

**‘Profound Understanding of Fundamental
Mathematics’ and Mathematical Life Histories**

**of some teachers teaching mathematics
in the intermediate phase in KwaZulu-Natal**

**A thesis submitted in fulfilment of the academic requirements for
the degree Master of Education**

By

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Abstract

This study had two components: 1) Investigating the conceptual understanding of teachers teaching elementary mathematics at primary schools in the province of KwaZulu-Natal, who had been successful in their mathematics modules in the National Professional Diploma in Education (NPDE) teacher upgrading program, and 2) Investigating the influence of their mathematical life-histories on their understanding and personal philosophies about mathematics. It firstly required the NPDE students from the University of KwaZulu-Natal to complete a questionnaire adapted from the TELT interview schedule used by Liping Ma (1999). This questionnaire was to assess whether these high scoring teachers had an understanding of basic mathematical concepts that could have been regarded as being profound. The second part of the study was designed in order to get these teachers to examine their mathematical life histories and then to look at how their life histories could have influenced their level of understanding. It was found that these teachers were procedurally capable and were aware of the algorithms that could be used to solve the problems posed, but they lacked deep understanding of the concepts and were thus conceptually weak. None of the teachers demonstrated an understanding of the fundamental mathematics concepts that were assessed, that could be regarded as been 'profound'. The mathematical life history portion of this study revealed that these teachers, having experienced mathematics education very differently due to their Apartheid influenced education, mentioned that there were definite influences that had a marked effect on their outlook on the subject and thus their belief in their ability to do basic/ fundamental mathematics.

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Declaration

I declare that project '**Profound Understanding of Fundamental Mathematics' and LIFE HISTORIES of some teachers teaching mathematics in the intermediate phase in KwaZulu-Natal**' is my own work, and that it is has not been submitted previously for degree purposes at any higher education institution.

Andre Mervyn Van Wyk

Ethical Clearance

Ethical Clearance for this project has been obtained from the University of KwaZulu-Natal. The Ethical Clearance certificate number is HSS/06635.

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List of Abbreviations

NED	National Education Department (White)
HOD	House of Delegates (Indian/Asian)
DoE	Department of Education and Training
HOR	House of Representatives (Coloured)
DET	Department of Education and Training (Black)
PUFM	Profound Understanding of Fundamental Mathematics
PCK	Pedagogical Content Knowledge
NPDE	National Professional Diploma in Education
PF	Fundamental Pedagogics
MkFT	Mathematical Knowledge for teaching
Tr	Teacher
TELT	Teacher Education and Learning to Teach Study (Michigan State University)

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Chapter 1: Introduction to the Study

This chapter aims to give an overview of the study. It gives a background to the study and that to some extent gives the context to the study. It reviews the rationale for the study and supplies the focus of the study. The chapter is divided into subsections with sub-headings in order to facilitate a clear understanding of the concepts to be dealt with in each section.

1.1. Teachers of the Rainbow Nation – Still in the Shadow of Apartheid

South African teachers are as diverse as the rainbow nation could allow. The country is made up of many different cultural groupings and these exist in a country that acknowledges eleven different official languages.

The socialisation of the teachers in South Africa is therefore an important consideration for any study that is carried out in this country. Teachers' socialisation influences their core responsibilities as educators and thus to a large extent it influences the education system within the country. Adler (2004, p.6) states that, "we in South Africa continue to work in a socio-cultural and political context deeply scarred by apartheid education."

Thomas J. Cooney (1995, p.170) makes the following statement with regards to such an issue:

Teachers operate in contexts, their knowledge framed and shaped by experiences many of which happen long before they formally enter the world of mathematics education. In the classroom, what the teacher knows is fused with her sense of purpose as a teacher of mathematics, her philosophy of teaching and learning, and her sense of responsibility to the community in which she teaches.

A. Brief Historical view

The following is an extract from a book by Simphiwe A. Hlatshwayo (2000). The purpose of this abnormally long extract is to give a brief view of from where South African Education is coming.

Karl Marx (1852, p.13) said “Men make their own history, but they do not make it as they please; they do not make it under circumstances chosen by themselves, but under circumstances directly encountered, given and transmitted from the passed.” Schooling in South Africa initially involved all races. As the country developed, a need arose to use the indigenous population in the mines and in agriculture as labourers. Schooling, which was dominated by the missionaries, was taken over by the government for the purpose of providing unskilled labour. The process of social control, involved denial of all political rights and physical control of the lives of all Black people in South Africa. This was accomplished by means of the pass laws, the reserve system and an encompassing net of repressive legislation. Although schooling in many capitalist countries is an important conduit for incorporating the young into the hierarchy of capitalist production, in South Africa the young were first and foremost divided into races; the Whites educated to be masters and Blacks educated to serve them. (p.50)

South Africa through its Apartheid system of governance set up 19 different education systems within the country, some of these being the DET (black schools); HOR (coloured schools); HOD (Indian/Asian schools); NED (white schools); and the homeland and traditional areas, like KwaZulu, Bophuthatswana, Ciskei and the Transkei.¹ Only in 1996 were all these various

¹ During Apartheid, the population was divided into four major population groups, with various sub-divisions: White [composed mainly of two groups: the Afrikaans speaking or Afrikaner group, chiefly descendants of Dutch settlers and the English-speaking which is chiefly of British extraction], Indian[broad name given to many Asians brought here in the second half of the nineteenth century from India as indentured servants to work in sugar plantations in Kwazulu-Natal], Coloured [are a product of miscegenation between Whites, Malay and the indigenous African population] , Black [also called Africans, Kaffirs, Natives, Non-whites, Plurals and Bantu, are the indigenous inhabitants of the country] – in order of decreasing privilege (Hlatshwayo, 2000). The classification of an individual was done on the basis of physical appearance but also behaviour, ancestry, etc. Thus, the groups are clearly not culturally homogenous. This creates problems of appropriate terminology in the post-Apartheid era. For instance, a person classified as ‘Black’ during Apartheid would consider herself Zulu, for instance. The idea of replacing ‘Black’ with ‘African’ raises other problems, because it creates the need for another term to refer to the inhabitants of the continent, etc. Since this thesis is set against the context of the Apartheid legacy, the Apartheid

departments amalgamated under the current Department of Education (DoE). However many of the schools in South Africa, are still to a large extent differentiated along racial and thus socio-economic lines, in some cases because they are in areas that are still predominantly populated by one specific racial grouping, in other cases because of preference of schools or in other cases because of financial constraint.

A dilemma arises in the South African Education System, in that whilst diversity must be acknowledged and embraced, the norms and standards of the mathematical fraternity have to still be acknowledged and upheld so that students of mathematics in this country will be internationally accepted. A balance has to be sought. The problem however is the different levels of education that the South African population find themselves at, due to the apartheid legacy. The government of the day (Apartheid Government) was not keen on, what was termed, Bantu education and thus did not place large sums of money into the training of Black, Coloured and Indian learners and teachers (Jansen & Taylor, 2003). It must however be acknowledged that many of these Colleges and Universities did still produce some very good teachers. I believe that this was mainly due to the commitment and dedication of these student teachers and their lecturers rather than because of the efforts of the Apartheid States Education system.

This is evidenced in the following tables which show the population distribution in schools from 1927 to 1977 and secondly the amounts of money spent on education for the different ‘official race groups’.

Table 1 School Population, 1927-1977 (Source: Hellman & Lever, 1980, p.160)

	1927		1977	
	Number	Percentage	Number	Percentage
White	352, 000	53,6	940,000	16,4
Coloured and Indian	78,000	11,9	890,000	15,5
African	225,000	34,5	3,900,000	68,1
Total	655,000	100,0	5,730,000	100,0

terminology has been used as a short-hand for the previous degrees of privilege, but of course with none of Apartheid’s values implied. As they are constructed names, they will be written with capitals.

Table 2 Expenditure on Education by Race, 1975-1985(millions of pounds)

Year	Africans	Coloureds	Indians	Whites	Total⁺
1975	131 (17%)	89 (11%)	39(5%)	536(67%)	795(100%)
1976	156	103	44	646	948
1977	191	133	56	816	1,196
1978	110	144	61	877	1,192
1979	245	179	75	1,000	1,498
1980	305(18%)	175(10%)	83(5%)	1,116(67%)	1,679(100%)
1981	298	247	123	1,361	2,029
1982	557*	294	155	1,688	2,695
1983	755	405	196	2,056	3,413
1984	122	451	225	2,032	3,932
1985	1,460(31%)	571(12%)	259(5%)	2,465(52%)	4,755(100%)

⁺ Totals show the effects of rounding

* excluding the “independent” Bantustans

Source: SAIRR 1976-1985

The following are a few views of prominent people in South Africa. The first is a late apartheid president, the second is the incumbent minister of education and the third is a prominent South African academic. It is through their words that we can get a sense not only of where the county comes from in terms of apartheid education but also where it finds itself at present.

B. A view of an Apartheid President of South Africa

A statement made by the then Minister of Native Affairs, Dr HF Verwoerd, in a speech delivered on the 17th September 1953 on the Second Reading of the Bantu Education Bill, encompasses and situates the feelings of the then Apartheid Governments views on Black Education. He stated,

When I have control over native education I will reform it so that the Natives will be taught from childhood to realise that equality with Europeans is not for them. People who believe in equality are not desirable teachers for Natives What is the use of

teaching the Bantu child mathematics when it cannot use it in practice? That is quite absurd. (Verwoerd, 1953, p.3585)

There is not much one can add to this. It clearly demonstrates how the Apartheid Regime viewed 'Black' education in South Africa. The consequences of this obviously still linger in the education system. For instance, the TIMSS tests not only place South African learners low in performance compared to the peers internationally, it also shows that this is largely due to historical disadvantage (Reddy, 2006).

C. The View of the current South African Minister of Education

Let me use a speech made by the Minister of Education to give an indication of the general state of affairs in education. Naledi Pandor (Minister of Education in South Africa from 2004) at the opening of the 2004 AMESA conference used the following quotes from a book (Getting Learning Right, 1999, p.138 -139):

...researchers report that during interviews with teachers from successful schools, success was attributed overwhelmingly to staff factors.

The study (Ibid, p.139) also reports that in two studies of mathematics teaching at grade four level, teachers were found to use mathematically incorrect or in appropriate language.

Both of these quotes clearly indicate that the ministry of education is aware of the importance of quality teachers to education and that there is at present a situation where there are many teachers who lack the basic knowledge required for the teaching of mathematics.

D. The View of a prominent South African Academic

Prof. Mamokgethi Setati (Associate Professor of Mathematics Education, Wits University) makes the following statement regarding the teaching of mathematics in South Africa:

The only reason why people think mathematics is difficult is because of the way they have been taught mathematics at school. To change people's perceptions and student's performance in mathematics we need to change the way it is taught. We need high quality teaching and high quality teachers in every classroom, for every child.High quality mathematics teachers are not just well qualified in

mathematics, but they are well motivated, professional and care for their students.What is critical for a mathematics teacher is to be able to understand why the student's solution is incorrectBeing a good mathematics teacher involves being more than being a good mathematician. It is more than just knowing and being able to do the mathematics for oneself. It is about being able to explain to students why the mathematical truths that we often take for granted work the way they do. (Setati, 2004)

Prof. Setati picks up on two important points in this article. The first point is the need for quality educators to teach mathematics. She states that students find mathematics difficult because of the way they were taught the subject. Here she clearly places the blame for the disappointing mathematics results at the foot of the teachers teaching this subject.

The second point that Prof. Setati emphasizes is that quality teachers of mathematics are not merely good mathematicians but teachers who are well-motivated to teach the subject; professional in their approach to the teaching of this subject; and are empathetic to the mathematical struggles that their students may be facing.

1.2. Teacher Training as it occurred in Apartheid South Africa and its consequences

Prior to 1995, there were approximately 150 state funded institutions providing teacher education (Parker, 2003). These institutions, operating under 19 different education authorities, offered a range of qualifications of varying quality. Colleges of education had the major responsibility for initial teacher training. Teacher educators in these institutions were state employees. Colleges operated much like high schools, with strong external framing of curricula and in most cases external examinations, full teaching timetables, little space for independent study, and little expectation that staff engage in research and become disciplinary experts (Parker& Adler, 2005).

In KwaZulu-Natal some of these colleges were: Bechet Teachers training College for the Coloured population; Springfield College of Education for the Indian/Asian population; Indumiso, Madadeni and Eshowe Colleges of Education for the Black population and Edgewood

and Durban Onderwys College for the White population. The colleges for Black, Indian and Coloured communities were poorly financed and thus lacked many essential resources required by the lecturers and academics who were employed to do the teacher training (Jansen & Taylor, 2003, p.7; Wildeman, 2003). The trainee teachers therefore often did not receive as good a standard of training as their White counterparts. What is equally important is that they had different curricula – and different methods of delivery (Jansen & Taylor, 2003, p.7). The lecturers at the Black training colleges were initially mainly white and thus had different educational backgrounds to their students (Bunting, 2006, p.75).

The curriculum design was linked directly to the beliefs of Christian National Education also known as CNE, which clearly promoted the principles of Fundamental Pedagogics. This, according to Khuzwayo (2000) not only had an influence on educational policy but also what was taught and how it was taught. Viljoen and Pienaar (1971, p.95) as proponents of fundamental pedagogics maintained:

Education is a particular occurrence in accordance with accepted values and norms of the educator and eventually also of the group to which he belongs. He is engaged in accompanying the child to self-realisation, but this realization must be in accordance with the demands of the community and in compliance with the philosophy of life of the group to which he belongs. In this way the South African child has to be educated according to Christian National Principles.

It must be noted that the term ‘group’ in the above quotation referred to the race group as perceived by the apartheid government. Although the quote looked at the education of children, the teachers were trained to fulfil the role of educator as seen by the likes of Viljoen and Pienaar (1971).

In most cases, the curriculum followed was simply a repeat of the high school curriculum with a slight lean towards ‘pedagogic’ methodology. This I experienced as a student at one of these colleges of education. The content knowledge base of many of these teachers was never extended and thus they found themselves limited when it came to sections that required deeper insight.

Bansilal (2002, p.23) states that the key assumption of studies that are concerned with mathematical knowledge from a teacher perspective concerns the fact that many teachers have had very poor exposure to quality learning experiences in Mathematics and that these experiences have left them scarred in terms of content knowledge.

1.3. Status of Mathematics Teachers in the Democratic South Africa

Where do South African mathematics teachers find themselves at present? The National Teacher Education Audit of 1996 and the Mathematics and Science Audit of 1997 produced factual and statistical revelations about teachers teaching in this subject. The Mathematics and Science Audit revealed that 50% of the mathematics teachers teaching this subject had no formal subject training (DoE, 2001a). The problem of inadequate training was particularly identified in the general education phase of the schooling system (grades 1-9). The Education for All (EFA) 2000 assessment also reported that, in spite of 85% of mathematics educators being professionally qualified, only 50% have specialized in mathematics in their training (EPA, 2005). Mji and Makgato (2006) quote a figure of 8000 mathematics teachers that will require in-service training to address their short comings in the subject. I am personally of the opinion that this figure is higher, as many of the primary school educators, some now teaching in higher grades, are generalist educators; they may be professionally qualified but lack specific training in mathematics (Reddy, 2006).

Although many of the teachers mentioned above have had no extra training, they are driven to teach subjects that they have no formal training for, in the absence of properly trained teachers.

1.4. The NPDE upgrading qualification

A. Rationale for the Development of the NPDE

The Department of Education and Training realized that there was need to upgrade the unqualified teachers' qualifications. However, it must be stated that some of the teachers deemed to be unqualified were in possession of a diploma in education or a degree, but had not studied mathematics beyond grade 12 level. The following table gives a good idea of what has been stated in this paragraph.

Table 3: The qualifications profile of mathematics teachers in KwaZulu-Natal, 1995

Subject Qualifications	Number of Mathematics Teachers	% of total Mathematics Teachers
Degree with three or more years of university mathematics courses	58	1,5%
Degree with two year mathematics course	18	0,5%
Degree with one year university mathematics course	60	1,5%
Higher Diploma in Education (Math's or Science)	38	1%
Secondary Teacher's Diploma (Math's or Science)	923	23%
Total Qualified	1097	28%
Total Unqualified	2850	72%
Total Number of Secondary Math's teachers employed	3947	100%

Source: Dempster (2000)

The old Apartheid Education recognized a teacher as being qualified if they were in possession of a Junior Certificate (JC), equivalent to grade 9, and had completed two years of formal teacher training. Many of these teachers went on to complete their matric (grade 12) certificates and were then regarded to have Matric plus one year of study (M+1). Sadly, there is still yet another group of teachers who are only in possession of a matric certificate. How can this be possible? The answer is relatively simple: since there are just not enough qualified teachers around to take up the new teaching posts that become available, others are taken in to fill the gap (reference).

Many of these under-qualified teachers were part of the education system and therefore could not be retrenched.

In order to address the issue of teacher training and the upgrading of teachers already in the system, the post-Apartheid DoE response was, firstly, to set in place the *Norms and Standards for Educators 2000* which sets out a normative framework for teacher education and training, which was meant to regulate the way teachers were trained and, secondly, in addressing the plight of the under-qualified and unqualified teachers, the DoE introduced the *National Professional Diploma in Education (NPDE)* (Wildeman, 2000).

The purpose of the NPDE qualification was to upgrade the qualifications of unqualified teachers who were already in the system and who were protected by the Education Labour Relations Council (ELRC) resolutions. The course was designed and introduced by the Standards Generating Body for Schooling and the Department of Education to allow these under-qualified teachers to attain REQV 13 (matric + 3 years training) status (Wildeman, 2000; Ngidi, 2005). This diploma was thus to allow the matric plus one year (M+1) of training and the matric plus two years of training (M+2) teachers to upgrade their qualifications to at least matric plus three years of training (M+3). This is the minimum qualification accepted by the DoE currently. The NPDE was registered by the South African Qualifications Authority (SAQA) in October 2000 (National Professional Diploma in Education, 2003). This course differs from other existing education or teaching programs in that it is a school-based educator skills program (Ngidi, 2005).

B. Rationale for Investigating the Mathematical Knowledge of Teachers on the NPDE

After being involved with teachers that were up-grading their qualification on the NPDE Course, run by the University of Natal, Pietermaritzburg (now merged with other institutions to form University of KwaZulu-Natal), I became very interested in finding out more about the *Mathematical Knowledge* that these teachers possessed.

What concerned me was that although many of these teachers have been teaching for many years, their basic mathematical skills were very lacking and in some cases close to non-existent.

This deficiency came through in the tutorial sessions and in the work that these students submitted for assessment.

The initial course-module was designed to in order to give all the students doing this course a basic mathematical knowledge. The level of the mathematics content was very comparable to the level of the curriculum of Intermediate Phase (grades 4-6) school mathematics. It was assumed that since many of these students were teaching at this level, they would have been familiar with the content and thus should have been able to cope comfortably.

The coordinators of the course were totally taken aback when the first assignment was sent for marking and many of the students had failed. This forced the coordinators to re-look at the mode of delivery of the course. The delivery mode required lots of discussion and group work. The tutors were expected to facilitate the learning process. It became obvious that many of the students could not participate in the discussions because they simply did not know what was going on. One student (a principal 62 years of age) wrote a letter stating that he had last done arithmetic thirty years ago and therefore had no understanding of the content.

It was agreed that adjustments had to be made to the mode of delivery. Tutors were asked to teach certain sections and also not to expect that students knew or were familiar with the content. Remedial sessions had to be arranged so as to assist the students who had failed. Students were given tutorials to work through at these sessions and the tutors moved around in the class assisting the students. The tutorials were then marked in the class with remedial work done when and where it was needed.

Two areas of concern came to the front at this point. One of the concerns was the fact that many – but not all – students seemed not to have an understanding of *mathematical content* that was strong enough for them to cope with basic mathematical problems from the intermediate phase scheme of work. The second concern looked at why it was possible for some students to succeed given that they had *mostly* come out of similar backgrounds. Were there perhaps other factors that contributed to their ability to do this level of mathematics? I decided to make these areas of concern the focus of my study.

1.5. Teachers' Mathematical Understanding

The first concern dealt with an issue that has been tackled in many studies and is almost common rhetoric in South Africa (see however the Literature Review for more on the mathematical knowledge of teachers). The Human sciences and Research Council in its review of the TIMSS study states the following regarding teacher preparation and retention:

On the whole, about half the teachers reported feeling ill prepared to teach content of either mathematics or the science curriculum. On inspection of the qualifications and experience of these teachers, this is not surprising. ...Those that qualified through the three year diploma from colleges of education probably did not go beyond the subject they did at school in Grades 10 -12. The lack of adequate preparation in terms of content knowledge in particular have left these teachers feeling poorly prepared to teach their pupils; the teachers are constrained by this in the classroom. Since resources are lacking in many schools and the teacher is often the pupils' only resource to learning, it is not surprising that the end result is so poor (Reddy, 2006)

The TIMSS report mentions teachers with three years of teacher training having problems with content, whereas the students doing the NPDE course had one or two years of teacher training. Thus, their situation was likely to be worse. One of the findings of the TIMSS report is that "27% of pupils, that were part of the study, were taught mathematics by teachers with no formal qualifications in mathematics." (Reddy, 2006)

Will this type of research merely lead us in the direction of a deficiency model since it points out that elementary teachers lack in their mathematical knowledge? This is an important question when considering the purpose of such a study. Cooney (1999), writing from a US perspective but presumably with a good knowledge of international research, states that much of the research on teachers' knowledge leads us in a direction of a deficiency model in that it points out what teachers (often elementary teachers) lack as mathematical knowledge. Cooney sites Graeber, Tirosh and Glover (1986) as an example of such research. I believe, like Cooney (1999, p.164) that such a study has merit in that it sets out to try to understand what the status of teacher knowledge is and thus hopes to build a case for increasing teacher's knowledge of mathematics.

However, in order not to repeat the common rhetoric that South African teachers have limited mathematical knowledge, I thought that another study looking at this and documenting what is common knowledge could not be of much value. I therefore moved my focus away from producing a deficiency type of study and focused on combining the study of teacher knowledge with that of a life-history type study so that I could explore reasons for the success of some of these teachers.

1.6. The Success of the Few – Against all Odds?

In looking at the reasons for the success of these students, I realized that I had to analyse what it was that they actually knew. Novak (1990, p.942) states that students may demonstrate high achievement as a consequence of “intensive rehearsal and rote learning” and notes that knowledge gained from such experiences is “soon lost or is not applicable to real-world contexts.” This statement by Novak, although written in context of science education, made me realize that the teachers’ results perhaps did not necessarily reflect their level of understanding of the concepts dealt with in the course. I had to find a way to ‘look deeper’ into the teachers’ understanding of mathematics.

Liping Ma’s (1999) book *Knowing and Teaching Elementary Mathematics*, where she reports on a comparative study that she did of US and Chinese teachers’ specific fundamental content knowledge of mathematics, gave me the idea to do a similar study on teachers teaching mathematics in the Intermediate phase.

The research area therefore had to look at the content covered in this course. As it was evident that there were key/fundamental areas in mathematics that were covered in the course, students’ knowledge of these fundamentals had to be tested. On this basis, I hoped to be able to identify students with a strong understanding of the relevant mathematical concepts and structures.

