM(ALF+1.)
      CALL KROPH (ALF,D1,RTZ,W,NX)
      DO 306 K=1,NX
      F(K)=0.
      DO 306 L=1,20
      ZED=L
C                              DEZ**))2(A/TT*)K(ZTR+.1(/TT*)K(ZTR+)K(F=)K(F  603
 306  F(K)=F(K)+       TT/(1.+RTZ(K)*TT/A(2))**ZED                   NEW KRPC
      DO 301 K=1,NX
 301  RA(I)=RA(I)+W(K)*F(K)
 6    ALF=ALF+1.
 500  CONTINUE
      RA(2)=RA(2)/A(2)
      RA(3)=RA(3)/A(2)**2.
      BARK=RA(2)/RA(1)
      SIGMA=(RA(3)-2.*BARK*RA(2)+BARK**2.*RA(1))/RA(1)
      AA=BARK**2./SIGMA-1.00000
      BB=BARK/SIGMA
      DO 501 I=1,NDATA
 501  P(I)=(BB/(BB+T(I)))**(AA+1.0)
C
C** OUTPUT SECTION
C
      WRITE (3,100) IS,ID,A,   AA,BB,BARK,SIGMA
```

```
      WRITE (3,101) (RA(I),I=1,3)
100   FORMAT (//30A2,5X,3I1,I2,I4,I1,I4/' ROUGHER ',5E20.9/' ROUGHER CON
     C ',4E20.9/)
      SUMSQ=0.
      DO 1001 I=1,NDATA
      SUMSQ=SUMSQ+(Y(I)-P(I))**2.
1001  WRITE (3,101) Y(I),T(I),P(I),SUMSQ
101   FORMAT (4E13.5)
      GO TO 98
99    CALL EXIT
      END
```

```
// XEQ CLEAN   FX
 .08672
 .08825      .004221                                3570.
 .09586      .0004376                               3570.
     BATCH RO CONC NO DEX 8/8/74 SULF                          311  080811645
04005
10000317019801480115
00000500100015002000300040000
  .02779
 .00409      .0001601                               3570.
 .05578      .000185                                3570.
     BATCH RO CONC NO DEX 8/8/74 GANGUE                        321  080811645
04005
10000746065306060564
00000500100015002000300040000
 .2722
 .8467       2.065                                  3570.
 .4336       .1347                                  3570.
     BATCH RO CONC NO DEX 8/8/74 COPPER                        331  080811620
04005
10000140009200730060
00000500100015002000300040000
 .1318
 .1486       .04277                                 3570.
 .1332       .0003526                               3570.
     BATCH RO CONC NO DEX 8/8/74 NICKEL                        341  080811620
04005
10000247014401130093
00000500100015002000300040000
 .1968
 .2468        .02787                                3840.
  .4024       .5815                                 3840.
    MINERAL BATCH RO CONC  INFINITE NWTST                      311  010311800
04005
10000247016101380126
00000500100015002000300040005000
```

```
// FOR ROCLF          RO TO CL FOR CF,CNF,ALPHA MODEL
*IOCS(CARD)
*IOCS(TYPEWRITER,KEYBOARD)
C
C** PROGRAM TO PREDICT ROUGHER CONCENTRATE FLOTATION FROM RATE PLUS NONFLOATING
C   PARAMETERS REGRESSED FROM A BATCH TEST ON THE ROUGHER FEED
C** IS,ID - TEST IDENTIFICATION (ROUGHER CONCENTRATE BATCH)
C** Y - OBSERVED NORMALIZED CONCENTRATIONS
C** T - OBSERVED FLOAT TIMES
C** P - PREDICTED CONCENTRATIONS
C** ALPHA - ROUGHER FEED FIRST ORDER RATE CONSTANT
C** S - SUM OF SQUARED DEVIATIONS
C
      DIMENSION Y(21),T(21),P(21),ID(7),IS(30)
 99   CONTINUE
      CALL RDCRD (1,NV,T)
      CALL RDCRD (1,NV,T)
      CALL RDCRD (1,NV,ALPHA)
      CALL BREAD (T,Y,IS,ID,NDATA)
      WRITE (3,2000) IS,ID,ALPHA
2000  FORMAT (30A2/7I5,F10.4)
      S=0.
      DO 1 I=1,NDATA
      P(I)=EXP(-ALPHA*T(I))
      S=S+(Y(I)-P(I))**2.0
1     WRITE (3,101) Y(I),T(I),P(I),S
101   FORMAT (4E12.4)
      GO TO 99
98    CALL EXIT
      END
```

```
// XEQ ROCLF   FX
 .08825      .004221                                3570.
 .09586      .0004376                               3570.
 .08672
     BATCH RO CONC NO DEX 8/8/74 SULF                          311  080811645
04005
10000317019801480115
00000500100015002000300040000
 .00409      .0001601                               3570.
 .05578      .000185                                3570.
  .02779
     BATCH RO CONC NO DEX 8/8/74 GANGUE                        321  080811645
04005
10000746065306060564
00000500100015002000300040000
 .8467       2.065                                  3570.
 .4336       .1347                                  3570.
 .2722
     BATCH RO CONC NO DEX 8/8/74 COPPER                        331  080811620
04005
10000140009200730060
00000500100015002000300040000
 .1486       .04277                                 3570.
 .1332       .0003526                               3570.
 .1318
     BATCH RO CONC NO DEX 8/8/74 NICKEL                        341  080811620
04005
10000247014401130093
00000500100015002000300040000
 .2468       .02787                                 3840.
  .4024      .5815                                  3840.
 .1968
    MINERAL BATCH RO CONC  INFINITE NWTST                      311  010311800
04005
10000247016101380126
000005001000150020003000400050000
```

```
// FOR ITIT                    H-R PRED FROM A,B
*IOCS(TYPEWRITER)
C
C** PROGRAM TO PREDICT BATCH FLOTATION THROUGH THE TWO PHASE MODEL GIVEN
C              Q - FROTH FLOW / FROTH VOLUME
C              T - FLOAT TIMES
C              A,B - FIRST ORDER FLOTATION AND FROTH RETURN RATE PARAMETERS
C** PRED - CONTAINS PREDICTED SPECIES CONCENTRATIONS AT TIMES T
C
      COMMON Y(13),T(13),PRED(13),ISA(50),ID(7),IS(30),ILC(7),AA(2),BB(2
      ),Q(13),NDATA,DB,DA,IWT(2),SG(13)
 23   CONTINUE
C
C** TYPEWRITER FREE FORMAT INPUT TO ARRAY Y
C
      CALL LARRY (13,NV,Y)
      IWT(1)=Y(1)
      IWT(2)=Y(2)
      A=Y(3)
      B=Y(4)
      NDATA=Y(5)
      Y(NDATA)=Y(6)
      CALL LARRY (13,NV,Q)
      CALL LARRY (13,NV,T)
      CALL MODSQ (A,B,S)
      WRITE (1,100) A,B,S
      WRITE (1,100) (T(I),PRED(I),Q(I),I=1,NDATA)
 100  FORMAT (3E12.4)
      GO TO 23
 99   CALL EXIT
      END
```

