AN INVESTIGATION OF THE USE OF MULTIPLE REPRESENTATIONS IN TEACHING FRACTIONS AT PRIMARY SCHOOL LEVEL IN SWAZILAND

by

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Declaration

I, Thab'sile Priscilla Dlamini, student number: 214583611, declare that, this dissertation is entirely my own work and that it has not been submitted for the degree in this or any other university. All sources used and quoted have to the best of my knowledge been properly acknowledged and indicated by means of references.

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Researcher Signature Date

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Supervisor Signature Date

ABSTRACT

This study aimed at identifying the kinds of representations primary school teachers commonly use in teaching fractions, how they use them and their reasons for using them. The study drew on the teaching model by Ball, Thames and Phelps (2008), who claim that representations play a crucial role in developing learners' understanding of mathematical concepts. Learners frequently make errors and teachers are required to identify the source of those errors and find ways of remediating them, usually by using multiple representations.

The study is framed by Vygotsky's (1978) social constructivism and Lesh, Post and Behr's (1987) typology of representations in primary mathematics; namely, verbal, pictorial or diagrammatic representations, concrete models, experience-based metaphors and symbols. Through classroom observations and interviews, the researcher sought to understand teachers' motivations for using particular representations in teaching the concept of fractions.

Findings from this study revealed that teachers use all the representations suggested by Lesh et al. (1987); however, it confirmed results from other studies that symbolic and spoken language tend to dominate in most classrooms. Teachers also preferred using the rectangular area model to the circle model. The study highlighted the need for teachers to exercise caution when using metaphors, so as to avoid the metaphor itself becoming the focus of the lesson. Teachers used the various representations available to them as scaffolds upon which to build learners' understanding of fractions, often through engaging them in group activities or demonstrations in which learners became active participants. Most of the representations were used to make the fraction concept concrete, to make the lesson interesting and exciting and to accommodate the different learning styles within the classroom.

The researcher recommends that teachers in the intermediate phase introduce operations on fractions using either concrete or virtual manipulatives or real-life problems. It is also suggested that teachers give learners opportunities to come up with the rules for performing operations on fractions themselves, using multiple representations which enable them to observe patterns and draw conclusions.

PREFACE

The work described in this thesis was carried out in the School of Science, Mathematics and Technology Education, University of KwaZulu-Natal, in **October 2014** under the supervision of **Ms Busisiwe B. Goba** (supervisor).

This study represents original work by the author and has not otherwise been submitted in any form for any degree or diploma to any tertiary institution. Where use has been made of the work of others, it is duly acknowledged in the text.

Thab'sile Priscilla Dlamini

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Firstly, I would like to thank the Lord Almighty for giving me the courage to embark on this study, and for guiding me throughout the research.

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Thirdly, I would like to thank the three teachers who participated in the study for their valuable contributions.

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DEDICATION

This work is dedicated to my grandparents, Manyatsi and Margaret Dlamini, who, even though they did not have great financial resources, ensured that their granddaughter received the best education they could afford.

Secondly, I dedicate this work to my daughters, Lindelwa and Temabhele, who supported and encouraged me throughout the study.

CHAPTER 1

INTRODUCTION

Fractions form the basis of many concepts taught at both primary and secondary school level; thus there is a need for learners to master fractions at the primary level and in so doing lay a solid foundation for concepts encountered at higher levels. It is imperative for teachers to devise strategies that enable learners to gain a thorough conceptual understanding of fractions. It is unfortunate that most teachers teach only rules and procedures for manipulating mathematical ideas. Sometimes when visual aids such as concrete representations are used, they are used for the wrong reasons. In this regard, Moyer (2001) observed that teachers sometimes used manipulatives as a reward for good behaviour. There is also empirical evidence that if visual aids such as concrete and semi-concrete aids (e.g. computer technology) and diagrams are used appropriately, they enhance conceptual understanding in mathematics (Barmby, Bolden, Raine, & Thompson, 2013; Naidoo, 2011). There is growing interest among educators in the use of multiple representations in the teaching and learning of mathematics.

Multiple representations refer to the various ways of presenting a mathematical idea with the aim of making it accessible to learners. Representations commonly used at primary school level are concrete materials, diagrams or pictures, symbols, spoken language, and experience–based metaphors (Lesh, Behr, & Post, 1987). Much research has been conducted on the use of these representations in the teaching and learning of mathematics, but it focuses mostly on learning. Since most studies show how beneficial these representations are to the learning process, this researcher set out to investigate the use of multiple representations by primary school teachers in teaching the concept of fractions.

1.1 STATEMENT OF PURPOSE

The researcher has observed a tendency among primary school teachers to focus mainly on symbolic representation when teaching mathematics, to such an extent that teachers disregard other representations that learners might use while working on a problem. The researcher sought to investigate the use of multiple representations by primary school teachers.

1.2 CRITICAL QUESTIONS

This study aimed at investigating the use of multiple representations in teaching fractions at the primary level in Swaziland. The key questions addressed in this study were:

1. What representations do teachers use in teaching fractions?
2. How do primary school teachers use these representations in classroom instruction?
3. What are the teachers' reasons for using or not using particular representations in teaching fractions?

1.3 RATIONALE

This study aimed at investigating the use of multiple representations by primary school teachers when teaching fractions. Based on anecdotal experience, the researcher has observed that some teachers in Swaziland tend to ignore learners' external representations, such as the diagrams they draw while trying to solve a given problem, and instead pay attention to the final solution in symbolic form. Skilled teachers tend to give careful attention to every piece of work, be it words, symbols or diagrams, to identify learners' conceptions and misconceptions. The researcher set out to identify representations used by teachers, how they used these representations and their reasons for using them.

This research could, firstly, inform teacher preparation at the pre-service level to ensure quality training for teachers. Secondly, this study could also benefit the Swaziland National Curriculum Centre, which is responsible for producing teaching and learning materials for schools. The study could reveal the way in which teachers in schools use representations in teaching fractions and, accordingly, help the centre to design appropriate materials to assist them. Thirdly, the in-service teacher-training department may also benefit, since the study provides ideas on how teachers may improve their instructional practice.

1.4 LITERATURE REVIEW

In this section, literature based on the teaching and learning of mathematics, especially fractions, is reviewed. Following this, common conceptions and misconceptions are described and finally, the use of multiple representations is discussed.

1.4.1 The teaching and learning of mathematics

Primary school mathematics forms a foundation for mathematical concepts taught at higher levels. Based on personal experience, secondary teachers tend to blame primary teachers when learners fail to understand basic concepts in mathematics. Researchers, argue that the main objective of teaching mathematics is to help learners understand and make sense of mathematical concepts (Galant, 2013; Gouws & Dicker, 2011). In study conducted by Galant (2013) in a study conducted in South Africa, teachers showed a lack of understanding of progression and of mathematical ideas. Forty six, Grade 3 teachers from a rural area in Cape Town took part in the study. This situation is likely to exist in the country since Swaziland has a similar context to South Africa.

Developing teaching strategies aimed at promoting learning with understanding should be every teacher's concern. However, some studies have shown that teachers' beliefs and discourse determine what constitute effective mathematics teaching (Stols, Ono & Rogan, 2015). According to Sfard (2001), discourse refers to the conversations within the classroom aided by the use of artefacts as communication tools. Hence, this study is focusing on the use of multiple representations as a tool for communicating mathematical ideas in the classroom.

It is common knowledge that most learners perform poorly in mathematics. Reform Curriculum 2005 in South Africa introduced outcome based education with the aim of improving education for all learners. In spite of the reform, learners continue to perform poorly in mathematics (Department of Basic Education, 2015). In 2015 for instance, matric pass average in mathematics was 49.1%. One of the concepts that have proved to be problematic is the concept of fractions. Similarly, in Swaziland there have been curriculum reforms and their impacts have not been explored.

1.4.2 The teaching and learning of fractions

Fractions, introduced as early as early as Grade 3 in most countries, continue to be a challenge to both teachers and learners. However, some researchers are of the opinion that fractions could be introduced as early as pre-school (Wilkerson & Gupta, 2015). Their study revealed that when fraction introduction is accompanied by the use of manipulatives, Grade 1 children gain a conceptual understanding of the fractions one-half, one-third and one-fourth. As a result, the children in their study had the ability to represent these fractions using either diagrams or symbols. Ball (1990a) and Lesh et al. (1987) are some of the seminal authors who have conducted studies in the teaching and learning of fractions.

1.4.3 Misconceptions in teaching fractions

Teachers encounter all kinds of errors during instruction. While preparing for instruction, teachers should be mindful of the mistakes they are likely to encounter. Skilful teaching involves being able to identify learners' errors and the source of those of errors and being able to correct them immediately (Ball, Thames, & Phelps, 2008). If learners' misconceptions are not dealt with immediately, they accumulate, resulting in discouragement. Many educators believe that the difficulties children experience with fractions are linked to a poor understanding of whole numbers. Many children conceive of fractions as two numbers; the reason is linked to the part–whole definition of a fraction, which can be eradicated by using a number line as a reference point for fractions (Wu, 2014). Another misconception is in relation to the use of area models. There is a belief that when using area models such as a circle to represent a fraction, the size of the divisions does not matter (Yearley & Bruce, 2014).

Other misconceptions are those related to operations on fractions. When adding fractions, for instance, learners often add numerators and denominators ($\frac{2}{3} + \frac{1}{2} = \frac{3}{5}$). The argument is that learners may get the correct answer using symbols but an entirely different answer using other representational media (Ball, 1990a). In such a case, Ball (1990a) argues that the problem is not the representation but a lack of understanding about how to use it. Another instance is that of multiplication and division with fractions, where learners confuse the process with that used for whole numbers, where multiplication yields a bigger number and division yields a smaller number. The misconception that multiplication always yields a bigger number is linked to the fact that whole number multiplication is another form of repeated addition; the idea that division always results in a smaller number is linked with division by partitioning (Lim, 2011). Therefore, for teachers to teach fractions effectively, they must have a knowledge of some representations used to mediate learning.

1.4.4 Multiple representations in teaching fractions

Representations play a significant role when students are learning about fractions (Cramer, Wyberg, & Leavitt, 2008). Through modelling, teachers should stress the importance of representing mathematical ideas in various ways (NCTM, 2000). For instance, in the teaching and learning of fractions, using diagrams or manipulatives can help learners visualise the size of fractions and hence help them to find equivalent fractions (An, Kulm, & Wu, 2004). Ball (1990a) also used contextual problems to help learners conceptualise division of fractions. In the same study, Ball emphasised the importance of listening to learners' discussions, as it offers the teacher opportunities to guide leaners on the use of correct terminology.

Representations have strengths and weaknesses. The circle model, for instance, can be problematic for learners when representing a fraction with an odd denominator, such as one-third (Ball, 1990a; Wu, 2014). Hence, teachers should be aware of such limitations and select suitable representations for particular situations. Since the researcher aimed at identifying representations used in the classroom and teachers used them to teach the concept of fractions, a theoretical framework was required.

1.5 THEORETICAL FRAMEWORK

Representations are tools used by teachers to help learners construct knowledge within each student's “zone of proximal development” (Vygotsky, 1978). Since this is the case, the researcher used Vygotsky's (1978) social constructivism as a theoretical framework. Vygotsky asserts that social interactions, where more knowledgeable people use tools, results in the acquisition of knowledge. The tools commonly used in the primary school classroom are concrete materials, diagrams or pictures, symbols, experience-based metaphors, and language (Lesh, Behr, & Post, 1987). The researcher therefore used Lesh et al.'s (1987) typology of representations as an analytical framework.

1.6 METHODOLOGY

This study adopted the qualitative approach because qualitative data are richer in meaning and detail compared with quantitative data (Babbie & Babbie, 2008). The researcher used the interpretive design aimed at understanding how and why primary school teachers prefer to use certain representations and totally ignore others when teaching the concept of fractions. The researcher conducted a multiple case study, focusing on the use of multiple representations by three primary school teachers who taught fractions in three different schools. The adoption of the case study design is particularly suited to research questions that require a detailed understanding of social processes because of the rich data collected in context (Cassell & Symon, 2004). A case study has the potential of addressing several issues, hence the use of more than one method.

The target population for this study was all primary school mathematics teachers teaching Grades 4, 5 and 6. The researcher selected Grade 4 because operations on fractions begin in Grade 4. Grade 6 was included because addition and subtraction of mixed fractions is taught at this level. A purposive sample of three teachers from three different schools participated in this study. The three teachers in this study were purposefully selected because they indicated that they used multiple representations when teaching fractions. Two of the selected teachers taught Grade 4, and one taught Grade 6. For data collection purposes, the researcher developed three instruments adapted from Naidoo (2011), namely: the interview schedule, observation schedule, and follow-up interview schedule. The researcher conducted all interviews and observations during the first term of the school calendar, and there was no interference with the school calendar or timetable. Each teacher was interviewed once before observations. Class observations were conducted in succession a week after each interview.

1.7 SUMMARY

This chapter began by giving a brief background on the teaching and learning of fractions. This description was followed by the motivation for doing the study, and the research questions that the study proposed to answer. A brief literature review on the teaching and learning of fractions using multiple representations was given, followed by a description of the theoretical framework and methodology. The next chapter gives a more detailed description of teaching and learning mathematics, and fractions in particular, according to the literature.

CHAPTER 2

LITERATURE REVIEW

There is an agreement among educators that the introduction of fractions heralds the beginning of a fear of mathematics (Wu, 2014). Fractions form the basis of many mathematical concepts taught at higher levels. Effective teaching for understanding using strategies that engage learners in meaningful learning is therefore of the utmost importance. Fractions can be represented in various ways, which, if used effectively, can make explicit the connections between the various representations and result in significant constructions in the minds of learners. This study aimed at determining the types of representations teachers use when teaching fractions and their reasons for using them or not using them. Were teachers aware of the importance of using multiple representations when teaching? Were teachers conscious of the importance of making explicit the connections between various representations? Through observations, the study determined how teachers used multiple representations in the classroom.

The first section of this chapter reviews primary mathematics in Swaziland, paying particular attention to the teaching and learning materials supplied by the National Curriculum Centre (NCC). A brief description of the kinds of representations used in the text books is then discussed. This leads to a review of the literature on representations and their role in teaching primary mathematics, including their strengths and weaknesses. Thereafter, the role of representations in teaching primary mathematics is discussed, paying particular attention to the teaching of fractions, and the misconceptions associated with fractions. The chapter concludes by discussing the role of multiple representations in learning fractions and its implications for teacher education and curriculum designers.

2.1 THE ROLE OF CURRICULUM MATERIALS IN TEACHING

NCC develops the curriculum materials used in the schools with the help of the Mathematics Panel. The materials include teachers' guides, pupils' books, and pupils' workbooks, supplied to all public schools free of charge. Textbooks play a vital role in instruction since they largely determine the content taught and what students learn in the classroom. For classroom teachers, books address three critical issues; the sequencing of topics, content to be taught and the activities to be used to engage learners in meaningful learning (van Garderen, Scheuermann, & Jackson, 2012). If curriculum materials substantiate learning, they can lead to high-quality instruction even for teachers with low mathematical knowledge (Hill & Charalambous, 2012). Typically, experienced teachers do not rely on textbooks, while teachers who lack knowledge of certain content areas tend to rely heavily on the curriculum materials (Son & Senk, 2010). Implementing the use of curriculum materials could be a problem for some teachers in primary schools since some have no formal training, and rely heavily on the curriculum materials. On the other hand, Swaziland also has graduates with formal training in education, specialised in content other than mathematics, yet teaching mathematics in primary schools.

Discernment of the mathematics requirements to suit the various grade levels plays an important role in preparing and shaping instruction (Ball, Thames, & Phelps, 2008). Such knowledge and the correct use of curriculum materials is acquired during training. Teachers who lack mathematical knowledge tend to lack skill at using curriculum materials effectively. Furthermore, the way in which mathematical ideas are presented in the textbooks seems to have a significant impact on learners' understanding of mathematics. This study explored the use of representations by teachers in teaching fractions, including their interactions with curriculum materials. What follow is a description of the presentation of fractions in the primary curriculum.

2.2. TEACHING AND LEARNING FRACTIONS IN SWAZILAND

In primary schools in Swaziland, fractions are introduced in Grade 2, using pre-partitioned paper strips, diagrams of area models and manipulatives. By the end of Grade 3, learners are expected to have acquired some knowledge about halves, quarters, fifths and tenths. The focus at this stage is on acquiring the correct fraction vocabulary and relating fraction symbols to area models. In Grade 4, a fraction is defined using sets, and learners are introduced to the idea of equivalent fractions using diagrams of area models and a fraction chart. Learners are then introduced to the addition of fractions using a fraction chart. In Grade 5, more addition and subtraction is taught. In Grade 6 multiplication involving common fractions is taught, as well as addition and subtraction of mixed numbers. Inspection of the text-books revealed that learners are not encouraged to generate their own diagrammatic representations of fractions; most of the area models are complete. The level of involvement in generating their own models is very low; however, when solving problems involving fractions they have to generate their own representations.

If teachers are to provide teaching and learning experiences for all learners in the mathematics classroom, teachers need to rethink their teaching strategies (Gouws & Dicker, 2011). One of those strategies, on which this study focuses, is using multiple representations in teaching fractions.