The next area of study was to look at the ‘mathematical’ life histories of a few of these successful teachers in order to try to identify any factors/characteristics that could have helped these successful teachers become successful in fundamental mathematics.

1.7. The Significance of the study

In investigating the *profound understanding of fundamental mathematics* (PUFM, see theoretical framework) of the teachers in the NPDE, it would be possible to give a more conceptually focused insight into the teachers' mathematical content knowledge and identify their relative areas of strength. Furthermore, it would make it possible to determine the extent to which the existing NPDE examination reflects PUFM or "intensive rehearsal and rote learning". Combining this with the study of the life histories of the more successful teachers may give some ideas as to how prospective teachers can better be supported in their career choice – of course taking into consideration that, statistically, the strongest determinant of educational success still is socio-economic background.

The value lies in the feedback that this study could give to the:

- Designers of teacher training courses like the NPDE Mathematics courses and the ACE mathematics courses;
- The writers of textbooks and teacher support materials;
- The Department of Education, so that the subject advisors would know how to assist teachers and also so that DoE could make funding available to teachers, in the form of bursaries, for the up-grading or training of in-service teachers.

The study could bring out possible oversights in the curriculum design of the NPDE and ACE courses. It could also inform the designers of the courses whether or not their key outcomes are being achieved and secondly whether their assessment tools are suitable and reliable.

The writers of teacher support materials could be informed of areas that could require greater emphasis. They could therefore write in such a way that their materials are useful to these teachers.

Many of the changes in education have been related to the method delivery. The big shift from the Christian National Education system to the Outcomes based system of education is a point to note. The change here focused mainly on the mode of delivery. Very little has been done in the line of content knowledge. The subject advisors to the schools seem to be so busy training

teachers at workshops that they have very little time for teacher support at the school level and sadly the training at these workshops is generic and is more focused on the nature of outcomes based education rather than on content. (Personal communication with two mathematics subject advisors revealed this.)

1.8. The focus of the study

The focus of this research project is two fold. Firstly, it seeks to establish whether teachers who scored well in NPDE examinations actually have a *profound understanding of fundamental mathematics* (PUFM).

Secondly it seeks to find possible common factors in the mathematical life histories of teachers that could have contributed to them both doing well in the mathematics examination and having PUFM.

This research project aims to establish factual knowledge about:

- The **mathematical content – knowledge (subject matter)** that these teachers possess, by investigating whether these teachers were able to demonstrate PUFM.
- The **mathematical pedagogic content knowledge** that these teachers have. Ma (1999) was successful in analysing teachers' content knowledge and pedagogic content knowledge through analysing their responses to the TELT (see methodology) interview questions.
- The **common factors/characteristics** of teachers who achieved well in both the academic test of the NPDE and the TELT test as used by Ma (1999). The focus at this point would be on teachers' backgrounds and the context in which they learnt mathematics.

1.9. Research Questions

1. Do teachers who scored high in the NPDE mathematics examinations have a 'Profound understanding of fundamental mathematics'?
2. Are there any similarities in these teachers' mathematical life histories that appear to have influenced whether or not they have developed a 'Profound understanding of fundamental mathematics'?

In order to answer the first question, I will need to identify:

- What is meant by fundamental mathematics?
- What is meant by a profound understanding?
- What are indicators of this profound knowledge?
- How have the teachers answered the TELT questionnaire?
- What conclusions can be drawn from the way they answered these questions?

This will be dealt with in the first part of the theoretical framework.

Some questions that follow from the second question are:

- Are similarities in their backgrounds:
 - The type of education that they were exposed to?
 - The level of mathematics that they completed at school?
 - The type of mathematical teaching that they were exposed to?
 - The level of school mathematics that they teach?
- What is their ability/affinity for mathematics based on? (What drives it?) Is their ability to do mathematics,
 - based upon natural ability and love for the subject? (What is their personal philosophy about the subject?)
 - based on the good foundation they received from their mathematics teachers at school?
 - based on their hard work and commitment to their studies?

The next chapter seeks to review the published literature available that deals with the concept of mathematical knowledge as it pertains to education, to review teacher identities, and lastly to look at studies that have reviewed life histories of teachers and how these studies could be compared to the this study as well as to review the outcomes of these studies.

Chapter 2: Literature Review

Even a strong belief of 'teaching for understanding' cannot remedy or supplement a teacher's disadvantage in subject matter knowledge (Ma, 1999)

2.1. Introduction

This study looks at a unique set of students, namely the students that did the NPDE mathematics modules through the University of KwaZulu-Natal, Pietermaritzburg. These students are all serving teachers who had been teaching for many years and had done some form of teacher training.

It secondly considers the aspect PUFM. This directs me to readings concerning the concepts of teacher knowledge and understanding. It is therefore imperative that I firstly consider what is written around these issues and then look what is written about the concept of Elementary/ Primary Education with regards teachers' knowledge.

Much has been written, in the field of mathematics educational research, about teacher knowledge. The first part of this chapter therefore aims to review the literature that deals with teacher knowledge by looking at pertinent literature around the concepts of Subject Matter; Teacher Knowledge versus Teaching Ability; Conceptual and Procedural Knowledge; PUFM and SUFM; Privileged Repertoires; Expert Knowledge; Pedagogic Content Knowledge and Mathematical Knowledge for Teaching. It seeks to base the study within these education knowledge discourses. The second part looks at some international and some South African points of view around the issue of teacher knowledge. The third part of this chapter deals with the 'life-histories' discourse and concept of teacher identities.

2.1.1. Subject Matter

There are basically two ways of defining subject matter knowledge. The definition could be given in qualitative terms as given by Dewey (1904) or it could be given by quantitative terms as

given by Wilson, Shulman, and Richert (1987). However, each of these definitions still proposes its own problems. The qualitative view of Dewey did not provide a straight forward way of measuring or evaluating knowledge (Even, 1990). Although the quantitative definition took the numbers of courses taken at college or teachers scores on standardized tests into account, this was still problematic as these ‘measures’ do not represent teachers’ knowledge of the subject matter. Does it mean that if a teacher obtained 50% in a particular mathematics course that the teacher has a mathematical knowledge that is solid (Even, 1990)? If both of these definitions are problematic then how can there be consensus on these definitions?

Even (1990) brings this debate into perspective when he states that:

Defining teachers’ knowledge not by the number of courses they have taken or their success on standardized tests, but by analyzing what it means to know mathematics, has some promise to contribute to the improvement of the quality of subject matter preparation for teachers and therefore the quality of teaching and learning. (p.522)

In the article, *Reaching for Common Ground in K-12 Mathematics Education*, Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid and Schaar reach agreement that:

Teaching mathematics effectively depends on a solid understanding of the material. Teachers must be able to do the mathematics that they are teaching, but it is not sufficient knowledge for teaching. Effective teaching requires an understanding of the underlying meaning and justifications for the ideas to be taught, and the ability to make connections among topics. Fluency, accuracy and precision in the use of mathematical terms and symbolic notation are also crucial. Teaching demands knowing appropriate representations for a particular mathematical idea, deploying these with precision, and bridging between teachers’ and students’ understanding. It requires judgment about how to reduce mathematical complexity and manage precision in ways that make the mathematics accessible to students while preserving its integrity. (Ball et al., 2005, p. 6)

In the above extract, Ball et al. speak of a *fluency, accuracy and precision* in the use of mathematical terms and symbolic notation. Though they write in a US context, this would be no different in South Africa.

Publishing in South Africa but also addressing international research, Van der Dandt and Niewoudt (2003) in their literature review propose that there are two approaches involving research that focuses on teachers' mathematical knowledge and that not only acknowledges the importance of the content of teachers mathematical knowledge but also the quality of the nature of the teachers' knowledge.

The first approach which focuses on characteristics of teachers, assumes that knowledge of and skills with mathematics content are essential to teaching. There are some researchers (e.g. Muijs & Reynolds, 2002) who indicate their disapproval of this approach as it is felt that formal mathematics qualifications cannot be linked to the student results.

The second approach focuses on the understanding of specific mathematical topics, procedures and concepts. It is this notion of quality that is more important than simple knowledge. It is noted in Ma (1999) that the American teachers with much higher 'qualifications' in mathematics, than their Chinese counterparts, did not necessarily possess a profound understanding of fundamental mathematics.

Teachers are unlikely to be able to provide an adequate explanation of concepts or to construct tasks that will facilitate learning of these concepts if they do not understand the concepts themselves. They can hardly engage their students in productive conversations about multiple ways to solve a problem if they themselves can only solve it in one way (National Research Council, 2001).

Weak subject knowledge is a consistent common feature in unsatisfactory teaching, restricting teachers' ability to respond effectively to pupils' difficulties and to make connections with other learning. It also affects the quality of planning and assessment (Ofsted, 2003, p.6)

The question is what "mathematical knowledge for teaching" may be. I explore that in the next section.

2.1.2. Conceptual Knowledge/ Procedural Knowledge/PUFM/SUFM

A common distinction is between conceptual and procedural knowledge (see chapter 3 for further discussions). Ball and Cohen (1999) link *conceptual knowledge* of mathematics to teaching. In that respect, they view it as being the knowledge of the subject matter in such a way that it will enable teaching. Included here are:

- deep conceptual understanding of topics and how they relate to each other; knowing what accounts as knowledge in particular fields and how knowledge is accounted for and justified;
- knowledge of controversies and contestations within fields; and
- knowledge of connections between the field and other fields.

This idea is further explored in the notion of PUFM (see below).

Procedural knowledge on the other hand provides a formal language and action sequences that raise the level and applicability of conceptual knowledge. Procedural knowledge is knowledge of the skills needed to carry out mathematical tasks and problems. There are two parts in procedural knowledge. One consists of knowledge of written symbols as representing some concepts. The other consists of the set of rules, formulas, and algorithms that are used to solve mathematics problems (Lee, undated).

A. PUFM

Liping Ma in her book “Knowing and teaching elementary mathematics” (1999) investigates *teacher content knowledge* and *teacher pedagogic knowledge*. Ma states that “a teacher’s subject matter knowledge of school mathematics is a product of the interaction between mathematical competence and concern about teaching and learning mathematics” (p.146). Liping Ma uses the phrase *Profound Understanding of Fundamental Mathematics* (PUFM). She describes this understanding of content knowledge as, “an understanding of the terrain of mathematics of fundamental mathematics that is deep, broad and thorough.” As I have chosen to use PUFM as part of my theoretical framework, chapter 3 addresses what this implies.

Ma suggested that teachers with PUFM make connections between mathematical concepts and procedures from the simple to the complex, appreciate different facets of an idea and various

approaches to a solution, are particularly aware of the simple but powerful foundational concepts and principles of mathematics, and are knowledgeable about the whole primary mathematics curriculum, not just the content of a particular age level. This is supported by an article by (Mooney, Fletcher and Jones, 2003).

Ma uses four questions that were developed for the TELT project to do her analysis. The methodology within her study is useful to this research in that it provides a definite instrument to measure teachers' *Profound Understanding of Fundamental Mathematics*. This methodology focuses on fundamental knowledge, which is the focus of primary/intermediate phase teachers.

However, her study was designed as a comparative study of the Chinese and American mathematics teachers. Her findings, whilst interesting, fundamentally indicate that the American teachers with high levels of education lacked knowledge of fundamental mathematics principles. She on the other hand found that the Chinese teachers, with lower education qualifications, had a more profound understanding of fundamental mathematics. This was partially a result of stronger conceptual focus in their own teaching, partially due to the professional self-development which the Chinese teachers considered an integral part of their work.

B. How did the Chinese teachers that Ma interviewed gain PUFM?

Ma (1999, p.129) makes the following statement based on her findings, "It seems to be that PUFM, which I found in a group of Chinese teachers, was developed after they became teachers – that it developed during their teaching careers." The following is a summary of her views about how PUFM was attained. She states that they could have attained it through:

- **Studying teaching materials intensely**

The Chinese teachers refer to *zuanyan jiaocai*. This term refers to the three main components namely, the Teaching Learning Framework, textbooks and the teachers' manual. The Teaching and Learning Framework is published by the Chinese National Department of Education. It is a document similar in some ways to the American National Council of Teachers of Mathematics' Standards for School Mathematics (NCTM, 1989). The quality of the textbooks is strictly controlled by the Chinese National Department of Education. Together with each set of books are teachers' manuals that are

meant to provide the background of the knowledge in the corresponding textbooks and how to teach this knowledge. The Chinese teachers study these three kinds of materials.

- **Learning Mathematics from colleagues**

Chinese teachers not only study the materials on their own. Through interaction with their colleagues they develop a deeper understanding of both the knowledge and teaching of school mathematics. The Chinese teachers are organised in *jiaoyanzu* or ‘teaching research groups’. These groups normally meet once a week to share ideas and reflections on teaching. Here the main activity is to study the teaching materials. Through these research groups and the interaction of the teachers, they learn from each other. Ma (1999, p.138) makes the following comment about this, “sharing ideas with colleagues, increases one’s motivation to study and make ideas clearer and more explicit. In addition, group discussion is a context where one is easily inspired.”

- **Learning Mathematics from students**

This is not an expectation in teaching in South Africa, where teachers are expected to be the authority within the classroom situation. Teacher Mao, one of Ma’s (1999) interviewee’s makes the following statement, “A good teacher can learn from his or her students to enrich himself. Sometimes the way of solving a problem proposed by a student is one I have never thought about, even though I have taught elementary school for several decades.” It is a given that teachers cannot hold all of the answers and that learners’ intuition and ingenuity must be acknowledged.

- **Learning Mathematics by doing it**

Ma stated that doing mathematics was a hot topic for the Chinese teachers whom she interviewed. For them, solving one problem in many different ways seemed to be an indicator of ability to do mathematics. Ma (1999, p.140) quotes one of the teachers as stating,

My knowledge of mathematics improved substantially after I became a teacher. When..., I had very little knowledge of elementary mathematics. – One way I have improved my mathematical knowledge is through solving mathematical problems, doing mathematics... To improve myself I first of

all did in advance all the problems which I asked my students to do. Then I studied how to explain and analyse the problems to the kids.

Teacher Wang in the quote above highlights the fact that many teachers have improved their mathematical knowledge by doing the mathematics. He did not stop at only being able to do the mathematics but also studied how to teach, i.e., the PCK linked to this mathematical knowledge. Ma (1999, p.141) comments that mathematics teachers should go back and forth between doing the mathematics and clarifying what it is that he is doing or teaching. This once again clearly demonstrates the link between a teacher's subject knowledge and the knowledge for teaching that is required by teachers.

Ma (1999, p.141) concludes this discussion by making the following statement about teachers' subject matter knowledge:

A teacher's subject matter knowledge of mathematics, which develops under a concern of teaching and learning, will be relevant to teaching and is likely to be used in teaching.

Here the concept of relevance is broached. Through the back and forth process of doing mathematics and then considering how to teach it, the teacher not only develops a procedural understanding of the topic but also, by having to look at teaching strategies and linking it to previous knowledge and future knowledge, forms a deeper understanding of the topic.

C. SUFM

The question that suggests itself is how profound the mathematical knowledge has to be. Writing in a British context, Mooney, Fletcher and Jones (2003) introduce the concept of SUFM in response to Ma's PUFM. Their study suggests that a minimum exit requirement for generalist primary teachers should be that they are able to demonstrate a *Sufficient Understanding of Fundamental Mathematics* (SUFM).

They use as their bench mark the proposal made in *Adding IT Up: Helping Children Learn Mathematics* (Kilpatrick, Swafford and Findell, 2001) that the definition of mathematical

proficiency for students contained there could be taken as mathematical sufficiency in trainees. These include the following:

- Conceptual Understanding – comprehension of mathematical concepts, operations and relations
- Procedural Fluency – skill in carrying out procedures flexibility, accurately, efficiently and appropriately
- Strategic Competence – ability to formulate, represent, solve mathematical problems
- Adaptive reasoning – capacity for logical thought, reflection, explanation and justification
- Productive Disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one own efficacy

(p.115)

Whilst I acknowledge that this is a possible form of understanding that the teachers in this research could possess, I am choosing to focus on the concept of PUFM.

D. Privileged Teaching Repertoires

As I am dealing with the effects of teacher education, I looked to study the links between initial teacher education and classroom teaching. Closer to home than the work discussed earlier, Paula Ensor from the University of Cape Town set out “to provide a theoretical account of the recontextualising of pedagogic practices by beginning teachers” (Ensor, 1999, p.2). Her starting point was that “it remains a recurrent concern within studies of teacher education and classroom teaching that teachers do not put into practice the repertoires they acquire, on teacher education courses.” (p.2)

Ensor (1999, p.11) states that access to recognition and realisation rules seemed to be the overwhelming factor that affected recontextualisation. In the classroom practices observed by Ensor and in interviews with the teachers, teachers were not able to demonstrate access to principles of selection, production or evaluation of the privileged repertoire. These teachers said they could not produce tasks like those introduced to on their mathematics course, tasks that embedded a particular view of mathematics teaching.

So what does this mean to my study? Let me firstly discuss the notion of a ‘privileged teaching repertoire’. Ensor takes the notion of a repertoire from Bernstein (1996). ‘Teaching repertoire’ refers to the set of practices from which teacher educators (and teachers) draw in the elaboration of their pedagogic practice. In the context of this study, I, like Ensor, will view it as privileged when it incorporates a particular selection of mathematical content and pedagogic resources for the production of mathematical tasks and the arrangement of these into lessons. Although her study looked at student teachers and their repertoires, it is clear that recontextualisation did not occur effectively when this repertoire was limited. This concept of a ‘privileged teaching repertoire’ also expands on the notion of PUFM by linking it to the ability to recognise what counts as mathematical knowledge and applies this to the context of teaching. This thus links to my study in that it asks how much effective teaching occurs when teachers possess a limited PUFM. For this study the relationship between a ‘privileged repertoire’ and PUFM is relatively strong. The question begs: if you do not have a ‘privileged repertoire’ can your understanding be profound?

E. The Role of Subject Expertise

As a demonstration of the fine line that distinguishes subject knowledge/content knowledge and pedagogic content knowledge as defined by Shulman (1986), I discuss a study done by Schempp, Manross & Tan (1998). These authors looked at the role of *subject expertise* and teachers’ knowledge. Their study explored the role of subject matter expertise in teaching. The purpose given by the authors for the study was that it was to ascertain the influence of content expertise on teachers’ pedagogical content knowledge. One of their findings and an important one for me was that “*When teaching subjects in which they were expert, the teachers were more comfortable and enthusiastic regarding their pedagogical duties and could accommodate a greater range of learner abilities.*” The relationship here is that if teachers do not have PUFM, then they will not be as comfortable in teaching mathematics. This then influences their attitudes and disposition towards the subject.

Let us now look further at what is written about knowledge required for the teaching of mathematics. This knowledge is given different names by different authors.

2.2. Pedagogic Content Knowledge / Mathematical Content Knowledge for Teaching & Expert Knowledge

The following discussion will look at what some authors have written around these issues. I have selected the following articles for the value I believe that they would add to this study.

2.2.1. Pedagogic Content Knowledge

Grossman & Yerian (1992) reviewed the international literature on pedagogical content knowledge and identified four kinds of studies in this area: (1) studies which tried to define and identify pedagogic content knowledge in teachers' thinking; (2) studies of the relationship between teachers' pedagogic content knowledge and classroom teaching; (3) studies of the relationship between teachers' pedagogical knowledge and student learning; and (4) examinations of the sources of pedagogical content knowledge. My study combines aspects of the first two kinds in that it seeks to identify the extent of teachers' mathematical understanding in relation to subject matter knowledge and pedagogical practice. In the life history part of my study, I deal with aspects which fall within the fourth kind of study.

Tall (2001) on the other hand looked at the concept of what mathematics is needed by teachers of young children. The focus of his article is not only on the kind of knowledge that teachers need for their own competence and confidence in mathematics, but also on how mathematics develops in the individual so that the teacher may be supportive in the long term development of the child. Tall reviewed the concepts of teacher confidence and teacher competence and noted that competence cannot be solely judged by the teachers' knowledge of mathematics itself but rather the quality of understanding that these teachers have for the task of teaching mathematics to the young. It is not only sufficient for these teachers to be able to do the processes in the mathematics that they teach but also to be able to engage the underlying principles.

2.2.2. Mathematical Knowledge for Teaching/ Mathematics for Teaching

A. Mathematical Knowledge for Teaching

The leading South African work in this area is dominated by the concept *Mathematical Knowledge for Teaching* (MKfT) (Kazima & Adler, 2006). Kazima and Adler go deeper into the concept of pedagogic content knowledge (PCK) as defined by Shulman (1986; 1987), namely that PCK “goes beyond knowledge of the subject matter per se to the dimension of subject matter for teaching” (1986, p.9). So instead of pedagogic knowledge for teaching, they look at the mathematical knowledge that is needed for teaching. This is a very broad section of work and therefore there has been many works that have been produced that look at specific sections of mathematics. Some of these are mentioned later.

They align the concept MKfT with the works of researchers such as Marks (1992), Even (1990), Ma (1999), Ball et al. (2004) and – locally - Brodie (2004) as these researchers seem to agree that teachers need a special kind of mathematical knowledge for teaching in order to teach well.

They argue that mathematical-knowledge-for-teaching might be regarded as a distinct branch of mathematics. Within the backdrop of the complexity science, they introduce four aspects of mathematics-for-teaching, namely,

- Mathematical objects
- Curriculum structures
- Collective dynamics
- Subjective Understanding

They believe that these concepts should be key principle around which teacher education should be organised. I will not delve into the explanation of these concepts as they do not have a bearing on this study.

B. Mathematics for Teaching

Expert knowledge systems provide a framework for differentiating relevant cues and attending to more salient information during planning and interactive decisions (Carter, Sabers, Cushing, Pinnegar & Berliner, 1987; Livingston & Borko, 1989). While obviously closely related to Bernstein’s notions of recognition realization, in my view this concept (Expert Knowledge) also

links closely to the principles of PCK, as it speaks of knowledge of mathematics that is more than the knowledge required by the non-educator. This form of knowledge is guided by the requirements of teaching. To me, teaching mathematics is a specialist field and thus it requires persons who teach this subject to have knowledge of mathematics that is specific to the teaching of mathematics.

Ebhart (1995) looked at how a US student teacher enrolled in an elementary mathematics content course came to terms with conceptual knowledge. This study was achieved through the study of one student's journal entries. Ebhart notes that throughout her journal, Elna, the student, makes note that mathematics should make sense and that doing mathematics should be largely a sense-making proposition. Ebhart states in her conclusion that the task of investigating the construction of conceptual knowledge and the subsequent transformation of that subject-matter into pedagogical content knowledge is extremely complex. It is not as tangible as a test or examination result and thus leaves this type of investigation open to author subjectivity. This study uses a different methodology to my study but it has a close resemblance in that it also uses the student voice through the medium of the journal as a means to accessing information about the students' experience, belief and views. Many of the authors who write around these concepts deal with specific sections of mathematical knowledge. In many of the following articles they speak of teacher knowledge and then relate it to a specific section in mathematics.

Even's (1990) study looked at subject matter knowledge that teachers needed to have to teach functions. There are many others who have carried out similar works some of these are: Marks (1992) worked on equivalent fractions; Stacey, Helme, Steinle, Butaro, Irwin & Bana (2003) have worked on 'decimal numeration'; Kazima and Adler (2006) worked on probability.