```
// FOR PRFWO
*IOCS(CARD)
C
C** DATA INPUT PROGRAM FOR SIMULATION OF TESTED CIRCUITS BY THE GAMMA FUNCTION
C   MODEL.  SEE PAGES 121-127
C** P - CONTAINS SPECIES PARAMETERS OF THE FRESH FEED AS ESTIMATED FROM OTHER
C   TESTING.
C** FG - CONTAINS SPECIES FRACTION IN FRESH FEED
C** FV - CONTAINS FRESH FEED VOLUME FLOW RATE (L/MIN)
C
      EXTERNAL PRFW1
      DIMENSION A(2),B(2),FG(16,2),TN(16,2),GA(16),R(16,2),P(3,2),FV(16)
      COMMON/INSKEL/MMM,A,B,FG,TN,GA,R,P,FV
      MMM=2
      MMM=16
      MMM=8
      MMM=3
      MMM=1
      DO 1 L=1,MMM
      READ (2,100) FG(L,1),FV(L)
      FG(L,1)=FG(L,1)/100.00
 1    FG(L,2)=1.000-FG(L,1)
C     UC                                                    )3.01F,X03,3.01F( TAMROF  001
 100  FORMAT (10X,F10.3,20X,F10.0)                                            SULF   C
C     IN                                                 )0.01F,X01,3.01F,X02( TAMROF  001
      READ (2,101) P
 101  FORMAT (6F13.9)
      CALL LINK (PRFW1)
      END
```

```
// FOR PRFW1                    CIRCUIT 57 AND 37 SIMULATIONS
      EXTERNAL PRFW2
C
C** PROGRAM TO PERFORM ACTUAL GAMMA FUNCTION MODEL PREDICTION OF TEST CIRCUIT
C   OPERATION.  SEE PAGES 121-127
C** SUBROUTINE IAIB GENERATES CIRCUIT CONFIGURATION  S/R KROPH GENERATES
C   QUADRATURE ROOTS AND WEIGHTS FOR INTEGRATION  S/R MINV5 PERFORMS INVERSION
C   OF THE CONFIGURATION MATRIX
C** HANDLES A MAXIMUM OF 16 FEED CONDITIONS TO THE SAME CIRCUIT
C
C** X - CONTAINS THE RATIO OF CONCENTRATE VOLUME FLOW TO TAILS VOLUME FLOW FOR
C   CALCULATION OF CELL MEAN RETENTION TIMES
C** RTZ,W - QUADRATURE ROOTS AND WEIGHTS FOR INTEGRATION
C** A,B - RATE DISTRIBUTION PARAMETERS
C** FG - SPECIES FRACTIONS IN FEED TO THE CIRCUIT
C** IA,IB - TAILS AND CONCENTRATE MATRICES RESPECTIVELY
C** RTG - PRODUCT OF THE TRANSPOSE OF THE R MATRIX AND THE G MATRIX.  SEE
C   PAGE 135
C** C - CONFIGURATION MATRIX OR ITS INVERSE
C** P - RATE DISTRIBUTION PARAMETERS AS MEAN, VARIANCE AND NONFLOATABLE FRACTION
C** FV - FRESH FEED VOLUME FLOWRATES (L/MIN)
C** V - EFFECTIVE VOLUMES (LITERS) OF FLOTATION UNITS
C** TAU - MEAN RETENTION TIME IN A UNIT
C** TN - MASS FLOW OF SPECIES IN FINAL CONCENTRATE (ASSUMING 100 UNITS OF FEED)
C** GA - SPECIES GRADE IN FINAL CONCENTRATE
C** R - SPECIES RECOVERY IN THE TEST CIRCUIT AS PREDICTED
C
      DIMENSION X(40),RTZ(15),W(15),A(2),B(2),FG(16,2)
      DIMENSION IA(40,40),IB(40,40),RTG(40),C(40,40),P(3,2),FV(16),V(40)
     ,TAU(40)
      DIMENSION TN(16,2),GA(16),R(16,2)
      COMMON/INSKEL/MMM,A,B,FG,TN,GA,R,P,FV,TAU,RTZ,W,X
      COMMON IA,IB
      DATA V /20*2488.,20*622./
      DATA NXX /15/
      DO 3000 K=1,40
      DO 3000 KK=1,40
      IB(K,KK)=0
3000  IA(K,KK)=0
      DO 8 M=1,2
```

```
      A(M)=P(1,M)**2./P(2,M)-1.00
      B(M)=P(1,M)/P(2,M)
      D1=RCGAM(A(M)+1.00)
      CALL KROPH (A(M),D1,RTZ,W,NXX)
      CALL IAIB (IA,IB,X)
      DO 4 L=1,MMM
      DO 50 I=1,40
      DO 50 J=1,40
50    C(I,J)=IA(I,J)+IB(I,J)*X(J)
      CALL MINV5 (C,40)
      DO 51 I=1,40
51    TAU(I)=V(I)/(-C(I,1)*FV(L))
      R(L,M)=0.
      DO 3 K=1,NXX
      DO 1 I=1,40
      RTG(I)=0.
      DO 2 J=1,40
2     RTG(I)=RTG(I)-IB(J,I)
1     RTG(I)=RTG(I)*RTZ(K)*TAU(I) /B(M)
      DO 6 I=1,40
      DO 6 J=1,40
6     C(I,J)=IA(I,J)+IB(I,J)*RTZ(K)*TAU(J)/B(M)
      CALL MINV5 (C,40)
      SUM=0.
      DO 7 I=1,40
7     SUM=SUM-C(I,1)*RTG(I)
      R(L,M)=R(L,M)+SUM*W(K)/D1
3     CONTINUE
      R(L,M)=(1.00-P(3,M))*R(L,M)
      TN(L,M)=FG(L,M)*100.*R(L,M)
4     CONTINUE
8     CONTINUE
      CALL LINK (PRFW2)
      END
```