2.3 REPRESENTATIONS

We cannot think about or communicate mathematical ideas unless they are represented in some form (Hiebert & Carpenter, 1992). Representations play a crucial role in developing learners' understanding of mathematical concepts. The word "representations" in this study refer to the various ways of expressing a mathematical idea (NCTM, 2000). Representations can be either internal or external and are effective in moulding, amplifying and generating mathematical ideas (Johnson & Lesh, 2003). To think about mathematical ideas we need to represent them internally in a way that allows the mind to operate on them (Hiebert & Carpenter, 1992). External representations such as concrete objects and manipulatives, and visual aids such as diagrams are designed and used to make abstract mathematical concepts more approachable to learners (Gravemeijer, 2010). Learners generate external representations to express how they have understood and represented information internally (Hiebert & Carpenter, 1992). There are five kinds of representations used in primary schools, namely; verbal, pictorial or diagrammatic representations, concrete models, experience-based metaphors and symbols (Lesh et al., 1987). For instance, the concept "fraction" can be described in words as part of a whole (verbal), a symmetrical object such as an orange can be cut (concrete model); part of a rectangle divided into equal parts can be shaded (picture); real life problems can be discussed (experience-based metaphor); or a fraction may be written using normal fraction notation (symbols) (Tripathi, 2008).

According to (Cuoco, 2001), learners develop their internal representations of mathematical concepts based on the external representations a teacher selects to introduce them. It is, however, not possible to see a learner's internal representation; it can only be deduced from the learner's external representations (Goldin & Shteingold, 2001). For this reason, it is important that teachers pay attention to every piece of work written by learners in order to understand their thinking.

Since no single representation can reveal all facets of an idea (Ball, 1990a), it is crucial that teachers expose learners to a variety of representations when defining a concept, in order to compound their understanding (Ainsworth, 2006; Bal, 2014; Bolden, Barmby, & Harries, 2013; Gagatsis & Elia 2004). The use of multimedia can act as a mind map and allows mathematical thinking to occur on both sides of the brain and accommodates all learning styles.

Seminal authors like Bruner (1966) and Vygotsky (1978) have also argued for the use of multiple representations when teaching. Bruner (1966) proposed that learning progresses through three stages, namely, the enactive stage involving active manipulation of concrete materials, the iconic stage, involving the manipulation of images, and finally the symbolic stage, involving abstractions. Vygotsky (1978), on the other hand, believes that social interactions involving a learner and a more knowledgeable person who uses tools, signs and language, lead to cognitive development in the learner. Since the study centred on teachers teaching in the intermediary phase, the use of such representations is of vital importance. This study focused on teachers using manipulatives, concrete models, diagrams and pictures (number lines and area models), metaphors and symbols (verbal and written) and spoken language in order to develop learners' understanding of fractions. What follows is a description of each mode of representation.

2.3.1 Concrete or virtual representations

Concrete manipulative materials play a vital role in the teaching and learning of mathematics (English & Halford, 1995) especially at lower primary school level. Using manipulatives appropriately can play a significant role in constructing meaning and communicating clearly in mathematics (Bolden et al., 2013; Fambaza, 2012; Moyer, 2001; Naidoo, 2011; Pape & Tchoshanov, 2001). As learners actively manipulate concrete materials, they develop a broad range of images that can be used to manipulate abstract concepts mentally (Moyer, 2001). For instance, giving two learners an apple to share or folding paper to show the concept of half would help students to develop the meaning of the fraction symbol $\frac{1}{2}$ by enabling them to associate the symbol with the other forms of representation. Whether teachers use or do not use manipulatives in the classroom is dependent on their beliefs about teaching and learning.

As learners interact with various external representations of mathematical concepts, they are in turn able to construct their internal representations of concepts (Goldin, 2002). Concrete learning aids and pictures or diagrams can help learners to visualise a mathematical concept and link it to their prior experiences (An, Kulm, Wu, 2004). For instance, in the teaching and learning of fractions, using manipulatives can help learners visualise the size of fractions and hence be able to identify equivalent fractions (An et al., 2004). When used effectively, visual representations can improve learners' problem-solving abilities (Ainsworth, 2006; Naidoo, 2011; Rajesh, 2009). Pape (2001) argues that while manipulating concrete materials, both teachers and learners are able to develop their understanding of mathematical operations and the steps involved. Through using play dough, learners observed by Caswell were able to learn and understand fractions and operations on them (Caswell, 2007). On the other hand, Mahn (1999) believes that using manipulatives does not guarantee conceptual understanding; this is, to a large extent, dependent on the teacher using them to teach. This statement is supported by Moyer (2001), who observed that some teachers allow leaners to use manipulatives not to build conceptual understanding but as a reward for good behaviour. When computer technology is used, it is considered to be a virtual or semi-concrete manipulative.

Virtual manipulatives like computer software have proved to be very useful in teaching mathematics. Computer technology does not only aid understanding of fractions by giving instant feedback, but also increases learners' enjoyment of lessons (Reiner & Moyer, 2005). When computers are used in the classroom as visual aids, there is a vast improvement in learners' performance, but the problem is that most schools lack computers, especially in rural areas (Fambaza, 2012). Even if the computers are present, most teachers need training in their use as visual aids (Fambaza, 2012; Naidoo, 2011). Although teachers in this study acknowledged the importance of technology in teaching mathematics, some felt they needed training in the use of computer software such as the geometer sketchpad. Naidoo (2011) conducted a study on how skilled teachers used visual tools such as computers in teaching mathematics. Teachers observed by Naidoo (2011) used visual aids to make mathematics more concrete and accessible to learners; make it interesting and fun; as an alternative strategy; and to help learners remember important concepts and procedures (p. 255). English and Halford (1995) argue that manipulating concrete representations is not enough if learners fail to conceive meaning and to grasp the associated symbolism. For instance, learners should be able to make connections between real-life problems and written symbols. However, learners can find feasible solutions to problems without receiving any formal instruction on rules and procedures, through the use of concrete materials (Carpenter & Fennema, 1996). Accordingly, teachers should give learners ample opportunities to grapple with a problem using manipulatives and other visual materials before introducing them to rules and procedures.

This researcher concurs with (Lee , Brown, & Orrill, 2011) that teachers tend to rely on symbolic notation, and that when other types of representations are used it is not for the purpose of constructing meaning but in order to demonstrate a solution. Some teachers, in their rush to finish the syllabus, feel the use of manipulatives is too time-consuming (Molebale, 2005) and others feel they are fun but not essential for teaching (Moyer, 2001).

2.3.2 Diagrams or pictorial representations

Studies have shown that pictures and diagrams in teaching mathematics improve the level of understanding (Barmby et al., 2013; Beckmann, 2004). However (Arcavi, 2003) argues that if learners have to visualise a diagram or picture which is conceptually rich, the cognitive demand on the learner could be very high, resulting in students shying away diagrammatic representations. He further states that the translation from diagrammatic representation to analytical representation, which is at the core of understanding mathematics, can be cognitively demanding. But the following studies demonstrate that the benefits of using visual representations far outweigh the disadvantages.

In one study, prompted by Singapore learners' good performance in the Third International Mathematics and Science Study (TIMSS) (1999), Beckman (2004), discovered a profound use of diagrams in their textbooks. The use of drawings of strips to represent quantities in mathematical problems made it easier for the learners to solve problems otherwise deemed challenging. In the case of learning equivalent fractions, it is simpler to compare two fractions using diagrams than using symbols. If learning materials such as textbooks have such an impact on learners’ understanding, how do our teachers feel about the materials supplied by the NCC? Do teachers feel there is a need to improve them or supplement them? This study determined how teachers used diagrams, including area models, number lines and fraction charts, when teaching fractions and operations on fractions.

2.3.3 Experience-based metaphors

According to Lakoff and Nunez (1997), every mathematical idea can be linked to an everyday experience. The use of experience-based metaphors helps to ground abstract mathematical concepts in our daily experience, making those ideas more accessible to learners. Metaphors are particularly useful in developing an understanding of abstract mathematical ideas and procedures that are not easy to represent using concrete representation (Presmeg, 2013). The metaphor can be understood by finding the relationship between the source (the real-life problem) and the subject (mathematical concept expressed in fraction symbols). In teaching fractions using a metaphor such as a real-life problem for the addition of fractions, the connection between the metaphor and the addition in fraction symbols should be clear. Presmeg (2013) further observed that failure to establish a relationship between metaphor and mathematical concept could result in the metaphor becoming the target, as learners try to understand the metaphor.

Through the use of contextual problems, Ball (1990a) was able to help her learners conceptualise the division of fractions. The real-life problem Ball used acted as the source and the division of fractions, the subject. Learners translated the problem to different diagrams, with Ball guiding them through questioning and probing as they worked on the problem. In the same study, Ball stressed the importance of listening to learners' expressed thoughts as they discussed their solution strategies. Ball used a real-life problem which learners tried to solve using different strategies; some reasoned through diagrams and some used concrete manipulatives.

Even though the use of real-life problems has proved to be useful in engaging learners in meaningful learning, some teachers have difficulty in composing problems based on real-life situations when teaching addition and subtraction (Austin, Carbone, & Webb, 2011)

Austin et al. (2011) conducted a comparative study of North American (USA) and South African (SA) student teachers' ability to compose acceptable word problems for addition and subtraction of fractions. The sample consisted of 13 USA students and 26 SA students. For 19 of the SA students, English was a second language. The students were instructed to write a story problem involving a real-life situation where Grade 4 to Grade 6 learners would add $\frac{1}{2}+\frac{3}{4}$ to solve the problem. More than 10% of both USA and SA students had difficulty formulating problems in real life. In cases where authentic problems were posed, the social and cultural differences were evident in their choices of reference units. For SA students, the most dominant unit was a loaf of bread, which was acceptable since it is a standard shape and size in South Africa, unlike in the USA, where bread comes in different shapes and sizes, making its use as a reference confusing. This implies that teachers should be conscious of learners' social and cultural backgrounds when constructing problems based on real-life situations. Ball (1990a) also observed that using contextual problems involving families could lead to revelations of sensitive personal information.

The studies discussed above are relevant to this study because they focus on the role of different representations in the teaching and learning of mathematics; with some paying particular attention to the use of real-life problems in teaching concept of fractions, which is the focus of the current study. The real-life problems are translated to symbols which are manipulated to find solution to the problems.

2.3.4 Symbolic representation

Symbolic representation in this study refers to both written and verbal symbols. In primary school mathematics there are two types of written symbols, namely, symbols that refer to quantity $(2, \frac{2}{3}, 2.5)$ and those relating to operations on quantities $(\times, +, \div)$(Hiebert, 1988). The introduction of symbolic notation should be done with other visual aids, like concrete materials. Hiebert and Carpenter (1992) observed that written symbols are informed by the multiple links learners have made with manipulative materials. As a result, through thinking and talking about the similarities and differences between the fraction symbol and the fraction bar representation, learners are able to make connections between different types of representations. It should be noted that the introduction of symbolic representations in an untimely fashion can have an adverse impact on the learning process (Sriraman & Lesh 2007), resulting in rote learning. Ball (1990b) observed in this regard that teachers tend to use manipulatives to capture and maintain learners' interest but not for building conceptual understanding.

According to Hiebert and Carpenter (1992), meanings of written symbols can evolve in two ways; through connecting with other forms of representation, such as concrete materials, and through establishing connections within the representation. Moreover, for symbols to acquire meaning learners must connect their mental representations of written symbols with their mental representations of concrete materials. For instance, the numeral $\frac{3}{4}$ takes on meaning when related to other representations like the area model with three parts out of four shaded. According to English and Halford (1995), there are three steps involved in naming a fraction using the area model. Firstly, the teacher should ascertain that the divisions are equal. Secondly, learners should identify the number of parts into which the whole is divided and relate it to the name of the fraction or the denominator (four equal divisions = fourths). Finally, they should identify the number of shaded parts and relate this to the total number of divisions. If three out of four parts are shaded, then the fraction's name is three-fourths. It is only when meanings of individual symbols are established that learners can be introduced to or think about creating meanings for rules and procedures that control actions on those symbols (Hiebert & Carpenter, 1992). For instance, adding fractions with the same denominator (e.g. two-fifths plus one-fifth).

2.3.5 Spoken language

Language is at the centre of all teaching interactions, be they written or verbal (Vygotsky, 1978). In Swaziland, the language of instruction is English, which is many learners' second language, particularly in rural areas. In most urban schools some learners speak English as their first language, while others speak languages other than English. In most multilingual classrooms in South Africa, the situation is the same; English appears to be the dominant language of instruction (Setati, 2005). Language in mathematics plays a crucial role, especially if the language of instruction is the learners' second language. Adler (2001) believes that when teachers are developing new meanings, the best language for instruction is the learners' first language. Khisty's (1995) study revealed learners benefited most from teachers who used mostly learners' first language in developing mathematical concepts and promoting student discussion within the classroom. Studies show that learners whose home language is English tend to perform better than learners who speak other languages in the home (Christiansen & Aungamuthu, 2012). In Christiansen and Aungamuthu's study carried out in South Africa which focused on misconceptions related to language, they analysed learners' responses to test items. In one of the items learners were instructed to compare $\frac{1}{2}$ and $\frac{1}{3}$ in terms of magnitude. Only

17.4% of learners whose home language was not English gave the correct response while 43.3% of those whose home language was English gave the right answer. In Swaziland, almost all the learners' home language is not English; as a result, they have difficulty interpreting real-life problems in English. Code-switching is done a lot at primary school level. In most cases, both concrete manipulatives and visual aids act as mediation tools when language becomes a barrier (Naidoo, 2011).

Both verbal and written communication are instrumental in assisting learners in understating the connections between concrete representations and symbolic notations (Cramer et al., 2008). It is through “teacher talk” while a teacher uses visual representations that learners are exposed to mathematical language (Naidoo, 2011). Therefore, the use of area models, number lines, or sets of objects should be accompanied by much discussion between the teacher and learners to enhance the development of fraction language. Cramer et al. (2008) further emphasised the importance of giving learners ample opportunities to describe fractions either verbally or through written language before they can use symbolic notation meaningfully. This was observed in their experiment with Grade 6 learners, in which they used the circle model to help learners develop a thorough understanding of addition and subtraction of fractions. This is supported by Ball (1990a), who suggested that teachers should be attentive to learners' discussions as they verbalise their thoughts, in a study in which she was trying to help her learners construct the meaning of the part-whole definition of a fraction. One of the learners, Betsy, verbalised the fraction $\frac{4}{2}$ as four "twoths". Ball guided the learners towards the correct use of mathematical language. Naidoo (2011) observed that one of the master teachers in her study (Penny), always accompanied her verbal explanations of mathematical concepts by diagrams, which helped make the concepts more concrete.

Classroom instruction is always situated in particular cultural contexts, implying that the demands on the teachers will differ (Ball & Forzani, 2010). For instance, learners from rural schools sometimes struggle to express themselves in the language of instruction (English), while students from urban schools express themselves freely in English. In some cases, teachers in rural schools use the vernacular to clarify important points.

Although language plays a significant role in mathematics, the use of correct mathematical terms does not always translate into sound mathematical thinking (Van Oers, 2010).

2.4 CHOOSING REPRESENTATIONS

Choosing appropriate representations is of critical importance for classroom teachers. As mentioned earlier, all representations have their strengths and weaknesses; teachers therefore, should be skilful at selecting representations to use for instruction. Teachers need skill in selecting and using content-appropriate multiple representations to facilitate instruction (Nichols, Stevenson, Heberg, & Gillies, 2015) in mathematics. Using a variety of colourful manipulatives, for instance, has a tendency of diverting learners' attention away from what they are supposed to learn, towards the manipulatives themselves (Uttal, Scudder, & Deloache, 1997). In this regard, (Brijlall & Niranjan, 2015) stress the importance of understanding the object of the lesson and selecting the manipulative with that purpose in mind. Choosing a representation to use for a task can be a challenge for teachers who have little experience, as they may lack a deep understanding of the task involved (Ainsworth, 2006). This could be the reason why some teachers viewed manipulatives as “fun” to use but not necessary for learning (Moyer, 2001). It is not only the manipulatives that are problematic; other representations, too, pose problems for teachers and learners.

The circle model has been widely used to demonstrate the part-whole concept of fractions; sometimes it can create problems for learners, especially when it is used to teach division by a fraction (Ball, 1990a). Ball observed that it was not easy for learners to partition a circle into three equal parts and in such cases, learners should use other shapes such as rectangles. Yearley and Bruce (2014) argue that too much reliance on the circle representation during instruction could lower students’ abilities to represent fractions that cannot easily be portioned, such as$\frac{2}{3}$. Wu (2014) also asserts that the circle model is awkward when it is used to represent fractions greater than one or to perform operations like multiplication with fractions. Another challenge observed by Yearley and Bruce (2014) in using representations such as the “part-whole area model”, which requires equal partitioning, is that the level of precision required was confounding for learners.

Another problem observed by (Ball, 1990a) was the use of commercially-produced fraction bars when comparing fractions. She argues that when teachers give learners opportunities to draw their own models, they struggle, resulting in fruitful discussions, which would never have come to the fore if ready-made bars were used. Uttal, Scudder and Deloache (1997) argue that using especially designed manipulatives to teach a specific concept can help learners not to focus on the manipulative per se but on its relation to the intended meaning. It is clear that the inappropriate use of representations, and the failure to use them at all, could lead to a lot of errors and misconceptions in the minds of learners.