Unlike the above mentioned articles, this study does not look at knowledge of a specific section of mathematics but is concerned with the knowledge that teachers need in order to teach mathematics *effectively* at an elementary level. It also seeks to establish whether the mathematical knowledge that they bring into teaching displays a profound understanding of key mathematical principles. It therefore examines the concept of quality of knowledge. This study however seeks to look at teachers' mathematical knowledge more than pedagogical content

knowledge. Although these concepts are very closely related, it is the difference in emphasis that distinguishes them.

2.3. The South African Point of View

The following sections look to review articles written by South African authors that are related or have a bearing on this study. I will firstly explore a few articles about teacher knowledge and then look at articles written about the NPDE program and lastly look at articles that although not directly related to the topic, have aspects in common with my study itself.

2.3.1. Articles about Teacher Knowledge

The majority of South African studies document teachers' lack of content knowledge for example (Legotlo, Maaga & Sebego, 2002; Mashile, 2001; Sibiya & Sibiya, 1996) or of pedagogic content knowledge in theory (Huckstep, Rowland & Thwaites, 2005; Mapolelo, 1999). Of these, none deal with the more profound understanding of content knowledge. Overall, the findings reveal lack or falling short of what is desirable and necessary for teaching.

Karin Brodie (2001) looked at what resources (material and knowledge) mathematics teachers need in order to work more confidently with the ideas of curriculum 2005. She considered a teacher's mathematical knowledge, the teacher's knowledge of learning and learners, and the teacher's knowledge of pedagogy in relation to the shift to learner-centred teaching.

In Brodie (2001) she argued that teachers' mathematical knowledge and their mathematics teaching practices are mutually constitutive; that is each one shapes and constrains the other, while remaining distinct analytical objects. The two concepts are then linked to form what Brodie termed *thinking practices*. She stated that the teachers' thinking practices include their mathematical knowledge and practices and their pedagogical content knowledge, which are resources for enabling their thinking practice. Her conclusions drawn in this article suggest firstly that we cannot think about mathematical knowledge in a vacuum, divorced from the notions of practice and secondly that we cannot teach mathematics to teachers without talking about practices; mathematical and teaching practices.

Suriza van der Sandt and Hercules D. Niewoudt (2003) investigated the geometry content knowledge of eighteen Grade seven teachers and one hundred prospective teachers in South Africa. Their results indicated that both the teachers and the prospective teachers failed to reach the level of geometric thinking and degree of acquisition expected from successful teachers. Although this study looked primarily at geometry it revealed the broader notion namely that there truly is a problem with the quality of teacher knowledge in the South African education system.

2.3.2. Articles about the NPDE programme

I will now look at a two articles written about the NPDE program specifically.

Wildeman (2000) looked at teacher training programs, specifically the NPDE. Although this article was written in 2000, it revealed the reasoning behind the establishment of such a qualification. This article also highlighted the role that this qualification was intended to play in reducing the number of under-qualified and unqualified teachers within the public schooling system. Wildeman also revealed the fact that this course was funded to the tune of R50 million (approximately 6, 2 million US dollars) for bursaries for 10 000 teachers in the rural areas to complete the NPDE. This money was however extended to students from the urban and peri-urban areas as well. Wildemans' (2000) article provided an important backdrop to this study as it gave a good explanation of what the purpose, function and reasoning for the NPDE are. However, it did not engage the knowledge of teachers or the sources of their learning.

David Ngidi's (2005) article evaluated the effectiveness of the competences of the NPDE program. He looked at the components set out in the Norms and Standards for Education (2000) document. The competences are related to each of these components. He concluded from his findings that the educators were satisfied with the help that the NPDE program offered them. He felt that they were equipped with competences they need in their teaching career. This is counter to my personal experiences with the program which motivated this study.

2.3.3. Articles which address Teachers' Knowledge Indirectly

Andile Mji & Moses Makgato (2006) looked at factors associated with high school learners' poor performance with a spotlight on mathematics and science. Their results indicated that there

were two main factors identifiable. The first factor is related to teaching strategies, content knowledge of teachers, motivation, laboratory use and non-completion of the syllabus in a year. The second factor identified associated with direct influences, was the role played by parents in their children's education and general language usage² together with its understanding in the two subjects.

2.4. Identities of Mathematics Teachers

The next part of the study looks at the identities of mathematics teachers. There seems to be a growing interest among researchers on the subject of teachers' identities. Among these researchers are Casey (1993), Coldron & Smith (1999), Connelly & Clandinin (1999), Middleton (1993), and Osler (1997). From the developing countries there are articles by Carrim (2001), Dhunpath (2000), Jansen (2001), Jita (1999, 2004), Samuels (2001), Matheson & Harley (2001), Olser (1997) and Soudien (2001).

In an attempt to look at researchers closer to home I looked at Parker (2004; 2006) and Jita and Vandeyar (2002). Parker (2004; 2006) identified three different mathematically related identities that novice teachers need to develop (2004). These she described as

- an identity as a *mathematician* (learning mathematics [content knowledge] – becoming a mathematician);
- an identity as a *student of mathematical education* (becoming someone who is interested in pursuing studies in mathematics education and learning from research) and
- an identity as a *mathematics teacher* (becoming someone who can utilise their knowledge of mathematics, mathematics education and education more generally to help learners develop productive mathematical identities that keep them interested in and motivated to learn the discipline at higher levels). (Parker, 2004)

It is implied that a teachers' *Self-Identity* has a major influence on his/her teaching. If a teacher identifies himself/herself more as a mathematician, emphasis will therefore be placed on his/her

² Teacher education generally was and is conducted in English or Afrikaans, which means that the majority of students learn in a language other than their mother tongue. This obviously adds to comprehension difficulties, etc.

content knowledge and the need for improving it. If the teacher identifies more with the identity of student of mathematics education he will spend more energy on developing his pedagogic content knowledge. The identity as a student of mathematical education speaks to the point also raised by Ma, that Chinese teachers continue to be students of mathematics who study textbooks, investigate mathematics with peers, solve mathematical problems, etc. (Ma, 1999, pp.148-9). The teacher that relates more to the identity of the mathematics teacher will incorporate his/her content knowledge and pedagogic content knowledge in innovative ways to make the teaching experience more meaningful. The ideal teacher needs to have a good balance of each of the above identities.

Jita and Vandeyar (2006) looked at teachers' identities through investigating their life histories and observing their classroom practices. Their study "examines the construction of two teachers' mathematical identities" (2002, p.39). By using the contradictions between the reformers (the designers of the Curriculum 2005 document) and the teacher' accounts of their lived experiences and identities, they constructed an account of why the teachers are still not engaging with the latest set of reform proposals. As stated, their article presented an empirical study of two primary school teachers of mathematics working in two former 'white-only' schools. Jita and Vandeyar explored the teachers' mathematics identities from the teachers' accounts of their life-histories and secondly sought to account for how it is that the classroom practices of many South African teachers in primary schools remain untouched by the recent reforms in the curriculum. It was found in this study that the two primary school teachers although having had comparable experiences of learning mathematics in their own schooling, which were generally weak and traditional, they constructed radically different classroom practices for their learners and also constructed fairly different identities from their early and somewhat similar experiences.

Parker's article and Jita & Vandeyar's article add the dimension of teacher identity to this study. Teacher identity as mentioned above plays an important role in helping researchers understand teachers 'knowledge' and 'understanding'. It can also therefore not be ignored when looking at teachers life histories. I however did not go to deep into analyzing teachers' identities as I believe that this would have shifted the focus of the study and also make the study to big. The teachers' identities will be looked at in relation to their answers to the interview. Although it is not the main focus of this study the identities clearly present themselves in the analysis of the

teacher interviews. I therefore mention the identities that revealed themselves although I did not specifically set out to find out how these teachers developed identities of themselves as teachers.

2.5. Life - Histories

2.5.1. An over view of Life -Histories

Life History research methodology falls under the broader Historical research methodology. Life History methodology has the potential to reveal much of what teachers believe and accept as good praxis in relation to their knowledge of content of mathematics and for the teaching of mathematics. Cooney (1999, p.170) argued on both empirical and philosophical grounds that what teachers learn is framed by the context in which that knowledge is acquired. Thus, deeper analysis of teachers' learning could reveal much about how teachers acquire knowledge. Here, two links can be made, firstly there is need to review how their learning took place, and secondly how they could have acquired PUFM.

This form of research is very broad so the goal of the research has to be clear from the outset. This study only looks to finding out about their *mathematical experiences* from their first memories to their most recent.

Plummer (1983, p.14) viewed life histories as being a full length book about one persons' life in his or her own words. Plummer stated that information is gathered over a number of years with the researcher providing gentle guidance to the subject, encouraging him or her to either write down episodes of life or to tape record them. These materials often are backed up by observations of the subjects' life, with interviews of the subjects' friends and acquaintances and with close scrutiny of relevant documents such as letters, dairies and photographs. Essentially, the life history is an "interactive and co-operative technique directly involving the researcher" (Plummer, 1983, p.139).

Goodson argued that life-histories "have the potential to make a far reaching contribution to the problem of understanding the links between 'personal troubles' and 'public issues', a task that lies at the very heart of the sociological enterprise" (2003, p.4). Their importance, he asserts, is

best confirmed by the fact that “teachers continually, most often unsolicited, import life history data into their accounts of classroom events” (Goodson, 1981, p.69.).

When it comes to the life histories of mathematics teachers in South Africa, I have not located a single study addressing the topic of PUFM or conceptual mathematical knowledge. The extent to which general studies of the life histories of teachers would be relevant to this study depends on whether or not they address the teachers’ disciplinary and pedagogical content knowledge. I have been unsuccessful in locating any South African studies of this nature.

There are a number of studies from abroad researching life histories of mathematics teachers, but they mostly deal with novice teachers, while I am interested in the knowledge of in-service teachers. An example of life history literature available is that of Goodson (2003) where he discussed the importance of doing research into the life histories of teachers but his article is too broad for the topic I am wanting to research.

I will now firstly review two South African articles and then review an Australian article dealing with life history studies that I believe are relevant to my study.

2.5.2. South African Articles about Life Histories

I found the interview between Herbert B. Khuzwayo and Dr. W.S Mpofana, which can be found in the thesis by Khuzwayo titled *Selected Views and Critical Perspectives: An account of mathematics education in South Africa from 1948 to 1994*, methodologically very similar to this study as it revealed much of the mathematical life history of Dr Mpofana. The interview covered his experiences from behind the desk as pupil to his present position. (At the time the thesis was written, he was senior subject advisor in the Southern KwaZulu-Natal region.) Khuzwayo used this interview to look at how apartheid education affected Dr Mpofana and how he managed to overcome these hurdles. This interview was not the main feature of the study but it played a significant role in helping Khuzwayo understand mathematics education as it occurred from 1948 to 1994. Thus, his study delves deeper into apartheid education, and this is where it deviates from my study.

Nkhoma (2002) aimed to learn from students and teachers in 'Black' schools, as to what classroom practices lead to success in school mathematics, in their impoverished contexts. This study has significance to me in that it not only looked at the views of teachers but also of successful Black learners. It tied in closely with the second question of this study, in that the study is looking at students who have achieved well in the NPDE examination.

His study revealed the following themes from the responses of the teachers and the students. They cited the following as being enabling: Extra classes; The teacher being friendly to us; Provided extra resources; Working in groups; The teachers' preparedness in class; The teacher used practical examples; Availability of the teacher; Encouragement/Motivation; The teacher demonstrating/exhibiting a profound understanding of school mathematics; Active participation allowed by teacher; Language used in class of suitable standard; Homework; Tests and Competition. This has informed the questions I asked the participating teachers about their life histories in the interview. These questions are discussed later in chapter four.

2.5.3. International Articles

An Australian Life History Study

In a paper titled *Understanding Professional Development through the Analysis of the Mathematical Life Histories of Primary School Teachers* (1998), Jean Carroll reported on an Australian study of the development of primary school teachers' views about mathematics teaching and learning. The study she reported on arose in response to widespread concerns about the teachers' knowledge and attitudes related to mathematics teaching and learning. This study analysed the mathematical life histories of five teachers in suburban Melbourne schools in order to identify the factors leading to professional development during their careers. A number of themes were identified in the life histories and these were used to make recommendations for effective planned professional development.

The study identified three factors:

- feelings about teaching mathematics (factor F)
- knowledge and feelings about doing mathematics (factor M) and
- knowledge of mathematics teaching (factor K).

Factor M and factor K differ in that factor M is about their ability to do mathematics (Parker’s Identity, teacher as mathematician) and factor K looks at their ability to teach mathematics (Parker’s identity, teacher of mathematics).

The teachers’ factor scores indicated whether they viewed themselves positively or negatively with respect to these factors. Teachers were considered to have a negative factor tendency if their score was less than the mean on each specific factor and a positive factor tendency if their score was greater than the mean score on the specific factor. A profile of each teacher was developed by combining the teachers’ factor tendencies. The following table was developed by Jean Carroll to clarify the Positive and Negative factor tendencies:

Table 4 Carroll’s Profile of teachers

	Positive	Negative
Factor F Feelings about teaching mathematics	F+ Positive feelings about teaching Mathematics including confidence, Enjoyment, excitement, challenging and finding it non- threatening.	F- Negative feelings about teaching mathematics including lack of confidence, lack of enjoyment and finding it threatening.
Factor M Knowledge and feelings about doing mathematics	M+ Knowledge and feelings about doing or studying mathematics are positive; have done well in mathematics; better in mathematics than other subjects and finds mathematics problems interesting and challenging.	M- Knowledge and feeling about doing or studying mathematics are negative; have not done well at mathematics; mathematics is not the best subject and find doing mathematics problems frustrating.
Factor K Knowledge of Mathematics teaching	K+ Knowledgeable about the methods and approaches for teaching mathematics to primary school children.	K- Lacking in knowledge about the methods and approaches for teaching mathematics to primary school children.

An example of this would be the classification F-M-K-. This classification type can be explained as following:

- F- Teacher experienced negative feelings about teaching mathematics including a lack of confidence, a lack of enjoyment and found it threatening
- M- The teacher's feelings about doing and studying mathematics are negative; has not done well at mathematics, mathematics is not best subject and finds doing mathematics problems frustrating.
- K- Experienced a lack of knowledge about the methods and approaches for teaching mathematics to primary school children.

It is therefore possible to categorise teachers into any of the Teacher Types as defined by Carroll (1998). She found that she could place teachers into one of eight different categories, namely F-M-K-; F-M-K+; F-M+K+; F+M+K+; F+M-K-; F+M-K+; F+M+K-; F-M+K-. Carroll's study is very close to this study firstly in that they both investigate life histories of primary school teachers teaching mathematics. Secondly they both seek to identify factors that could have led to *professional* development and competence in the doing and teaching of mathematics. They differ in that I have not tried to categorise the teachers as Carroll did, but instead looked to the factors behind their feelings, attitudes and knowledge development. I however did use the themes she used in her analysis to assist me with my analysis as I felt these themes seemed to be very relevant and they came through naturally within the teachers' responses.

Conclusion

The literature reveals that although there may have been similar studies to this one done earlier, there is still a definite place for this study in that its uniqueness lies in its link between PUFM and Mathematical Life Histories. This study draws from the views of numerous authors and researchers.

The next chapter aims to place the theory and concepts dealt within this study. It reveals these and discusses their significance to the questions asked by this study.

Chapter 3: Conceptual Framework

This chapter will aim to ground the study in conceptual theory. It will review the concepts dealt with and show their significance to the study. Many of the concepts dealt with are those mentioned by Ma (1999). The first of these concepts I would like to deal with is that of PUFM. Two elements of PUFM will then be further expounded upon: Depth and Complexity. The third area to be looked at is the linked concepts of Procedural and Conceptual Understanding. The fourth concept is the concept of elementary mathematics as it applies in this context. Lastly, I draw out from the literature framework for analyzing the mathematical life histories of mathematics teachers.

3.1. A profound understanding of fundamental mathematical knowledge

Liping Ma in her book *Knowing and Teaching Elementary Mathematics* uses the phrase *Profound Understanding of Fundamental Mathematics*. She described this understanding as, “an understanding of the terrain of mathematics of fundamental mathematics that is deep, broad and thorough.” (p.120) Liping Ma links the concepts of depth, vastness and thoroughness to Duckworth’s views namely that “we should keep learning of elementary mathematics and science ‘deep’ and ‘complex’ ” (1987, p.44).

A further look at the term ‘profound’; it is usually considered to mean intellectual depth. Ma (1999) mentions that this lead to three connotations namely deep, vast and thorough. She viewed these terms as been interconnected. Ma defined ‘understanding a topic with depth’, as connecting it with more conceptually powerful ideas of the subject. She defined ‘understanding a topic with breadth’ as connecting the topic with topics of similar or less conceptual power and views ‘thoroughness’ as the ‘glue’ that ‘glues’ knowledge of mathematics in a coherent whole (p.121). There seemed to be two schools of thought around depth of teachers’ knowledge. Researchers like Grossman, Wilson & Shulman, 1989; Marks, 1987; Steinberg, Marks, & Haymore, 1985;

Wilson, 1988, all agree that teachers' understanding should be deep. Ball (1989) however views depth slightly differently from Ma and the others in that to her depth is "vague" in that it is "elusive in its definition and measurements". Ball proposed that we look at depth through linking it to *correctness; meaning; and connectedness*.

Ball (1990) noted the following about depth and flexibility of teachers' knowledge. She stated that,

Teachers should understand the subject in sufficient depth to be able to represent it appropriately and in multiple ways – with story problems, pictures, situations and concrete materials (Ball, 1990).

They need to understand the subject flexibly enough so that they can interpret and appraise students' ideas helping them to extend and formalize intuitive understandings and challenging incorrect notations (Ball, 1990).

Ma (1999) extended this idea and said that a teacher with PUFM should have:

- **Connectedness.** These teachers have a general intention to make connections among mathematical concepts and procedures, from simple and superficial connections between individual pieces of knowledge to complicated and underlying connections among different operations and sub-domains. Such teachers, Ma says, help their learners to see mathematics as a unified body of knowledge rather than fragmented through learning isolated topics.
- **Multiple perspectives.** These teachers appreciate different facets of an idea and various approaches to a solution, as well as their advantages and disadvantages. Such teachers lead their students to developing a flexible understanding of the mathematics discipline.
- **Basic Ideas** (or rather, awareness of central concepts and principles)
These teachers are teachers that display mathematical attitudes and are particularly aware of the "simple but powerful basic concepts and principles of mathematics" (Ma 1999, p.122). These teachers tend to revisit and reinforce these basic ideas.
- **Longitudinal coherence.** These teachers are not limited to the knowledge that should be taught in a certain grade. They have a fundamental understanding of the whole elementary mathematics curriculum, (Ma, 1999, p.122). They are ready at any time to review crucial concepts that students have studied previously. They also know what

students are going to learn later, and take opportunities to lay proper foundation for this knowledge.

If teachers are using algorithmic methods in their teaching, that is, giving learners a finite set of rules and never explaining the origins of these rules nor the reasons for using the rules, then the learners will never be exposed to the depth of the subject. Such teachers never allow their learners to be exposed to multiple perspectives. Learners learn one method only.

Secondly, learners will begin to view the subject as consisting of series of rules which need to be learned in order to achieve. These rules (algorithms) will have little or no connection with other topics. Learners will not see the interconnectedness of the subject and thus never note the complexity of mathematics. I note, many times the mastery of algorithms is perceived to be the attainment of mathematical knowledge – this is reflected in examination papers which call for calculations rather than explanations, generalizations, proofs, etc. This fallacy is perpetuated by the examination system that demands that learners are able to achieve a certain amount of points/marks in order to pass. This normative type of assessment still exists in South African Education today. What this type of examination fails to test is whether a learner has attained mathematical knowledge which is (deep, complex knowledge) or simply knowledge of algorithms. In many cases this knowledge of algorithms is mastered and reproduced.

In order to draw conclusions about teachers' PUFM, an investigation of the teachers' ability to make interconnections between mathematical concepts had to be done. Liping Ma states: "When it [mathematics] is composed of well-developed, interconnected knowledge packages, mathematical knowledge forms a network solidly supported by the structure of the subject." (p.113)

These knowledge packages could demonstrate the ability of the teacher to make links with past, present and future elements of a specific aspect of fundamental mathematics knowledge. These links should demonstrate the ability of teachers to use knowledge of concepts previously dealt with (background knowledge) in the concept that they are presently teaching and in sections that may require this knowledge in the future. The ability to make these links could demonstrate what Ma termed 'longitudinal coherence'.

Whilst it is accepted that the designing of knowledge packages – as Ma (1999, p.21) got her participants to do – will reflect coherence, this study did not set out to do such but looked simply at whether the participants were able to make connections with both past and present knowledge that linked to the given topics. It was felt that getting the participants to draw up knowledge packages would be time consuming and the related analysis beyond a Masters project.

This study therefore aimed to highlight the concepts of depth, vastness and thoroughness by using Ma's four concepts, namely Connectedness, Multiple Perspectives, Basic Ideas and Longitudinal Coherence as the key concepts that will determine whether one could, through analysing the teachers' responses, determine whether they have PUFM.

3.2. Procedural and Conceptual Knowledge

Ma also entertains the concepts of procedural and conceptual knowledge. How can these be defined? Hiebert and Leferve (1986) defined conceptual knowledge as knowledge that is “rich in relationships” and a “connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information” (p.6). Conceptual knowledge is thus closely linked to meaning and meaningful learning.

Procedural knowledge on the other hand encompasses knowledge of the syntactic structures or symbolic systems of mathematics, and the rules, algorithms or procedures that are used to manipulate symbols in order to solve mathematical tasks. Even (1990), stated that procedural knowledge can be learned with or without meaning. Skemp's (1976) study referring to relational and instrumental understanding is similar to the above, in that to him instrumental understanding is knowing ‘what to’, “rules without reasons” , while relational understanding is knowing both what to do and why you are doing it. It should be noted here that whilst these forms of knowledge (procedural and conceptual knowledge) can be distinguished analytically, they are inextricably intertwined (Hiebert, 1986).

The distinction between procedural and conceptual understanding was used in analyzing the answers given on the questionnaires by the participants as they could be indicators of the type of

knowledge that these teachers have. It forms part of the analysis in that it would lead to assessing whether a participant displayed a form of PUFM. This will be discussed in a chapter 5.

3.3. Elementary Mathematics/ Foundational Mathematics

Ma viewed elementary mathematics as being *fundamental mathematics*. Ma (1999, p.124) made the following comments about Elementary Mathematics. Elementary she stated can be viewed as basic mathematics – a collection of procedures – or as fundamental mathematics. Fundamental mathematics is elementary, foundational and primary.

- It is elementary because it is at the beginning of mathematics learning.
- It is primary because it contains the rudiments of more advanced mathematical concepts.
- It is foundational because it provides a foundation for students' further mathematics learning.

Historically, mathematics was based around the study of geometry and arithmetic. Even though today the field of mathematics has advanced and expanded, it is still accepted that the foundational fields are arithmetic and geometry. Ma stated that elementary mathematics that is composed of arithmetic and primary geometry is foundational of the discipline on which advanced branches are constructed (Ma, 1999, p.116).³

3.4. Life Histories

Plummer (1983, pp.108 and 109), discussed the three main types of life history writing suggested by Allport (1942), these being the comprehensive, the topical, and the edited. He extended these to include:

- The *comprehensive life document*, which purports to grasp the totality of a person's life.
- The *limited life document*, which aims to confront a particular issue in the person's life.
- The *comprehensive topical personal document*, which organises the material around a special theme that is not related to an overall life.