```
// FOR PRFW2                OUTPUT FOR PRFW1
*IOCS(1443 PRINTER)
C
C** PROGRAM TO OUTPUT RESULTS FROM PRFW1
C** ZZ - RECOVERY OF VALUABLE COMPONENT
C** ZZZ - RECOVERY OF REMAINING MATERIAL (NONVALUABLE)
C** SS - RATIO OF RECOVERIES - VALUED/NONVALUED
C** Z11 - TAILS GRADE IN VALUES
C
      DIMENSION IA(40,40),IB(40,40),A(2),B(2),FG(16,2),TN(16,2),GA(16),R
      (16,2),P(3,2),FV(16),TAU(40),RTZ(15),W(15),X(40)
      COMMON/INSKEL/MMM,A,B,FG,TN,GA,R,P,FV,TAU,RTZ,W,X
      COMMON IA,IB
      DO 9 L=1,MMM
      GA(L)=TN(L,1)/(TN(L,1)+TN(L,2))*100.
      SS=R(L,1)/R(L,2)
      ZZ=R(L,1)
      ZZZ=R(L,2)
      Z11=(FG(L,1)*100.-TN(L,1))*100./(FG(L,2)*100.-TN(L,2))
 9    WRITE (3,101) ZZ,ZZZ,TN(L,1),TN(L,2),GA(L),SS,FV(L),Z11
 101  FORMAT (8E12.4)
      WRITE (3,103) ((IA(I,J),J=1,40),I=1,40),((IB(I,J),J=1,40),I=1,40)
 103  FORMAT (1H1,' TAILS MATRIX A'//40(40I2/),1H1,' CONCENTRATE MATRIX
      B'//40(1X,40I2/))
      CALL EXIT
      END
```

```
// FOR IAIB                          S/R FOR CIRCUIT 36
      SUBROUTINE IAIB (IA,IB,X)
C
C** USED TO GENERATE TAILS AND CONCENTRATE MATRICES FOR A ROUGHER,CLEANER AND
C   CLEANER MIDDLING CIRCUIT.  USED BY PRFW1.
C** SEE PAGES 121-123
C
      DIMENSION IA(40,40),IB(40,40),X(40)
      DO 1 I=1,39
      X(I)=.011
      K=I+1
      IA(K,I)=1
      IA(I,I)=-1
 1    IB(I,I)=-1
      IA(40,40)=-1
      IB(40,40)=-1
      IA(21,20)=0
      IA(1,40)=1
      DO 2 I=21,40
 2    X(I)=.025
      DO 3 I=1,20
 3    IB(21,I)=1
      DO 4 I=27,40
 4    IB(21,I)=1
      RETURN
      END
```

```
// FOR                          CIRCUIT 48   1ST RO 4 CL
      SUBROUTINE IAIB (IA,IB,X)
C
C** USED TO GENERATE TAILS AND CONCENTRATE MATRICES FOR A ROUGHER, SCAVENGER,
C   CLEANER, CLEANER MIDDLINGS CIRCUIT WHICH WAS TESTED.  USED BY PRFW1.
C** SEE PAGES 121-123
C
      DIMENSION IA(40,40),IB(40,40),X(40)
      DO 1 I=1,39
      K=I+1
      IA(K,I)=1
      IA(I,I)=-1
      IB(I,I)=-1
 1    X(I)=.011
      IA(40,40)=-1
      IB(40,40)=-1
      IA(21,20)=0
      IA(1,40)=1
      DO 4 I=21,40
 4    X(I)=.025
      DO 2 I=2,20
 2    IB(21,I)=1
      DO 3 I=25,40
 3    IB(21,I)=1
      RETURN
      END
```

```
// FOR                          CIRCUIT 53   NO CL MIDS
      SUBROUTINE IAIB (IA,IB,X)
C
C** GENERATES TAILS AND CONCENTRATE MATRICES FOR A ROUGHER-CLEANER CIRCUIT
C   WHICH WAS TESTED.  USED BY PRFW1
C** SEE PAGES 121-123
C
      DIMENSION IA(40,40),IB(40,40),X(40)
      DO 1 I=1,39
      K=I+1
      IA(K,I)=1
      IA(I,I)=-1
      IB(I,I)=-1
 1    X(I)=.011
      IA(40,40)=-1
      IB(40,40)=-1
      IA(21,20)=0
      IA(1,24)=1
      DO 2 I=21,24
 2    X(I)=.025
      DO 3 I=1,20
 3    IB(21,I)=1
      RETURN
      END
```

```
// FOR                          CIRCUIT 61   RO CL RECL
      SUBROUTINE IAIB (IA,IB,X)
C
C** GENERATES TAILS AND CONCENTRATE CONFIGURATION MATRICES FOR A ROUGHER,
C   CLEANER AND RECLEANER CIRCUIT WHICH WAS TESTED.  USED BY PRFW1
C** SEE PAGES 121-123
C
      DIMENSION IA(40,40),IB(40,40),X(40)
      DO 1 I=1,39
      K=I+1
      IA(K,I)=1
      IA(I,I)=-1
      IB(I,I)=-1
 1    X(I)=.011
      IA(40,40)=-1
      IB(40,40)=-1
      IA(21,20)=0
      IA(1,40)=1
      DO 4 I=21,40
 4    X(I)=.025
      DO 2 I=1,20
 2    IB(25,I)=1
      DO 3 I=27,40
 3    IB(21,I)=1
      RETURN
      END
```

```
// XEQ PRFWO   FX
 .4769          .466              0.              .002638        .0002154        0.
  .065          .38               .164            1527.          3438.
 .077           .61               .264            1474.          3485.
 .071           .45               .182            1505.          3458.
 .068           .42               .18             1503.          3590.
 .075           .44               .195            1563.          3455.
 .063           .40               .185            1402.          3197.
 .073           .43               .166            1462.          3314.
 .069           .42               .168            1482.          3381.
 .078           .48               .187            1438.          3288.
 .096           .41               .174            1455.          3325.
 .074           .46               .214            1476.          3413.
 .076           .56               .187            1339.          3064.
 .074           .43               .135            1458.          3333.
 .084           .46               .198            1414.          3288.
 .093           .45               .219            1394.          3195.
 .098           .52               .228            1395.          3245.
```