2.5 MAKING CONNECTIONS BETWEEN REPRESENTATIONS

Making connections between ideas, facts or procedures is at the centre of understanding mathematics (Hiebert & Carpenter, 1992). Galant (2013) also notes that the difference between those who demonstrate a deep understanding of mathematics and those who lack such understanding is the former's ability to discern the connections between various types of representations of the same mathematical concept. When forms of representation other than symbolic representations are used, there is often no clear link between the representations. In this regard, (Berthold, Eysink , & Renkl, 2009) observed that learners usually encounter difficulties when attempting to relate multiple representations to one another; they tend to concentrate on one representation only. Accordingly, when links are made between representations, those links must be mathematically relevant. Translation ability amongst various representations is usually associated with success in mathematics, especially in problem solving (Gagatsis & Shiakalli, 2004; Lesh et al., 1987). Translation ability pertains to the thought processes required in moving from one type of representation to another (Lesh , Post, & Berh, 1987); for instance, from an area model to a fraction symbol.

The fundamental role played by visual representations in aiding conceptual understanding in mathematics education makes it imperative for teachers to give learners enough practice in using visual representations for them to acquire the skill (Gagatsis & Elia 2004). According to Lesh et al. (1987b), understanding a mathematical concept implies being able to realise the idea embedded in forms of qualitatively different representations, flexibly manipulate the idea within given representational systems, and correctly translate the idea from one representational system to another.

Gagatsis and Shiakalli (2004) conducted a study focusing on the translating ability of university students as far as the concept of functions is concerned. Their emphasis was on three types of representational systems, namely, algebraic, graphic and verbal. One hundred and ninety-five students wrote a test in which they were required to perform translations. In one task, the students had to translate from verbal representation to graphic and algebraic representation. In another task, students had to translate a graphic representation to verbal and algebraic representations. Data were analysed using the Statistical Package for Social Sciences (SPSS). The results of the study revealed that the students found it easy to translate from verbal to algebraic representation, but there were challenges whenever graphic representation was involved. The students also failed to recognise that the graphic representation and the verbal representation were depictions of the same function.

The current research is relevant in that it also seeks to examine the types of representations used by classroom teachers when they teach fractions to primary school learners. In addition, it also seeks to determine if teachers can make explicit the connections between the different representations (concrete, diagrammatic, verbal, contextual and symbolic). For the current study, data were collected through interviews and observations. Through observations, the researcher gained first-hand information on how teachers engaged learners in developing an understanding of the fraction concept. For teachers to engage learners in productive activities, they must have some knowledge of the various representational forms and be able to create classroom environments that promote optimum learning.

2.6 THE ROLE OF REPRESENTATIONS IN TEACHING FRACTIONS

For teachers to teach mathematics effectively, they need a thorough understanding of the content so that such knowledge can be accessed easily during instruction (NCTM, 2000). Furthermore, a vast knowledge of the various ways of representing mathematical ideas and the associations between them is needed. Representations such as manipulatives play a vital role, acting as mediating tools in developing a conceptual and procedural understanding of mathematical ideas (Brijlall & Niranjan, 2015). Teachers need efficient ways of representing algorithms to show the meaning of each step in the procedure (Ball et al., 2008). For instance, to understand the algorithm for the division of fractions, “invert and multiply” teachers need to know the principle behind the procedure in order to help learners create meaning. Teachers usually achieve this with multiple representations, such as realistic problems or visual aids. The kinds of representations teachers use during instruction determine the representations learners will use in problem solving (Gagatsis & Shiakalli, 2004). Teachers in a traditional classroom normally use only symbolic representations; when other representations are used, they are used to clarify solutions, not for developing an understanding of mathematical ideas (Lee et al., 2011).

Teachers should have a deep understanding of the mathematics they teach at grade level so that they can represent it in multiple ways (Ball, 1990b; Schoenfeld & Kilpatrick, 2008). For instance, the fraction concept can be expressed using the circle model or as a point on a number line or as part of a collection of objects. The circle model has proven to be the most efficient way of helping learners build mental images of fractions (Tripathi, 2008).

Other researchers have argued that the fear of mathematics begins with the introduction of fractions, which have no reference point for the learners (Wu, 2014). Learners tend to view fractions as two whole numbers instead of one number. For learners to develop an in-depth knowledge of fractions, they must be exposed to a variety of representations to facilitate their understanding (NCTM, 2000). The use of multiple representations in teaching and learning reduces cognitive load on working memory (Lesh & Doer, 2003) and facilitates learners' development of the fraction concept. Cognitive load may be defined as the number of mental resources, mostly working memory, required for performing a particular task (Woolfolk, 2010). It is common knowledge that many learners do not perform well in mathematics for various reasons. The apparent limitations in some learners' understandings are not intrinsic but rather because of partially-developed internal representations that leave long-term cognitive obstacles (Goldin & Shteingold, 2001). Furthermore, as long as cognitive barriers persist, learners will be unable to create useful models for solving problems.

Representations should be conceived as instruments used in the classroom for explaining and justifying arguments (Pape & Tchoshanov, 2001). Multiple representations allow learners to realise that there are other ways to present and solve mathematical problems. However, teachers should be aware of the relative strengths and weaknesses of those representations (NCTM, 2000), so as to make the right choice for a particular situation. The circle model, for instance, even though the most commonly-used model to represent the part-whole relationship, has its limitations. The circle model can be divided easily into an equal even number of parts, but it is a challenge to divide it into an odd number of parts (Caswell, 2007). In such cases, other shapes such as the rectangle or other regular shapes are ideal to use.

Naiser et al. (2003) conducted a study that focused on strategies used by higher primary school teachers with the aim of improving the teaching of fractions. Data were collected using interviews and observations of video recordings. The results of the study suggest that teachers failed to connect fractions to real-life situations, hence making the content less accessible to learners, and instead resorted to rote learning. Naiser et al. (2003) further argue that strategies like connecting fractions to real-life situations, using manipulative representations, conducting open discussions with learners to identify misconceptions and putting more emphasis on improving lesson preparation and instruction can improve learners' cognition of fractions. Furthermore, they contend that teachers are not using manipulatives as much as they should, possible because of a lack of confidence or experience in using them. If connections between models or diagrams and symbols representing fractions are not stated clearly, learners are compelled to make their own conclusions about the fraction notation (Osana & Pitsolantis 2013). In this study, the researcher was interested in ascertaining whether teachers were making connections between the different representations they used during instruction.

Osana and Pitsolantis (2013) conducted a study focusing on the importance of connecting concepts and procedures during mathematics instruction. Their sample comprised Grades 5 and 6 learners, divided into two groups; control group and treatment group. One group received instruction involving treatment that made connections between representations explicit. Learners in this treatment group showed great improvement in their knowledge of the fraction concept and were able to make connections between fraction symbols and conceptual meanings. Further observation revealed a failure on the part of the teachers to demonstrate clearly the connections between fraction symbols and the models and pictures, resulting in students drawing their own conclusions. This researcher, a teacher educator, concurs with Osana and Pitsolantis (2013), in having observed that teachers tend to put a great deal of emphasis on symbolic manipulation without any consideration for conceptual understanding. One of the reasons for conducting this study was to uncover why teachers use or do not use particular representations, other than symbols, in teaching fractions.

Barmby et al. (2013) conducted a study focusing on helping teachers to develop the use of diagrammatic representations in teaching mathematical concepts in primary school classrooms through professional development. The sample were ten teachers from ten primary schools teaching Grades 3 and 5. Data were collected using semi-structured interviews and observations. From their findings, it was evident that the use of diagrams benefited both teachers and learners. The use of diagrams boosted learners' confidence; hence, conceptualisation of mathematical concepts was enhanced. Teachers' knowledge and instructional practice improved in the sense that they were able to use a broad range of diagrams. Barmby et al. (2013) and (Pape & Tchoshanov, 2001) agree that understanding fractions is not just a matter of being able to use multiple external representations, but being able to construct meaning from diagrams and concrete materials and to make connections with the symbols related to them. When introducing new mathematical tools such as virtual models of fractions, teachers should spend more time helping learners interpret and reason with the models they create than in learning how to use technology (Mendiburo, Hasselbring, & Biswas, 2014).

Mendiburo et al. (2014) designed a computer software system that delivered virtual fraction strips aimed at helping learners solve problems involving the ordering of fractions. In this study, learners were able to create fraction models using virtual fraction strips, but some failed to use their models to engage their reasoning in arranging the factions from smallest to largest.

Brijlall and Niranjan (2015) investigated the use of manipulatives in teaching trigonometry in a South African school; their findings revealed that concrete representations improve thought-processing skills and enable a smooth transition from concrete to abstract. Since they used Lesh’s (1979) translation model, their study proved that concrete representations effectively merge the model. This study also uses the Lesh et al. (1987) model but focuses on teaching fractions at primary school level.

2.7 SUMMARY

This chapter began by giving a summary of the primary mathematics curriculum in Swaziland, paying special attention to the breakdown of the concept of fractions using curriculum materials. The next section discussed the significance of curriculum materials in teaching and learning. This was followed by a review of the literature on the role of representations in teaching and learning fractions. It ended by giving a few summaries of studies done on the use of the various representations. The next chapter will discuss the theoretical framework.

CHAPTER 3

THEORETICAL FRAMEWORK

The theoretical framework of a study helps to locate the broad understanding of reality (Moodley, 2012). In this study, the researcher used Lev Vygotsky's educational theory of social constructivism. Many teaching and learning theories developed over the past century have aimed to improve mathematics education. Russian psychologist Lev Vygotsky is one of the major contributors towards improved teaching and learning, through theories such as social constructivism, the main construct of which is “the zone of proximal development” (Vygotsky, 1978). In this chapter social constructivism and the zone of proximal development, which form the theoretical basis underpinning this study, are discussed. In addition, in this study Lesh et al.’s (1987b) model of multiple representations is used as an analytical tool, and is also discussed below.

3.1 SOCIAL CONSTRUCTIVISM

Social constructivism is a learning theory which has its roots in constructivism. It underscores the crucial role played by culture and context in society in constructing knowledge. Social constructivists see both the context in which learning takes place and the social contexts that learners bring to their learning environment as important (Kim, 2001). Kim further asserts that proponents of social constructivism agree that knowledge, meaning and understanding of the world around us are dealt with within the classroom from either the point of view of one learner or the point of view of the whole class.

Creswell (2009) cited in McKinley (2015) asserts that social constructivism serves as a useful theoretical framework as it allows necessary qualitative analysis to reveal insights on how people interact with the world. Vygotsky (1978) believes that it would be difficult to gain social meanings of symbol systems and learn how to use them without social interactions with more knowledgeable others (Kim, 2001). The Swiss psychologist, Jean Piaget, was one of the first psychologists to venture into studying cognitive development in children. His theory of cognitive development forms a foundation for all other developmental theories that followed thereafter. According to Piaget, learning occurs in an individual’s mind through the processes of assimilation and accommodation. Piaget’s theory was widely accepted, although some psychologists such as Vygotsky questioned the exclusion of the social aspect of learning (McLeod, 2009). Vygotsky’s learning theory emphasised the role of culture and social context in cognitive development (Vygotsky, 1978). According to Vygotsky, an individual develops cognitively through participating “in various forms of social interactions using tools and signs, which are social in their nature” (Lourenco, 2012;pg.282). Tools and symbols therefore play a vital role in developing knowledge and understanding. The central construct of Vygotsky's social constructivism is the zone of proximal development (ZPD).Vygotsky describes the ZPD as the difference between what learners can achieve on their own, and what learners can achieve with the help of their teachers (Vygotsky, 1978). Vygotsky further describes learning in terms of social and cultural elements mediated by the teacher and tools mediated through the zone of proximal development and internalised by an individual. In Vygotsky's view, the ZPD is crucial in explaining how learning occurs (Dahms, Geonnotti, Passalaqua, Schilk, Wetzel, & Zulkowsky, 2007). The use of artefacts characterises most human activities related to thinking and learning; and the most salient “are semiotic tools such as language, specialised symbolic systems and educational models” (Sfard & McClain, 2002;pg.154). Since learning occurs within the zone of proximal development, a description of the ZPD is given in the next section together with the various tools used as scaffolds.

3.2 THE ZONE OF PROXIMAL DEVELOPMENT

Vygotsky defines the zone of proximal development as follows:

> … the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (Vygotsky, 1978, p.86)

Supporting and assisting children as they actively engage in the learning process is fundamental to Vygotsky’s ZPD (Verenikina, 2008). Vygotsky believes that most effective learning occurs within the learner's ZPD, when, like a flower, the learner is at the flowering stage (Murphy, Scantlebury, & Milne, 2015). Vygotsky used the ZPD to distinguish two levels of psychological development: actual development achieved through independent problem solving; and potential development gained through collaboration with an adult or a more capable being (Mahn, 1999). Learners working within the ZPD with teachers' guidance can construct an understanding of concepts which they would fail to do on their own (van Compernolle., 2012). What a learner can achieve when working independently on problems reveals the development that has already taken place (Stott, 2016). Expanding on Vygotsky's work, Zarestkii (2009) observed that during classroom interactions (teacher-learner or learner-learner interactions), the ZPD does not disappear but instead expands. The teacher is a mediator in the learning process and is responsible for creating classroom environments conducive to learning to ensure that all learners are active participants in the learning process. What students learn in the classroom is mostly determined by the classroom environment (NCTM, 2000). Vygotsky states that symbolic tools mediate higher-order thinking, and the most vital tool is language (van Compernolle., 2012). Since social interactions, language and tools or representations play a vital role in creating learners' ZPDs, these are discussed further in the next section.

Vygotsky likened development to a fruit harvest, where a farmer, in assessing his harvest, would not only consider the fruit that has ripened but also the fruit that is in the process of maturing (Zaretskii, 2009). This implies that within the ZPD some processes will be fully developed and some in the process of developing. In Fig. 1, the innermost region represents what the child can do independently and the outermost region represents what the child cannot do, even with assistance. The middle region is the ZPD, representing what the child can achieve with the help of an adult or more capable peer. In teaching fractions, it is therefore the teacher's responsibility to determine what learners can do independently and where facilitation is required by giving learners problems with several levels of difficulty. This is achieved if learners actively participate in the learning process.

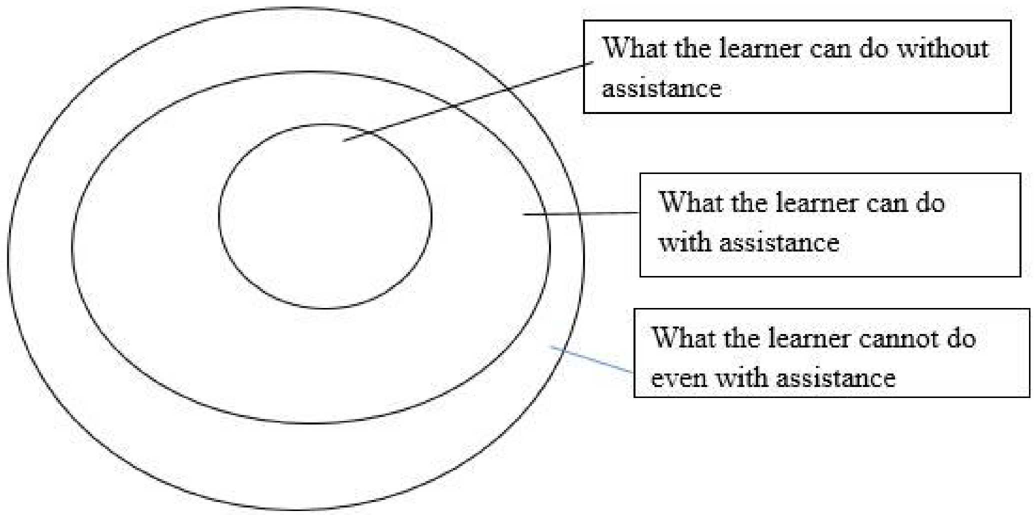

Figure 3.1: Adapted from Vygotsky's zone of proximal development.

The idea of learning through social interactions using various tools as scaffolds within the ZPD is the theoretical framework for the current research. It proposed to identify the different tools used by teachers in teaching fractions and the meanings they attached to those tools.

Scaffolding, defined figuratively in educational terms, refers to useful interventions utilised by a teacher to assist learners to progress (Silver, 2011). Vygotsky did not use the term "scaffolding", which has become synonymous with the ZPD, but Wood, Bruner and Ross (1976) cited in (McLeod, 2012) introduced it. McLeod (2012) further states that Vygotsky believed that assisting learners while they work on a task, either by general encouragement, specific instructions or the use of particular tools, encourages students to complete tasks. Silver (2011) suggested the following guidelines for scaffolding teaching within the ZPD: 1) assess the learner's current knowledge and experience; 2) relate current knowledge to what learners can do without assistance; 3) simplify tasks and give occasional feedback; 4) use verbal cues and prompts to guide learners; and 5) emphasise specific vocabulary that comes to the fore during the course of the lesson,

3.2.1 Social interactions within the classroom

Vygotsky argues that the development of the mind is influenced by society (Dahms et al., 2007). As children interact with people in society through questioning, they develop the ability to communicate and solve problems (Vanderburg, 2006). Kim (2011) states that without social interactions with adults or more knowledgeable others in society, children would find it difficult to develop symbol systems or know how to use those systems. Higher-order mental functions first develop through social interactions before they are internalised by an individual (Verenikina, 2008). Some of the interactions observed in the classroom are between the teacher and the learners, and others are learner-to-learner interactions. In the classroom environment, a teacher assumes the position of an agent of a particular culture. Hence they make choices and judgements on what and how mathematics will be taught (Nickson, 1992). Vygotsky further asserts that the most crucial aspect of children's psychological development is the attainment of the culture to which they belong (Verenikina, 2008). These include all the tools they use such as artefacts, symbols and language.