³ From a South African perspective it is noted that Ma speaks of elementary mathematics, but this does not imply the Elementary phase of the South African schooling system. Ma's elementary mathematics would certainly also be relevant content in the Intermediate and Senior phase.

- The *limited topical personal document*, usually aims at throwing light on a highly focused area of life.

This study falls into the definition of a limited topical personal document since it focused primarily on the teacher's mathematical life histories.

As discussed earlier in this study (Literature Review Chapter), Jean Carroll (1998) identified themes from the teachers' life histories that I believe that are very relevant to this study. Carroll's aim in her analysis was to identify events, experiences and individuals that contributed to the professional development of the teachers.

Carroll (1998) identified the following themes in her analysis of the teachers' life experiences:

- **Experiences as Students:** This theme is significant as it reveals much about the teachers' attitudes towards the mathematics, their views of their knowledge of mathematics, and assists in determining their teacher types.
- **Teacher training:** This theme reviews the type of training that these teachers received as students and how it has impacted on their teaching.
- **Knowledge of mathematical teaching:** This theme plays an important role in that it exposes the teachers' views about teaching the subject, both positive and negative aspects. It also highlights strengths and weaknesses that the teachers perceive in their ability to teach mathematics.
- **Personal Philosophies:** Can be explained to be the expression of positions that convey a sense of coherence in self-understanding regarding teaching practice and personal history.
- **Significant Influences:** This concept refers to anyone who played a significant role in the teachers' development and understanding of mathematics.

3.5. Conclusion and Conceptual framework

This study was framed on three major educational conceptual frameworks. The first concept is teacher knowledge (discussed earlier). This concept is used to answer the first part of this study in that it allows for the investigation of PUFM. The teachers' PUFM will be analysed through

using the four indicators used by Ma (1999, p.122) namely Connectedness; Basic Ideas; Longitudinal Coherence and Multiple Perspectives. These indicators will be used in analyzing the teachers' responses to the four different scenarios as used in the TELT study. This process is underpinned by whether the teachers' understanding is procedural or conceptual.

The other key aspect to this study is the analysis of the teachers' mathematical life histories. This process is lead by the themes identified by Carroll (1998), forming the second concept package. The themes as mentioned earlier are:

- Own school experiences
- Teacher training
- Personal Philosophy
- Knowledge of mathematics teaching
- Significant influences

Third significant concept is that of teacher identity. This notion of identity leaned heavily of the studies of Parker (2004; 2006). The three main identities Parker identified were mentioned previously: that of teacher as mathematician; teacher as student of mathematical education; and as teacher of mathematics. Although these identities seem limiting I believe that they together with the identities mention by Jita and Vandeyar (2006) give a sense of the type of identity that these teachers have developed over their years of teaching. This concept of teacher identity began to reveal itself as the study grew. I did not set out specifically to study teacher identities.

The next chapter will reveal how these concepts are used in this study. It will look at how methods were applied in order to bring these and other concepts out of the teachers that were interviewed.

Chapter 4: Methodology

4.1. Introduction

This chapter provides the detail of the research design and the methods used in the research process. The chapter will look firstly at the research process that was used in this research and secondly discuss in detail the research instruments used by the researcher. The advantages and disadvantages of the instruments are discussed and reasons for their use are given. The research procedure and the sampling process are described in order to contextualize the research design.

The methodological process was guided by the main questions asked in this research study. The process had to reveal answers to these questions.

4.2. Case Study as a Research Method

This study falls into the broader definition of a case study. Although this study does not necessarily follow the structure of a case study per se I have included aspects of this methodology in the design of this study. The following is a brief discussion of Case Study Methodology.

Case Study research has been in use for many years. A look at the history of Case Study Research reveals that the earliest use of this form of research can be traced to Europe, predominantly France (Tellis, 1997). The methodology was used by the University of Chicago, Department of Sociology from early 1900's to 1935. Several problems however were raised by researchers in other fields and this thus led to researchers then adopting a more scientific approach to sociology research. As the use of quantitative methods advanced, there was less use of the case study methodology (Tellis, 1997). Strauss and Glaser (1967) noted that in the 1960's researchers were becoming concerned about the limitations of quantitative methods and this thus brought about a renewed interest in Case Study Methodology.

Tellis (1997) points out that literature contains numerous examples of the applications of case study methodology. Some of these are found in the fields of Medicine and Law, where 'cases' make up the large body of student work. Schools of business have also been aggressive in the implementation of case based learning, or "active learning" (Boisjoly & DeMichiell, 1994).

In looking a bit closer at what case studies are and where this study falls in terms of this literature, it becomes necessary to define types of case studies and to look at their designs.

Robert K. Yin defined the case study research method as an empirical inquiry that investigates a contemporary phenomenon within its real-life context; when the boundaries between phenomenon and context are not clearly evident; and in which multiple sources of evidence are used (Yin, 1984, p.23). The phenomenon here is the PUFM of these teachers.

Yin (1993) listed several examples of case studies together with appropriate research design in each case. He gave suggestions for a general approach to designing case studies, and also made recommendations for exploratory, explanatory and descriptive case studies. A further look at what Tellis (1997) said about these types of case studies revealed the following:

Exploratory Case studies

It is found that fieldwork and data collection may be undertaken prior to the definition of the research question and the hypotheses. The framework of this type of study must still however be created ahead of time. This type of study has been considered as a prelude to some social research. Pilot projects are very useful in determining the final protocols that will be used.

Explanatory Case Studies

This type of study is useful for doing causal studies. In very complex and multivariate cases, the analysis can make use of pattern-matching techniques.

Descriptive Case studies

These studies require that the investigator begin with a descriptive theory. Pyecha (1988) used this methodology to study special education, using a pattern matching procedure. Several states were studied and the data about each state's activities were compared to one another, with

idealized theoretical patterns. What is implied here is that this type of study is the formation of hypotheses of cause-effect relationships. The selection of cases and the unit of analysis are developed in the same manner as in the other types of case studies.

This study lies closest to this last category of case studies.

4.3. The Research tools

The instrument used two basic research tools, namely the questionnaire and the interview. There is need therefore to look more formally at what these tools are and what their strengths and short falls are. The questionnaire was to collect information about the teachers' PUFM, while interviews were used to interrogate the more successful teachers' life histories.

4.3.1. The Questionnaire

Wilson and Mclean (1994) considered the questionnaire to be a widely used and useful instrument for collecting survey information, providing structured, often numerical data, being able to be administered without the presence of the researcher, and often being comparatively straight forward to analyze.

The attraction to this research tool may be because it is perceived to be easier to administer and also generally easier to analyze. This perception however must be counterbalanced against the time taken to develop, pilot and refine the questionnaire, by the possible unsophisticated and limited scope of the data that are collected, and from the likely limited flexibility of response, as observed by Wilson and Mclean (*ibid.*: 3). Thus, whilst it is easier to administer, there is little guarantee that the desired results will be achieved.

I chose to use the method of incorporating the TELT questions (discussed in more depth in chapter 6) into a questionnaire as I felt that they were open-ended type questions. They leave much room for teacher interpretation and thus to some extent can be seen as a written interview. In doing this, I deviated from Ma's (1999) methodology. She used these questions rather as interview questions and then analysed the interviews. Whilst I agree that this process could lead

to a much deeper analysis as the interviewer will be able to probe the teachers' knowledge by adjusting the questioning technique as he wishes in response to the answers he receives - there is still the possibility that probing will influence the response. When I chose to use questionnaires, it was however strongly informed by practical concerns; the teachers lived far apart and it was possible to drive hundreds of kilometres for an interview only to be told that unexpected events have made it impossible to conduct the interview that day.

4.3.2. The Interview

The research interview has been defined as “a two-person conversation initiated by the interviewer for the specific purpose of obtaining research-relevant information, and focused by him on content specified by research objectives of systematic description, prediction, or explanation” (Cannell and Kahn, 1968, p.527). It is through interaction that participants, interviewers and interviewees, discuss their interpretations and points of view with regard to the common topic.

It becomes clear that through the interview process, the interviewer is able to extend the question in keeping with the response received. This makes the process dynamic and thus allows for much deeper probing of the interviewee. This is specific to the unstructured interview.

The other three kinds are the structured interview, the type used in this research, the non-directive and the focused interviews. As the structured interview was used in this research project, it is necessary for there to be a clear understanding of this type of interview. It is one in which the content and procedures are organized in advance. The sequence and wording of the questions are determined by means of the schedule and thus the interviewer is left very little room to make modifications to the set questions. However the responses are still influenced by the questions asked and other behaviour by the interviewer.

The reason for using such a structured interview was to ensure that the interviews achieved their aims and that there was similarity between the interviews. There was however room for extension and for exploration of the interviewees' responses. The questions acted to a large extent as a guide to probe the teachers' mathematical-life histories and to some extent their knowledge of mathematics.

4.4. The Sample

4.4.1. The Sampling strategy

The sampling strategy used for this study could be classified as **non-probability** sampling. This form of sampling sees the research targeting a particular group, in the full knowledge that it does not represent the wider population. This form of sampling is in line with the case study method of research.

Built into non-probability sampling is convenience sampling and purposive sampling. Here are the definitions of these types of sampling:

Convenience sampling involves the choosing of the nearest individuals to serve as respondents. The researcher employing such a strategy normally chooses the sample from those to whom he has easy access. This is convenient and removes many of the problems associated with travelling to do research. The problem however with this strategy is that the sample does not represent the population and therefore it makes it difficult to generalize about the wider population.

Purposive sampling involves the researcher handpicking the cases to be included in the sample on the basis of their typicality. This is done in order to build up a sample that is suitable to their specific needs. Whilst this form of sampling may satisfy the needs of the researcher it cannot pretend to represent the wider population as it is selective and biased.

This sampling strategy, *non-probability convenience sampling*, was employed in this study in the following way. As the study was to investigate whether high scoring teachers on the NPDE had a form of PUFM, the study was limited to students who obtained scores on the NPDE mathematics course in excess of 80%. For practical reasons, I only worked with teachers attending the NPDE at the University of KwaZulu-Natal, Pietermaritzburg. This selection process could thus be deemed to be a type of convenience sampling since it ignores the fact that numerous institutions across South Africa also ran this course. There were differences between the courses, as each institution used their own material and their own form of delivery.

The NPDE course was run at nine different Learning Centres across the province. These centres stretched from Matatiele in the south of the province to Vryheid and Newcastle in the north of the province. It therefore became imperative that students who could easily be contacted were invited to be part of this study. This is again a form of convenience sampling.

4.4.2. The Specific Sample

The research project is a case study of a few of the teachers that studied mathematics on the NPDE course run by the University of KwaZulu Natal, Pietermaritzburg. The Course has run successfully for the last four years. Three groups of students had completed this course by the time this study was completed..

The reason for choosing students from this particular group has been discussed in Chapter 1.

I obtained the results of these students from the university and highlighted the top achievers from the second and third intake of students who completed this course. The group was made up of students who obtained marks in excess of 80% for the final mark. This mark was made up of the marks obtained for the two assignments and the examination set for the course.

I identified twenty students initially, but due to travel constraints and students not showing interest, the sample was eventually reduced to five students. I handed out twenty questionnaires but only received 15 back. This figure was achieved mainly, in many cases, by me pushing for these forms to be returned. Many of these were half completed and many simply left sections out. This in itself could be an indication of 1) lack of interest, 2) their lack of knowledge about the issue that was asked, or 3) simply time constraints. I therefore had to work with those students who showed an interest, had completed enough of the questionnaire to allow for analysis, and who availed themselves to be interviewed.

The final sample, of five students, is rather small when we consider that 148 students completed this course at the time the study was started. It is however in line with the total number of students who obtained results above 80% for their course mark.

To get a true view of the students on the NPDE Course, I believed that I needed to include teachers with different life experiences. I therefore chose candidates that had varied backgrounds even though some had belonged to the same 'racial grouping', see footnote on page 2. All of the candidates come from totally different social contexts. The candidates are;

- A Coloured teacher from the city of Pietermaritzburg;
- A Black teacher from the Pietermaritzburg township of Sobantu;
- A Black teacher from the rural area around the town of Greytown in the Midlands of KwaZulu-Natal;
- An Indian teacher from the city of Pietermaritzburg, but who was raised in a small farming town initially and then completed his schooling in Pietermaritzburg;
- A White, Afrikaner, teacher from the town of Ladysmith.

Their life experiences differed vastly and thus their experiences presented a wider view of the South African context.

4.5. The Instrument

The instrument used in this study is broken into three parts namely:

Questionnaire 1 (Appendix 1)

The TELT questions as used by Liping Ma, dealing with mathematical content knowledge and looking for PUFM, but adopted to the questionnaire method.

Questionnaire 2 (Appendix 2)

Dealing with pedagogic content knowledge of multiplication of two digit numbers and checking whether any level of PUFM can be seen through the interpretation of these questions.

Interview (Appendix 3)

This deals with the Mathematical Life Histories of the teachers.

It is my aim to discuss each part of the instrument separately. I will look at the purpose of the instrument and the role it plays in answering the questions posed by this research. I will also look at how each part of the instrument reveals different aspects of interest to the study.

4.5.1. Questionnaire 1

The first questionnaire was based on the TELT questions developed by Debra Ball for her dissertation research (Ball, 1988), discussed previously and used by Liping Ma in the research for her book. These four questions covered both content and pedagogic content knowledge. It is therefore important that there is a clear understanding of what the TELT questions were.

A. The TELT Questions

In her research, Ma used four questions designed by Deborah Ball for her dissertation research (Ball, 1998). These questions were designed to probe teachers' knowledge of mathematics in the context of common things that teachers do in the course of teaching (Ma, 1999), in all the aspects mentioned previously: longitudinal coherence, basic ideas, multiple perspectives and connectedness. The interview tasks were structured by weaving a particular mathematics idea into a classroom scenario in which the idea played a crucial role. For instance, teachers were asked how they would respond to a (false) learner claim about the link between perimeter and area.

TELT instruments cover a broad field of mathematics. They are not limited to one topic only. The TELT instruments for mathematics are concerned with four common elementary topics: *multidigit subtraction, multidigit multiplication, division by fractions, and the relationship between area and perimeter* (Ma, 1999), but also addresses much deeper aspects of mathematics like inverses, understanding of the positional number system, proving versus empirically verifying, the concept of the distributive law, to name a few. These are very fundamental concepts and principles in mathematics in that they lead on to so many different concepts and theories within mathematics.

Here is Ball's reasoning for the development of scenario one and two:

Since place value (and its root idea, grouping) is a fundamental mathematical idea and since pupils often find it difficult, it seemed a critical area of teachers' knowledge to investigate. I embedded place value concepts in two different interview tasks; one a classroom scenario focused on student difficulties with the multiplication algorithm, the other a structured planning-teaching assessment exercise and subtraction with regrouping ("borrowing"). Although place value is the

underlying foundation for these conventional procedures, adults can perform the procedures competently without thinking about place value at all. I wanted to examine how well the prospective teachers' adult operational knowledge of numbers equipped them to help make sense of the meanings of these operations with them.
(Ball, 1988, p.49)

I therefore mimicked the procedure that Ma used by making use of the TELT interview questions. However, unlike Ma's research, this is not a comparative study. It is not looking at whether teachers who are highly qualified academically have a strong PUFM or how the South African teachers compared to the American or Chinese teachers who Ma used in her study. The focus in this section of the study is more on ascertaining whether high scoring teachers on the NPDE course actually possess a PUFM.

The extent of the teachers' PUFM will be analysed through firstly deciding whether the knowledge is conceptual or only procedural and secondly by looking for the properties of PUFM as described in the conceptual framework in chapter 3, namely Connectedness, Multiple Perspectives, Basic Ideas and Longitudinal Coherence.

4.5.2. Questionnaire 2

The second questionnaire looked more at the pedagogic knowledge of the teachers involved in this study. It sought to expose the aspects of Connectedness; Multiple Perspectives; Basic Ideas and Longitudinal Coherence through the process of analyzing the way the teacher teaches and assess one particular mathematical concept.

The second questionnaire was designed to reveal the specific pedagogic knowledge required by teachers who teach fundamental/primary mathematics. Although this is not a major emphasis in this study, it is hoped that by looking more in-depth at one scenario, the teachers' mathematical knowledge would be revealed in their answering of questions pertaining to their teaching of the topic. This questionnaire looked more specifically at the second TELT question: multiplying two two-digit numbers.

4.5.3. The Interview Schedule

The interview was primarily designed to probe the mathematical life-histories of the interviewees. In designing and analyzing the interviews, I chose to a large extent to use themes covered by Carroll (1998) as discussed in the literature and the conceptual framework. I believed that these themes were relevant and thus provided a tested method for analyzing the teachers' life histories. I also considered the research of Parker (2004) and Jita and Vandeyar (2007) that looked at teachers' identities. Both of these were discussed in-depth in chapter 2 and 3.

The interview questions were divided into sections, namely:

Personal information/Personal Philosophy/Significant influences

This section aimed to present the teacher as they saw themselves. It firstly aimed at getting a view of how the interviewee saw themselves through asking probing questions about their personal lives. Secondly, it sought to establish whether any person (parent, grandparent, teacher, mentor, and etcetera) could have had any influence on their education in general and specifically on their mathematical education.

The Influence of the NPDE and formal teacher training

This section of the study is aimed at probing the influence of the course on both the mathematical and the pedagogical knowledge of the teacher – though obviously the personal impression of competence may deviate from what has actually been learned. It also sought to probe the impressions that these teachers developed about the course. And for the purpose of informing the course organizers, it looked at the compliments and complaints that the teachers raised about the course.

Life at School/The influence of teachers and teaching methods

The main goal here was to establish from the teachers' life at school as learners, what influence their schooling had on their mathematical knowledge – again, the subjectivity must be recognised. It looked at what teachers did in their school that had a lasting impression on these teachers. It also looked at the personality of their teachers that had a lasting influence on them as learners.

Life out of School/The period from student to teacher

This section of the interview aimed at looking into the development of the teacher from novice to professional. It hoped to achieve this by questioning the teachers about the changes that they experienced as they developed as teachers. This also helped to determine the teachers' identities.

The final interview schedule is reproduced in Appendix 3. When initially designing this interview schedule, there were only four areas of focus. The four areas looked at who the teachers are; their experiences at school; their experiences on the NPDE/teacher training; and their experiences as they progressed from novice teacher to professional teacher. It however became evident in the reading of the transcripts that there were more themes noticeable. It was therefore decide to use a thematic approach to the analysis of the interviews, inspired by Carrol (1998).

4.6. Conclusion

The methodology reported on above aimed to look at two distinct areas of knowledge, namely the teachers' mathematical knowledge or rather the quality/profoundness of the mathematical knowledge that the teachers possess, and secondly the mathematical life-histories of these teachers.

There are short-comings to these methodologies, in that the use of the questionnaires limits the candidates' freedom to express their views about aspects that pertain to the questions but are not asked on the questionnaire. Also, prompting is not possible on questionnaires, and thus it is highly likely that the questionnaires give less inclusive data than asking the same questions in interviews would have done.

The use of Life-History methodology opens the path for teachers to give fictitious stories. The interviewer has to trust the sincerity and integrity of the person being interviewed to a large extent in order to overcome this problem. Even the most sincere responses are likely to reflect retrospective reconstruction of events – we live life forwards, but understand it backwards.

Chapter 5: Analysis of Teachers' PUFM

The aim of this chapter is to analyse the responses to the questionnaires dealing with teacher knowledge and PUFM, namely questionnaire one and two, in order to draw conclusions about the teachers' PUFM. The analysis will review the findings of the first and second questionnaires by looking at each scenario separately before making general inferences.

Before actually doing the analysis, I believe that I need to introduce the main candidates that were selected to participate in this research. The teachers' names have been changed to protect their identities. The analysis that follows will look at these teachers individually but will at times also look at the broader group.

5.1. The Selected Candidates

The first teacher is **Tr. Belinda**. She is a Coloured teacher who was born and raised in the city of Pietermaritzburg. She has been teaching for the last 35 years at the time of the study. Many of these years she has been at the same school. She teaches at a school in the city of Pietermaritzburg. This school was originally exclusively for the 'Coloured community'. It is not well resourced and the buildings are very old. The school is in a relatively decent state. The class sizes however are very big and range from between forty and fifty in a class. Tr. Belinda studied at the Zonneblom Teacher College in Cape Town. This College was set up for the training of Coloured teachers. Tr. Belinda had to study at this college because there were no colleges that trained Coloured teachers in the Province of KwaZulu Natal at the time.

Tr. Dumisani is a Black principal of a school in a rural area.. This area of the province is amongst the poorest areas of the province. There is very little development that has happened in this area. He had been teaching for 29 years at the time of the study. He schooled in the area where he is now teaching and completed his schooling at the local high school where he studied mathematics up to grade twelve. He did his initial teacher training at Madadeni Teacher Training College in Newcastle in KwaZulu-Natal.

Tr. Barend is a White teacher, teaching in a school in Ladysmith. This school is regarded as a Model C school. This type of school was a White school under the apartheid era. It is now a public school but is to a large extent self-sustaining in that the parents pay for most of the expenses of the school while the government pays mainly for the teachers' salaries. Tr. Barend had been teaching for 4 years only at the time of the study, as he was the school sports director before this. Tr. Barend schooled at a small school in the Ladysmith area. He regarded his education as being privileged when compared to the education received by many of the race groups in the country. Tr. Barend states that he enjoyed his primary school career but battled through high school. He did his initial teacher training at Durban College of Education, also known as Dokkies, for two and a half years. He was diagnosed much later in life as having ADD (Attention Deficit Disorder). After two and a half years of battling at college he left to pursue a career in the Prison Services. He decided later in life to return to teaching. This is when he joined the NPDE course and graduated as a qualified teacher.

Tr. Margaret is a Black teacher teaching in the township of Sobantu in Pietermaritzburg. She had been teaching for 30 years at the time of the study. Tr. Margaret grew up and schooled in the area she is now teaching in. She completed her Junior Certificate at the local high school where she studied mathematics up to grade ten. She then went on to do her initial teacher training at Eshowe Teachers Training College. After many years in teaching she decided to study again and registered for the NPDE at the University of KwaZulu-Natal.

Tr. Daya is an Indian teacher teaching mathematics at a predominantly Indian Primary School in the Pietermaritzburg area. He is also teaching in the area where he grew up and completed his schooling at a local secondary school where he studied mathematics up to grade twelve. He completed his schooling in 1996 and is thus relatively young when compared to the other teachers in the sample.

5.2. Teachers' PUFM

5.2.1. Questionnaire 1 and Questionnaire 2

The aim here is to review each topic and then decide whether the knowledge displayed is procedural only or conceptual and whether it can be categorised as PUFM. It is noted that a conceptual understanding should contain procedural knowledge.

Firstly I will consider,

- if the procedures (algorithms) are mathematically correct and sound.
- if the teachers show signs of understanding these mathematical concepts and whether their understanding is solid or merely pseudo-conceptual as Ma(1999) puts it.

Secondly, I deepen the analysis by looking for the aspects that Ma sees as the indicators of PUFM, namely, *Connectedness, Multiple Perspectives, Longitudinal Coherence, Basic Ideas*.