```
// FOR PRF22                    CIRCUIT SIMULATION FOR GENERAL 25 UNITS
C
C** SIMULATION OF VARIOUS GENERAL FLOTATION CIRCUITS BY THE GAMMA FUNCTION FIRST
C   ORDER MODEL.
C** THE METHOD USED IS OUTLINED IN PAGES 133-140
C** DD - CONTAINS TYPICAL PARAMETERS MEAN, VARIANCE AND NONFLOATING FRACTION FOR
C   A SPECIES. I.E. RATE DISTRIBUTION PARAMETERS.
C** BY SUITABLE ARRANGEMENT OF THE OVERTURNED CARDS THE PROGRAM MAY BE USED TO
C   EVALUATE A CIRCUIT AT FIVE TONNAGE RATES OR VARIATIONS OF THE SAME CIRCUIT
C   AT A FIXED TONNAGE, I.E. A ROUGHER-SCAVENGER CIRCUIT WITH 1,2,3,4, AND 5
C   CELLS AS FINAL CONCENTRATES.
C*************************************************************************
C** PROGRAM IS NOW SETUP SO AS TO PERFORM AN EVALUATION OF SEVERAL CIRCUIT
C  VARIATIONS AT A FIXED TONNAGE THROUGHPUT.
C*************************************************************************
C** X - CONTAINS THE RATIO OF CONCENTRATE VOLUME FLOW TO TAILS VOLUME FLOW FOR
C   EACH FLOTATION UNIT.
C** RTZ,W - CONTAINS THE ROOTS AND WEIGHTS FOR THE QUADRATURE FORMULA.
C** A,B - CONTAINS SPECIES DISTRIBUTION PARAMETERS IN THE FORM A=MEAN**2/VARIANC
C   E-1.  B=MEAN/VARIANCE
C** GA - CONTAINS THE CIRCUIT FINAL CONCENTRATE GRADES
C** R - RECOVERY OF THE TWO SPECIES
C** P - ALSO CONTAINS RATE CONSTANT DISTRIBUTION PARAMETERS
C** CONTAINS FEED GRADE OF THE SPECIES
C** IA,IB - TAILS AND CONCENTRATE MATRICES RESPECTIVELY
C** RTG - CONTAINS THE PRODUCT OF THE R MATRIX TRANSPOSED AND THE G MATRIX.  SEE
C   PAGE 135
C** C - CONTAINS THE CIRCUIT CONFIGURATION MATRIX OR ITS INVERSE
C** V,TAU - EFFECTIVE VOLUME OF EACH FLOTATION UNIT AND MEAN RETENTION TIME AS
C   CALCULATED FOR EACH UNIT.
C** - DUPLICATE P ARRAY
C
      EXTERNAL PRF23
      DIMENSION X(25),RTZ(15),W(15),A(2),B(2),GA(5),R(5,2),P(3,2),FG(2)
      DIMENSION IA(25,25),IB(25,25),RTG(25),C(25,25),FD(5),V(25),TAU(25)
      DIMENSION DD(3,2)
      COMMON MMM,X,RTZ,W,A,B,FD,GA,R,P,TAU,RTG,V,FG
      COMMON IA,IB,NO
      DATA NXX /15/
C     UC                                                    100.=)1(GF
```

```
C     UC                              /.0,4512000.,836200.,.0,664.,9674./ DD ATAD
      DATA DD/.08643,.01329,0.,.002612,.0002175,0./                        NI     C
      FG(1)=.002                                                           NI     C
      NO=25
      MMM=5
      MMM=1                                                                       C
      FG(2)=1.0000-FG(1)
C                                                          .01-I*.01+.08=)I(DF   18
C                                                                5,1=I 18 OD
      DO 82 I=1,25
 82   V(I)=2488.
      DO 83 I=1,3
      DO 83 J=1,2
 83   P(I,J)=DD(I,J)
      FD(1)=106.                                                                  C
      DO 8 L=1,2
      L11=L*5+14
      DO 3000 K=1,25
      DO 3000 KK=1,25
      IB(K,KK)=0
 3000 IA(K,KK)=0
C
C** CALL TO THE SUBROUTINE TO GENERATE CIRCUIT CONCENTRATE (IB) AND TAILS (IA)
C   MATRICES.
C
      CALL IAIB (IA,IB,X,L11)
      DO 8 M=1,2
      A(M)=P(1,M)**2./P(2,M)-1.00
      B(M)=P(1,M)/P(2,M)
      D1=RCGAM(A(M)+1.00)
C
C** CALL TO THE SUBROUTINE TO GENERATE THE ROOTS AND WEIGHTS NECESSARY FOR A
C   GAUSS-LAGUERRE QUADRATURE INTEGRATION FORMULA ON PAGE 140
C
      CALL KROPH (A(M),D1,RTZ,W,NXX)
C                                                        MMM,1=L 4 OD
      DO 50 I=1,25
      DO 50 J=1,25
 50   C(I,J)=IA(I,J)+IB(I,J)*X(J)
      CALL MINV5 (C,25)
```

```
      DO 51 I=1,NO
C                                   )766666.61*525./)L(DF*)1,I(C-(/)I(V=)I(UAT   15
C
C** CALCULATION OF EACH FLOTATION UNITS MEAN RETENTION TIME.
C
 51   TAU(I)=V(I)/(-C(I,1)*FD(1)/.525*16.66667)                                  C
      R(L,M)=0.
      DO 3 K=1,NXX
      DO 1 I=1,NO
      RTG(I)=0.
      DO 2 J=1,NO
 2    RTG(I)=RTG(I)-IB(J,I)
 1    RTG(I)=RTG(I)*RTZ(K)*TAU(I) /B(M)
      DO 6 I=1,NO
      DO 6 J=1,NO
 6    C(I,J)=IA(I,J)+IB(I,J)*RTZ(K)*TAU(J)/B(M)
      CALL MINV5 (C,25)
      SUM=0.
      DO 7 I=1,NO
 7    SUM=SUM-C(I,1)*RTG(I)
      R(L,M)=R(L,M)+SUM*W(K)/D1
 3    CONTINUE
      R(L,M)=(1.00-P(3,M))*R(L,M)
C                                                  )2,L(R*)L(DF*)2(GF=BT
C                                                  )1,L(R*)L(DF*)1(GF=AT
                                                                          C
      TA=FG(1)*FD(1)*R(L,1)                                               C
      TB=FG(2)*FD(1)*R(L,2)
      GA(L)=TA/(TA+TB)*100.
 4    CONTINUE
 8    CONTINUE
      CALL LINK (PRF23)
      END
```

```
// FOR PRF23                OUTPUT FOR PRF22
*IOCS(1443 PRINTER)
C
C** OUTPUT PROGRAM FOR GENERAL FLOTATION CIRCUIT SIMULATION PROGRAM PRF22.
C** ARRAY DESIGNATIONS THE SAME AS FOR PRF22.
C
      DIMENSION X(25),RTZ(15),W(15),A(2),B(2),FD(5),GA(5),R(5,2),P(3,2),
      TAU(25),RTG(25),V(25),FG(2)
      DIMENSION IA(25,25),IB(25,25)
      COMMON MMM,X,RTZ,W,A,B,FD,GA,R,P,TAU,RTG,V,FG
      COMMON IA,IB,NO
      MMM=5
      WRITE (3,100) P,A,B,FG,TAU
  100 FORMAT (1H1,' PARAMETERS  '/1X,10E11.4/' FEED GRADE '/2E12.2/' RES
       TIME '/3(10E11.4/)/)
      WRITE (3,101) (FD(L),R(L,1),R(L,2),GA(L),L=1,MMM)
  101 FORMAT (4E16.6)
      WRITE (3,103) ((IA(I,J),J=1,NO),I=1,NO),((IB(I,J),J=1,NO),I=1,NO)
  103 FORMAT   (//' TAILS MATRIX A'//25(1X,25I2/)/' CONCENTRATE MATRIX B
      '//25(1X,25I2/))
      CALL EXIT
      END
```