Hiebert and Capenter (1992) believe that, as with concrete materials, it is likely that social interaction within the classroom influence the kinds of relationships that students construct. Sharing discussions about regularities and patterns in a written symbol system may support the personal construction of relevant relationships. They believe that manipulation of concrete and visual representations without reflection is unlikely to stimulate a construction of the relationships that lead to understanding. This notion is supported by Gouws and Dicker (2011) who posit that the interactions among learners during group work and manipulating different types of representations does not necessarily guarantee conceptual understanding of concepts. However, what is learned depends on how the teacher facilitates the learning process, that is, the quality of teacher–learner interactions. It is therefore imperative that as learners manipulate the different representations of fractions, they are allowed enough time to reflect and discuss any emerging patterns and regularities, and that the teacher is available to control the level of frustration. Vygotsky stressed that language and cultural tools mediate social interactions (Stott, 2016).

3.2.2 The role of language in teaching and learning

Vygotsky (1978) cited in (Vanderburg, 2006) posited that language is at the heart of all interactions, be they written or verbal. Classroom teachers are constantly interacting with learners through speech, giving them instructions and facilitating the learning process. It is therefore a necessity for teacher to use a language that every learner understands. If the language of instruction is not the learners' first language, communication becomes a problem. Naidoo (2011) observed that visual tools, when used appropriately, bring clarity when verbal communication is a problem. Contrary to the popular belief that learning mathematics through one's native language is the best way to learn mathematics, using one's native language can be perceived negatively by society, resulting in derogatory names being given to the group taught in their native language (van Laren & Goba, 2013). Interestingly, students in Van Laren and Goba's (2013) study felt it was easier to understand mathematical concepts when taught through the medium of isiZulu.

Mathematics is a language on its own with many technical terms, and translating it into a language which may have a limited mathematical vocabulary can be a challenge. Teaching fractions effectively means using the correct language and technical terms, otherwise misconceptions can arise. For example, verbalising $\frac{2}{3}$ as "two over three" instead of "two-thirds" could lead to the misconception that a fraction is two numbers. On the other hand, adopting correct mathematical language does not translate to actual mathematical thinking; mathematical thinking is overly dependent on the use of symbols, but cannot be identified with the symbol as such (Van Oers, 2010).

Other symbolic tools or representations used in teaching fractions are concrete manipulatives, virtual manipulatives, pictures or diagrams and written symbols. Teachers and representations act as mediators in the learner's ZPD in the teaching and learning of fractions.

3.2.3 How tools are used in developing higher-order thinking

It is typical of human activity, in particular of thinking and learning, to be accompanied by especially designed tools (Sfard & McClain, 2002). Both cognitivists like Jean Piaget and socio-culturalists like Vygotsky are of the view that tools play a vital role in the construction of meaning and the development of higher-order thinking. Teachers use various tools within the classroom to develop mathematical understanding and higher-order thinking. In the case of teaching fractions, it is the teacher's task to determine what learners already know or can do without a teacher's assistance and where guidance is needed. Guided by curriculum materials and experience, the teacher selects the tools to use to mediate the learning of fractions. Some of the tools used in teaching fractions are concrete materials (such as fraction bars, symmetrical objects, counters, paper folding), virtual models, pictures or diagrams (area models). How these tools are selected and used and the strategies used to promote mathematical thinking is influenced by the teacher's experience and beliefs about the nature of mathematics (Nickson, 1992). As stated earlier, some teachers use concrete materials to make abstract mathematical concepts more concrete to learners, but others use them as a reward for good behaviour.

Sfard and McClain (2002) also argue that the importance attributed to these tools changes depending on the user's philosophical views and their view of cognition. Higher-order thinking is developed using educational tools which are unique to the society and context within which learning is located (Murphy et al., 2015). Using the teaching and learning of fractions as the context, one of the teachers in the current study mentioned that she usually targets the guava season to teach fractions in Grade 4 since there are plenty of guavas, which learners can manipulate to construct the meaning of fractions. She posited that her students found it easy to show fractions on a number line after manipulating the guavas. It is therefore important for teachers to use objects learners are familiar with, taken directly from the learner's world.

Through observations, this study aimed to identify the mediation tools used by teachers in teaching the concept of fractions, and to uncover how the teachers used the tools in classroom instruction. The researcher also had to find an analytical tool for the study and came up with that proposed by Lesh et al.'s (1987) multiple representational model, as shown in Figure 2 in the next section.

3.3 ANALYTICAL FRAMEWORK

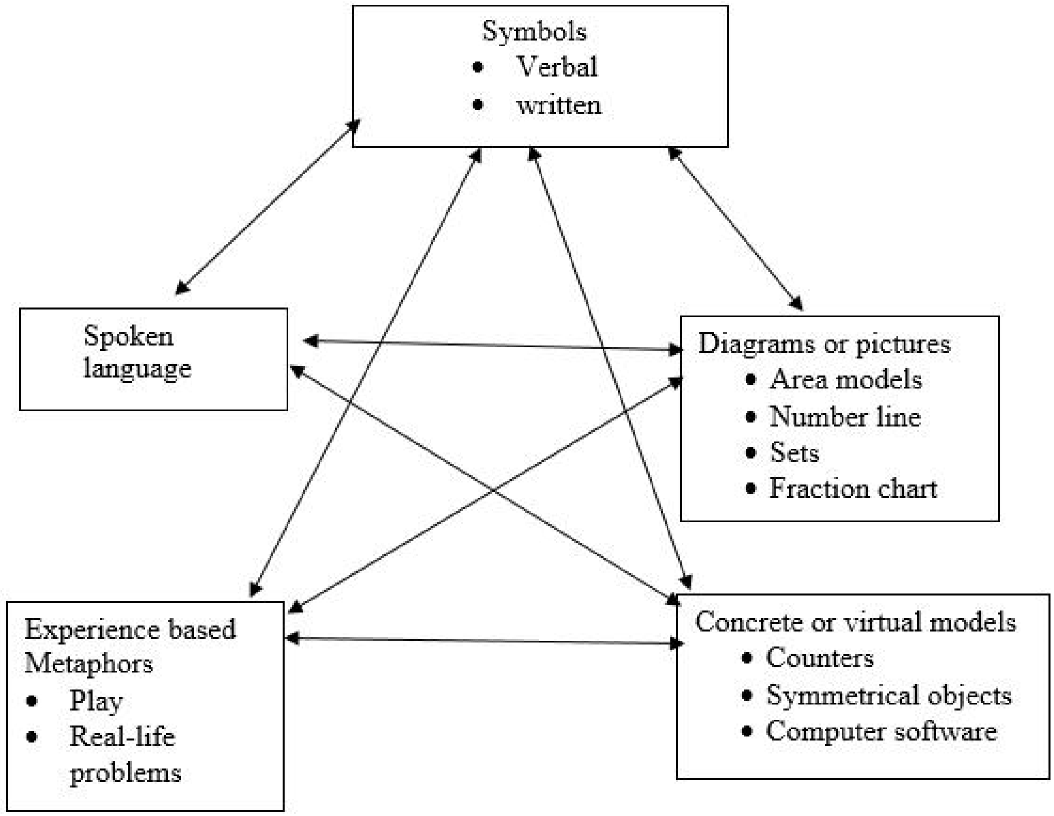

Figure 3.2: Representations used at primary school level. Adapted from Lesh, Post, and Behr's model (1987, p. 33).

The analytical framework used in this study is adapted from (Lesh et al., 1987) who identify five distinct forms of representation used in learning mathematics and problem solving; see Figure 2. Lesh and Doer (2003) differentiated between representations used at primary school and those used at secondary school levels, and later improved their model of representations. The representations commonly used at primary school level are written symbols, concrete manipulatives, diagrams or pictures, experience-based metaphors and spoken language. For instance, the concept of a fraction can be described in words as part of a whole (verbal), a symmetrical object like an orange can be cut (concrete model); part of a rectangle that has been divided into equal parts may be shaded (picture); real-life situations may be described (experience-based metaphor); or a fraction may be written down using fraction symbols (Tripathi, 2008). According to this model, if a learner understands a mathematical concept he/she should have the ability to translate the concept into different representations. Teachers should, therefore, be aware of the various ways of representing fractions or operations on fractions in order to expose learners to the various forms, hence making a concept more accessible to all students.(Hiebert & Carpenter, 1992) emphasised the need for learners to be given enough opportunities to manipulate blocks for them to understand the symbolic representation of whole numbers. Using Lesh et al.'s model in this study enabled the researcher, through classroom observations, to identify and determine the extent to which teachers were using the different types of representation. Furthermore, classroom observations ascertained how they were using these representations during instruction.

According to Lesh et al. (1987 b), understanding a mathematical concept implies being able to: Realise the idea embedded in forms of qualitatively different representations, flexibly manipulate the idea within given representational systems, and correctly translate the idea from one representational system to another. Teachers use the different types of representations for various reasons. According to Lesh et al. (1987), as learners learn mathematical ideas, the translation and transformation networks become more complex, and it becomes necessary for teachers to use representations to "simplify, concretise, particularise, illustrate, and paraphrase these ideas, and embed them in familiar situations". (Lesh et al., 1987, p. 36). For instance, if learners are given the task of adding two fractions, the teacher can make it more concrete by translating it to a real-life problem.

Furthermore, Lesh et al. (1987b) believe various representational systems can be used to discover learners' strengths and weaknesses, which can be used to identify instructional opportunities. For example, a teacher can pose questions and instruct learners to represent them using various representations. If the question is presented as a real-life problem and the learner is unable to translate it to written symbols, the learner can begin by translating it to a diagram, and thereafter to written symbols. Research has shown that some representations are problematics for learners. Ball (1990b) observed that the circle model posed a lot of challenges for learners if given a fraction like two-thirds, requiring them to represent it using a diagram. Teachers should therefore be cognisant of some of the representations that may pose problems for learners while preparing for classroom instruction.

According to (Cuoco, 2001), learners develop their own internal representations of mathematical concepts based on the external representations that a teacher selects to introduce them. It is, however, not possible to see a learner's internal representation; it can only be deduced through the learner's external representations (Goldin & Shteingold, 2001). In teaching fractions, the teacher can ascertain whether his or her teaching has been effective through the learners’ external representations. When forms of representations are used other than symbolic representations, there is often no link between the representations. Accordingly, when making links between representations, those links must be mathematically significant. In teaching fractions, sometimes teachers use concrete manipulatives but fail to link them to the rules for operations on fractions. Sometimes teachers use false representations of mathematical ideas, as was observed in one of the lessons.

Translation ability amongst various representations is usually associated with success in mathematics education, especially in problem solving (Gagatsis & Shiakalli, 2004; Lesh et al., 1987).Translation ability pertains to the thought processes required in moving from one type of representation to another (Lesh et al., 1987); for instance, from an area model to a fraction symbol. Because visual representations play a fundamental role in aiding conceptual understanding in mathematics education, learners should be given enough practice in the use of visual representation for them to acquire the skill (Gagatsis & Elia 2004).

In teaching fractions, teachers can use different forms of representation to identify learners' strengths and weaknesses by presenting fractions in one form such as area models and instructing learners to write them in another form such as written symbols. This way conceptions and misconceptions are unveiled (Lesh et al., 1987). Furthermore, the following observations were made: Translations to diagrams are easier than from diagrams; translations dealing with written language are easier than translations dealing with written symbols, and the easiest is the one requiring a child to read a fraction in different forms.

3.4 SUMMARY

In this chapter, the theoretical framework and analytical tool for the study were discussed. The researcher reviewed the literature on Vygotsky's educational theory, social constructivism, and addressed its main constructs, that is, the ZPD and the idea of scaffolding. The research emphasises the importance of tools, language and social interaction in the learning process. The chapter ended with a description of the analytical framework used in the study. The next section describes the methodology used to collect data.

CHAPTER 4

METHODOLOGY

This chapter focuses on methodology, which includes the research design, methods of data collection, a description of instruments used for data collection, methods used for data analysis and ethical considerations. Since the researcher intended to investigate how teachers used multiple representations in teaching fractions in the intermediate phase, the qualitative method was used. A multiple-case design was employed to increase validity. Interviews and observations were used as data-collection methods. Issues of ethics were considered and necessary steps were taken to protect participants.

4.1 RESEARCH APPROACH

The research design provides a framework for data collection and data analysis (Bryman, 2008). This study used the qualitative approach. Qualitative research is a research strategy that emphasises words rather than quantities in both data collection and analysis (Bryman, 2008). The qualitative method was chosen because qualitative data are richer in meaning and detail compared to quantitative data (Babbie & Babbie, 2008). The three main purposes of qualitative research are to explore, describe and explain (Babbie & Babbie, 2008). Babbie and Babbie summarise each of these purposes as follows:

Exploration is carried out when the phenomenon the researcher is studying is relatively new or persistent. Exploratory studies usually yield new insights into the topic of interest in research. The limitations of exploratory studies are that they rarely provide satisfactory answers to research questions.

Descriptive studies refer to studies that seek to describe situations or events; most qualitative studies aim at describing situation or events. The researcher makes an observation of a situation and then gives a clear description. Descriptive studies seek to answer questions of what, where, when and how.

Finally, explanatory studies respond to the question of why certain phenomena occur.

In relation to these three purposes of qualitative research by Babbie and Babbie (2008), this study was a descriptive study, since it intended to identify representational tools used in the classroom when teaching, which may be considered the 'what' of the phenomenon. In addition to this, observations determined how the representations were used. On the other hand, it is also an explanatory study because it sought to understand the reasons why teachers used or did not use particular representations, thus addressing the question of "why".

4.2 PARADIGM

In this study, the researcher adopted the interpretive paradigm. The interpretive paradigm used here aimed at understanding how and why primary school teachers prefer to use certain forms of representation and totally ignore others when teaching the concept of fractions. The interpretive design is based on a concept from ontology; that reality is subjective and interpreted by people in various ways based on their beliefs (Darke, Shanks, & Broadbent, 1998). The researcher tried to gain an in-depth understanding of how teachers use representations in the mathematics classroom, and further sought to understand the meanings teachers attach to those representations.

4.3 RESEARCH DESIGN

Research strategies associated with qualitative research are phenomenology, ethnography, grounded theory, critical studies, and the case study. The researcher conducted a case study, focusing on the use of multiple representations by three primary school teachers in teaching fractions in three different schools. A case study may involve one case or several cases. A case study relating to a single case is appropriate if it is unique and the purpose is to test a theory (Darke et al., 1998). On the other hand, in multiple-case designs, the investigation of a phenomenon is conducted in various contexts. This study adopted the multiple-case design with the aim of investigating the use of multiple representations by primary school teachers in diverse settings when teaching fractions. The adoption of the case-study design is particularly suited to research questions that require a detailed understanding of social processes because of the rich data collected in context

(Cassell & Symon, 2004). A case study has the potential of addressing several issues, hence the use of more than one method. Some of the methods associated with this strategy are participant observation, direct observation, ethnography, interviews (semi-structured or unstructured) and sometimes questionnaires (Cassell & Symon, 2004). Many case study researchers, in their pursuit of the delicate and intricate interactions and processes occurring within organisations, tend to use a combination of methods, and deliberately triangulate data and theory, thereby improving validity. Case studies can be useful for exploring new or emerging processes or behaviour.

4.4 TARGET POPULATION

Target population refers to the entire population to which the study aimed at generalising its results and findings. The target population for this study was all primary school mathematics teachers teaching Grades 4, 5 and 6. These grades were selected because operations on fractions are introduced and developed in these grades.

4.5 SAMPLE AND SAMPLING PROCEDURE

A purposive sample of three teachers participated in this study. In purposive sampling the researcher selects subjects that are informed about the topic of interest (Schumacher & McMillan, 2010). The researcher aimed at identifying representations used by teachers in teaching factions, establishing how they used those representations and finding out their reasons for using or not using them. Hence it was important to select only teachers who used a variety of representations in their teaching. Initially, the researcher described the purpose of the study to more than ten teachers with the aim of identifying teachers who were using multiple representations in their lessons. The three teachers in this study were selected because they indicated that they used multiple representations when teaching fractions. Through observations and interviews, the researcher hoped to answer these questions: What kind of representations do teachers use when teaching fractions? How do teachers use multiple representations in the classroom? What are their reasons for using or not using certain representations?

Two of the selected teachers taught Grade 4, and one taught Grade 6. As mentioned above, the researcher chose these teachers because of their enthusiasm pertaining to the use of multiple representations in classroom instruction. The three were from three different schools, each school representing the types of schools found in the Manzini region of Swaziland. Convenience sampling was used to select the schools, in terms of accessibility. All schools are located close to public roads. The following is a description of the three schools:

School A

School A is a rural school situated about 30 kilometres from Manzini city centre. The school has double streams from Grades 1 to 7. Some of the children come from poor socio-economic backgrounds. All learners' first language is SiSwati. The school is old, with some of the classrooms having leaking roofs. There is only one computer in the school, used by the school secretary, and one photocopier. Even the principal lacks a personal computer. The school secretary types tests and exams. All classrooms have chalkboards. More than half of the teachers commute from Manzini every day, traveling approximately 60 km a day. All the teachers are diploma-holders, except for the principal, vice-principal and one teacher, who have university degrees.