Below is a summary of the performance of the sample group that returned their questionnaires. I have taken each section and looked at how each teacher answered the question.

A. Scenario 1: Subtraction with regrouping

Tr. Dumisani

Tr. Dumisani proposed the use of 'regrouping' numbers into the correct place values, first subtracting their tens values and then their units.

He reduces the subtraction problem to that of subtraction of a single digit number from a two digit number.

$$83 - 57 = \text{-----}; \text{ subtract the tens } (83 - 50 = 33) \text{ then their units } (33 - 7 = 26)$$

$$91 - 79 = 91 - 70 = 21 - 9 = 12^4$$

⁴ This notation is making use of '=' as a procedural sign, and is obviously mathematically incorrect. Calculations reproduced here are as the teachers wrote them in their questionnaires.

The first stage of his method is relatively simple. He simply subtracts from the value the highest possible multiple of ten. The problem arises for me in that he does not mention how he would deal with subtraction where the minuend is a value greater than the unit value of the subtrahend for example (33 – 7). It seemed that he would simply rely on counting in order to solve this section of the problem. He stated that his learners are taught to write the numbers in columns and then to do the subtraction. He feels that this ensures that they subtract the correct digits. This he believed reinforced their knowledge of place value. He did not address borrowing or regrouping *per se*.

PUFM

Basic Ideas:

For him the basic principle of place value is most important. He drove home the point that learners need to be able to ‘split’ numbers into units, tens and hundreds. His only link to the problem here is that it allows the subtraction of the tens first and secondly the units. The ability to subtract a single digit number from a two digit number is crucial to his algorithm. He however never discussed it and thus did not engage the idea of regrouping or decomposing so as to assist with subtraction. This method demonstrated a very simple understanding of subtraction, but it fails to discuss the main idea.

Connectedness:

No evidence of links to other areas or concepts was evident as the reference to place values was not linked to regrouping, which remains implicit.

Longitudinal coherence: The only evidence of this was given in his mention of the subtraction of four digit numbers.

Multiple perspectives:

This was only evidenced in his statement, “now-a-days there are so many methods, since learners are made to think and formulate their findings ...”. However, he did not discuss what this may mean in this case.

Tr. Dumisani seemed to be aware of the common algorithm used to solve such a problem. He however altered it by getting his students to subtract the highest multiple of ten that is possible and then to use counting to determine the unit value. His understanding was based primarily on the concept of place value. It was mainly procedural and reinforced a fixed algorithm, albeit not the common one – thus making it harder to extend to subtraction of numbers with more digits. To me, he offered very little to indicate that he has a deep understanding of the concept of regrouping. Secondly there was very little in the way of links to previous knowledge or even to future related topics, the only extension being subtraction with four digits. This cannot be regarded as indicating breadth in his understanding. If we consider the complexity of his understanding it is noted that his understanding of the concept is limited to the understanding of the algorithm. He did mention that he is aware of other methods to solve this problem but does not mention any. In considering the above analysis I do not believe that his understanding of this scenario can be considered to be profound.

(It must be recognized again that the absence of the possibility of probing does limit the analysis in the sense that it is possible that the teachers would have exhibited a more profound understanding when probed. Thus, I can only talk about the likelihood of the teachers' understanding being profound. The same would to some extent have applied to interviews as well, as it is not possible to know what the teachers know.)

Tr. Daya

The method used by this teacher involves working with multiples of five or ten. He added to or subtracted values from the original value in order to obtain values that were multiples of five or ten. He speaks of regrouping numbers into more convenient numbers. He believed that working with numbers that are multiples of five or ten are more convenient to subtract. He used the concept of place value (broke values into tens and units) but never mentioned it at all. He focused on the simpler example of subtracting when the units value of the minuend was less than the units value of the subtrahend, for example $78 - 56 = 22$, and thus fundamentally ignored the essence of the question with its focus on regrouping. He proposed that the pupils do the following to solve this problem (notice the absence of brackets in the second line):

$$\begin{aligned}
78 &= 70 + 8 \\
- 56 &= - \underline{50 + 6} \\
&= 20 + 2 \\
&= 22
\end{aligned}$$

The second method to solve it is as follows:

$$\begin{aligned}
&80 - 2 - 50 - 6 && (80 - 2 = 78 / \text{converting } 78 \text{ to a multiple of } 10) \\
&= 80 - 50 - 2 - 6 \\
&= 30 - 8 \\
&= 22
\end{aligned}$$

The problem in particular with the second method lies in the explanation of what ultimately appears to be negative numbers to intermediate phase pupils, that is why is $-2 - 6 = -8$.

In his view of prior knowledge that he expected pupils to have, he merely mentions simple subtraction, subtraction of 10's; and subtraction of numbers with identical units. All these topics he reinforces using purely procedural methods. They merely lead to the method that he proposed teaching to the pupils. He did mention that learners needed to understand these concepts of mathematics. He however did not elaborate on what he means by understand the concept of subtraction.

Tr. Daya proposed that in order to get pupils to understand the concept of subtraction, the following order to teach the concept:

- | | | |
|---------|-------------------------------------|-----------------------------------|
| Step 1. | Subtraction of tens | (done by oral drill and homework) |
| | Example: $21 - 10 = 11$ | |
| Step 2. | Subtraction with identical units | (done by oral drill and homework) |
| | Example: $31 - 11 = 20$ | |
| Step 3. | Rewrite to a more convenient number | (oral and homework) |
| | Example: $78 = 80 - 2$ | (used above) |
| Step 4. | Subtraction using rewriting numbers | |
| | Example: $80 - 2 - 50 - 6$ | (the example he used above) |

Each step mentioned above is taught to get pupils to understand the algorithm. He however never mentioned the mathematical principles underlying the algorithm.

He did not mention any other links but that of solving word problems. Whilst this would be a logical extension of this topic it is merely application of the method that he proposes.

PUFM

Basic ideas: For him the basic ideas were simple subtraction and changing numbers into more ‘convenient’ numbers. He stated that pupils must regroup numbers into more convenient numbers which was simply rearranging the numbers so that the calculation is simpler.

Longitudinal coherence: The teacher’s only form of longitudinal coherence was given by the steps 1 – 4 that he used to get pupils to understand subtraction. Whilst this method maybe considered longitudinal, it is conceptually very weak.

Connectedness: He made no real links to other mathematical topics, but only mentions basic subtraction and the use of word problems.

Multiple Perspectives: He used a method that was different to the one he was taught at school. He however only covered the method he felt was convenient for him. He believes that it is sufficient for learners to know one method.

When considering the above analysis, it is impossible to say anything about Tr. Daya’s understanding of subtraction with regrouping, as he does not engage in regrouping. The closest he came was the rewriting the number as a subtraction, which does not reveal any notions of regrouping. Thus, on the basis of his responses on the questionnaires, it is not possible to characterize his understanding as profound. The steps he suggested are meaningful but appear to have a procedural focus and he did not provide an explanation of the conceptual underpinnings. His only extension of this topic is the link he makes, as mentioned earlier, to word problems. This is not an indication of complexity.

Tr. Margaret

Tr. Margaret mentioned the concept of place value and the importance of understanding it. She saw the understanding of subtraction as being vital to the concept. She also saw that an understanding of the value of a number was 'key' to the understanding of subtraction. She demonstrated four methods.

The method she proposed was that of reconstruction of numbers (she calls this breaking numbers down). The example she gave is,

$$52 = (10 + 10 + 10 + 10 + 10 + 2) - (10 + 10 + 5).$$

This method showed her understanding of place value but it failed because after subtracting the two 10's she does not explain how to subtract (32 - 5). We still have the situation where the unit value of the minuend is bigger than the unit value of the subtrahend but the procedure remains unexplained. In other words, it did not involve regrouping.

She spoke of regrouping and performed this calculation:

$$\begin{aligned} 50 + 2 - 20 - 5 &= 50 - 20 + 2 + 5 \\ &= 20 + 2 + 5 = 27 \end{aligned}$$

Tr. Margaret did not explain the change in sign from - 5 to + 5. If we assume that she sees the calculation as $52 - (+ 25) = 50 + 2 - (20 + 5) = 50 + 2 - 20 - 5$ but then the next step is unclear, and the regrouping that must have taken place remained implicit. Using the distributive law and the regrouping/rearranging of the numbers we note that her method worked. Thus, we can make no assumption of her conceptual understanding of regrouping.

She called the third method, the add-on method. This was similar to the regrouping method. In this method she proposed that we add values onto the given numbers so that subtraction becomes easier. She gave the example of $91 - 79$. Here she believed that by adding 9 onto 91 and 8 onto 79 that the subtraction would be easier since to her it is easier to subtract (100 - 88). She did not, however, explain how to complete the calculation and so again the regrouping itself was not addressed.

The fourth method she called the counting down method. She did not explain this method in detail.

The example given $65 - 58 = 65 - 50$
 $= 15 - 8 = 7$

Here she subtracted by the largest multiple of 10 and then subtracts the units. She reduced the subtraction to subtraction of a value between 10 and 20 and a value less than 20. By counting down, regrouping is simply avoided.

Tr. Margaret criticized the traditional methods of ‘borrowing’. She felt that the concept was “totally wrong”, but did not explain why.

PUFM

Basic ideas: Tr. Margaret mentioned the following as basic to the understanding of this concept.

- Place Value
- Subtraction (simple subtraction)
- Knowing the values of numbers
- The rounding-off of numbers
- Graphical representation of numbers [I assume she meant the use of number lines]
- Discovery (as a method of learning)
- Addition and subtraction as inverse operations

Longitudinal Coherence: Tr. Margaret produced a flow chart that resembled the following:

Start with Digits - move to place value - teach the breaking down and expanding of numbers - then let the learners work on the number line - teach the importance of estimation and the rounding of numbers and number names. Regrouping does not figure on her flow chart.

A few questions could be asked:

- Why is a link made between estimation and subtraction? My only guess is that estimation plays an important role in developing the sense of correctness.
- Why are number names separate from place value? The teacher does not make a strong link between value of a number and the value of the digits of the number. The digits of a number must not be seen as three independent numbers but as part of a collective value.

Connectedness:

Tr. Margaret mentioned many concepts in her questionnaire. She however never mentions how these topics interlink and how they relate to each other. She used the distributive law but did not explain why she uses it; she used the concept of rounding so as to simplify her subtraction. She tends to round to multiples of five and ten. She linked her method to decomposition of numbers and proposed two methods of decomposition that she would use to solve such a problem. She used the number line as a form of graphical representation

Multiple perspectives:

Tr. Margaret was aware of various procedures and it seems she was prepared to teach these various procedures. She presented many of these methods in answering her questionnaire.

Tr. Margaret revealed much about her understanding of this topic. She demonstrated depth of understanding in that she was not only aware of algorithms that can be used to solve this calculation but was aware of the deeper rooted mathematical concepts [like number names and values and expanding numbers] that are related to this topic. She also showed breadth in that she draws on other concepts like the graphical presentation (number line) and estimation which she linked to subtraction. It is easy to see the level of complexity of her understanding as she not only revealed one algorithm but also showed that she has thought out and worked through numerous algorithms. However, it was again problematic to assess her understanding of regrouping, as her methods generally are directed at avoiding it – in this case, a deliberate choice.

Tr. Belinda

Tr. Belinda offered very little in the way of an explanation of how she teaches this section and of how she understands the related topics. The method she explains was the traditional algorithm and she used the concept of ‘borrowing’ as a means to expanding the given values.

Her method uses:

$$\begin{array}{r} 52 \quad 5 \text{ tens and } 2 \text{ units} \\ - \underline{25} \quad 2 \text{ tens and } 5 \text{ units} \\ \underline{27} \end{array}$$

She said, you must take away 1 ten from the 5 tens (leaving 4), because 5 is larger than 2.

And the 2 units become 12 units (10 + 2)

We then have $12 - 5 = 7$ units

And 4 tens minus 2 tens = 2 tens.

We thus have 2 tens and 7 units which gives a value of 27.

The method she teaches was a method that was taught in most South African schools. I was also taught this method. There are a few mathematical misconceptions in her method. The first misconception that I want to pick up on is the one that states that, “We cannot subtract a bigger number from a smaller one”. Although pupils at this level have not yet exposed to this type of subtraction it is incorrect to state that one cannot subtract a bigger number from a smaller number. A pupil’s future learning must not be confused by emphasizing a misconception (Ma, 1999, p.3). The second misconception that needs to be discussed is that ‘borrowing from the tens’. Here Ma (1999, p.4) argued that to treat the two digits of the minuend as two friends, or two neighbours living next door to each other is mathematically misleading as it suggests that the two digits are two independent numbers rather than part of one number and secondly that the value of a number does not have to remain constant in computation, but can be changed arbitrarily – if a number is ‘too small’ and needs a large one for some reason.

PUFM

Basic ideas:

She mentioned the following:

- Place value
- Counting/ counting in tens and then in fives

This was in accordance with her method and explanation.

Connectedness:

She did not link this section to any previous or future topics knowledge except those mentioned as being part of the basic ideas.

Longitudinal Coherence: The only longitudinal coherence evident was the link she made between place value, the expansion of a number and subtraction.

Multiple perspectives:

The teacher teaches the method she was taught. She believed that you must teach the learners at least one method and then allow them to do their own exploration. She however felt that there is far too little time to allow for much of this exploration.

Tr. Belinda answered this portion of the questionnaire very scantily. She wrote the bare minimum and left some of the questions blank. This could be because of a lack of interest or because she felt she stated everything she needed to about the topic. Tr. Belinda's knowledge was based purely on her knowledge of the algorithm. A means to an end sought of knowledge as she is of the opinion that if she teaches at least one method then it is sufficient for pupils as they will be able to cope with this type of calculation. It was therefore difficult to analyse the depth, breadth and complexity of her knowledge. It was clear that her knowledge she demonstrated was procedural as it was based on the algorithm. Teachers' who expected their students merely to learn the procedure tended to have a procedural understanding (Ma 1999, p.3).

Tr. Barend

Tr. Barend prefers to use the traditional column method. He stated that he drills this algorithm so that his learners are able to do it. He used the concept of 'borrowing' to explain the expansion of values. In his introduction to the concept, he stated that he prefers to use practical examples

(manipulatives) like marbles and stones to assist him. He however failed to explain how he will use these manipulatives to explain the concept of multi-digit multiplication.

Tr. Barend mentioned that he teaches the section as he was taught it. He however asked one of his more experienced colleagues to assist him in answering this question, thus what is mentioned on the questionnaire, in Afrikaans is not his answer but that of his colleague. For the analysis process I ignored all the Afrikaans statements and concentrated on Tr. Barend's contribution. The following is an assessment of his PUFM.

PUFM

Basic Ideas: He regarded the following concepts as basic to the understanding of this concept, Place value and order of numbers, multiplication tables and 'bonds' (basic addition facts), and the basic meaning of subtraction. It is unclear why Tr. Barend considers multiplication a basic idea for subtraction.

Connectedness: He failed to make links to any future knowledge that the learners may need, but continually mentions the link to bonds which is obvious and multiplication tables which is not.

Longitudinal Coherence: Tr. Barend drew up a flow chart that demonstrated that he will move from simple subtraction (subtraction without borrowing, as he puts it) and then he moves to subtraction with borrowing.

Multiple perspectives: Tr. Barend continually mentioned that he is open to learners being innovative, and using their own methods, yet he himself only used one method and drills this method which is likely to convey the opposite message to the learners.

Tr. Barend's understanding of this topic was based on the way he was taught it. His understanding is not deep as there is very little in his answer to convince one that there is breadth

since he did not make any links to other related topics. He spoke of other methods but only teaches one. His understanding therefore cannot be regarded as being complex.

The concept of subtraction with regrouping is so basic that it is hard to imagine that teachers would not have a deep, broad and complex enough understanding of this topic. A few basic methods are noted from the discussion of teacher methods above. The first method to be discussed that of “borrowing”. This is not a real mathematical explanation. Thus these teachers understanding appeared conceptual, but like Ma (p.22) states it was in fact too faulty and fragmented to promote learners conceptual learning. Their understanding was limited to what Ma (p.27) calls “surface aspects” of the algorithm. The second method that arose is the method of using manipulatives. The problem with using manipulatives is that their use is dependent upon the knowledge of the teacher using them.

B. Scenario 2: Multiplication of two digit numbers

The following is an analysis of questionnaire 1 (appendix 1) but the topic was covered in greater depth in the second questionnaire (Appendix 2). Since this questionnaire was only given to the selected candidates the discussion below only reflects the replies of these teachers. The findings have been placed in table format as it should highlight the differences and similarities that exist between these teachers.

Tr. Daya

Teachers Daya’s method was based totally on the column algorithm. He stated that he insists that the learners use the ‘correct arrangement’ (questionnaire 2). The learners have to draw lines that go down the page in order that they write the correct digits in the correct column. He insists that they add zeros. Strangely, he insists that these are added before the calculation starts. This is in keeping with what he was taught at school.

If we look at the aspects he considers important part of the teaching process required for this concept we note the following: he firstly considers simple multiplication and also considers multiplication tables as vital foundations for the teaching of this topic. His progression is to consider two-digit multiplication that is, calculations like 47×7 .

He extended this topic to solving word problems [application as he calls it], to division [used to check the quotient] and to squaring. Looking at this in relation to PUFM, I note the following:

PUFM

Basic Ideas: He mentioned that the following concepts, could be regarded as basic to the understanding of this section, single digit multiplication first and then single digit and two digit multiplication.

Longitudinal Coherence: He mentioned that he progressed from simple single digit multiplication to single digit multiplied by double digit multiplication and then to two by two and more multiplication.

Connectedness: He mentioned that he let them test their quotients through multiplication thus linking to division. He also mentioned squaring and cubing but however did not discuss how he will make the links to these topics.

Multiple perspectives: The teacher did not mention any nor did he seem to tolerate other methods. He mentioned that he drills one method so that the pupils will at least know one method. He was only prepared to allow his learners to attempt other methods once they have mastered his method. He claimed that time restrictions do not allow for too much investigative work.

The analysis of this PUFM reveals clearly that his knowledge was mainly procedural. Whilst he is aware of the links to single and double digit multiplication, he never really covers the connection to multi-digit multiplication and also never discusses the main aspect of the scenario namely what was wrong with the calculation and why he used the zeros in his calculation.

Tr. Dumisani

Tr. Dumisani sees the problem in this scenario as being a one caused by an insufficient understanding of the concept of place-value. He believed that it is important that the learners are

firstly taught to place the zero in its place as it eliminates the problem that could arise with the lack of understanding of place-value. He teaches the standard algorithm that he was taught at school. This could once again be because he was comfortable with the procedure. However, much of the discussion in his response was about general teaching methodology, rather than methodology specific to this concept. I do not necessarily believe that this was due to confusion with the questions asked on the questionnaire as I worked through the questionnaire with him and then left him to answer it on his own.

When using the instrument to assess his understanding the following was noted:

PUFM

Basic ideas: He regarded the following concepts as basic to the understanding of this topic Place-value, Estimating, Working with money (he made no mention of where the link is to this topic), ordering of numbers (here it is assumed that he was talking about the value of numbers)

Longitudinal Coherence: He did not discuss the process of teaching this specific topic. In his explanation he only made links to place-value and the use of zero to ensure that pupils remember the place value.

Connectedness: He only mentioned the topics listed above but did not demonstrate how they were connected to the given topic.

Multiple perspectives: He did not mention any other methods and states that he teaches this method because he was taught it. He stated that he allowed the learners to use other methods once they have mastered the method that he had taught them. He stated that in doing this it allowed them to be 'good mathematicians'. He did not discuss this any further so it can only be assumed that he meant that by allowing them to apply their minds to such a calculation this will allow them to become better mathematicians. He also stated that these learners 'must formulate their rules as thinkers so that self dependent could be promoted', (sic).

Tr. Dumisani was definitely aware of the standard algorithm and was able to do this type of calculation. He, as noted earlier, believed that the problem that the learners had within this scenario is that they did not have a sufficient understanding of place-value. His knowledge of this scenario seemed too focused around the algorithm and knowledge of place-value. I do not believe that it is deep enough. His links to other concepts are not strong support links or natural progressions from this topic. It is therefore important to critique these concepts, the first being estimating, this is a basic skill required for all types of calculations and whilst necessary it is not vital for the understanding of this topic. The second concept 'comparing and ordering numbers' is very basic to arithmetic, again whilst this concept is foundational it is the knowledge of place-value that is more necessary. The third concept he mentioned is that of 'working with money'. This skill whilst important, I do not believe that it is a natural extension of this topic. When doing multiplication calculations dealing with money, there is a greater link to the multiplication of rational values. I therefore do not believe that his knowledge is more than procedural.

Tr. Margaret

Tr. Margaret mentioned that she discourages the use of the standard algorithm (which she called the column method) as she was of the opinion that learners needed to develop their own rules. She felt that learners must be exposed to the discovery method of teaching. The method she encouraged was the rounding off and breaking up method. Here she gave the example of 125×645 . She broke up 125 into 100 plus 25. She then multiplied 645 by 100, as she believed that was a basic skill and that this would give the learners an idea of what the answer should be (estimation). She would then multiply by 25 and add this value to the answer to get the final answer. This could also be done by dividing the answer obtained from 645×100 by 4 and adding the results so as to obtain the final result. When using the instrument to review her understanding the following is noted:

PUFM

Basic ideas:

The following concepts are mentioned as basic to the understanding of this topic, Place-value; Breaking up method (decomposition); Estimation; Basic multiplication

Longitudinal Coherence: This is evidenced by her mention of factorization, a future topic and repeated addition as well as the mention of the use of fractions such halving and the concept of doubling and the process of rounding. These are however not all related topics.

Connectedness: Tr. Margaret made definite links to both future and previously taught topics. She also used these to assist her in her teaching of this section of this section of mathematics.

Multiple perspectives: She showed clear knowledge of other methods to do deal with this concept.

Tr. Margaret's depth of understanding of this concept was evidenced by her mention of the basic topics place-value, decomposition (which she referred to as breaking up), estimation and basic multiplication. It is noted that the basic multiplication she refers to is single digit multiplication. She mentioned that she felt that factorization should be taught with multiplication but did not state why. I do believe that factorization could be done but factorizing values as big 645 and 125 takes us into the process of multi-digit multiplication. Tr. Margaret's complexity of understanding could be rated by her willingness to introduce different computational procedures (algorithms) in her teaching of this topic. She definitely had a procedural understanding of this topic and is aware of the basic concepts that support this concept.

Tr. Belinda

Tr. Belinda has taught this method using the standard algorithm. She taught it using the following steps:

Step 1 Multiply with the digit

Step 2 Multiply with the ten.

When we multiply with the ten, we put down a '0' in the unit/digit column

The example she gives is;

	22
	<u>x 11</u>
Step 1	22
Step 2	<u>220</u>
Step 3	<u>242</u>
Step 4	Check with calculator

When using the instrument to review the teachers understanding, the following is noted:

PUFM

Basic ideas: The following is regarded as basic to the understanding of this topic, Number tables from 1 to12, basic addition facts (bonds), and 1 digit multiplication.

Longitudinal Coherence: The teacher made the link between single digit multiplication and repetitive addition. She spoke of moving from bonds to single digit multiplication and then multiplying large numbers.