```
// FOR IAIB
      SUBROUTINE IAIB (IA,IB,X,N2)                                          C
C
C** IN THIS GENERAL CICUIT SIMULATION SUBROUTINE A MAXIMUM NUMBER OF TWENTY-FIVE
C   FLOTATION UNITS IS AVAILABLE, EACH CIRCUIT OR CIRCUIT VARIATION USES ALL OF
C   THE CELLS.
C
      DIMENSION IA(25,25),IB(25,25),X(25)
      DO 1 I=1,25
      IA(I,I)=-1
      IB(I,I)=-1
 1    X(I)=.011
C
C*************************************************************************
C
C** SUBROUTINE TO GENERATE CONCENTRATE (IB) AND TAILINGS (IA) MATRICES FOR A
C   GENERAL ROUGHER SCAVENGER CIRCUIT.  SEE PAGES 116-119
C** X - CONTAINS THE RATIO OF CONCENTRATE VOLUME FLOW TO TAILS VOLUME FLOW FOR
C   EACH CELL.  ESTIMATED TO CALCULATE CIRCUIT RESIDENCE TIMES.
C** N2 - NUMBER OF ROUGHER CELLS, I.E. FINAL CONCENTRATE CELLS.
      NN=N2                                                         ROSCAV C
      NX=NN+1                                                       ROSCAV C
      DO 2 I=1,24                                                   ROSCAV C
      K=I+1                                                         ROSCAV C
 2    IA(K,I)=1                                                     ROSCAV C
      DO 3 I=NX,25                                                  ROSCAV C
 3    IB(1,I)=1                                                     ROSCAV C
C
C*************************************************************************
C
C** CARDS FOR ROUGHER-CLEANER-RECLEANER CIRCUIT.
C** N2 - NUMBER OF RECLEANER CELLS  NUMBER OF CLEANERS IS FIXED AT FOUR.
      NN=25-N2-4                                                    RCREC  C
      NX=NN-1                                                       RCREC  C
      NO=N2                                                         RCREC  C
      NY=NN+NO+1                                                    RCREC  C
      NZ=NN+1                                                       RCREC  C
      DO 2 I=1,NX                                                   RCREC  C
      K=I+1                                                         RCREC  C
 2    IA(K,I)=1                                                     RCREC  C
```

```
      DO 3 I=1,NN                                                    RCREC   C
 3    IB(NY,I)=1                                                     RCREC   C
      DO 4 I=NY,25                                                   RCREC   C
 4    IB(NZ,I)=1                                                     RCREC   C
      DO 5 I=NZ,24                                                   RCREC   C
      K=I+1                                                          RCREC   C
      X(I)=.025                                                      RCREC   C
 5    IA(K,I)=1                                                      RCREC   C
      IA(1,25)=1                                                     RCREC   C
      X(25)=.025                                                     RCREC   C
C
C*****************************************************************************
C
C** CARDS FOR ROUGHER-CLEANER-CLEANER MIDDLINGS CIRCUIT.
C** N2 - NUMBER OF CLEANER CELLS  NUMBER OF CLEANER MIDDLINGS CELLS IS FIXED AT
C    FOUR.
      NN=25-N2-4                                                     RCM     C
      NX=NN-1                                                        RCM     C
      NZ=NN+1                                                        RCM     C
      NO=N2                                                          RCM     C
      NY=NO+NZ                                                       RCM     C
      DO 2 I=1,NX                                                    RCM     C
      K=I+1                                                          RCM     C
 2    IA(K,I)=1                                                      RCM     C
      DO 3 I=1,NN                                                    RCM     C
 3    IB(NZ,I)=1                                                     RCM     C
      DO 4 I=NZ,24                                                   RCM     C
      K=I+1                                                          RCM     C
      X(I)=.025                                                      RCM     C
 4    IA(K,I)=1                                                      RCM     C
      IA(1,25)=1                                                     RCM     C
      X(25)=.025                                                     RCM     C
      DO 5 I=NY,25                                                   RCM     C
 5    IB(NZ,I)=1                                                     RCM     C
C
C*****************************************************************************

C** CARDS FOR ROUGHER-CLEANER CIRCUIT.
C** N2 - NUMBER OF CLEANER CELLS.
      NN=25-N2                                                       ROCL    C
```

```
      NX=NN-1                                                    ROCL    C
      NZ=NN+1                                                    ROCL    C
      DO 2 I=1,NX                                                ROCL    C
      K=I+1                                                      ROCL    C
2     IA(K,I)=1                                                  ROCL    C
      DO 3 I=1,NN                                                ROCL    C
3     IB(NZ,I)=1                                                 ROCL    C
      IA(1,25)=1                                                 ROCL    C
      DO 4 I=NZ,25                                               ROCL    C
4     X(I)=.025                                                  ROCL    C
      IF(N2-1) 6,6,7                                             ROCL    C
7     DO 5 I=NZ,24                                               ROCL    C
      K=I+1                                                      ROCL    C
5     IA(K,I)=1                                                  ROCL    C
6     CONTINUE                                                   ROCL    C
      RETURN
      END
```

```
// FOR RESTT                              RESIDENCE TIME VOL ESTIMATION
*IOCS(TYPEWRITER,KEYBOARD)
C
C** PROGRAM TO BEST ESTIMATE THE EFFECTIVE VOLUME OF A KNOWN NUMBER OF STIRRED
C   UNITS WHICH ARE ASSUMED TO BE PERFECTLY MIXED
C** Y - OBSERVED TRACER CONCENTRATIONS AT TAILS EXIT
C** T - CORRESPONDING TIME OF OBSERVATION WITH THE INPUT PULSE AS TIME ZERO
C** V - INITIAL EFFECTIVE VOLUME ESTIMATE FOR ALL UNITS.  ON EXIT V IS THE BEST
C   ESTIMATED EFFECTIVE VOLUME
C** P - PREDICTED TRACER CONCENTRATION AT TIME T
C** S - SUM OF SQUARED DEVIATIONS
C
C** COULD BE SIMPLIFIED BY COMBINATION WITH USEIN
C
      DIMENSION T(25),Y(25),P(25)
       CALL LARRY (25,NV,Y)
       CALL LARRY (25,NV,T)
      N=25-NV
      ST=100.
      W=10.**10.
      V=37000.
      V=17000.
      V=10500.
3     V=V+ST
      CALL SSQS (T,Y,P,N,V,S)
      IF(S-W) 1,2,2
1     W=S
      GO TO 3
2     V=V-2.*ST
      ST=ST/10.
      IF(ST-.2) 99,99,4
4     CALL SSQS (T,Y,P,N,V,S)
      W=S
      GO TO 3
99    CALL SSQS (T,Y,P,N,V,S)
      WRITE (3,100) (T(I),Y(I),P(I),I=1,25),V,S
100   FORMAT  (25(3E14.4/)/2E12.4)
      CALL EXIT
      END
```