School B

School B is a semi-urban school located about nine kilometres from the city centre. This is a big school with a high enrolment. On average, there are forty-five students per class. The number of streams per class ranges from three to four. All classes are in good condition and fitted with chalkboards. The principal holds a master's degree, three other teachers have degrees, and an additional five were pursuing degrees on a part-time basis. The school does not have a computer laboratory and does not offer computer technology. The learners come from various backgrounds, but mainly from low-income or middle-income families. There are other nationalities other than Swazis, particularly Asians.

School C

School C is an urban school situated about 1.5 kilometres from the city centre. It is a very well-maintained and secure school. Classes double stream from Grades 1 to 7. Children come from various socio-economic backgrounds, including many from various other African and Asian countries. All are encouraged to speak English all the time. Some of the teachers have either recently upgraded their qualifications to degrees or are working towards degrees, since they have easy access to institutions of higher learning, where they learn part-time. Classrooms are fitted with whiteboards. The school offers computer technology as a subject to the learners, and most of the teachers are computer literate. The school is well equipped; even the principal has a computer in his office.

4.6 INSTRUMENTS FOR DATA COLLECTION

For data collection purposes, the researcher developed three instruments, namely, the interview schedule, the observation schedule and the follow-up interview schedule. The following is a description of these tools, adapted from Naidoo (2011).

4.6.1 The interview schedule

The researcher prepared an interview guide (Appendix A) consisting of three main parts or questions: Question 1 was the teacher profile; question 2 covered the teaching and learning materials they used; question 3 examined whether teachers valued the use of multiple representations or not. Of these, question 2 was designed to answer the first research question; i.e. What representations do teachers use in teaching fractions?

4.6.2 The observation schedule

The researcher constructed an observation schedule (Appendix B) used simultaneously with video recording, consisting of four parts or questions. Part 1 consisted of a frequency table aimed at answering the first research question, that is, What were the tools used in classroom instruction and what was their frequency of use? Part 2 looked at the social interactions within the classroom as teaching and learning progressed. In Part 3, the researcher wrote observations on how each teacher used multiple representations during instruction. Both Parts 2 and 3 answered research question 2: How are multiple representations used in teaching fractions? Part 4 was for noting any other observations.

4.6.3 Follow-up interview schedule

After each lesson, the researcher interviewed teachers to ascertain their reasons for using or not using particular representations. This helped to answer the third research question, i.e. What are teachers' reasons for using or not using particular representations?

The schedule (Appendix C) comprised four parts. The main purpose of the follow-up interview schedule was to help the researcher answer the “how” and “why” of the research questions. Part 1 and Part 2 aimed to answer how and why teachers use multiple representations in teaching fractions, and whether they valued the various representations available to them. Part 3 sought to discover how representations were used in various contexts. In Part 4, the researcher sought information on the kinds of help teachers felt they might need in order to use multiple representations more effectively. The questions required teachers to give explanations on exactly how and why they used multiple representations in their teaching.

4.7 DATA COLLECTION METHODS

As stated earlier, data were drawn from a case study of three primary school teachers who used multiple representations to teach fractions. A research method is the technique used for data collection (Bryman, 2008); for this study, the researcher used interviews and observations data collection instruments using a semi-structured interview schedule and an observation schedule respectively. As suggested by Naidoo (2011), people's behaviour varies in different situations, hence the need for using different data collection methods or triangulation. Interviews with teachers took place before observations and after observations (follow-up interviews). Descriptions of these methods and their application in this study are given in the next three sections.

4.7.1 Interviews

An interview is a “face-to-face engagement between two people where questions are asked by the interviewer in order to elicit responses that can be analysed within qualitative situations” (Dakwa, 2015). Interviews are the most common method used for gathering information in qualitative research (Hartley, 2004). There are three types of interviews, namely, structured, semi-structured and unstructured. The purpose of any qualitative interview is to enable the interviewer to see the research topic from the perspective of the interviewee, and to enable an understanding of how and why they came to have their particular perspective. In an interview, the researcher asks questions and the interviewee responds orally to questions. The interviews with the teachers were semi-structured. In conducting a semi-structured interview, the researcher prepared an interview guide (see Appendix A). As suggested by Bryman (2008), the interview guide consisted of a list of questions on specific topics to be covered, giving the teachers much leeway in answering. During interviews, as the researcher picked up new things, more questions emerged. The development of the interview guide did not end with the beginning of the first interview; the researcher continuously modified it as the collection of data progressed (Cassell & Symon, 2004). The interview guide helped prevent deviation from the topic of interest and the collection of unnecessary data. An audio recorder captured all interviews, decreasing misinterpretations and making transcription easy.

All interviews took place during the first term of the school calendar. Interviews with Simon and Pam occurred in the head teacher's office, and with Dan, in his Grade 4 classroom. All interviews were carried out a week before classroom observations. They helped build rapport and created rich profiles of participants.

4.7.2 Non-participant observations

Research has shown that observations are a feasible way of determining effective strategies aimed at improving mathematics instruction in primary classrooms (Thompson & Davis, 2014). An observation affords the researcher opportunities to analyse both verbal and non-verbal communication, as well as their interaction with the environment (Chitiyo, Taukeni, & Chitiyo, 2015). The role played by the researcher or observer in a study varies depending on the aim of the study. There are different types of observations, but this study employed non-participant observation.

The researcher in this study was a complete observer, that is, she did not take part in any activities in the classroom, but the participants were fully aware of her role. Non-participant observation used in this study allowed the researcher to study first-hand the day-to-day experiences of teachers using multiple representations in teaching the concept of fractions. The researcher observed teachers in their natural setting, the classroom, using tools such as visual aids, real-life problems and symbols to teach the concept of fractions. Throughout the lessons, there was no interference with the teacher and the learners. Instead, the researcher sat quietly at the back observing and taking notes. Two observations per teacher were conducted in succession, to ensure consistency. In addition to field notes, all lessons were video recorded. The following is a description of classroom observations with each of the three teachers, with pseudonyms used:

Simon

The researcher paid Simon a visit on 4 April 2014 for an interview aimed at developing a profile and establishing rapport, and to schedule a time for observation. The meeting took place in the principal's office. The researcher did not interfere with the school timetable. The first observation was scheduled for 11 April and the second for 13 April. Each lesson was one hour long. The first lesson, according to the timetable, began at 8 a.m. and the second lesson began at 9 a.m. The first lesson was on defining a fraction using a discreet set, and the second lesson was on the addition of fractions with a common denominator.

Dan

An appointment was set with Dan a week earlier for an interview and to schedule time for classroom observations. The interview took place on 6 April 2014 in Dan's Grade 4 class, after school. The two observations were scheduled for 14 and 15 April and took place as scheduled. Both lessons were one hour long; the first lesson beginning at 8 a.m. and the second at 9:30 a.m. The first lesson was on the addition of fractions with a common denominator and the second lesson was on the subtraction of fractions with a common denominator.

Pam

Pam’s interview took place on 26 April in the vice principal’s office. Pam and the researcher scheduled interviews for 2 and 5 of May 2014. Both lessons were an hour long, the first beginning at 9 a.m. and the second at 12 noon. The first lesson was on the addition of fractions with different denominators, and the second lesson was on the addition of mixed numbers. The researcher did not interfere with the school timetable.

4.7.3 <u>Follow-up interviews</u>

One follow-up interview after the second observation was conducted with each of the participants. The researcher used the schedule for follow-up interviews (see Appendix C).

All interviews took place in participants' classrooms. Interviews with Simon and Dan happened during tea break, at about 10.30 a.m. and with Pam at lunch time, 1.15 p.m.

4.8 DATA ANALYSIS PROCEDURE

Data were collected using interviews and observations, using instruments such as an audio recorder for interviews and a video recorder for classroom observations. Information from audio and video recordings were transcribed. Data were organised according to Lesh et al.'s (1987) model of representations and combined with the research questions. Data were presented per case studied. Pictures were used as the researcher wanted to show how teachers used representations to represent fractions.

4.9 ETHICAL CONSIDERATIONS

Qualitative research is more likely to intrude into personal issues compared to quantitative research; hence there are guidelines for conducting qualitative research. These guidelines include policies regarding informed consent, deception, confidentiality, anonymity, privacy and caring (Schumacher & McMillan, 2010). A brief description of each and a further explanation of the ethical process embarked on follows:

4.9.1 Informed consent

According to Schumacher and McMillan (2010), before one can conduct research, each participant is required to sign an informed consent form. The Director of Education granted permission for conducting this study in the three schools. An informed consent form was prepared and given to each teacher to read and sign (Appendix D). In the consent form, the time required for participation was stipulated. Teachers were assured of confidentiality and anonymity about information gathered. The researcher established a trusting relationship with participants, allowing them to select both interview and observation times.

The researcher sought permission from parents and guardians to allow their children to take part in the study. Letters were written to parents and guardians describing the nature of the study. All the children who took part received permission from parents or guardians.

The learners were not part of the study per se but because they were part of the environment where observations were conducted, permission was sought.

4.9.2 Confidentiality and anonymity

Schumacher and McMillan (2010) state that the participants in a study should not be identifiable. They further declare that it is the "researcher's responsibility to protect the individuals' identities from other persons in the settings and to safeguard the informants from the general reading public" (p. 339). To ensure that the teachers in this study were not identifiable, the researcher used pseudonyms. The schools were identified with the letters A, B and C. However, anonymity and confidentiality cannot be guaranteed if the Director of Education were to request the findings of the research and the names of the schools concerned.

4.9.3 Privacy and empowerment

According to Schumacher and McMillan (2010), researchers negotiate with participants, making them understand the power they have over the research process. The researcher assured teachers that findings would be used purely for research purposes, and would be made available to them. In addition to that, the results would be used to improve pre-service training in mathematics, and teaching and learning materials in the schools.

4.10 TRUSTWORTHINESS

The researcher addressed the issue of trustworthiness by focusing on credibility, transferability, dependability, and confirmability.

Credibility has to do with internal validity. Internal validity refers to variables other than those studied that may affect the outcome of the research. To ensure credibility, data collection involved more than one instrument, that is, observations and interviews (triangulation). Before the commencement of data collection, teachers were asked to sign consent forms in which it was clearly stated that teachers were at liberty to withdraw from the study, to ensure that they were genuinely willing participants. Furthermore, the researcher established rapport before each interview by engaging the participants in light discussion about current news. Transcriptions of audio recordings of interviews were double-checked to verify interpretations.

Transferability was ensured by proving rich descriptions of context under which both interviews and observations were conducted.

To ensure dependability, a rich description of research design and data collecting instruments was provided to participants. The researcher kept records of all data collection processes, that is, the audio recording of interviews, notes and video recordings, for easy access and verification purposes.

Confirmability has to do with ensuring that the experiences of the participants are presented accurately and are not influenced by the researcher's preferences. To ensure confirmability, a detailed description of research methods, data collection strategy and data collection instruments was prepared.

4.11 SUMMARY

This study used a case study design, and data collection was done through interviews and observations. A clear description of data collection tools and procedures was given. This was followed by a description of how interviews and observations were carried out, including methods of data analysis. Finally, issues of ethics and trustworthiness were considered. The next chapter presents data analysis sourced from interviews and non-participant observations.

CHAPTER 5

DATA PRESENTATION AND ANALYSIS

This chapter presents data collected throughout the course of the study. Data is presented according to the individual teachers who participated in the study. Data collection began by acquiring a rich profile of each of the three participants involved, as well as a description of their teaching styles. The researcher used pseudonyms for all three participants to protect their identities, namely, “Simon”, “Dan” and “Pam”. As stated in the previous chapter, data were sourced from interviews and classroom observations with each participant. Detailed transcripts of interviews and classroom observations are accessible in the appendices, and the relevant sources stated as data are presented. This is followed by data on the kinds of representations teachers used, how they used the various representations and their reasons for using those representations.

5.1 PARTICIPANT ONE: SIMON

Simon teaches in a rural school. There are 38 learners in his classroom. The school has no computer laboratory and only one computer, used by the school secretary, who types all tests and exams.

5.1.1 Simon’s profile

Simon was the most experienced of the three teachers, with 30 years’ experience in teaching mathematics and science. He is a diploma holder who specialises in mathematics and science, having taught these subjects in the same school for the past thirty years. For most of this time he has been teaching Grades 5, 6 and 7; this was his third year teaching Grade 4. He is an active member of the Primary Mathematics Panel. Simon is passionate about teaching and believes that all lessons should be learner-centred.

Although Simon used curriculum materials supplied by the Ministry of Education and Training, he felt there was a need to supplement these materials. He showed me a book he used which he said was good as a source of mental mathematics problems. His main language of instruction is English, but every now and then he uses SiSwati, which is the learners' first language. He believes that all lessons should have visual aids, and move from the concrete to the semi-concrete (pictures and diagrams) and finally to numerical symbols. This was evident in both his lessons. He begins all his lessons with an activity involving the learners, calling this "human activity".

5.1.2 Simon's teaching style

Simon said that he was passionate about teaching mathematics, having developed a love for it while in school. He said he was good at mathematics but struggled in the languages. All his lessons from classroom observation were learner-centred. The learners were actively involved from the beginning to the end of the lessons. When asking questions, he discouraged learners from answering questions in unison, and was quick to identify learners not paying attention.

When introducing a lesson, he checked prerequisite skills, connecting the new concept to other established concepts. For instance, when introducing division of fractions, he asked questions on division of whole numbers. Discussions within the classroom were either whole group or small group (four learners) discussions. He moved around the classroom ensuring that each member of a group was participating fully in any given activity. He guided his learners as they drew diagrams of sets and found fractions of given sets. He seemed to understand the importance of active participation by learners in constructing knowledge and, in addition, he clearly demonstrated the connections between different representations.

5.1.3 Kinds of representations used by Simon

5.1.3.1 Concrete representations

In the interview, Simon said he used more than one kind of representation when teaching. Classroom observations confirmed this. In his first lesson, Simon used stones as counters for learners to form sets in his first lesson, although he did not use them in the second lesson.

Figure 5.1: Simon distributing stones to be used as counters.

5.1.3.2 Diagrams or pictures

From class observations, the researcher observed that Simon did not draw diagrams of sets on the board but instead instructed learners to draw diagrammatic representations of fractions of sets they had previously manipulated using stones. In the second lesson he used rectangular area models to demonstrate the addition of fractions with a common denominator.

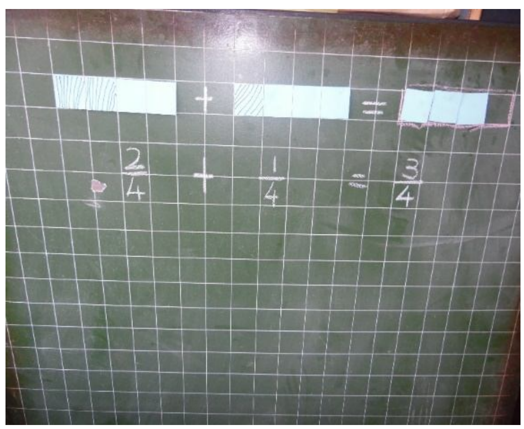

Figure 5.2: Simon's diagrams of area models showing addition of fractions.

5.1.3.3 Experience-based metaphors

Simon introduced each new concept with an activity involving the learners, which he called "human activity". In the first lesson, he used learners to illustrate the part-whole definition of a fraction using sets, and in the second lesson to demonstrate the addition of fractions with a common denominator. In the second lesson a desk was used to represent a whole, and a taxi was mentioned as a real-life whole. The following is a transcript of part of this lesson:

T: *Now let me make an example of a kombi (mini-bus). How many people can fit in a kombi?*
L: *Fifteen people.*
T: *How many people can be seated on this desk?*
L: *Four*.
T: *Is it possible for a kombi to carry thirty people?*
L: *No*.
T: *Is it possible for a desk to carry eight people?*
L: *No*.

In the same lesson, he used a loaf of bread as an example in real-life. He used a loaf of bread as a unit, which was familiar to all learners:

A whole … think of something that we normally buy and like. What do we like? Bread. Our whole is a loaf of bread. Then we buy what, if it is not a loaf of bread? (Appendix E).

5.1.3.4 Symbolic representations

Simon used both written and verbal symbols. In lesson observations, he emphasised correct verbalisations of numerical fraction symbols. For instance, Simon kept stressing that pupils should not verbalise the fraction $\frac{2}{4}$ as "two over four" because according to him, "over" means something that has gone past. Instead they should say "two-fourths" or "two out of four":

> *Two-fourths, not two over four. Or you can say two out of four. You have taken out two out of four. What else can you say? Two of the four, not two over four. That means it is finished or overflowing.* (Appendix E).

5.1.3.5 Spoken language

Simon used both English and SiSwati, but mainly English. Simon probed and asked questions as learners worked in groups translating a symbolic problem into an area model. In most cases learners did not respond verbally but through actions.