Connectedness: The teacher made links to working with money, time (days and hours and minutes, which is multiplication by 24 and 60) and division (long and short division).

Multiple perspectives: The teacher believed that pupils need to be taught a method and once they are familiar with it then to allow them to explore other methods.

Tr. Belinda's understanding of this concept seemed very procedurally orientated. She taught the standard algorithm as she believed that learners should at least know one procedure correctly. Her depth of understanding was linked to the concepts she mentions as basic to the understanding of this topic, namely counting, tables and bonds and single digit multiplication. She also repeatedly mentioned the link between multiplication and repetitive addition. Whilst these are important basic concepts, the teacher seems to overlook the problem created by the lack

of understanding of place-value. It is difficult to comment about the teachers' complex understanding of this topic as she failed to review any other method but simply stated that she allows learners to use other methods once they have mastered the method that she had taught them.

Tr. Barend

Tr. Barend stated that he would use place-values and thus teach using the zeros as place values in these calculations. This is the standard algorithm. His method is as follows:

$$\begin{array}{r}
 125 \\
 \times 645 \\
 \hline
 625 \qquad (125 \times 5) \\
 5000 \qquad (125 \times 40) \\
 75000 \qquad (125 \times 600) \\
 \hline
 80625
 \end{array}$$

He stated that by using brackets he gets his learners to 'breakdown a number'. This was a way of using the distributive law. The teacher however did not make this link but merely showed it as a way of how each step in the algorithm is calculated.

PUFM

Basic ideas: The teacher believed that the following concepts are important to the understanding of this concept, Place-values, decomposing of the number, and multiplication of single digit numbers

Longitudinal Coherence: The teacher started with multiplication of single digit numbers then linked to two digit numbers and then to three digit numbers and more.

Connectedness: He did not link the topic to any other mathematics topics but merely spoke of the progression from single digit to multiple digit multiplication.

Multiple perspectives: He used the long multiplication method but interlinks it with the concept of distribution (consciously or not one is not sure). Tr. Barend was open to other methods but he did not suggest any in his questionnaire. He stated that he felt that he could improve in this regard.

Tr. Barend's understanding of this concept was based clearly around the algorithm. He taught it and the concepts that he believed would make using this algorithm easier. His understanding is not very deep. He recognized place-value as being important but he did not discuss its importance to the concept. His first port of call when teaching this section is a review of single-digit multiplication. He believed that his complex understanding of this topic is weak and possibly needed some work. I would therefore not regard his knowledge of this concept as being profound.

Conclusion

The teachers recognized that the problem that the learners were experiencing was due to a misunderstanding of place-value and thus they did not line up the partial products correctly. The teachers described the methods that they would use and then discussed why they felt that the learners made the mistake. Some the teachers felt that it was a problem with the learners not knowing the procedure, whilst others felt that it was due to a lack of understanding of place-value. Like Ma (p.54) states the teachers' perspectives on the problem paralleled their subject matter knowledge of the topic.

C. Scenario 3: Generating Representations of Division by Fractions

In this scenario the teachers were required to accomplish two tasks:

- To compute $1\frac{3}{4} \div \frac{1}{2}$
- To represent meaning for the resulting mathematical sentence.

When looking at the large group, it is noted that some of the teachers simply did not answer this question. Of the fifteen returns received, three did not write anything on the questionnaire. This could be because they did not understand the question or that they could not understand what it meant to divide by a half or that they could not find a good story that would depict this scenario.

To review the teachers' responses to this scenario I used a combination of Ma's and my own categories. The analysis of this scenario will be dealt with slightly differently to the first two topics as this scenario called for more than just an analysis of a procedure as the first two scenarios did. I have therefore placed the teachers' responses into categories and then tried to relate their responses to the PUFM instrument. I have also opted to use responses from teachers who answered this section of the questionnaire but were not part of the sample of five. Here are the categories:

Wrong strategy and no answer

There were two responses that I categorized as having 'wrong strategy and no answer', Ma (p.58), because although the teachers wrote something in response to the question, they did not show much or any understanding of the procedure to obtain the answer nor did they provide a story that could be used to show some understanding of the question asked. To demonstrate this here is a response from one of the teachers:

"I don't think that there can be a model for this. Except that we can change that mixed fraction to an improper fraction." (Tr. Mahlongo, he was one of the teachers who responded but was not part of the final five that formed the sample.)

Tr. Mahlongo showed that his knowledge of division of fractions was almost non-existent. He felt that there was no way of calculating this and that the only thing that could be done to this calculation was to change the mixed number to an improper fraction.

Confounding division by $\frac{1}{2}$ with division by two

Three of the teachers that I placed in this group confused dividing by $\frac{1}{2}$ and dividing by two or halving. Their stories concerned dividing cakes and pizza's equally between two people. Tr. Barend mentioned in his story that he wanted to divide the one and three quarter pizzas equally between himself and his friend Jimmy. Tr. Mahlongo wanted to divide one and three quarter loaves of bread in half. They both are clearly confusing multiplication by a half or dividing by two with dividing by a half. Tr. Belinda also confused it with the division by 2. She makes the statement, "I am now dividing it between 2". Although she got the correct answer, namely $3\frac{1}{2}$ she was more concerned with finding out how much each person would get after the division.

Correct Algorithm but no or an improper representation

This group of teachers managed to do the calculation but failed to provide a story that reflected what was meant by division by a half. Tr. Belinda also fell into this group since she was aware of the algorithm but could not get a story that represented this mathematical situation. Looking at her story brings out a few glaring inconsistencies. Her story reads as follows:

I have two pieces of timber. I cut one into 4 quarters ($1 = \frac{4}{4}$). I cut the other into 4 quarters and take away one leaving $\frac{3}{4}$. I now have $\frac{7}{4}$ (Tr. Belinda notes that she changes mixed numbers into improper fraction) I take another piece of timber and cut it into four. I take away 2 and now I have $\frac{2}{4}$ but 2 quarters = $\frac{1}{2}$ I invert this and it becomes a whole number, I am now dividing it between 2.

Solution: $\frac{7}{4} \times \frac{4}{2}$ or it could stay $\frac{7}{4} \times \frac{2}{1} = 3\frac{1}{2}$ quarters

$$\frac{7}{4} \times \frac{4}{2} = \frac{7}{2} = 3\frac{1}{2} \text{ quarters}$$

Each gets 3 quarters and a $\frac{1}{2}$ quarter.

It is clear that she did not understand that in dividing by a half you are actually finding out how many halves there are in this given fraction. Tr. Belinda is able to deal with the concepts of inversion of fractions, multiplication of simple fractions and converting between mixed numbers and improper fractions. She was clearly aware of the basic algorithm and is able to use it. She however definitely had a lack of understanding of the concept of dividing by a $\frac{1}{2}$.

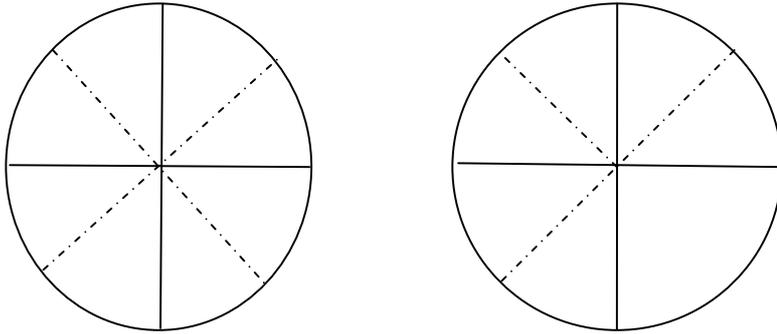
Tr. Daya also falls into this group, since he confused the inverting with multiplication of two. He simply multiplied his number of pizza pieces by two and thus ends up with 14 pieces and says that because each pizza is made up of 4 pieces the 14 pieces must be divided by four to give the result $3\frac{1}{2}$. He did not divide the pieces that he has by half but multiplied by two thus doubling the number of pizza pieces he has. In his illustration he drew two complete pizzas and two pizzas with each one missing one quarter. Therefore the value $3\frac{1}{2}$ will indicate that there are now three and a half pizzas, which is double the original number. He thus could also be placed into the above category as well.

In summary, these teachers seem to be able to use the algorithm but they battled with the word illustration. They may be classified as being procedurally strong but conceptually weak. They are aware of the basic skills related to fractions. They can convert to improper fractions and can do multiplication of fraction but they could not illustrate the meaning of the scenario using an everyday example. It was through doing this that their knowledge of basic concepts was found to be lacking. I could therefore not regard their knowledge of division of fractions as profound as it is not deep or complex enough for this.

The Measurement Model of Division

Tr. Dumisani used the notion of the Measurement Model of Division in that he asks ‘how many halves are there in $1\frac{3}{4}$ ’. He found the correct answer but did not demonstrate how he comes to this answer. He simply mentioned that he would get the pupils to do a practical demonstration using the $1\frac{3}{4}$ loaves of bread of which he spoke.

Tr. Margaret also used this Measurement Model. She however demonstrated this by drawing the following circles:



Her story was as follows:

I bought $1\frac{3}{4}$ m of black material and I wanted to divide this material into halves. How many halves am I going to get?

This is a correct representation but it falls apart below.

She explained her process as follows:

Assume the pieces of material are circular. Divide these circles into quarters so as to depict $1\frac{3}{4}$. There should be seven quarters in total at the moment. Each quarter is then to be divided in half as depicted by the dashed lines. This creates fourteen pieces. Each of these pieces is half of a quarter. This can be written as $\frac{14}{4}$. This is then reduced to $3\frac{1}{2}$.

Tr. Margaret also demonstrated what she called the traditional algorithm. Her calculation using the traditional algorithm was correct and set out well. She thus showed that although she knew the algorithm she needed another method to explain her story.

To critique the methods used by the above mentioned teachers we have to unpack the errors that they seem to be making. To me the errors are due to their lack of the understanding of division by a $\frac{1}{2}$. The method used by Tr. Dumisani and by Tr. Margaret uses the idea of dividing each quarter in half and then counting the number of quarters that they have. This gave them 14 quarters. They then wrote 14 quarters as $\frac{14}{4} = 3\frac{1}{2}$. Since these teachers new that the algorithm

would yield such answer they were happy that their answer was correct. These teachers are aware of how to convert from mixed numbers to improper numbers, of how to invert fractions and how to do multiply simple fractions. They therefore have some understanding of fractions but they lack an understanding of what it meant to divide by a half. This was the main idea of this scenario.

Conclusion

This scenario was therefore not well done and all the teachers answered it poorly. There seemed to be two main problems. The teachers (all but one) knew the algorithm but struggled to create a story that described what was meant by division by a half. The second main problem was the teachers' understanding of division by a half. The conceptual understanding of this concept was lacking all the teachers'. They thus were all procedurally knowledgeable but conceptually very weak.

D Scenario 4: Exploring New Knowledge: The relationship between Perimeter and Area

This scenario will be analysed by grouping the replies of the teachers based upon their strategy used or not used. Ma (p.85) mentions three strategies employed by the American teachers namely,

- Consulting a book
- Calling for more examples
- Using Mathematical Approaches (indicating rejection or acceptance)

What follows is also a grouping of the replies. These categories are not necessarily the same as those of Ma's.

Group 1: No answers, non-mathematically informed answer

Two replies were blank. This could have again been because of a lack of understanding and thus a way of opting out; or it could be that because it was the last scenario that the teacher lost interest or ran out of time and thus did not attempt it. Tr. Dumisani is placed into this group since his replies were more of a pedagogical nature. He spoke of praising the pupil for showing insight and thought. He did not state whether he accepts or rejects the pupil's answer and did not try to

check whether the solution is correct or not. Tr. Barend also left this section blank but attached some notes he received from one of his more experienced colleagues.

Group2: Getting the pupil to explain the findings

Tr. Mahlongo, Tr. Daya and Tr. Mofokeng were not convinced to accept the claim with only one example.[Tr. Mahlongo and Tr. Mofokeng were part of the population but not part of the final sample of five] They wanted the pupil to explain the method he used to come to this conclusion. It could be that they themselves were not sure of his method and required further explanation of his method or it could be that they themselves could not draw a conclusion about whether this was true or not, depicting uncertainty on their behalves.

Group 3: Explanations of the scenario

Two teachers, Tr. Zulu and Tr. Kheswa [were part of the population but not part of the final sample of five] both simply explained what the scenario was about. They calculated the area and the perimeter of the given shapes. Their calculations seemed to be done as though they were checking the pupil's results. They offered no explanation nor did they offer any acceptance or rejection of the pupil's claim.

Group 4: Acceptance of the claim

Two teachers namely Tr. Thembu and Tr. Belinda stated that they would accept the claim. Tr. Thembu stated; "I would tell her that yes it is true." Tr. Thembu's reason for accepting this claim is not mathematical because he stated that since the perimeter was being extended there should be more space and thus the area should increase. He failed to explore the relationship between the sides and the perimeter and the sides and the area. Tr. Belinda also gave a similar reason for her acceptance of this claim. She stated; "... if the perimeter is longer or shorter the area becomes larger or smaller."

Group 5: Diverting the student

Tr. Hani [also part of the population but not part of the sample] opted to divert the pupil. He stated, "...we are going to learn about that next week" This could have been done to buy himself time so that he could go and investigate this claim for himself or because he was not sure of how to do the calculation to disprove this claim, or it could simply be to stay with his planned topic.

Group 6: Making an exploration/investigation

Tr. Margaret was the only teacher that actually investigated the claim made by the pupil. She investigated this claim by getting pupils to try out numerous dimensions. She herself drew up two shapes and determined the perimeter and the area of these shapes. She then drew a table so as to show the results obtained when she used different dimensions. Sadly she did not draw any conclusions and it is thus not possible to conclude whether she agreed or disagreed with the pupil.

Conclusion

This scenario investigated the teachers approach to a mathematical idea that was new to them. It looked particularly at the relationship between perimeter and the area of a rectangle. Ma stated that two aspects of subject matter knowledge contribute to a successful approach: knowledge of topics related to the idea and mathematical attitudes. The teachers mostly showed knowledge of the related topics. Many of them could calculate the perimeter and the area of these shapes yet all but one could not make a link between the perimeter and the area. It was in their attitudes to the challenge posed by such learner that we can see that they lacked a profound understanding of this concept.

Chapter 6: Analysis of Teachers Mathematical-life Histories

6.1. Introduction

This is the second part of the analysis and will review the interviews with the teachers. This part looks at what was seen as the second stage of the research. The second stage began once the teachers who answered the first questionnaire well were identified. I then set up a meeting with each of them so that I could give them the second questionnaire and also to interview them about their mathematical life-histories.

The aim of this chapter is to analyse the life histories of the teachers that were interviewed.

6.2. The Thematic Approach to analysis of the data collected in the interviews

6.2.1. Theme 1: Experiences as Learners

This theme became important as many of the teachers spoke extensively about their experiences as scholars in mathematical classrooms. In many cases it was these experiences that had marked influences on their attitudes towards mathematics, their views towards their knowledge of mathematics and their perception of the type of teacher that they wanted to be (Carrol, 1988; Parker 2000).).

The teachers make the following comments about the above:

Tr. Margaret

Before we had to remember arithmetic. We ... (pause). It was wrong. It was wrong. We did not know what it means. They just say $9 \times 9 = 81$. We could recognise it not knowing the concept. What does it mean?

We used to recite our tables everyday. It was boring. I didn't like that teacher. I wanted different methods.

It was boring. It was boring because we had to cram it. This is this, this is this and for the fact we didn't make it for ourselves. This was the teacher's idea so we have to follow. If we forget one sentence or one method we forget everything. Lets take an algebra sum, if you forget something so every thing is wrong. All because I crammed it.

It was obvious that Tr. Margaret's experiences as a student were not enjoyable as they mainly recited and recalled methods given by the teachers. She did not feel challenged or part of the education process since to her the learning process merely demanded her to memorise facts and procedures. From what she states below it is noted that she distances herself from this approach.

Tr. Belinda

I teach you learn. Lots of the time was spent on teachers teaching and you listening. We were not allowed to think and work out processes; we were just given (these). These methods stood us in good stead. I still use some of these processes with my children (pupils/learners). For him it was straight forward text book method.

Tr. Belinda enjoyed this method and saw it as still being an effective way of teaching. She felt that if she was shown one method then she could apply it and then develop other methods from her understanding of this one method. She still uses this methodology in her teaching.

The teachers above differ in that Tr. Margaret believed that being given methods does not involve the learner enough and that the process simply demands recall and not understanding, whereas Tr. Belinda believed that you cannot leave young learners to fathom out methods without introducing them to at least one approach or method.

Tr. Dumisani

Tr. Dumisani and Tr. Margaret on the other hand brought up the point of making mathematics practical. Tr. Dumisani stated that he appreciated the way his teacher made the subject very practical. He made the following comment about this:

“So that motivated me because there was lot of practicality he used when teaching mathematics. He could bring oranges, slice them amongst and then... when dealing with fractions or even apples, bread to. After that he would give to each and every learner in his class.”

Tr. Margaret also made a very similar comment about her primary school teacher. She felt that the teachers’ actions not only made the subject practical but were also enduring to the students in that she would bring things from home to assist her students understanding and then give them the food to eat.

The teachers all had different experiences at school. Tr. Belinda’s recall of her mathematics teachers’ approach is one of I teach you learn. She enjoyed this as to her there was method and she felt comfortable that the teacher knew what he was talking about. This sought of approach however did not go down well with Tr. Margaret as she felt it did not challenge her as a learner. Tr. Dumisani and Tr. Margaret on the other hand also enjoyed the fact that their teachers tried to make the subject practical by using manipulatives to assist in their teaching. It is thus important to note that there seems to be approaches that are acceptable to some but are not enjoyed by others. What seems to come through is that these teachers’ school experiences have definitely influenced the teaching.

6.2.2. Theme 2: Teacher Training

The interview set out to probe the teachers’ recall of this area of the study. It was aimed at getting a broad view of the experiences that these teachers had as student teachers both during their initial training and their training on the NPDE course.

Tr. Barend stated the following about his initial training at Dokkies:

“The teaching was very strict. It was very conservative. It was, more information given. At school we were allowed to participate but in lectures at Dokkies you just sat and you listened. You very seldom discussed things. There was no group work - nothing of the kind. The lecturer standing in front, he was the boss and you just followed.”

Tr. Dumisani noted the following about his initial training as a teacher at Madadeni College of Education:

It was very wonderful. It was well staffed. I can say that it was not (so) nice. Why I say it was not so nice because educators were from different races. The Principal of the school was Mr. ...who was also a mathematics examiner by those years. He was exemplary. Even his colleagues were very keen to assist...

Tr. Margaret noted the following about her initial training at Eshowe College of Education:

According to this Eshowe was a very big school and most of our teachers were Afrikaners and they were so racist. They don't like the Blacks. They had no time to teach the children. You end up teaching yourself. Because we Blacks we know nothing about mathematics. They even change their medium of instruction to Afrikaans, so that we all fail. The teacher that was responsible for mathematics, use to sell clothes, eggs and everything. He really did nothing.

Tr. Belinda noted that the college which she initially attended was very similar to a high school. They were expected to wear a uniform like at school. They were in many ways treated like high school pupils. She made the following comments about her College:

"It was like a high school. It was the same thing as going to '..... High School' The Coloured high school she attended. There wasn't a Biology lab or a science lab. There was nothing. It was a very run of the mill school."

To conclude this theme, let me first mention that there seemed to very varying views about training as it occurred at all these teacher training institutions. One teacher enjoyed the strict approach but others like Tr. Margaret felt that her time at the College was wasted. Secondly it must be mentioned that all these teachers did mathematics as part of a generalist training course and not as specialist teachers of mathematics. This impacted on their mathematical knowledge as they merely repeated school mathematics and then combined it with didactics or teaching methodology. From the level of mathematics covered at these 'Colleges' it is noted that the mathematical knowledge of these teachers was not extended beyond the grade ten level as this was the level at which many of them left school to start their lives as student teachers.

6.2.3. Theme 3: Personal Philosophy

Carrol (1998) explained personal philosophy to be the expression of positions that convey a sense of coherence in self-understanding regarding teaching practice and personal history.

Tr. Belinda felt that mathematics comes easily to her. She believed that she has a natural inclination for the subject. She made the following statement in her interview,

“I (have) always loved mathematics. Math’s has always been my pet subject.”

Tr. Barend expressed a very similar view about mathematics. He stated:

“...it’s because I am math’s orientated. That’s why I might find it easy”

“I got excellent results for mathematics because I love the subject. It was an extremely intriguing, fascinating subject.”

Tr. Margaret on the other hand liked the subject because she felt she did not have to study it. She made the following comments about mathematics during her interview:

“There is no study in mathematics, I just practice.”

Tr. Dumisani has a very practical philosophy of mathematics. He believed that people are not aware that mathematics is everywhere and it incorporates all aspects of their lives. He makes the following statement in this regard:

“One cannot say mathematics is difficult because it is within ‘ourselves’ (us) and that is our lives. Everything we have or think of is mathematics. Money, if you think of money, then that is mathematics. How much do you want to earn? You could say the figures. That is mathematics. What do you want to possess? It is mathematical. And your dreams are mathematical. You cannot live without mathematics.”

Three basic philosophies are obvious from the analysis. The first being that they do well because they have a natural ability/ affinity for the subject, secondly because mathematics is logic and you therefore do not need to study it, and thirdly mathematics is part of our modern lives as it is incorporated into everything we do therefore we should be able to do well in this subject.

6.2.4. Theme 4: Significant influences

In Carrol (1998) she described this theme as meaning the descriptions of experiences that the teacher would view as having informed an on-going change in their teaching practice and consequent self-understanding. The teachers interviewed mentioned influences that came from parents, teachers and lecturers.

Teacher Belinda makes the following statement in this regard:

“My mother was a teacher and of course in those years there was not much for Coloured ladies to go into. They either became teachers or nurses.”

Tr. Belinda mentioned two major influences, namely the factor that her mother was a teacher and secondly the fact that during the Apartheid era there were not many options available for young Coloured women to take up as many did not have the finances to study at Universities and there were very few bursaries available. Thus many ‘Black’, Coloured and Indian young women went into teaching rather because they could get a bursary to study rather than because it was a career choice.

Tr. Barend makes the following comment:

“..partly (because) my wife, being a student teacher. I met her at Dokkies training college in Durban.”

Here we see that he became a student teacher because of the influence of a girl friend who was studying at the college of education.

Tr. Margaret brings yet a third aspect to this discussion in that she states that she became a teacher because of her love for children. When asked about what influenced her to become a teacher she stated, *“Love (for) children.”* Here we have the career path option coming forward. Tr. Margaret did not mention any other external influences that helped her make her decision to become a teacher and it should be noted that none of her parents had any form of education.

Tr. Dumisani introduces the fourth aspect namely that of a teacher as a major source of influence. He states the following,

“It was because of my teacher. Amongst my teachers in the primary schools, there was a mathematics teacher. Fortunately he was related to my family. So it could be said that his each and every step he took was observed. He was also teaching us mathematics”

This aspect has been mentioned by many of the teachers. Let us now look at statements made by the other teachers in this regard.

Tr. Belinda made the following comment:

“.....my high school teacher, Mr. J..... He definitely made an impression., it was his way of expressing himself and his way of explaining things to us. Mathematics was enjoyable. Mathematics to him was a breeze. He made it so that we could enjoy it.”