```
// FOR USEIN              USE OF INTEGRATION QTFG
*IOCS(TYPEWRITER,KEYBOARD)
C
C** MAINLINE DATA INPUT PROGRAM FOR INTEGRATION BY TRAPAZOIDAL RULE
C** USED TO OBTAIN AREAS UNDER RESIDENCE TIME DISTRIBUTIONS
C** SEE PAGES 37-41
C
      DIMENSION X(25),Y(25),Z(25)
 1    CONTINUE
      CALL LARRY (25,NV,X)
      CALL LARRY (25,NV,Y)
      NDIM=25-NV
      CALL QTFG (X,Y,Z,NDIM)
      WRITE (1,100) Z
 100  FORMAT (5E12.4)
      GO TO 1
 99   CALL EXIT
      END
```

```
// FOR BATPL
*IOCS(1443 PRINTER)
*IOCS(TYPEWRITER,KEYBOARD)
C
C** PROGRAM TO CALCULATE NORMALIZED BATCH TEST CONCENTRATION VERSUS TIME CURVES
C   FROM CONCENTRATE AND TAIL ASSAYS AND WEIGHTS
C** IF SULFUR ASSAYS ARE ENTERED A NON-SULFIDE CONCENTRATION CURVE IS ALSO
C   OUTPUT
C** WT - CONTAINS CONCENTRATES AND FINAL TAIL WEIGHTS
C** A - CONTAINS CONCENTRATE AND FINAL TAIL ASSAYS
C** CONTAINS NORMALIZED CONCENTRATION VALUES
C
      DIMENSION IALFA(40)
      DIMENSION A(15),WT(15),W(15),S(15)
C**   CALCULATES MINERAL AND GANGUE CURVES
 100  FORMAT (I3)
 101  FORMAT (80A2)
 102  FORMAT (8F10.4)
 7    READ (4,100) N
      IF(N) 99,99,8
 8    READ (4,101) IALFA
C
C** FREE FORMAT TYPEWRITER INPUT TO ARRAY WT
C
      CALL RINP (IALFA,1,78,15,NVRD,WT)
      READ (4,101) IALFA
      CALL RINP (IALFA,1,78,15,NVRD,A)
 5    SUMS =0
      DO 1 I=1,N
      W(I)=A(I)*WT(I)/100.
 1    SUMS = SUMS + W(I)
      K=N-1
      DO 2 I=1,K
      X=0.
      DO 3 J=1,I
 3    X=X+W(J)/SUMS
 2    S(I)=1.0-X
      WRITE (1,102) (S(I),I=1,N)
      IF (A(1)-50.)6,7,7
6     DO 4 I=1,N
```

```
4     A(I)=100.-A(I)
      GO TO 5
99    CALL EXIT
      END
```

```
// FOR PYRHH                CALCS PYRRHOTITE CURVE FROM CU,NI,S
*IOCS(1443 PRINTER)
*IOCS(CARD)
C
C** PROGRAM TO GIVE PYRRHOTITE CURVE FROM CU,NI,AND S ASSAYS.  PROGRAM ASSUMES
C   CHALCOPYRITE TO BE CUFE2, PENTLANDITE TO BE (FE,NI)9S8, AND PYRRHOTITE TO BE
C   FE7S8.  34.6 CU, 30.4 FE, 35.0 S IN CHALCOPYRITE $ 39.1 FE, 41.2 NI, 19.7 S
C   IN PENTLANDITE $ 60.4 FE, 39.6 S IN PYRRHOTITE.
C
      DIMENSION A(13,5),S(5),WT(13),T(20)
 97   READ (2,100) T
      DO 10 L=1,5
 10   S(L)=0.
      WRITE (3,103)
      WRITE (3,100) T
      WRITE (3,103)
 100  FORMAT (20A4)
 101  FORMAT (5F10.4)
C
C** READ IN KNOWN WEIGHTS IN SAMPLE AND KNOWN SULFUR, COPPER, AND NICKEL ASSAYS
C
      CALL RDCRD (13,NV,WT)
      CALL RDCRD (13,NV,A(1,1))
      CALL RDCRD (13,NV,A(1,2))
      CALL RDCRD (13,NV,A(1,3))
      N=13-NV
      DO 1 I=1,N
      X=WT(I)*A(I,1)
      XX=35./34.6*WT(I)*A(I,2)
      XXX=19.7/41.2*WT(I)*A(I,3)
 1    A(I,4)=(X-XX-XXX)/WT(I)*60.4/39.6
C
C** OUTPUT CALCULATED IRON AS PYRRHOTITE ASSAYS
C
      WRITE (3,101) (WT(I),A(I,1),A(I,2),A(I,3),A(I,4),I=1,N)
      IF(A(N,4)) 40,41,41
 40   A(N,4)=0.
 41   CONTINUE
      DO 2 J=1,4
      DO 2 I=1,N
```

```
 2      S(J)=S(J)+A(I,J)*WT(I)
        DO 6 I=1,N
 6      S(5)=S(5)+WT(I)*(100.-A(I,1))
        DO 3 I=1,N
        A(I,5)=(100.-A(I,1))*WT(I)/S(5)
        DO 3 J=1,4
 3      A(I,J)=A(I,J)*WT(I)/S(J)
        L=N-1
        DO 4 J=1,5
        DO 4 I=1,L
        K=N-I
 4      A(K,J)=A(K+1,J)+A(K,J)
        WRITE (3,103)
 103    FORMAT (//)
C
C** OUTPUT NORMALIZED CONCENTRATION CURVES FOR SULFUR, NONSULFIDE, COPPER,
C    NICKEL AND PYRRHOTITE SPECIES
C
        WRITE (3,102) ((A(K,J),J=1,5),K=1,N)
 102    FORMAT (5F10.4)
        GO TO 97
 98     CALL EXIT
        END
```