5.1.4 Simon’s reasons for using multiple representations

Quizzed about his motivation for using more than one representation, Simon stated that he did this based on the knowledge that people have different learning styles. Therefore, using different representations catered for the different learning styles in the classroom:

> *It is very important to use more than one visual aid. But there are some topics where it is hard to come up with a visual aid. I have noticed that some people do not easily understand if there are no visual aids because we learn in different ways; some people learn visually, and some hear when you talk* (see Appendix H).

5.1.5 How Simon used representations

5.1.5.1 Concrete representations

In Simon's first lesson, the learners were actively involved as they worked in groups creating and partitioning sets to form fractions. In Figure 5, Simon instructed learners to divide a set of eight stones into four equal parts. He moved around ensuring that all members of each group were participating. Then he asked the learners, “Each set is what of the whole group?” The learners responded in unison, “One-fourth”. Simon instructed a learner to write on the board $\frac{1}{4}$ of 8 = 2. This activity continued for about 15 minutes, as learners worked on finding fractions of different sets of stones.

Figure 5.3: Simon’s learners manipulating stones

5.1.5.2 Diagrams or pictures

In the first lesson, Simon first involved learners in an activity in which they created sets using stones. Simon instructed each group to draw on plain paper a diagram representing the sets of stones they had just manipulated. He then gave them mathematical sentences to complete using a diagram, for example $\frac{1}{2}$ of $12 = ...$

Simon used two area models to demonstrate addition of fractions with a common denominator (Figure 4). The picture shows Simon's demonstration of two-fourths plus one-fourth using diagrams of area models. The answer had already been determined using "human activity", the use of area models being an alternative method. As shown in Figure 4, Simon aligned area models with symbolic representations.

5.1.5.3 Experience-based metaphors

In Simon's lessons, the learners were excited and most of them wanted to be part of the group of volunteers used for demonstrations. Based on both lesson observations, Simon introduced new lessons using an experience-based metaphor that he called "human activity". This was an activity involving the learners. In the first lesson, learners formed members of a set. There were six learners altogether in the set. The set then divided into two equal parts to represent halves, shown in Figure 6.

Figure 5.4: Simon using learners to demonstrate half of six.

Another group of volunteers formed a group of eight, which was then divided into four equal parts, to show quarters of eight (see Figure 5.5).

Figure 5.5: Simon using learners to demonstrate a quarter of eight.

In the second lesson, the use of a desk to represent a unit was slightly confusing. When asked about the desk in follow-up interviews, he said the desk or taxi represented capacity. Figure 5.6 below shows pupils sitting at two desks, which, according to Simon, represented two wholes or two units.

Figure 5.6: Simon demonstrating two wholes.

He then removed two learners from one desk and three from the other desk, showing how the remaining two, at one desk, and the remaining one, at the other desk, represented two-fourths and one-fourth, respectively. Below is Figure 5.7, showing the remaining pupils.

Figure 5.7: Shows two learners in one desk representing two-fourths and one learner in the other desk, representing one-fourth.

Simon then instructed the remaining learner in the desk on the right to join the two learners on the left. He was demonstrating “putting together” or combining: “Let us combine the people in the two desks. So how are we going to do this? You on the second desk move to the first desk.” Figure 5.8 shows learners seated in one desk after Simon had combined them.

Figure 5.8: Simon demonstrating the sum of two-fourths and one-fourth.

The following is the ensuing conversation that took place between Simon and the learners:

T: *We have added two fractions, what is the denominator?*

L: *Four.*

T: *How many are they now?*

L*: Three.*

T*: Three is the numerator. If we say two-fourths plus one-fourth, the answer is three-quarters* (see Appendix E.)

5.1.5.4 Symbolic representations

When defining a fraction, Simon wrote the numerical symbol for the fraction and the name for each component of it (see Figure 5.9).

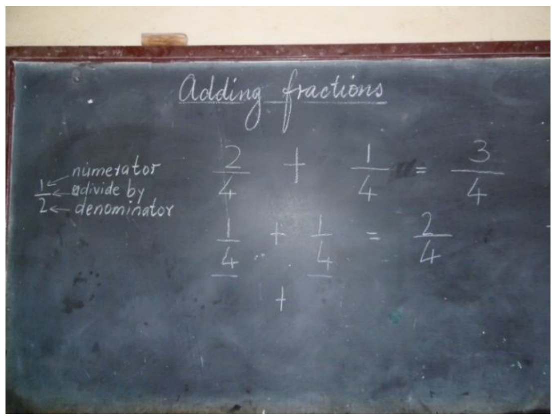

Figure 5.9: Simon’s symbolic representation of fractions.

Simon put a lot of emphasis on verbal representation of fractions. Whenever he wrote a fraction symbol on the board, he emphasised correct verbal representation. He ensured that other forms of representation, such as area models or concrete manipulatives, always accompanied symbolic representation.

5.1.5.5 Spoken language

As stated earlier, Simon used both official languages, SiSwati and English. Instruction was mainly in English; SiSwati was used to clarify certain points when he felt learners were not following. He further used SiSwati when giving examples familiar to learners, such as buying bread in halves and quarters.

5.1.6 Interactions within Simon's classroom

As shown in Table 5.1, the most common interactions observed in Simon's classroom were whole group discussions, with the teacher leading discussions, and learner-to-learner interactions in small groups, sometimes in pairs.

TABLE 5.1: Showing interactions within Simon's classroom

Type	Not at all	Sometimes	Frequently	All the time
Whole group			✓	
Small group			✓	
Pairs		✓		
Individual			✓	

Table 1: Interactions within Simon's classroom.

5.1.7 Simon's reasons for using or not using certain representations

5.1.7.1 Computer technology

The researcher wanted to know if Simon would have used computer technology if the school had computers:

> *I think if you have computers, you can use them as long as you know how to operate a computer. Teaching with computers can mean less work for teachers because in a computer you prepare your lesson and put in the visual aids you want to use. Even if you have a challenge in drawing accurate diagrams, using computer technology, you can draw perfect diagrams. Computers are also helpful in terms of storing information; I can easily retrieve stored information* (see Appendix H).

He further mentioned that even if they had computers, they would still need to use them efficiently.

5.1.7.2 Experience-based metaphors

When Simon was asked in the follow-up interview why he began his lessons with an activity involving learners, he stated that an activity involving a real situation made it easier for learners to understand the concept of a whole:

> *As you have seen I started with an activity involving the learners themselves. If you introduce the visual aids first, it creates confusion. If I had started with the chart and paper strips, it would have been difficult for them to understand the meaning of a whole. So you have to start with real life first. That is why I used the desk and taxi as examples in real life* (see Appendix H).

5.1.7.3 Symbols

When Simon was asked why he emphasised that numerical symbols should be verbalised correctly his response was:

> *It is a misconception to say "over" because in English when you say over you mean something that has gone past or is overflowing, it is more than what is necessary* (see Appendix H).

5.1.7.4 Spoken language

As stated earlier, Simon used both official languages, SiSwati and English. Instruction was mainly in English; SiSwati was used to clarify certain points when he felt learners were not following. As he put it:

> *I use both English and SiSwati because if I use English only, the language becomes a barrier. When I see they do not understand this word, I give an example in SiSwati* (see Appendix H).

5.2 PARTICIPANT 2: DAN

Dan teaches in an urban school, which offers more than two languages in the curriculum. In addition to SiSwati and English, they also offer French and Portuguese. There were 42 learners in his classroom, of various nationalities.

5.2.1 Dan’s profile

Dan is a male and the least experienced of the three participants, with 15 years’ teaching experience. He has just completed his bachelor's degree in inclusive education, specialising in mathematics. At diploma level, Dan specialised in languages. He has very little experience in teaching mathematics; since starting at his current school six years ago, he has taught languages to Grades 5, 6, and 7. This was his first year of teaching Grade 4s at his current school, having done so at a previous school, a private, urban school where he taught mathematics to Grades 5 and 6. Prior to that, he taught in a rural school for three years, covering all subjects, including mathematics.

5.2.2 Dan’s teaching style

From classroom observation, Dan used the traditional style of teaching, where learners are passive recipients of knowledge. Dan used demonstrations in all his lessons. All concrete or visual aids which he used were done so as demonstrations by him, with learners as observers, although he did get learners to draw a diagram to represent a fraction. Whole group discussion dominated in his lessons, with the teacher leading discussions. Most of the time learners responded together to questions or they would finish the sentence together with the teacher. Learners participated actively only when they were called upon to write mathematical sentences or symbols on the board.

Classroom interaction was either whole group–teacher or learner–teacher interaction observed during an assessment. Dan did not give learners opportunities to explain their answers.

5.2.3 Representations used by Dan

5.2.3.1 Concrete representations

Dan used pens of different colours as counters during the second observation period to demonstrate the subtraction of fractions. There was no use of concrete manipulatives during the first observation period. Figure 5.10 shows the red and blue pens used by Dan as concrete representations:

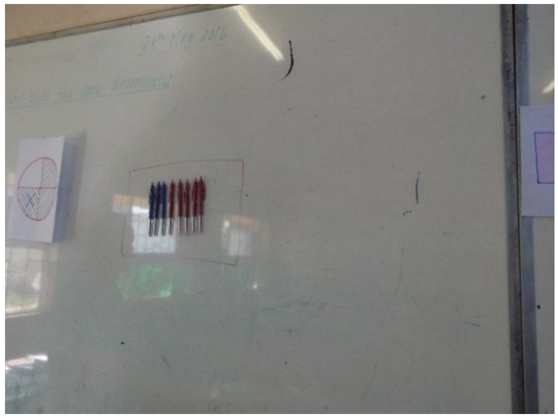

Figure 5.10: Red and blue pens used by Dan as concrete manipulatives.

5.2.3.2 Diagrams or pictures

During the first class observation period, Dan used diagrams of both circles and rectangular areas, which he brought to class already drawn on sheets of papers. In addition, Dan also used a set of objects to demonstrate the part-whole definition of a fraction and a fraction chart to show the addition of fractions with a common denominator. In the second observation, he used a number line in addition to diagrams of geometric shapes to illustrate subtraction of fractions.

5.2.3.3 Experience-based metaphors

Dan used real-life problem to explain the part-whole definition of a fraction and further used the same problem to demonstrate the addition of fractions with a common denominator. The following is a problem used by Dan:

> *Mr Thwala has five sweets. He gives three sweets to Thabo and two sweets to Gugu. What fraction of the sweets did Gugu get?*

In the second observation, he used the sharing of a bar of chocolate to emphasise that the denominator depends on the number of parts into which the whole has been divided:

This is a whole. Like a complete bar of chocolate, it has not been shared by anyone, but for some reason if the whole can be divided for two people, what will each person get?

5.2.3.4 Symbolic representations

Dan did not seem to mind how learners verbalised the symbols, paying little attention to correct terminology. It was only when he was about to use the fraction chart that he wrote the fraction names, from halves to tenths, on the board. Learners were then instructed to write the numerical symbols next to each name.

5.2.3.5 Spoken language

In the interviews, Simon said that he used both English and SiSwati during instruction, which was confirmed during observations. The reason given in the follow-up interview was that if the only language of communication in the classroom is English, some learners who cannot speak English might well be scared to ask questions:

> *I use both English and SiSwati because if I use English only, the language becomes a barrier. When I see they do not understand this word, I give an example in SiSwati.*

5.2.4 Dan's motivation for using multiple representations

Dan believes that using more than one representation captures the learners' interest and caters for the different learning styles:

> *You know we learn every day as you try this and that, you must have noticed that at the beginning, there are some students that I kept calling on at the back but today as we kept trying this and that they were able to concentrate and grasp. You learn that one should not stick to one teaching style; try to diversify your teaching. You find that one method will win even that child whom most teachers had written off* (see Appendix H.2).

5.2.5 How Dan used various representations in his lessons

5.2.5.1 Concrete representations

Dan used concrete representations only once, in the second lesson when he was demonstrating the subtraction of fractions with a common denominator. He stuck eight pens onto the whiteboard, five red and three blue, and described it as a set, shown in Figure 5.11.

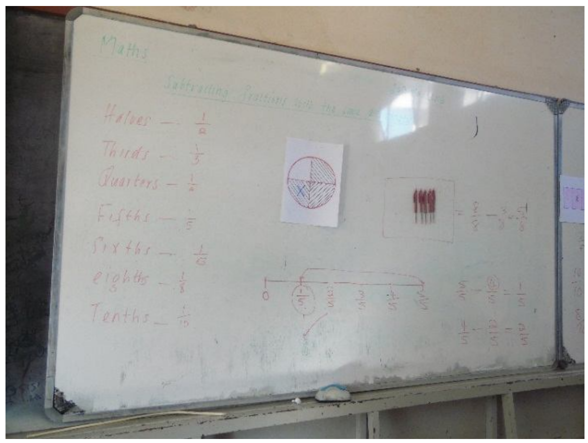

Figure 5.11: Pens, area model and number line used by Dan to demonstrate subtraction.

The following is a question and answer dialogue between Simon and his learners:

T: *If I decide to remove these (removes all blue pens), what do I have?*

L: *Five.*

T: *What fraction is that?*

L: *Five over eight*

T: *I am remaining with 5/8. What happened for me to remain with 5/8. What did I take out?*

L: *Learner writes* $\frac{8}{8} - \frac{3}{8} = \frac{5}{8}$ (Appendix E.2).

5.2.5.2 Diagrams or pictures

Dan translated a real-life problem to a diagram in which the small circles represented the sweets, as in Figure 5.12:

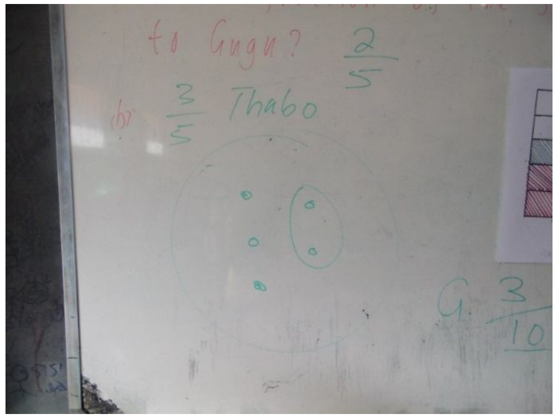

Figure 5.12: Dan’s translation of the word problem.

Dan explained the word problem as follows:

Mr Thwala has five sweets. He gives three sweets to Thabo and two sweets to Gugu. What fraction of the set of sweets has been given to Gugu?

He used the diagram to explain that the set of five sweet represented a whole and that once a person starts sharing them, fractions are the result. Then he asked learners, “What fraction of the set of sweets has been given to Gugu?” The learners responded, “Two over five.” Simon drew an area model to show the fraction of sweets received by Gugu, shown in Figure 5.13. This time, he used a rectangle, which he divided into five equal parts, shading the two-fifths received by Gugu:

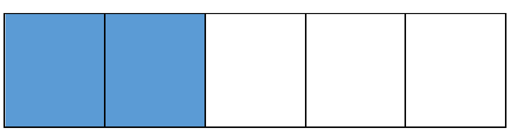

Figure 5.13: Area model showing two fifths.

Dan used the diagram to explain that the unshaded area represented the fraction of sweets received by Thabo and the shaded part, those received by Gugu. Then he produced an area model of a circle, which he stuck onto the board to show the learners another fraction, in this case, three-quarters; Figure 5.14:

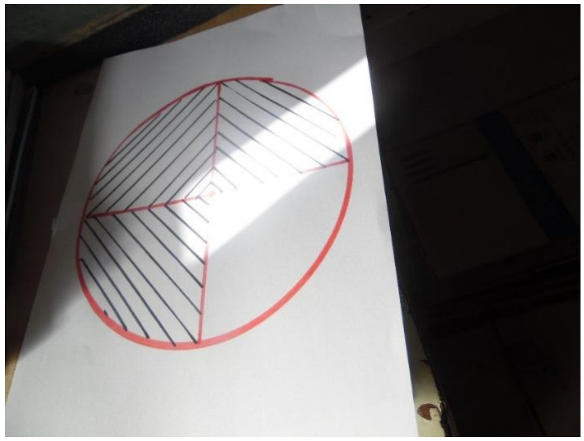

Figure 5.14: Dan’s area model of three-quarters

He emphasised while using diagrams that the number of parts into which the whole is divided represents the denominator, and that the shaded area represented the numerator. He eplained this while writing the fraction symbol on the board. He then used area models with more than one colour shaded to show addition of fractions with a common denominator (see Figure 5.15).

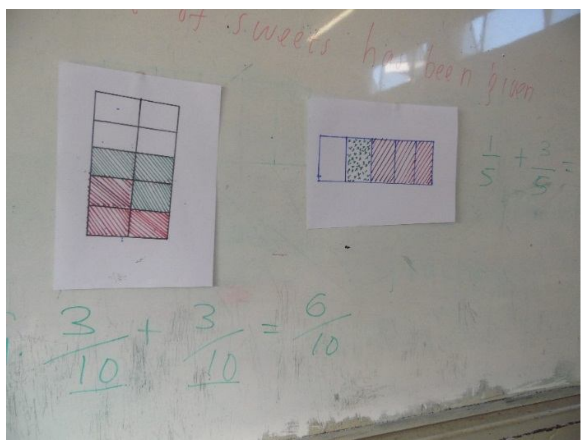

Figure 5.15: Dan’s area model used for demonstrating addition of fractions with a common denominator.