Tr. Barend made the following comment in this regard:

“There have been some teachers and I can name them.was one of them, he taught me Business Economics; Mrs. Mac.... she taught me math's..... But to answer, teachers I can remember had a huge influence on my life.”

Tr. Margaret also spoke of a teacher who made mathematics very practical. He would bring chickens to school to use as a teaching aid for fractions she states. She also mentioned that he used his arms to show the differences in angles. The teachers' practical method of teaching has made a lasting impression on her.

The influence that these teachers had seemed to come about due to the perceptions their students formed about them. They viewed them as being enthusiastic, caring, knowledgeable and enduring towards them. It could be these characteristics that remained engraved in the minds of these teachers.

6.2.5. Theme 5: Mathematics Teaching

This theme is slightly different to the others as all of the teachers have been teaching many years and have thus developed good pedagogical skills over this period. All of them stated that they teach mathematics as a subject at their schools and enjoyed teaching the subject. They felt that their problems with teaching mathematics was brought on by pupils who did not have the 'basics' in place and they therefore found that they had to keep going back to re-teach sections of work that these pupils should have known. Their opinions about teaching therefore did not differ much. Tr. Daya brought up the fact that after doing the NPDE mathematics course he felt that he could do a much better job at teaching mathematics than his high school teacher. This to him was a challenge.

Table 5 (on the next two pages) Summary of the above themes

	Experience at time of interview	Training	Teacher Training	School Experiences	Maths Level
Tr. Belinda	30 years	Zonneblom College of Ed	From the Teachers comments we can read that she did not find College totally enjoyable as she saw it as being a repeat of high school.	Loved maths at school. Enjoyed the method used by her teacher.	Grade ten. Maths at College as part of generalist PTD course. NPDE Maths.
Tr. Dumisani	32 years	Madadeni College of Ed.	Enjoyed – Lecturers were enthusiastic as they were national examiners and thus insisted that their students worked hard.	Loved maths and enjoyed the way his teacher made the subject practical.	Grade ten maths(arithmetic) Maths at college as part of generalist PTD course and the NPDE Maths courses.
Tr. Daya	6 years	SACOL, MSTP & other Computer Qualifications	Enjoyed the NPDE. He did not have any other formal training as a teacher. He taught unqualified and then joined the NPDE to get his qualifications.	Hated the subject because of a specific teacher at his high school.	Grade 12 And then the general mathematics course taught on the NPDE(senior phase)
Tr. Margaret	31 Years	Eshowe College of Ed.	Found her initial training frustrating. She ended up teaching herself most of the time. She gained little in the way of mathematical knowledge from her time at this college.	Enjoyed the subject. Did not like the fact that recall and memorisation were the main features of their mathematics learning.	Grade ten Maths and 1 st year maths at College – not specialist course and the NPDE (Intermediate Phase)
Tr. Barend	5 Years	Durban College of Ed.(Dokkies)	The teaching was very strict. It was very conservative. It was a case of information given rather than interactive learning.	Enjoyed maths at primary school but battled at high school.	Grade Ten (JC) Maths and maths at College as part of generalist PTD course – not specialist course and the NPDE intermediate phase course.

	Maths Knowledge	Maths Teaching	Significant Influences	Personal Philosophy
Tr. Belinda	Confident in her ability to handle mathematics knowledge at primary school level. Feels that it has been to long since she has done any of the high school maths.	Loves teaching the subject and believes that it is her duty to give the pupils one method to stimulate understanding and then to let them explore other methods. Feels confident in her ability to teach this subject at this grade.	Mother was a teacher. Her high school maths teacher she was very good and thus made a significant impact in her understanding of mathematics.	Feels that she has a natural ability for mathematics. "I (have) always loved mathematics; maths has always been my pet subject."
Tr. Dumisani	Confident to handle the maths at the grades he is teaching but feels intimidated by high school maths because he last studied arithmetic.	Loves teaching this subject as he feels it is part of him and his everyday life.	A teacher who was a family member. A Principal who was also a mathematics Teacher.	Believes that maths must not be seen as being difficult as 'it is part of us', it is part of our everyday lives.
Tr. Daya	Confident in his ability to handle maths content up to and including matric work.	He developed a belief that he was capable of teaching this subject and felt confident that he could do a better job than his high school teacher.	He had no significant influences but always wanted to become a teacher. His motivation was more intrinsic.	Only a that after leaving school and doing the NPDE maths course did he believe that he was capable of doing well at this subject.
Tr. Margaret	Tr. M is confident that she can do and teach the level of mathematics that she is teaching now. She claims that taught herself much of the mathematics.	Loves teaching this subject. She felt initially that she was not competent but this changed after doing the NPDE maths modules. Tr. M felt alone in her as a teacher of maths initially as she received very little support from	No member of family was a teacher or had studied previously. She enjoyed the way her primary school principal made his lessons very practical.	Mathematics is not a subject to study. It must be practiced and through repetition you will become competent. Work hard and you will achieve good results.
Tr. Barend	Teaches Grade seven maths and feels confident about his knowledge of the mathematics that is taught at this level.	Loves teaching the subject but feels restricted by the older more experienced teachers in his school. They insist that sections are taught they way they have always been taught.	Father was a magistrate and mother was an administration clerk at court. His primary school teacher was his favourite teacher he taught maths and coached him rugby.	Believes that he is 'maths orientated' thus implying that some people are not and therefore they battle with the subject.

6.3. Conclusion

This chapter aimed at presenting the teachers life histories and an analysis of the themes that were analysed. These findings have been presented and they now therefore need to be discussed further. This will be dealt in detail in the next chapter. I will also seek to draw conclusions from the evidence presented and discussed in Chapter 5 and Chapter 6.

Chapter 7: Discussion and Conclusions

7.1. Introduction

This chapter summarises the findings brought up in the last two chapters of analysis and then draws conclusions from these findings. The first part of the Chapter will deal with the discussion of the individual teachers. It will look at both the teachers' performance in terms of PUFM and their Life-histories.

The second part of the chapter will draw conclusions by revisiting the research questions and assessing whether they have been answered, and if not look for reasons why this occurred. Secondly, it will look at possible short comings to the study and possible extensions. Finally I make recommendations to various concerned participants in the education system at both Governmental and Institutional level.

7.2. Part One – Teacher Understanding (PUFM)

In order to present this discussion I will look at the findings on each teacher and then draw conclusions about whether they possess PUFM or not. I will examine the teachers questionnaires and then use the four aspects namely Knowledge of Basic Concepts, Ability to make Connections (Connectedness), Ability to see the Longitudinal Coherence of topics (Longitudinal Coherence) and lastly check if there is any mention or link to other methods (Multiple Perspectives) to judge if their understanding is only procedural or procedural and conceptual and finally to state whether they could be considered to possess PUFM or not.

Tr. Belinda

This teacher is one of the most experienced teachers at the school at which she teachers. She is also seen to be very diligent with mathematics. She scored 89% for her final paper on the NPDE mathematics course. It would be expected that her understanding of mathematics at the Intermediate Phase level is be well developed.

When examining her response to the first scenario, it is noted that Tr. Belinda mentions only two basic concepts and did not make any connections to other related topics. She did however demonstrate a form of longitudinal coherence when she mentioned that she will expect the learners to know place-value, be able to expand numbers (uses the concept of borrowing) and then be able to do basic subtraction. Tr. Belinda mentioned that whilst she sees the need for learners to be able to apply different methods, she is prone to teaching just one method as it helps her to move through the expected work scheme quicker. She stated that there is very little time to complete what is expected of them as teachers. Self discovery type exercises are time consuming, she added.

Her response to the second scenario revealed a bit more. Here, she mentioned the basic concepts of place-value, multiplication tables and single digit multiplication. She stated that the learners battle to do multiplication if they do not know their multiplication tables and therefore drills this section and then moves to single digit multiplication and then multiplication of large numbers. This could be regarded as of longitudinal coherence. Tr. Belinda stated that she would then move to the concepts of money and time and will link these to multiplication. This could also be seen as the teacher making connections to other topics (connectedness). She however only mentioned these and did not elaborate on the type of links that she would make nor did it imply connections to other mathematical concepts. The once again reiterates that she teaches one method and lets the pupils do their own discovery in their own time.

Tr. Belinda's response to the third scenario revealed limitations in her conceptual knowledge. Her response fell into the group who confounded the concept of multiplying by two, dividing by two and dividing by a half. Although she was aware of the algorithm and used it correctly, she spoke about dividing the piece of plank into two equal portions. This clearly demonstrated a lack of understanding of the concept of dividing by a half.

Tr. Belinda's response to the fourth scenario was even more surprising. She accepted the learners claim and based her reason on the fact that if either the length or the breath is increased then the shape must have a bigger area. This is true but she did not consider the fact that although the perimeter may increase there may not necessarily be an increase in the area. Consider a rectangle with sides 4cm by 3cm. The perimeter is 14cm^2 and the area is 12cm^2 . If the sides are now altered

to be 6cm by 2cm, then the perimeter is 16cm but the area remains 12cm^2 . She did not investigate or try to show the link between perimeter and area.

To conclude I do not believe that this teachers' understanding of these concepts can be regarded as being 'Profound'. There is clear evidence of procedural fluency, strategic competence and to some extent adaptive reasoning, but her conceptual knowledge has limitations. She tends to view concepts and procedures in isolation (limited longitudinal coherence and connections). The later could partially be the result of her teaching the same grade for many years, but that does not change the fact that it has the potential to impact negatively on the learning of her learners.

Tr. Dumisani

At the time of the interview, Tr. Dumisani had been teaching for 32 years, besides being principal of a school in the area. It would therefore be expected that with he should have developed a good understand of the mathematics subject knowledge for the area of mathematics that he was teaching. Let me now look at his responses to various scenarios.

When analyzing his response to scenario one, the first thing that becomes apparent is that he did not mention the standard algorithm nor did he mention the notion of 'borrowing'. His method is to subtract the tens from the initial amount to get a new value and then to subtract the units in order to get a final answer. He effectively reduced his calculation to subtraction of a one digit value from a two digit value. However, he never addressed the problem of how to subtract when the unit value of the minuend is larger that of the subtrahend. The two basic concepts that he mentioned are place value and regrouping of numbers. He also mentioned that it is important that the learners can subtract single digit values. This could be seen as some very rough form of longitudinal coherence and thinly demonstrated. Lastly, Tr. Dumisani mentioned that he is open to numerous methods and bears in mind that pupils will come up with different methods since they are taught to "*think and formulate their own findings.*"

Tr. Dumisani stated, in his reply to the second scenario, that he liked to make links to prior knowledge as it allowed the pupils to assess how much they knew and this would result in a smooth link between the knowledge already taught and the knowledge that was still to be taught. He mentioned that for this topic he would review the mathematical tables and the concept of

place value. He stated that his method of teaching this topic is linked to the way he was taught it. He initially insists that the learners use zeros so that they are aware of the true value of the digits and once they have reached a stage where they are no longer making mistakes, he let them use their own methods. He was thus open to different views or multiple perspectives with regard to this concept. Lastly his very loose links to the concepts of place value and regrouping makes it difficult to believe that he has the ability to connect to other related topics in an adequate manner.

Tr. Dumisani asked the very relevant question in scenario three, namely, 'how many halves are there in $1\frac{3}{4}$ '. This thus placed him into the group that uses the Measurement Model of Division. His procedural understanding of the algorithm was clear but he showed a lack of understanding of the concept of dividing by a fraction.

Tr. Dumisani did not really answer the question in the fourth scenario. He spoke of praising the pupil for showing insight and thought. I could be easily interpreted that he accepted the learner's view by the type of praise he gives to the learner.

In summary, let me state that I do not believe that this teacher had a strong enough understanding of these concepts to call it 'profound'. He has a definite knowledge of the basic algorithms and is procedurally fluent in using these. He however lacks the depth of understanding and his inability to make relevant links to other related topics is an indication of the limitation he experiences with regard to the breadth of understanding. He mentions that there are other methods and algorithms, whilst noble, he never discussed any. This could also indicate a limitation in his complex understanding of these concepts.

Tr. Barend

Tr. Barend has not been teaching for many years. He initially studied at a teachers training college and then left to pursue a career in the prison services. He then left the prison services and took up a position as a sports coordinator at the school at which he is presently teaching. He worked for a few years in this position before joining the NPDE course. His knowledge of mathematics seems to be based upon his recall of his school work. He recalls many of the

algorithms he was taught at school but displays very little conceptual knowledge of the related concepts. When examining his responses to the various scenarios this becomes very obvious.

For the first scenario he mentions three concepts that he regarded as basic, namely place-value, 'bonds' (addition tables) and simple multiplication. He continuously mentioned place-value as being vital to this process. In his reply he never mentioned how these are linked to the topic. The only form of longitudinal coherence in relation to the topic came in the form of a flow chart that simply indicates that he will move from simple subtraction (subtraction without borrowing, as he puts it) to subtraction with borrowing. We could also say that there is some form of longitudinal coherence evident in his relating place-value, bonds and simple multiplication to this topic. On the last aspect, multiple perspectives, he spoke about letting learners explore other methods but did not mention any of these methods himself. We can therefore not be sure if he is aware of any of these.

Tr. Barend showed some link to previous knowledge when he mentions the concepts, place-value; decomposing a number; distribution and multiplication of single digit values as being basic to the understanding of this section. He failed to link this topic to any future topics. His expression of longitudinal coherence is restricted to a progression from single digit multiplication to two digit multiplication and then to three and more digit multiplication. His only link to other methods of dealing with this topic is the fact that he used distribution to assist him in applying the traditional method. He definitely displayed the procedural knowledge to deal with this topic. In using the aspects that Ma uses to judge his conceptual knowledge of this topic it is found to be limited, since he did not make solid links to other topics or displays any strong sense of awareness of the longitudinal coherence around this topic. I therefore cannot say that he has a 'profound understanding' of this topic.

For the third scenario Tr. Barend was placed into the group that confused multiplication by $\frac{1}{2}$, division by $\frac{1}{2}$ and division by two. He seemed to confuse these concepts and therefore it can be concluded that he did not have a 'profound' understanding of division by $\frac{1}{2}$.

Tr. Barend's response to the fourth scenario was the most surprising; he stated that he did not know what this question was about and so he got one of his colleagues to assist him with it. He did not respond to the question posed but merely attached some notes from his colleague. This was a clear indication that he had no knowledge of this topic or that he could not understand the question. He was aware of the concepts of area and perimeter but could not make the link between the two. This again shows that he did not have a 'profound understanding' of this topic.

In summary, it must be stated that Tr. Barend's knowledge of these topics limited to procedure and is lacking conceptual understanding. He therefore does not display PUFM. He displays some procedural knowledge (although limited); he has and displays productive disposition as he sees mathematics as sensible useful and worthwhile but his conceptual understanding is very weak.

Tr. Daya

Tr. Daya is a relatively new to mathematics teaching. He is the youngest of the group. He also completed mathematics up to grade twelve. He however stated in his interview that he hated mathematics at school because of his mathematics teacher. He did not believe that he was any good as a teacher of mathematics. Let me now discuss his responses.

Tr. Daya in his response to the first scenario introduced the concept of regrouping values into more convenient numbers. He stated that the method he now teaches is not the method he was taught at school as he was taught the traditional column-borrowing method only. His method entails getting pupils to change the values of numbers to multiples of ten or identical units. He believed that it is easier to do the subtraction if you are subtracting from zero or when the digits are identical as the result will be a multiple of ten. He then added or subtracted the amount he took off or added on in order to get his more convenient value. To get the pupils used to this method he used a process of getting pupils to be able to change values to more convenient values followed by drill and practice, and then he got the pupils to change values so as to get identical units, again followed by drill and practice, and lastly he got them to change values to multiples of ten.

In dealing with this scenario he mentioned simple subtraction, subtraction of tens, and subtraction of numbers with identical units as being the basic ideas that he believes that pupils

should know in order to do subtraction with regrouping. He did not touch on the concept of place value as such. His only inclination to other link to related areas was his mention that he would get pupils to solve word problems. His position of multiple perspectives was that it is good, but he only teaches his method.

Whilst his procedure is based on the concept of regrouping he fails to mention the reason for regrouping. To him, the process of regrouping is done so as to make subtraction easier. So to be brutal I would classify his knowledge of this section as being more procedural than conceptual. If he has a conceptual understanding of these individual concepts he does not display this in his teaching or discussion of this concept.

In his response to the second scenario, Tr. Daya stated that he used the traditional method. He recognized that pupils need to be given time to explore other methods for themselves but stated that time for this sort of learning was not available. He pushed the pupils to master this one method and drills them to get the 'layout' of this type of calculation right. In order to achieve this, he progresses from single digit multiplication to two digits multiplied by one digit and then to two digits multiplied by two digits. This could be regarded as longitudinal coherence within the topic of multiplication. He connected this type of multiplication to division in that he stated that he will get his pupils to "*test the quotient through multiplication*". He also linked this topic to squaring and the solving of word problems. I therefore feel that his response to this scenario, more than the first, indicated more of a conceptual understanding with connections and clear longitudinal coherence. His procedural knowledge of the algorithm is good but he however again fails to explain the link to place-value and distribution.

Tr. Daya fell into the group of teachers that confounded the division by a half; multiplication by a half and division by two. He got the right answer but multiplied by two. He indicated in his diagram that he would have two full pizzas and two three quarter pizzas. He thus doubled the amount instead of dividing the existing pizza into halves. His confusion seemed to be around the concept of multiplying by two when inverting. He thus had the procedural knowledge but lacked the conceptual understanding of this topic.

For the fourth scenario, Tr. Daya was not happy just to accept the pupil's claim. He stated that he would want the pupil to explain his method to the full class and then let the class evaluate whether this was true or false. Whilst this is pedagogically strong it does not indicate whether he himself actually understood the claim and could refute it. As he previously indicated that he preferred to teach one method, this is in contrast, pedagogically, and may reflect on his uncertainty about the correct answer. The only inclination that could indicate that he seemed to accept the pupils claim comes in his statement that he would praise and reinforce the effort of the pupil. I am therefore of the opinion that he did not possess the conceptual knowledge of the link between perimeter and area to actually refute this claim.

In summary, I believe that Tr. Daya is overall very knowledgeable of the procedures required to teach these sections. His conceptual knowledge is not deep enough for me to state that his knowledge is profound. His conceptual knowledge of the easier topics was stronger but that of the more involved topics was lacking. I therefore do not feel that he has 'PUFM'. Still, he demonstrated good procedural knowledge, adaptive reasoning and a productive disposition. All of these are evidenced in the above paragraphs.

Tr. Margaret

Tr. Margaret is the last of the sample group that I will discuss. She is regarded as an experienced teacher as she had been teaching for 30 years at the time of the interview. She indicated that she loved the subject but much of what she learnt about the subject she taught herself. She did not have a good experience with her mathematics lecturer at her College of Education and thus worked on her own quite a lot.

Tr. Margaret's responses to the questions made for some interesting reading. Let me now discuss her responses.

Tr. Margaret's response to the first scenario revealed much and she explained a good deal. She firstly indicated that she disliked the traditional algorithm as she found the concept of rounding up and rounding down to be, as she says '*totally wrong*'. She is also against the idea of 'borrowing'. She considered the concepts of place-value; understanding of what subtraction means; the knowledge of the mathematical terminology used in subtraction; and the ability to

expand numbers, as being basic ideas that pupils need to master in order to have a thorough understanding of this section of mathematics. Tr. Margaret gave two methods that she teaches, the first being the expansion of the numbers into tens and units and then doing the subtraction and secondly what she calls the add-on method. This clearly indicated that she has developed multiple perspectives of this topic. The flow chart she drew up indicated that she is aware of the longitudinal coherence of the various interlinking concepts. Tr. Margaret displays the strongest conceptual understanding of this topic.

Once again it can be seen, when looking at her response to scenario 2, that Tr. Margaret has a strong sense of what this topic entails. She mentioned place-value, breaking up numbers (decomposing values); estimation and basic multiplication as basic ideas behind the teaching of this topic. All of these aspects have a marked effect on the level of understanding of this type of subtraction if they are not in place before trying to teach subtraction with regrouping. Tr. Margaret mentions factorization; repeated addition; the use of fractions; the concepts of halving and doubling and the process of rounding but she does not discuss the nature of these links. This demonstrates that she is knowledgeable about these topics but it cannot be concluded that she is able to make the link between multiplication of multi-digit numbers and these mentioned topics.

Tr. Margaret displayed a form of longitudinal coherence in that she believed that pupils need to be taught place-value first then the rounding-off and decomposition methods must be used. She stated that she would like the students to estimate their answer using rounding and repeated addition to calculate the value. Tr. Margaret did not like the traditional algorithm and preferred learners to be given a chance to develop their own rules and methods. She mentioned two different methods of solving such a problem and thus is able to view such a topic from multiple perspectives.

Tr. Margaret's response to the third scenario demonstrated an understanding of the topic. She used what Ma (1999, p.87) calls the Measurement Model of Division. She mistakenly states that that she needed to divide the sectors in half. This clearly indicated her lack of understanding of the concept of dividing by a fraction. She showed that she is aware of the traditional algorithm, but still opted to use another method to solve the problem. She is thus procedurally capable but conceptually weak.

Tr. Margaret was the only teacher that investigated the claim in the fourth scenario. She did not just accept the answer as some did but she sadly fails to draw any conclusions from her findings. She did not take a stand as to whether or not she agreed or disagreed with the pupils claim.

So what can we conclude from Tr. Margaret's response? She clearly is procedurally very capable and demonstrates this well in her application of the standard algorithms. She also demonstrates that she is open to new methods and likes investigative work. Of the teachers interviewed she demonstrated the best understanding of these topics. She however cannot be said to have a profound understanding of fundamental mathematics as she at times was procedurally proficient but conceptually very weak.

7.2.1. Conclusion Part One

To conclude the first part of the discussion section of this chapter let me revisit the first key question asked,

Do teachers who scored high in the NPDE mathematics examinations have a Profound Understanding of Fundamental Mathematics?

In answering this question I had to establish what was meant by fundamental mathematics. This we did by looking at Ma (1999; p.116) where she concludes that Elementary mathematics is Fundamental mathematics. Secondly I looked at the concept of 'Profound Understanding of Fundamental Mathematics'. I then used Ma's indicators of PUFM as an instrument to judge whether the sample of teachers in their responses to the four TELT scenarios demonstrated PUFM.

The findings are rather dismal but are they in line with the general view of the education system in South Africa? There is definitely a trend noticeable among these teachers. They are all aware of and can apply the basic algorithms required in scenario one and two. They are procedurally competent with regards these two concepts. They however seem to lack the conceptual knowledge that supported these concepts and the algorithms that are applied to these concepts. Even (1990), notes that procedural knowledge can be learned without meaning. This to me is what has happened with these teachers.

The teachers battled to deal with the third and fourth scenarios. All the teachers could do the algorithm to obtain the correct answer for scenario three but they could not find a suitable story that reflected what it meant to divide by a half. They confused division by half with division by two. Their stories reflected this. Two of the teachers divided the quarters in half and obtained the correct answer. This however showed a lack of understanding of the concept of dividing the shape into halves. These misunderstanding clear highlighted the teachers' conceptual limitations around this concept.