```
// FOR QTFG                    TRAPIZOIDAL RULE INTEGRATION
      SUBROUTINE QTFG (X,Y,Z,NDIM)
C
C** IBM SUPPLIED SUBROUTINE FOR INTEGRATION OF TABULATED FUNCTIONS * USED BY
C   USEIN.
C** X - CONTAINS ABCISSA DATA
C** Y - CONTAINS ORDINATE DATA
C** Z - CONTAINS STEPWISE AREA UNDER CURVE ** Z(NDIM) CONTAINS TOTAL AREA
C
      DIMENSION X(1),Y(1),Z(1)
      SUM2=0.
      IF(NDIM-1) 4,3,1
 1    DO 2 I=2,NDIM
      SUM1=SUM2
      SUM2=SUM2+.5000*(X(I)-X(I-1))*(Y(I)+Y(I-1))
 2    Z(I-1)=SUM1
 3    Z(NDIM)=SUM2
 4    RETURN
      END
```

```
// FOR SSQS                                WITH RESTT
      SUBROUTINE SSQS (T,Y,P,N,V,S)
C
C** SUBROUTINE FOR RESTT
C** C - NUMBER OF STIRRED UNITS
C** SV - FLOWRATE THROUGH STIRRED UNITS IN UNITS OF T AND V
C
      DIMENSION T(1),Y(1),P(1)
      S=0.
      C=20.
      SV=870.
      DO 1 I=1,N
      E=C*SV/(V*RFACT(C-1.))*(C*SV*T(I)/V)**(C-1.)*EXP(-C*SV*T(I)/V)
      P(I)=E*27.64
C
C** THE ABOVE CONSTANT (27.64) IS THE AREA UNDER THE OBSERVED CONCENTRATION
C   VERSUS TIME CURVE IN UNITS OF Y AND T.  CAN BE CALCULATED FROM PROGRAM USEIN
C
 1    S=S+(Y(I)-P(I))**2.
       RETURN
       END
```

```
// FOR BREAD                              READ IN BATCH DATA
      SUBROUTINE BREAD (T,Y,IS,ID,NDATA)
C
C** SUBROUTINE FOR INPUT OF CONCENTRATION CURVE DATA * FIRST SECTION OF THESIS
C   PART TWO
C** T - AN ARRAY OF LENGTH NDATA CONTAINING TIMES
C** Y - AN ARRAY OF LENGTH NDATA CONTAINING NORMALIZED CONCENTRATIONS OF A
C   SPECIES CORRESPONDING TO ARRAY T
C** IS - AN ALPHANUMERIC ARRAY OF 60 CHARACTERS IDENTIFYING THE TEST
C** ID - AN INTEGER ARRAY OF SEVEN WORDS CONTAINING THE TEST CODE
C** NDATA - NUMBER OF PAIRS OF CONCENTRATIONS AND TIMES
C
      DIMENSION T(1),Y(1),ID(7),IS(30)
      READ (2,201) IS,ID,NDATA
 201  FORMAT (30A2,6X,3I1,I2,I4,I1,I4/2X,I3)
      READ (2,202) (Y(I),I=1,NDATA)
 202  FORMAT (20F4.3)
      READ (2,203) (T(I),I=1,NDATA)
 203  FORMAT (20F4.2)
      RETURN
      END
```

```
// FOR MINV5                    MATRIX INVERSION
      SUBROUTINE MINV5 (C,M)
C
C** MATRIX INVERSION SUBROUTINE FOR THE GAMMA FUNCTION MODEL CIRCUIT SIMULATIONS
C    AND ANYFT REGRESSIONS
C    ALSO USED BY GCONT
C
      DIMENSION C(40,40)
C     DIRECT MATRIX INVERSION LIB. PROG.  5.0.CE003
      DO 5014 I = 1,M
      W = C(I,I)
      C(I,I) = 1.0
      DO 5011 J = 1,M
 5011 C(I,J)= C(I,J) / W
      DO 5014 K = 1,M
      IF (K - I) 5012,5014,5012
 5012 W = C(K,I)
      C(K,I) = 0.
      DO 5013 J = 1,M
 5013 C(K,J) = C(K,J) - W * C(I,J)
 5014 CONTINUE
      RETURN
      END
```

```
// FOR RCGAM                    IBM GAMMA FUNCTION
      FUNCTION RCGAM (XX)
C
C** IBM SUPPLIED FUNCTION SUBPROGRAM FOR THE EVALUATION OF THE GAMMA FUNCTION
C
      IF(XX-31.) 6,6,4                                                  MACHSPEC
 4    RCGAM=10.**10.
      RETURN
 6    X=XX
      GX=1.0
      IF(X-2.0) 50,50,15
 10   IF(X-2.0) 110,110,15
 15   X=X-1.0
      GX=GX*X
      GO TO 10
 50   IF(X-1.0) 60,120,110
 60   IF(X-1.0E-6) 62,62,80
 62   Y=FLOAT(IFIX(X))-X
      IF(ABS(Y)-1.0E-6) 130,130,64
 64   IF(1.0-Y-1.0E-6) 130,130,70
 70   IF(X-1.0) 80,80,110
 80   GX=GX/X
      X=X+1.0
      GO TO 70
 110  Y=X-1.0
      GY=1.+Y*(-0.5771017+Y*(0.9858540+Y*(-0.8764218+Y*(.8328212+Y*(-.56
      84729+Y*(.2548205+Y*(-.05149930)))))))
      GX=GX*GY
 120  CONTINUE
 130  CONTINUE
      RCGAM=GX
      RETURN
      END
```

```
// FOR KROPH                    ROOTS/WEIGHTS GEN LAQUERRE QUADRATURE
      SUBROUTINE KROPH (ALF,CC,X,A,NN)
C
C** GENERATES ROOTS AND WEIGHTS FOR GENERALIZED GAUSS-LAGUERRE QUADRATURE.  THE
C   MAXIMUM DEGREE OF POLYNOMIAL POSSIBLE IS MACHINE SPECIFIC AS WELL AS PARTLY
C   DEPENDENT ON THE VALUE OF ALF
C** SOLVES FOR ROOTS AND WEIGHTS IN THE GENERAL INTEGRATION PROBLEM PRESENTED ON
C   PAGE 140
C** COURTESY OF DR. I. J. KROPHOLLER, LOUGHBUROUGH UNIVERSITY
C** ALF - VALUE OF THE POWER OF X IN THE GENERAL INTEGRAL FORM
C** CC - GAMMA FUNCTION OF ALF+1.
C** X - ROOTS OF THE QUADRATURE FORMULA
C** A - WEIGHTS OF THE QUADRATURE FORMULA
C** NN - DEGREE OF THE POLYNOMIAL REQUIRED
C
      DIMENSION X(15),A(15),B(15),C(15)                                    C
C                                      )52(C,)52(B,)52(A,)52(X NOISNEMID
C                                      )23(C,)23(B,)23(A,)23(X NOISNEMID
      EPS=1.E-9
 3    DO 1 I=2,NN
      XI=I
      B(I)=(ALF+2.*XI-1.)
 1    C(I)=(XI-1.)*(ALF+XI-1.)                                    KRP    C
      CSX=CC                                                      TEMP
 2    CALL LAGUR (NN,X,A,ALF,B,C,EPS,CSX,CSA,TSX,TSA)
      RETURN
      END
```