After instructing learners to identify the fractions represented by each shaded region and writing those fractions on the board, he then asked them to identify the fraction represented by all the shaded area. Then he wrote $\frac{3}{10} + \frac{3}{10} = \frac{6}{10}$. He kept emphasising that when you add two fractions with a common denominator you add only the numerators:

We do not add the denominators; that is the rule because look here (referring to diagram*). If we were to add 10 and 10, it will give you six over 20. Is this true*?

Then he stuck another diagram of an area model onto the board, divided into five equal parts; one part shaded blue and three red parts red, as shown in the diagram above. Dan then used the diagram to create a mathematical sentence:

T: *How many parts do we have here?*

L: *Five parts.*

T: *Five parts, which means every fraction we will talk about concerning this diagram, will be over what?*

L: *Five.*

T: *Can you identify two fractions from this diagram?* (Appendix E.2).

He wrote the two fractions next to the diagram; $\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$. He then asked learners for the sum of the two fractions. Three learners did not respond, but the fourth learner gave the correct answer. Dan used the diagram to explain to the other learners how the fourth learner arrived at the correct answer. Satisfied that every learner understood, he stuck a fraction chart on the board, using it to clarify the concept of a whole (see Figure 5.16).

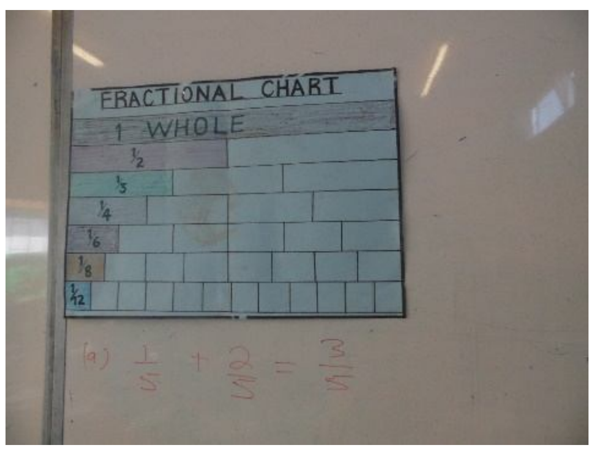

Figure 5.16: Dan's fraction chart.

He continued to explain:

> *We can get the concept of whole from whole. This is a whole, like a complete bar of chocolate. It has not been shared by anyone, but for some reason, if the whole can be divided for two people, what will each person get?*

The learners responded: *One over two.*

> *One over two. We call these halves. If now the same bar is divided among three people, we will now have three parts. What do we call these?*

One learner responded: *Thirds.*

> *If the same whole (one bar of chocolate) can be shared among four people. What does it create?*

Learners: *Four parts.*

> *Remember it is equally shared (*emphasising that all pieces are equal*). If the whole is shared amongst six people equally, these are sixths. The same applies if we have eight people; we get eighths. We can divide based on the number of people. There are so many fractions depending on how many parts you want to divide the whole. They all come from a whole.*

Then he used the chart to demonstrate the addition of fractions: "*Now suppose we were adding quarters, one-quarter plus one-quarter*," (pointing at chart). The learners responded, "Two-quarters". Dan responded affirmatively, linking fraction chart and area model, "Two-quarters, based on how parts many are shaded or unshaded."

In the second observation, Dan used both circular and rectangular area models to demonstrate subtraction of fractions. In addition to area models, a number line was used, shown in the figure below. This part of the lesson was on the addition of fractions with a common denominator. For demonstration, Dan stuck a diagram of a circle onto the board, divided into four equal parts, with three parts shaded:

> Teacher: *How many parts are shaded?*

Learners: *Three*

T: *If I remove one of the shaded parts, how many parts would remain?* (Putting a cross in one shaded area)

L: *Two over four.*

T: *Can someone come forward to write a mathematical sentence for what we just did?*

The pupil wrote $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

Then he stuck a similar area model onto the board, this time crossing out two parts of the shaded region. Again learners were instructed to write a mathematical sentence corresponding to the procedure. One learner attempted to write the mathematical sentence but failed. Dan used the area model to guide the learner until he wrote the correct sentence:

$$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$$

Dan then decided to show addition on a number line. He drew a number line on the board showing fifths, shown in figure 5.11. He demonstrated how to find the answer to this incomplete number sentence:

$$\frac{5}{5} - \frac{2}{5} =$$

5.2.5.3 <u>Experience-based metaphors</u>

Dan used real-life problem to illustrate the part-whole relationship of a fraction, and further used the same problem to demonstrate the addition of fractions with a common denominator. The following problem was used by Dan:

Mr Thwala has five sweets. He gives three sweets to Thabo and two sweets to Gugu. What fraction of the sweets did Gugu get?

5.2.5.4 <u>Symbolic representation</u>

Dan did not seem to mind how learners verbalised the symbols, paying little attention to correct terminology. He did not discourage learners from verbalising a fraction such as $\frac{3}{5}$ as "three over five" instead of "three parts out of four parts" or "three fifths". Dan used correct fraction names just before he introduced the fraction chart, which was intended to show addition of fractions. Dan wrote the fraction names on the board and instructed learners to write the numerical symbols next to the names. He wrote the fraction names from halves to tenths, shown in Figure 5.11. After he had used area models to demonstrate the addition of fractions with the same denominator, he introduced the fractions chart, which he used to show the addition of fractions.

The only thing Dan cautioned his learners against was using a slanted line (/) to separate numerator from denominator, telling them instead to use the horizontal line (—).

5.2.5.5 Spoken language

Dan used both formal and informal language, explaining the link between the two:

When we are subtracting, we are decreasing. The word decrease is confusing others. Let us use a simple term "taking away". When subtracting you are taking away. If for example you have twenty sweets and I come and take ten Are you going to have more or less sweets? (Appendix E. 2).

5.2.6 Interactions within Dan's classroom

Interactions observed were mostly between the teacher and the whole class. As shown in Table 5.2 below, learners either worked together in whole group discussions led by the teacher or worked individually during evaluations.

TABLE 5.2: Showing interactions within Dan's classroom.

Type	Not at all	Sometimes	Frequently	All the time
Whole group				✓
Small group	✓			
Pairs	✓			
Individual			✓	

5.2.7 Dan's reasons for using multiple representations

Dan stated that he used multiple representations because he felt that if one used multiple ways of finding solutions it captured the interest of the learners, even those who easily get bored:

> *Yes, you know we learn every day. As you try this and that, you must have noticed that at the beginning, there are some students that I kept calling on at the back but today as we kept trying this and that they were able to concentrate and grasp. You learn that one should not stick to one teaching style, try to diversify your teaching. You find that one method will win even that child whom most teachers had written off* (see Appendix H. 2).

5.2.7.1 Computer technology

Dan’s school was the only school with computers, and where learners were taught computer technology.

Asked why he did not use computer technology to teach mathematics, his response was that the school had not considered that idea, but revealed that he had used computers for teaching at his previous school:

That is an idea we have never explored. But when I was teaching in a private school, we used computers for teaching.

5.2.7.2 Diagrams and pictures

Dan used both circle and rectangular area models. Asked why he used those shapes, he said that the learners were familiar with those shapes and it was easy to divide them: "… because they are familiar with those shapes”.

5.2.7.3 Symbols

When the researcher asked Dan why he emphasised that learners should use a horizontal line instead of a slanted line when separating numerator from denominator; he said, though it is not wrong to use a slanted line, however learners at Grade 4 level were encouraged to use a horizontal line.

5.2.7.4 Spoken language

Dan used only English as a medium of instruction. He attributed this to the culture of the school and his experience as a teacher in a private school, an English medium school with diverse nationalities:

When I came here, I found that it is a norm to speak English. Coming here, I discovered that learners understand English easily. Coming from a private school, an English medium school, it is always English.

Then he mentioned that when he had taught in the rural school, he had used both SiSwati and English because English was a challenge to the learners:

In the rural areas I used both languages because I could see that English was a challenge, so I had to come down to their level.

5.3 PARTICIPANT 3: PAM

5.3.1 Teacher profile

Pam, a female, has 19 years teaching experience in mathematics. She holds a diploma with majors in mathematics and science. Pam teaches mathematics and science to Grades 5, 6 and 7. She had been teaching in this school for eight years. Pam taught mathematics and science for eleven years in a rural school before transferring to the current semi-urban school, where she continued to teach these subjects. She is also a trained marker for external science examinations for Grade 7. The lessons discussed below were presented to a Grade 6 class.

5.3.2 Pam's teaching style

Pam is a very active teacher, and one can tell that she is passionate about teaching mathematics. She believes learners should be actively involved in the learning process. Even though all discussions were whole group discussions, the learners were given opportunities to volunteer to do certain special tasks. For instance, even though there were very few apples to manipulate, she made a point of getting learners to handle the apples, and not herself. By using a fraction chart, she also gave learners a chance to prove their understanding by having them demonstrate certain ideas to the rest of the class.

When learners gave incorrect responses, Pam used those responses to clarify a point and guide the other students. For instance, when a learner responded “three out of ten” instead of “seven out of ten” to a certain question, she asked the learner to demonstrate how he arrived at that answer and further showed him how to correct his procedure.

She competently built on learners’ prior knowledge. Pam showed the importance of prior knowledge when adding mixed numbers in which the fractions had different denominators. After converting mixed numbers to top-heavy fractions, the new fractions had different denominators; she referred learners to the previous lesson in which they had added such fractions with different denominators.

5.3.3 Kinds of representations used by Pam

5.3.3.1 Concrete representations

Pam did not use concrete materials in her first lesson on adding fractions with different denominators, although she did make use of apples as manipulatives when adding mixed numbers. Figure 5.17, below shows a learner using the apples as manipulatives:

Figure 5.17: Pam demonstrating addition of mixed numbers with two learners as volunteers.

5.3.3.2 Diagrams or pictures

Pam did not use any diagrams during the first observed lesson when teaching the addition of fractions with different denominators. She used a fraction chart and fraction strips during the second observation, in a lesson on the addition of mixed numbers.

5.3.3.3 Experience-based metaphors

Pam used a game to help learners accumulate multiples of numbers. The title of the game was, "There is a fire on the mountain." Learners formed a circle and ran around, with the teacher leading the song. It goes:

Teacher: *There is a fire on the mountain.*

Learners: *Run, run, run.*

This was repeated three times, and then an accumulation of multiples began.

Teacher: *In threes* (Said when teaching multiples of three.) The learners responded by standing in groups of three. Those learners who did not form a group of three stood aside. The teacher then instructed each group of three learners to count in threes, and so on, with several different numbers called out by the teacher.

5.3.3.4 Symbolic representations

Since Pam was teaching a higher grade, symbolic representation dominated her lessons.

5.3.3.5 Spoken language

In Pam's classroom, the main language of instruction was English, although she used a little SiSwati every now and then.

5.3.4 Pam's motivation for using multiple representations

Pam used multiple representations in teaching fractions because it benefited the learners by improving their understanding:

They help a lot; they help the learner's understanding. However, you have to prepare a lot, sometimes use your own money to buy apples. You have to think what visual aids you need, and how can I use them to benefit the learners?

5.3.5 How Pam used the various representations

5.3.5.1 Concrete representations

Pam used apples which some of the learners manipulated as she demonstrated the addition of mixed numbers (see Figure 5.18):

Figure 5.18: Pam using apples to demonstrate addition of mixed numbers.

Pam started by writing a symbolic problem on the board; $2\frac{1}{4} + 1\frac{2}{4}$. Then she asked for two volunteers to come forward. With the help of the volunteers, she cut one apple into four equal parts, followed by a distribution of the apples according to the problem, while pupils watched with interest. She then asked for the third volunteer to add the two mixed numbers using the apples. The third volunteer first took the whole apples, then the fractions of apples. The teacher then wrote the answer on the board. Pam wrote another problem and went through the same procedure.

5.3.5.2 Diagrams and pictures

Pam did not use diagrams during the first observation period but did use the fraction chart during the second observation. She started with a symbolic problem, adding two fractions with different denominators, first finding the lowest common denominator. Then she used a fraction chart (see Figure 5.19):

Figure 5.19: Pam using a fraction chart to demonstrate addition of mixed numbers.

Pam continued to explain as follows:

T: *You can also add fractions with different denominators by using a fraction chart. What did we get when we added 1/5 and ½?*

L: Seven out of ten.

T: *Now, how do we use the fractional chart? We take a strip that is one-fifth and another one that is one-half* (sticks both strips onto the fraction chart). *We said our LCM is 10; we will look at the fraction with denominator 10. Then we take the strips and stick them adjacent to each other. Then we will compare the answer we get with the first answer, 7/10.* (Sticks strips on chart). *What is the answer*?

L: *Seven out of ten.*

L: *Three out of ten.*

T: *How did you get the answer seven out of 10?*

Learner demonstrates on the chart.

T: *How did you get the three out of ten?*

Another learner demonstrates, counting the parts not covered by the strip.

T: *The correct answer is seven out of ten. You count the part covered by the strip. Where it end, is your answer. The part not covered by the strip is not your answer.* (Appendix E.)

Pam also asked another volunteer to demonstrate the addition of one third and one half using the fraction chart.

5.3.5.3 <u>Experience-based metaphors</u>

Figure 5.20 below shows learners playing a game for generating multiples of three:

Figure 5.20: Learners accumulating multiples of three.

Pam's strategy of using a game to generate multiples seemed to work, since learners did not struggle to produce multiples. Whenever working on examples on the board requiring learners to produce multiples, she would remind learners about the game.

5.3.5.4 <u>Symbolic representations</u>

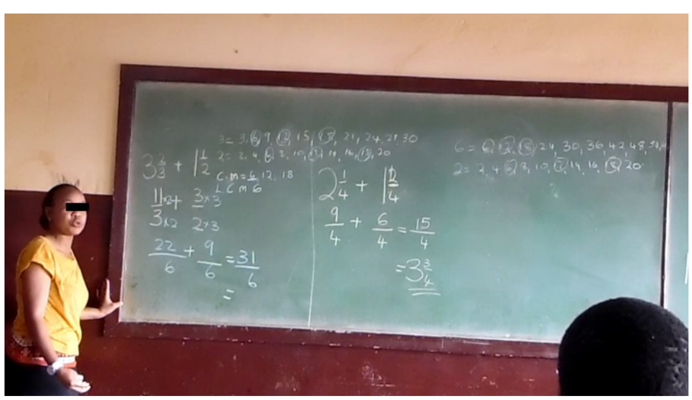

Figure 5.21: Pam demonstrating addition of mixed numbers.

Although Pam used representations other than symbols, symbolic representations dominated her lessons, as shown in Figure 5.21. While using symbolic representation, she would verbally connect the symbols to other representations. For instance, when finding the lowest common denominator during the second observation, she would remind learners of the game they had played for generating multiples:

We start by accumulating multiples of the denominators. Remember, fire on the mountain, fire on the mountain? You accumulated multiples of the denominator.

During the second observation, Pam wrote an addition problem involving mixed numbers, then used manipulatives to estimate an answer. The estimation turned out to be the same as the correct solution arrived at through symbolic manipulation. Pam also cautioned learners against writing mixed numbers where the fraction and the whole number were different sizes:

Be careful how you write a mixed number. The whole number must be the same size as the fraction (see Appendix E).

5.3.5.5 Spoken language

Pam used English throughout the lesson during classroom observation and her learners were comfortable with that, responding well in English. Pam used language very well, reminding learners of concepts learned in earlier grades or in previous lessons, connecting them with the current lesson (the first observation period):

> *Then we will come back and add common fractions with different denominators. In Grade 4, you added fractions with the same denominator. In Grade 4, it was as good as adding the numerators, denominator remains the same.*

During the second observation, she said:

This takes us back where we started, fractions with different denominators. We start by accumulating multiples of the denominators. Remember, fire on the mountain, fire on the mountain? You accumulate multiples of the denominators (see Appendix E).

5.3.6 Interactions within Pam's classroom

From non-participant observations, it was evident that Pam's most preferred teaching strategy was using whole group discussions and demonstrations led by the teacher, as shown in Table 5.3, below. Learners worked individually during an assessment. Although teacher demonstration dominated Pam's lessons, students actively participated in demonstrations, and were given opportunities to explain their answers.

TABLE 5.3: Showing interactions within Pam's classroom.

Type	Not at all	Sometimes	Frequently	All the time
Whole group				✓
Small group	✓			
Pairs	✓			
Individual			✓	

5.3.7 Pam's reasons for using multiple representations

5.3.7.1 Concrete representations

Pam stated that using concrete objects that learners could manipulate was very interesting to the learners and helped them to draw diagrams:

Using the concrete first helps them to draw the diagrams easily because they are able to relate the diagram to the concrete. Each time the pupil draws a diagram he will remember the concrete objects he has manipulated.

She also stated that another reason for using concrete manipulatives was that it helped learners to represent fractions on a number line:

> *Because when you are representing a number sentence on fractions on a number line, from the concrete, it will help the child to divide the number line accordingly. For example, when multiplying three by a fourth, when using apples each apple will be divided into four equal parts* (see Appendix E).

5.3.7.2 Diagrams or pictures

Pam said that she used diagrams or pictures in her lessons, and stated that the introduction of diagrams without the prior use of concrete manipulatives seemed confusing to the learners:

I use diagrams and pictures but sometimes they are a bit confusing. But when you bring the concrete, they enjoy the manipulation, they understand more clearly. (See Appendix H.)