The fourth scenario was handled the worst as only one teacher actually tested the learners' proposition. Sadly this teacher did not draw any conclusions from her investigation. They either accepted the statement as is or they got the learner to explain the statement to the class and then got the class to draw conclusions about the validity. Whilst this was pedagogically acceptable it does not give a clear view of the teachers understanding of this topic. Again there is no clear indication that the teachers' understanding of the relationship between perimeter and area is profound.

It can thus be concluded that although these teachers scored high on the NPDE assessments their understanding of the fundamental mathematics dealt with in this study is not profound. These teachers still have a limited conceptual understanding of mathematics for teaching, are not able to link topics, and have limited perspectives on the topics. This shows that the NPDE mathematics course has failed in two respects; 1) to provide teachers with PUFM, and 2) to assess them for PUFM.

7.3. Part Two: Teachers Mathematical-life histories and influences

The second part of this discussion and conclusion chapter looks at the second key question namely,

Are there any similarities in these teachers' mathematical life histories that appear to have influenced whether or not they have developed a 'Profound understanding of fundamental mathematics'?

In order to do the second part of this chapter I am going review the analysis in terms of the secondary questions asked on page 16 of this document.

7.3.1. Type of Education they were exposed to

All these teachers where initially exposed to very different education systems. Tr. Belinda's school fell under the House of Representatives (Coloured); Tr. Barend's school fell under the House of Assembly (White); Tr. Daya's school fell under the House of Delegates (Indian); Tr. Dumisani's fell under the DoE schools (Blacks under the KwaZulu Government) and Tr. Margaret's school fell under the DET which look after schools not covered by the KwaZulu government. All these departments had completely different funding and were run in total isolation. The standard of education created by these various departments was completely different with, as mentioned earlier, the least amount of money going to the black schools especially those in the rural areas.

It is surprising that the teacher that showed the best understanding (although regarded as not having PUFM) came from one of these poorly funded schools. The teacher from the HOA schools, having had the best schools, the best qualified teachers and the best resources did not display a good understanding of these topics. It must be noted that this teacher however has been diagnosed as having ADD and is on medication for it. He was thus not one of the top pupils to come out of these schools. His results from these scenarios go against the accepted notion that the 'white' population should have the best understanding of the basic mathematical concepts.

It must be stated here that there could be other circumstances that could have led to both teachers achieving as they did. Tr. Margaret's self motivation and her drive to succeed seemed to have led her to work extra hard and she managed to develop a strong understanding of these topics.

7.3.2. Type of mathematical teaching/ Influence of Teachers

A notable observation is that all the teachers irrespective of the schools they went to remembered a teacher or lecturer (but not necessarily both) that made a significant impression on their mathematical knowledge. Most of the comments made by the teachers about their teachers were in keeping with the findings of Nkhoma, (2002), discussed on page 46. Let me now look at some of the common responses from the teachers.

The teachers spoke of teachers that used practical examples to make their lessons interesting. They appreciated this as they felt it made the understanding easier.

Tr. Dumisani indicated the following in a quote about his mathematics teacher:

He was respectful because he would come to our side when ever we encountered problems. He was a father figure by the way he could guide and assist. He was helpful and then he was fully prepared because he was knowledgeable.(sic)

He was not he only teacher that felt this way. Tr. Belinda, Tr. Daya and Tr. Margaret all felt the same. In reading the above quote two main themes are revealed. These were firstly that these teachers were friendly, they were available and they motivated and encouraged their pupils, and secondly that these teachers were prepared in class and demonstrated a good understanding of school mathematics, (Nkhoma, 2002).

The next common factor from the interviews was that most of the teachers enjoyed the subject and projected this in their teaching. This is indicated in the responses by Tr. Barend and Tr. Belinda and Tr. Dumisani. Here are quotes from all of these teachers:

“He was a professional. He was a disciplinarian. He was exemplary too. He preached what he lived.... He did not like anyone to be left behind because (to) him mathematics was within himself. He wanted to produce effective learners or better mathematicians compared to himself.” (Tr. Dumisani)

“(It was) his way of teaching, his way of expressing himself and his way of explaining things to us. Mathematics was enjoyable. Mathematics to him was a breeze. He made it so that we could enjoy it. He didn’t make it a high ‘fluted’ thing. To him mathematics was mathematics and it was something we had to learn and it was something that we get on with it.” (Tr. Belinda)

“His whole manner – he had absolute love for teaching in general but when came up he opened that math’s book and I will never forget his face. He would say this is wonderful. ... His enthusiasm just flowed over to us. It rubbed off on us.” (Tr. Barend)

It is clear from the above quotes that such attitudes were important influence in the teachers’ mathematical life histories. It covers the fact that these teachers as pupils believed in and were motivated by the fact that their teachers were confident in what they were teaching. They thus were perceived to know their subject well. The second key concept here was the teachers’ enthusiasm for the subject. This the teachers believed rubbed off onto them and thus some of them enjoyed the subject.

7.3.3. The Influence of their Personal Philosophies

This concept played a significant role in their development and understanding of mathematics. Many of the teachers under this theme indicated that they felt that they were inclined towards mathematics or had a natural ability to do this subject. Here are a few quotes from the teachers that highlight this:

“I think it was just my in-born joy I get from the subject. A mathematics flair.” (Tr. Belinda)

“One cannot say mathematics is difficult because it is within ourselves and that is our lives. Everything we have or think of is mathematics.” (Tr. Dumisani)

“I felt that is part of me. I was just not learning from him. I was actually learning from math’s. That was the way we felt.” (Tr. Daya)

“The math’s can be more difficult, more challenging. But on the other side maybe its (sic) because I am math’s orientated. That’s why I might find it easy.” (Tr. Barend)

A second common theme that arises here is that of hard work. Tr. Margaret’s response to the question, what do you think motivated you to do well in the NPDE course, was that

“You must practice it. There is no other way. You must practice it... I did a lot of practice. I have got four 72 page books.”

Tr. Barend also made a similar comment about his work ethic, when questioned about his results he achieved on the NPDE mathematics courses. He stated:

“Hard work, also I must be honest, the back ground was that..”

Here it must be noted that these teachers worked hard so as to extent their subject knowledge. Tr. Margaret also mentions that because of her lecturer at college she had to teach herself much of the work. The notion of ‘enjoyment’ must be considered to have had a remarkable influence on these teachers results on both the NPDE and in the answering of the Questionnaire. Thus the teachers drive to work hard and their belief that they are naturally mathematically inclined are definite factors that can be regarded as having influenced teacher understanding.

7.3.4. Teacher Identities

In drawing conclusions about these teachers’ identities I relied on the identities proposed by Parker (2004) and Jita and Vandeyar (2006). The three identities of Parker are once again, Teacher as Mathematician, Teacher as Student of Mathematical Education and Teacher as Mathematics teacher/educator. Jita and Vandeyar introduced us to the identities of ‘Master of the Basic’s’ and ‘Learner and Teacher of Mathematics’.

When considering the teachers that were interviewed I found that many of the teachers (Tr. Belinda, Tr. Dumisani, and Tr. Barend) felt that they were ‘mathematically inclined’. This presented the idea that they viewed themselves as mathematicians. I however am of the opinion

that this came about because they felt that their procedural knowledge of these concepts was good. For me this is not sufficient to say that these teachers' identities should be viewed as Parker states, teacher as mathematician. Teacher Belinda, Tr. Barend and Tr. Daya give the impression that they view themselves as 'masters of the basics' and thus to them the teaching of method (mastery of the basic algorithms) is a major part of their teaching. Tr. Margaret like teacher Sharon in the study of Jita and Vandeyar (2006) sees mathematics as something that learners can work on and master. She also sees herself as a continuous (lifelong) learner of mathematics. Tr. Margaret's identity can thus be likened to that of teacher Sharon, namely Learner and Teacher of mathematics.

To conclude, I believe that all these of teachers grounding in mathematics were inadequate, their levels of teacher training were poor and their formal qualifications did not necessarily indicate competence (Jansen, 2001). I cannot see them as being able to justifiably identify themselves as mathematicians or as Students of Mathematics education. They fall closer to the identity, teacher of mathematics.

7.3.5. Conclusion to Part Two

In conclusion it is noted that although these teachers all had significantly different school backgrounds this was not as strong an influence as it is perceived to be. Most of the teachers were capable of doing the basic algorithms. None of the teachers showed PUFM. Therefore the differences in the education systems did not play a big role in their understanding. It is as though the method of teaching 'procedure' was the main focus across the schools.

A stronger influence to me seemed to come from their personal philosophies towards the subject. The hard work mentioned by these students relates to learning the methods and algorithms that were required in order to solve the calculations that the course required of them. They therefore merely revised diligently the work covered in the contact sessions and the assignments. Tr. Margaret states that she used four books for revision for the examinations.

What is evident is that these teachers to a large extent still teach the way they were taught. Thomas & Pedersen, 2003 call this notion "a common maxim". Hiebert, Morris & Glass (2003: 201) allow me to extend this by quoting them:

People learn to teach, in part, by growing up in a culture – by serving as passive apprentices for 12 years or more when they themselves were students. When they face the real challenges of the classroom, they often abandon new practices and revert to the teaching methods their teachers used.

It has however been reported that outdated teaching practices and lack of basic content knowledge have resulted in poor teaching standards and that these poor standards have also been exacerbated by a large number of under-qualified teachers who teach in overcrowded and non-equipped classrooms (DoE,2001). It is therefore obvious that those who teach the way their teachers taught them are merely continuing this cycle of mediocre mathematical education.

From a teaching perspective it seems that factors like, teachers making use of practical examples and manipulatives; teachers that showed interest in their pupils; teachers being perceived to have a thorough knowledge of the subject and teachers' being enthusiastic about the subject, have a great effect on influencing the pupils like or dislike of the subject.

To draw Part 1 and Part 2 together is now important. The teachers' achieved high on the NPDE but failed to indicate an understanding of the fundamental concepts that could be regarded as being profound. The question therefore arises, why did they manage to score high if their understanding of these fundamental concepts cannot be regarded as being profound? To answer this I believe I will need to look more intensely at the NPDE assessment process. This however was not the focus of the study and I therefore did not focus to heavily on reviewing the assessment tasks and the examination papers to intently. It however seems obvious that these assessments required the students to have a strong procedural knowledge of the course work as these teachers demonstrated this in answering the questionnaires.

Whilst I am aware that the NPDE course is merely a generalist teaching qualification and not a specialist qualification like the Advanced Certificate in Education (ACE), there is still a greater need for these teachers to be taught the basic mathematical skills required to teach up to and including the Senior Phase of the GET. I believe that it is still important that this course strives to develop the teachers PUFM.

7.4. Final Observations

This study reveals that there is need for greater teacher support from the DoE subject advisors. Tr. Margaret mentioned that she felt totally on her own for a long period before she began the NPDE course. This indicates that she is not being supported enough by the department officials. Like the Chinese National Department of Education the DoE could produce a set of Teaching and Learning Framework textbooks and teachers' manuals that could be used as guidelines by the teachers for the teaching of the different sections of the work at each specific level. Whilst it is accepted that the New Curriculum Statement documents have this intention, they are not specific enough and the textbooks that are available at present in many instances differ with regard to emphasis and thoroughness of sections. By the department controlling this more there will be greater uniformity and thus the standard and quality of education will be better controlled. This may seem as a very top down approach to teaching but I believe that because of the inherent problem of lack of understanding that exists within our teachers and pupils, there needs to be some form of control.

The Chinese teachers in Ma's study indicated they learnt much from their colleagues. I believe therefore that the Cluster system which is used by the DoE for moderation needs to be extended so that these clusters can also become forums where these teachers could teach each other, support each other and share views about their understanding of concepts.

7.5. Short Comings of the Study

This study aimed to find out whether teachers who scored high on the NPDE course had PUFM. This was done by using questionnaires as the main form of gathering information about teacher knowledge. This process was to some extent limiting as it was found that some teachers were reluctant to answer another questionnaire. Some answered the questionnaire very vaguely and showed very little interest. Other teachers answered very well but felt limited. This could be due to the space provided or because they wanted to say more than what the questions were asking. I designed the questionnaire based purely on the scenarios of Ma and Ball's TELT project. Since I was not going to interview the teachers about their knowledge around these concepts I opted to break up scenario one and two into numerous questions. These two scenarios thus provided greater amounts of information than scenario three and four. I also found that I could not easily

use Ma's instrument to analyse scenarios three and four and had to look for common themes. This to me was a short fall in Ma's analysis as well, as she also did not necessarily use the instrument to analyse scenarios three and four.

Ma interviewed all her candidates and did not use a questionnaire as I did. She was thus able to probe the teachers more about their understanding of these concepts. Ma also had the teachers' draw up knowledge packages which she used to assist her in her analysis of these teachers knowledge. This process was however time consuming and difficult as it meant travelling to various part of the province on numerous occasions to get this done properly.

The method of sampling used at the time was the most convenient and I believed the most effective way to get the teachers with differing school experiences. The approach however I believe was limiting and could have been extended to include more students. This could have assisted me in drawing broader conclusions about the knowledge of the teacher's that completed the NPDE mathematics course through the University of KwaZulu Natal, Pietermaritzburg campus.

The approach used I believe was a very noble attempt at looking at what teachers new and whether it could be linked to how they were taught. The approach however was far to open ended. It brought up the concepts of teacher belief and teacher identity. These two concepts are very well researched and thus much has been written about them. It became clear to me that you cannot look at teachers mathematical life histories with looking at the beliefs and their identities. I tried to incorporate the concept of identities but to me this was not the main focus of the study. I think that an investigation into the teachers' beliefs and identities could be a possible extension to this study.

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Appendix 1

**University of KwaZulu Natal
Pietermaritzburg Campus
School of Education
Masters Program**

Background

Dear participant

This questionnaire is part of a Masters study that is reviewing the mathematical knowledge that students gained on the NPDE Intermediate phase mathematics courses. It will also look at the educational life histories of the students that participated in this course.

Please note that this process is two fold. It asks you to complete a questionnaire and secondly to participate in an interview that is plus minus 30 minutes long.

Whilst your participation is completely voluntary it is hoped that you would participate fully in this investigation as such a study will be helpful in the review process of this course and the investigation into teacher knowledge.

I would like to ensure you that all information given in either the interview process or on the questionnaire would be kept strictly confidential. I undertake not disclose any information that can be used against any participant without the written permission of the participant.

Any reference from the questionnaire or the interview that is used in the dissertation would be coded. No actual names or places would be used.

All information would be locked up in either the filing cabinet in my office or in the office of my supervisor. Both these places have very good security.

Lastly your participation is really vital to the betterment of this course and ultimately to the betterment of the teaching process within our province.

Yours sincerely
Andre Van Wyk
Student/ Coordinator NPDE Mathematics Course

Personal information

Name: _____

Student No. _____

Name of school at which you are presently teaching.

Contact details:

Telephone numbers (Home) Code _____ No. _____

(Cell) _____

(School) _____

Home Address

Number of Years teaching _____

Grades which you teach _____

Subjects which you teach _____

Last school year in which you did mathematics

Grade _____

Year _____

Name of the last school you attended

Questionnaire 1

Consider the following scenarios and then answer the questions that relate to each of the scenarios' below. You will notice that each scenario presents a different teaching challenge.⁵

- Scenario one (Section A) deals with how you would go about teaching a specific topic.
- Scenario two (Section B) deals with correcting mistakes that students may be making.
- Scenario three (Section C) deals with generating representations that could assist you the teacher to relate the mathematics to other things.
- Scenario four (Section D) deals with how teachers deal with students who present challenging or unconventional methods to prove concepts in mathematics.

Please write on the paper provided. If any additional paper is required please staple it to the others and making sure that your name is on the top of it.

⁵ In the versions provided to the teachers, there was more space provided for answers and each section started on a new page. This has been changed for the purpose of reproduction of the thesis.

Section A

Subtraction with regrouping:

Scenario

Let's spend some time thinking about one particular topic that you may work with when you teach, subtraction with regrouping. Look at the following questions:

$$\begin{array}{r} 52 \\ - 25 \\ \hline \end{array}$$

$$\begin{array}{r} 91 \\ - 79 \\ \hline \end{array}$$

$$\begin{array}{r} 65 \\ - 58 \\ \hline \end{array}$$

$$\begin{array}{r} 83 \\ - 57 \\ \hline \end{array}$$

- 1.1. Describe the method you would use to find the answers to the calculations above.

- 1.2. How would you teach a class of primary school learners to do such subtraction?

- 1.3. Is the method you would teach your pupils the same method that you were taught in primary school.

- 1.4. What would you say pupils would need to understand or be able to do before they could start learning subtraction with regrouping.

1.5 Draw up a rough flow chart of what you believe should be taught before a lesson on subtraction with regrouping.

1.6 Give a brief discussion of each of these sections (concepts). Use examples to assist you with your discussions.

1.7 What do you believe would be an extension of this topic?

Section B

Multidigit Number Multiplication; Dealing With Students Mistakes

Scenario

Some sixth- grade teachers noticed that several of their students were making the same mistake in multiplying large numbers. In trying to calculate,

$$\begin{array}{r} 125 \\ \times 645 \\ \hline \end{array}$$

the students seemed to be forgetting to "move the numbers"(i.e the partial products) over on each line. They were doing this:

$\begin{array}{r} 125 \\ \times 645 \\ \hline 615 \\ 492 \\ \underline{738} \\ 1845 \end{array}$	instead of	$\begin{array}{r} 125 \\ \times 645 \\ \hline 615 \\ 492 \\ \underline{738} \\ 79335 \end{array}$
--	-------------------	---

While these teachers agreed that that this was a problem, they did not agree on what to do about it. What would you do if you were teaching sixth grade and you noticed that several of your students were doing this.

Section C

Generating Representations Division by fractions

Scenario

People seem to have different approaches to solving problems involving division with fractions. How do you solve a problem like this one?

$$1\frac{3}{4} \div \frac{1}{2} =$$

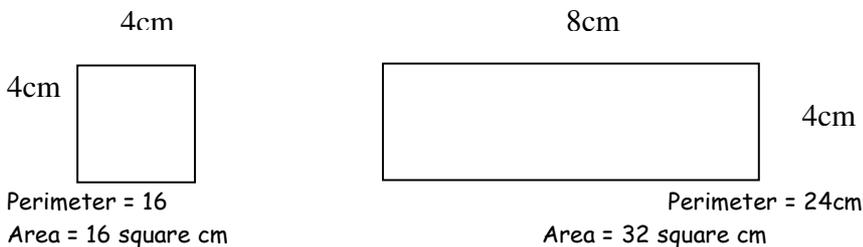
Imagine that you are teaching division with fractions. To make this meaningful for kids, something that teachers do is relate mathematics to other things. Sometimes they try to come up with real-world situations or story problems to show the application of some particular piece of content. What would you say would be a good story or model for $1\frac{3}{4} \div \frac{1}{2}$?

Section D

Exploring New Knowledge; The Relationship between Perimeter and Area

Scenario

Imagine that one of your students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she discovered that as a perimeter of a closed figure increases the area also increases. She shows you this picture to prove what she is doing;



How would you respond to this student?

Thanks for your responses. They will be used and kept safely. All the above responses will be kept confidential unless you give written permission for their use.

Appendix 2

Questionnaire Two

Multiplication of two digit numbers

Do you make links to prior taught knowledge in your preparation for new lesson preparation?

Discuss the prior knowledge that you would call on when preparing a series of lessons on multiplication of two digit numbers?

Describe the process you would use after teaching them an introductory lesson on two digit multiplication?

In your teaching of multiplication of two digit numbers what steps would you get them to follow? List these below.

Do you as a teacher believe in giving students a set of rules to follow when you teach a section? Explain your answer.

Many teachers get their students to either put down a zero value or leave a blank space before multiplying by the second digit.

- What method do you use?
- Why do you believe that you use this?

What are your views on reinforcement/ drill/ practice in the teaching process?

How would you assess whether a learner has attained a suitable level of understanding of multiplication of two digit numbers? Describe your assessment strategies and evaluation processes.

Mention three sections in mathematics that this section could link to. An example being, the multiplication of three and two digit numbers.

Appendix 3

The Interview

The Interview Process

The interview is divided into four sections.

Personal: This strata aims to present the teacher to the reader. It looks at who the teacher views themselves to be. It looks at how the teacher came to be the person they are. It aims to get the teacher to disclose whether any person (teacher; parent or mentor) could have influenced the teacher's career choice.

The NPDE: This strata looks to investigate the influence the Course had on the teacher. It hopes to probe the teacher/student to discuss their impressions, excitements and disappointments they experienced in the mathematics course.

Life at school: This strata forms the major portion of the interview. This is the phase of the teacher's life that should have the greatest influence on the student's career choice and mathematical ability. It is hoped that the questions in this strata will reveal much about life at school and in the mathematics classroom in particular.

Life out of school, from student to teacher: This strata looks at the development of the teacher. It will probe the changes the teacher experienced in their content knowledge and pedagogic knowledge in the journey from novice teacher to professional teacher.

Interview Schedule

Personal

1. A few personal questions.
 - Who is
 - What type of employment were your parents involved in?
 - Did your parents or any of your siblings study at an institution for higher learning?
 - What motivated you to become a teacher?
 - What are your earliest memories of mathematics?

The NPDE Program

2. How is it that you became involved in studying on the NPDE program?
3. What was your experience of this like whilst studying on this program;
 - was it enjoyable;
 - was the course design demanding or could you cope;
 - was it the level of work that was covered suitable to the level at which you are presently teaching;
 - was the assessment process suitable and appropriate for this level of study?
4. You achieved good results in the mathematics courses you studied on this program. What factors do you believe contributed to this success?
5. Would you regard yourself as a hard working student, who is determined to obtain good results? If your answer is yes, then what do you believe motivates you to work hard?

Life at School

6. What do you think were the school experiences that motivated you to follow the path you followed.
7. Many people regard mathematics as being a difficult subject. Why do you think you succeeded when many others did not?
8. When you recall the teachers that taught you mathematics from primary school through to high school, do you believe that there was a specific teacher that made a lasting impression on your mathematical knowledge and ability?
9. If yes, what is it that you remember the teacher doing that made this impact on you.
 - i. Was it his approach to you as a pupil?
 - ii. Was it his sense of enthusiasm for the subject?
 - iii. Was it because he was a strong disciplinarian?
 - iv. Was it because of his method of teaching?
10. What was the format of a standard mathematics lesson when you were at school?
11. What method of instruction was used more frequently, individual instruction or small group work.
12. Did this method of instruction make the mathematics lessons interesting or boring/tedious? Explain your answer.
13. What resources were made available for the learning of mathematics at school.
14. Can you remember some of the textbooks you used in school?
15. If you had to rate them by allocating a score out of ten, what would you score them and why?

Life out of school, from student to teacher

16. Where did you first study to be a teacher?
17. Describe the institution at which you studied?
 - Was it a big institution?
 - Was it well maintained?
 - Was it well stocked with resources?
18. Did you study mathematics when you first studied as a teacher?
19. What do you remember about your first mathematics lesson?
 - Were you nervous?
 - When preparing for this lesson, were you confident in your knowledge of the content that you had to teach?
20. Describe your transition from novice educator to professional.
 - Name some of the factors that could have influenced this transitional process?
 - Describe how your knowledge of the mathematical content you teach has changed over the years.
 - Is the way you teach presently in any way similar to how you were taught mathematics in school?