```
// FOR LAGUR                    S/R FOR QUADD
      SUBROUTINE LAGUR (NN,X,A,ALF,B,C,EPS,CSX,CSA,TSX,TSA)
C
C** SUBROUTINE TO FIND ROOTS AND WEIGHTS FOR THE GENERALIZED GAUSS-LAGUERRE
C   QUADRATURE INTEGRATION DESCRIBED ON PAGE 140
C** USED BY SUBROUTINE KROPH
C** COURTESY OF DR. I. J. KROPHOLLER, LOUGHBUROUGH UNIVERSITY
C** NOTE THAT THIS ROUTINE IS MACHINE SPECIFIC AND WILL GENERATE REAL NUMBER
C   OVERFLOW ON A MACHINE WITH A CAPABILITY OF A SMALLER RANGE THAN 10**-39 AND
C   10**38
C
C                                          )52(C,)52(B,)52(A,)52(X NOISNEMID
      DIMENSION X(32),A(32),B(32),C(32)                                    C
C                                          )51(C,)51(B,)51(A,)51(X NOISNEMID
      FN=NN
      CC=CSX                                                      TEMP
      CSX=0.
      CSA=0.
      TSX=FN*(FN+ALF)
      TSA=CC
      DO 7 I=1,NN
      IF(I-1) 6,2,3
 2    XT=(1.+ALF)*(3.+.92*ALF)/(1.+2.4*FN+1.8*ALF)
      GO TO 6
 3    IF(I-2) 6,4,5
 4    XT=XT+(15.+6.25*ALF)/(1.+.9*ALF+2.5*FN)
      GO TO 6
 5    FI=I-2
      R1=(1.+2.55*FI)/(1.9*FI)
      R2=1.26*FI*ALF/(1.+3.5*FI)
      RATIO=(R1+R2)/(1.+.3*ALF)
      XT=XT+RATIO*(XT-X(I-2))
 6    CALL LROOT (XT,NN,ALF,DPN,PN1,B,C,EPS)
      X(I)=XT
      A(I)=TSA/DPN
      XXX=ABS(PN1)**(1.00000/FLOAT(NN-1))
      DO 1 J=2,NN
 1    A(I)=A(I)*C(J)/XXX
      A(I)=A(I)*(PN1/ABS(PN1))
      CSX=CSX+XT
```

```
      CSA=CSA+A(I)
7     CONTINUE
      RETURN
      END
```

```
// FOR LROOT                    S/R FOR QUADD
      SUBROUTINE LROOT (X,NN,ALF,DPN,PN1,B,C,EPS)
C
C** SUBROUTINE TO CALCULATE ROOTS OF THE QUADRATURE POLYNOMIAL  USED BY S/R
C   KROPH.
C** COURTESY OF DR. I. J. KROPHOLLER, LOUGHBUROUGH UNIVERSITY
C
      DIMENSION  B(32),C(32)                                                C
C                                                          )52(C,)52(B NOISNEMID
C                                                          )51(C,)51(B NOISNEMID
      ITER=0
 1    ITER=ITER+1
      CALL LRECR (P,DP,PN1,X,NN,ALF,B,C)
      D=P/DP
      X=X-D
      IF(ABS(D/X)-EPS) 3,3,2
 2    IF(ITER-20) 1,3,3
 3    DPN=DP
      RETURN
      END
```

```
// FOR LRECR                    S/R FOR QUADD
      SUBROUTINE LRECR (PN,DPN,PN1,X,NN,ALF,B,C)
C
C** SUBROUTINE TO CALCULATE VALUE AND DERIVATIVE OF THE GENERALIZED LAGUERRE
C   POLYNOMIALS USING THE RECURSIVE RELATIONSHIP.  USED BY S/R KROPH
C** COURTESY OF DR. I. J. KROPHOLLER, LOUGHBUROUGH UNIVERSITY
C
      DIMENSION  B(32),C(32)                                                C
C                                                          )52(C,)52(B NOISNEMID
C                                                          )51(C,)51(B NOISNEMID
      P1=1.
      P=X-ALF-1.
      DP1=0.
      DP=1.
      DO 1 J=2,NN
      Q=(X-B(J))*P-C(J)*P1
      DQ=(X-B(J))*DP+P-C(J)*DP1
      P1=P
      P=Q
      DP1=DP
1     DP=DQ
      PN=P
      DPN=DP
      PN1=P1
      RETURN
      END
```

```
// FOR RFACT
      FUNCTION RFACT (N)
C
C** ROUTINE USED TO CALCULATE FACTORIAL EXPRESSIONS
C
      IF(N-1) 4,4,3
 4     RFACT=1.
      GO TO 99
 3    RFACT=N
      K=N-1
      DO 1 I=1,K
 1    RFACT=RFACT*FLOAT(N-I)
 99   RETURN
      END
```

```
// FOR MODSQ
      SUBROUTINE MODSQ (A,B,S)
C
C** SUBROUTINE FOR USE WITH ITIT OR SEAR2
C
      COMMON Y(13),T(13),PRED(13),ISA(50),ID(7),IS(30),ILC(7),AA(2),BB(2
     1),Q(13),NDATA,DB,DA,IWT(2),SG(13)
      S=0.
      W1=Y(NDATA)
      W2=.5*(1.-Y(2))
C                                                                    )I(GS/B=B
      PRED(1)=1.
      DO 1 I=2,NDATA
      Z=A+B+Q(I)
      X=4*A*Q(I)
      P=SQRT(Z**2.-X)
      E=(-Z+P)/2.
      F=(-Z-P)/2.
      Z=E-F
      ZEDD=T(I)-T(I-1)
      PRED(I)=(E*EXP(F*ZEDD)-F*EXP(E*ZEDD))/Z
      PRED(I)=PRED(I-1)*PRED(I)
1     CONTINUE
      DO 2 I=2,NDATA
      PRED(I)=(1.-IWT(1)*W1)*PRED(I)+IWT(1)*W1
2     S=S+(Y(I)-PRED(I))**2.
      RETURN
      END
```