5.3.7.3 Experience-based metaphors

The game Pam used to accumulate multiples of numbers was a reminder of how to generate multiples. Pam did this because she had observed that learners tend to confuse factors and multiples.

5.3.7.4 Symbolic representations

Pam's reason for using mainly symbols in her lessons was that generally, at higher primary level, there is less use of concrete manipulation. According to Pam, using concrete manipulation is common from Grades 1 to 4.

5.3.7.5 Spoken language

Pam said she used English in her lessons because the external examination papers were written in English, and she wanted the learners to get used to English:

In upper grades, we have to use English because the examination paper is written in English.

Another reason given by Pam was that when doing problem solving, learners fail to interpret problems if they are not accustomed to English.

5.4 SUMMARY

From the data presented, it is evident that teachers use multiple representations when teaching fractions at primary schools. What was also evident was that teachers who did not specialise in mathematics during pre-service training had some challenges in using multiple representations in teaching. The idea of using virtual manipulatives like computer technology is still a strange idea since most schools have no computers, except a few in urban areas.

CHAPTER 6

DISCUSSION OF FINDINGS AND RECOMMENDATIONS

The previous chapter presented data derived from both interviews and classroom observations. Rich descriptions of each participant's profile were presented, together with their teaching style and a review of the various representations they used in teaching operations with factions. The study proposed to answer the following questions:

1. What kinds of representations do teachers use in teaching fractions?
2. How do primary teachers use the representations in classroom instruction?
3. What are the teachers' reasons for using or not using particular representations in teaching fractions?

This chapter presents a discussion of the findings and recommendations of how multiple representations can be used more effectively to teach fractions in the intermediate phase.

6.1 REPRESENTATIONS USED BY PARTICIPANTS

All participants in this study used multiple representations in their lessons on fractions. Teachers selected multiple representations to facilitate instruction on fractions. Participants used representations supplied by the National Curriculum Centre for teaching fractions, while Simon and Pam, as the most experienced teachers, also relied on experience. The findings confirmed observations by Bal (2014), whose study revealed that teachers' experience determined the choice of representation in problem-solving.

The emphasis of one representation over another depended on the preferences of the teacher. The representations observed during classroom observations were manipulatives (counters, symmetric models, paper strips), diagrams (area models, fraction chart, sets, number line), metaphors (a game, real-life problems), symbols (verbal and written) and spoken language. Spoken language and symbolic representation appeared to be most favoured by all participants Although Pam used all representations, verbal and written symbols dominated her lessons, as she taught Grade 6. According to English and Halford (1995), the use of manipulatives tends to decrease as the grade levels increase.

The area model appeared to be the most dominant visual aid used by participants who taught Grade 4 when teaching addition and subtraction of fractions. Dan used both the circle and the rectangular area, while Simon used only rectangular shapes. Simon preferred to use rectangular area models instead of circle models, because he felt there was a lot of mathematics involved in drawing a circle, since one would need a protractor and a pair of compasses. Simon mentioned that partitioning a circle into equal parts would not be easy, as also observed by (Wu, 2014), since one would have to calculate angles at the centre. By avoiding the circle model, Simon displayed his knowledge of some of the challenges associated with the circle model, which were also noted by Ball (1990a). Yearly and Bruce (2014) also argue that too strong a reliance on the circle model could lower learners' abilities to represent fractions with odd denominators, such as two-thirds.

Some studies, however, show that using the circle model is the most effective way of helping learners create mental images of fractions, and for representing the addition and subtraction of fractions (Cramer et al., 2008). Participants in this study favoured the rectangle model. The pizza, normally used as a tool for describing fractions in countries such as the United States of America, is a new concept for older teachers and for most of their learners. Instead of a pizza, participants favoured the loaf of bread, which learners are familiar with and which is rectangular. The chocolate bar was also used, as were Pam’s game and word problems.

The use of spoken language varied, depending on the location of the school. Learners in rural and semi-urban schools do not usually converse in English with one another, even when in school. Participants from both semi-urban and rural schools used both English and SiSwati for instruction, but Dan (in the urban school) used only English, with learners expressing themselves equally fluently in English. This in is agreement with Ball and Forzani (2010), that language used in instruction is situated in a cultural context. Nevertheless, even Dan admitted to using both English and SiSwati when he used to teach in a rural school.

6.2 HOW PARTICIPANTS USED REPRESENTATIONS

The next section describes how participants used the different representations in class.

6.2.1 Using concrete representation

All participants in this study used concrete manipulation, but they differed in the way they used them. In Simon's lessons, the learners handled the materials themselves as they actively engaged in group activities. Dan, on the other hand, used the manipulatives as a demonstration tool in teacher-led whole class discussion. Pam used concrete representations as demonstration tools during teacher-led whole group discussions, but her learners were actively involved in constructing knowledge.

The way Simon used concrete manipulatives to model the fraction concept, moving from the concrete to the abstract, was backed up by his beliefs about mathematics teaching and learning. Simon stated in follow-up interviews that learning should always move from the concrete, using manipulative or metaphors, to the semi-concrete, using diagrams, and finally to the abstract, using symbols. Simon moved around the classroom assisting and guiding learners as they actively manipulated sets to form fractions of sets. All participants used concrete manipulative as scaffolds to help them progress from a concrete to an abstract understanding of the fraction concept, which is consistent with findings from Naidoo (2011).

Dan used concrete materials as a demonstration tool, depriving learners of the chance to construct knowledge through manipulation. The reason given in follow-up interviews was that the lesson did not warrant grouping of students and that if he felt there was a need he would have grouped them. According to Dan, the learning process progresses through three stages: the teacher talks about the concept, demonstrates concepts using a visual aid, and finally asks learners questions to assess their understanding. This is the traditional approach to teaching and learning.

Pam started with a symbolic problem on the addition of mixed numbers and then demonstrated multiple ways of finding a solution to the problem, one of which was manipulatives. The solutions found using the various methods were, of course, all the same. By using concrete objects, Pam guided learners towards finding a solution to a symbolic problem that could otherwise have seemed very abstract and complex. Pam did not use the concrete manipulatives merely to confirm a solution to the problem, but used them in such a way that they helped learners to understand the steps in the procedure and make sense of the whole procedure. This is in line with Pape and Tchoshanov (2001) observation that the use of concrete representation improved learner's understanding of abstract concepts.

6.2.2 Using diagrams or pictorial representations

Simon introduced diagrams of sets only after learners had used sets of counters (stones) to find fractions of sets. Working in groups of four, learners translated concrete models to diagrammatic representations. By doing this, Simon demonstrated his understanding that learners must be actively involved in the learning process, and that learner-to-learner interaction, according to Vygotsky (1978), plays an important role. Simon and Dan both used area models, but differed in the way they used them. Simon introduced area models after learners had manipulated concrete models. This showed his understanding of the stages of learning, according to Bruner (1966), that is, that it occurs from the concrete to the visual and finally to the abstract. On the other hand, Dan introduced concrete models after area models. There was no specific reason given for this sequence except that by using different visual aids, he was helping to keep learners interested.

Simon used rectangular area models, and made sure that connections were drawn between area models and symbols. According to Galant (2013), making connections between different representations shows a deep understanding of mathematical ideas. Simon gave learners opportunities to represent concrete models using diagrams, which neither of the other participants did. Simon did not only use them, he made sure that learners understood how the various representations related to one other, giving them opportunities to translate diagrammatic representations of fraction addition to symbols and vice versa, which, according to Ainsworth (2006) and Lesh et al. (1987) is critical for understanding mathematical ideas.

Dan used a diagram of a set, rectangular and circular area models, a fraction chart and a number line. All diagrams were used in the traditional way, where the teacher talks, demonstrates and writes on the board and the learners watch. In order to explain the addition of fractions, he translated the word problems into a diagram of a set. No time was spent focusing on the diagram, so that learners missed out on an opportunity to reflect on how it related to the word problem. Area models (rectangle and circle) immediately followed the set diagram. In my observation, Dan used the various diagrammatic representations as scaffolds to build an understanding of fractions with a common denominator. By displaying area models with differently-coloured shading on the board and instructing learners to write the accompanying symbolic representations next to them, Dan afforded learners opportunities to make connections between the area models and the symbols. Translation ability is key to understanding mathematical concepts (Ainsworth, 2006; Lesh et al., 1987). Dan used one area model with two different shadings to show the addition of two fractions. This eliminated the confusion that might have been created by two diagrams, where learners might have ended up doubling the denominator when adding fractions with a common denominator. However, the sequencing of the lesson was questionable; in the middle of the lesson, Dan explained the proper naming of fractions, using the fraction chart. In this researcher’s opinion, the fraction chart should have been used to teach learners the proper names of fractions at the beginning of the lesson, and then reinforced later, when showing how to add fractions with a common denominator.

Pam used only the fraction chart together with fraction strips. First she demonstrated the addition of fractions with different denominators using the fraction chart and fraction strips. Then she gave learners opportunities to demonstrate their understanding by allowing them to be become actively involved in working out solutions, using the fraction chart and strips, guided by the teacher. Comparisons were made between the two solutions found; one using the concrete manipulation of the fraction strips and one using the fraction chart.

6.2.3 Using metaphors

Metaphors are a powerful tool in developing an understanding of mathematical ideas that are difficult to represent using concrete objects (Presmeg, 2013). All participants used experience-based metaphors in their lessons: Simon used what he termed "human activity", a loaf of bread and a taxi; Dan used word problems and a bar of chocolate, and Pam used a game. All teachers used metaphors to aid learners' understanding of fractions by relating them to everyday experiences.

During the first observation period, Simon used learners themselves as members of a set to develop their understanding of a fraction being part of a set, which seemed to work, from the way learners responded to questions. The activity helped to make the definition of a fraction as part of a set more concrete to the learners. In addition, the activity generated much excitement.

During the second observation, the metaphor Simon used was quite confusing at the beginning. He used a desk, which he said in the follow-up interview represented a whole, with learners acting as parts of the whole by siting at the desk. The referent unit was not clear in this particular case, and could have been challenged by a different group of students. For instance, the number of students who could fill up a desk might vary, depending on the size of the learners. The same applies to the other example of a taxi, which he said could take a maximum of fifteen people. With a different group of learners, less afraid of questioning the teacher, the examples could have ended up being the target, and not the source, as intended. In this regard, Presmeg (2013) observed that if the proposed mathematical connections are hard to distinguish, then the metaphor becomes the target of student's learning.

Dan used a simple word problem based on real-life experience as an introduction to the addition of fractions, aimed at making the definition of a fraction as part of a set concrete to the learners. The teacher translated the word problem to a diagram of a set and finally to symbols. The use of the word problem to represent the concrete was in line with Bruner's (1966) description of the stages of learning, progressing from enactive, to iconic and ultimately to symbolic. In the same lesson, Dan mentioned a bar of chocolate to emphasise the part-whole definition of a fraction. Since urban learners are familiar with chocolate bars, they responded positively to questions asked in relation to it.

The game used by Pam to generate multiples proved to be very useful in helping learners remember how to generate multiples when finding the lowest common multiple when adding fractions with different denominators. The game was played during the first observation, but was referred to several times in the second lesson, enabling learners to establish firm connections between the metaphor and the symbolic representation of the fractions.

6.2.4 Using symbolic representation

In all classroom observations, participants used symbolic manipulation side-by-side with other representations. Simon paid close attention to both verbal and written symbols. drawing learners' attention to correct verbal representations of fraction symbols. For example, Simon stated that the symbol $\frac{1}{3}$ represents one-third or "one out of three", not "one over three". Dan, on the other hand, seemed to disregard the verbal representation of fractions by learners. For instance, the fraction $\frac{3}{5}$ was assigned the meaning "three over five" by both teacher and learners, instead of "three parts out of five parts" or "three-fifths". According to English and Halford (1995), the naming of fractions using the part-whole definition should follow a sequence; first emphasise that the whole divisions are equal, then point out the number of parts into which the whole is partitioned, giving the denominator's name. Finally, the number of shaded parts should be identified, resulting in the full fraction name. In their study, Yearley and Bruce (2014) found that such inaccurate naming of fractions by learners confounded learners' construction of meaning when building an understanding of fractions as numbers. Simon, on the other hand, demonstrated his awareness of students' misconceptions in naming fractions, discouraging learners from using "over" to refer to the division line, and insisting on accurate verbal representations.

Symbolic manipulation dominated Pam's classroom instruction. Having used concrete manipulation and diagrams to help learners create mental images for the addition of fractions, Pam introduced algorithms for adding fractions with different denominators. This was in accordance with the findings by Bal (2014) that symbolic manipulations tend to dominate classroom instruction.

6.2.5 Using spoken language

English was the primary language of instruction in all classroom lessons observed, although some participants used SiSwati (learners' first language) to clarify certain points or to give examples. Two of the participants were not as comfortable with English as Dan, who specialised in languages at diploma level. The use of learners' home language was more evident in the rural schools where learners tended to struggle to express themselves in English. One participant stated that compelling learners to use English only resulted in learners withdrawing and avoiding asking questions when they did not understand.

Two of the participants omitted to give learners opportunities to discuss amongst themselves while working on problems, while Simon organised learners into groups to work on problems together. Even in Simon's class, the learners would clam up as soon as he approached, depriving him of an opportunity to listen to their reasoning.

Both Simon and Pam used mathematical language to create scaffolds for learners' understanding of the addition of fractions and various mathematical terms. For instance, Simon took the time to define fractions carefully, and throughout the lesson emphasised the proper verbalisation of fractions. Pam, also emphasised the correct verbal representation when describing terms like factor, highest common factor and lowest common denominator. Emphasising the correct verbal mathematical language, according to Hill and Charalambous (2012), is typical of teachers with a high mathematical knowledge for teaching.

All participants would pause and study learners' facial expressions after defining a mathematical term, and ask learners if they understood the explanation. For instance, Pam gave a verbal description of the procedure for converting a mixed number to a top-heavy fraction and wrote it on the chalkboard. Simon described the fraction symbol regarding numerator and denominator and wrote that on the chalkboard. Dan's description of addition and subtraction using both formal and informal language accompanied his written explanations. Spoken language emerged as the dominant representations in all classrooms (Bal, 2014).

6.3 INTERACTIONS WITHIN CLASSROOMS

Interactions between teachers and learners help learners to develop mathematics language. Interactions observed within the different classes were mostly between teachers and learners, occurring mainly during whole class discussions. The teacher would describe a concept with the aid of visual aids like manipulatives and diagrams, then ask questions based on the explanations. The learners would respond either in unison or individually.

Only one participant encouraged learner-to-learner interactions, having learners work in groups of four to translate a symbolic mathematical sentence into a diagrammatic representation using area models, and vice versa. Simon moved around the class encouraging discourse among members of each group. The researcher observed that learners in most groups engaged in vibrant discussions until the teacher approached the group, whereupon they would fall silent. Discussions among learners help the teacher to identify learners’ conceptions and misconceptions (Ball, 1990a). Although these learners ceased speaking when their teacher approached Simon was nevertheless able to follow their thinking to some extant from observing their manipulation of the area models (Naiser et al., 2003).

6.4 MOTIVATION FOR USING PARTICULAR REPRESENTATIONS

This section is organised according to the themes that arose in the follow-up interviews.

6.4.1 Interesting and exciting

All participants alluded to the fact that multiple representations play a significant role in the teaching and learning of fractions at primary school level. Teachers felt that the use of multiple representations ensured that all learners' learning styles were accommodated, resulting in all learners being attentive and showing an interest in the lesson (Moyer, 2001). In follow-up interviews, participants cited lack of resources as an obstacle when using multiple representations in teaching mathematics; this is consistent with findings from research by Nichols et al. (2015). Some participants asserted that teachers used their personal resources for buying learning aids, since the schools lacked commercially-made learning aids.

6.4.2 For conceptual understanding

Simon stated that he used both concrete and diagrammatic representations in all his lessons. Although practical activities tend to take a lot of time, Simon felt he would rather not finish the syllabus than have learners only partially grasp mathematical concepts, and thus activities formed an integral part of his approach. Simon's assertion is in contrast with Molebale's (2005) finding that teachers do not use manipulatives because of time constraints and the rush to finish the syllabus. Simon felt that teaching Grade 4 learners without concrete manipulatives made mathematics concepts too abstract for learners to understand. Pam, on the other hand, stated that she rarely used manipulatives in higher grades, but used them a lot when teaching the foundation phase.

Simon avoided using circular area models when teaching the addition of fractions although it is the most commonly-used shape, according to research. In follow-up interviews, he posited that the circle model required a lot of mathematics to construct, hence his avoidance of it. By doing this, Simon demonstrated his knowledge of the limitations posed by the circle model (Ball, 1990a; Yearley & Bruce, 2014). Simon wanted the learners to draw their area models of fractions; therefore, the rectangle model was the most suitable because learners were familiar with drawing straight lines. Dan, however, used both circle and rectangular area models; he did not have a specific reason for using them other than that learners were familiar with circles and rectangles:

> *They are familiar with these shapes. More so, using the circle is to accommodate the divisions, when you go up to eight. It is not easy with the triangle. There is no specific reason, just to show them that you can use different shapes* (Appendix H.2).

However, a closer look at both the learner's textbook and teacher's guide revealed that those were not the only shapes used. Fraction representations in these curriculum materials also involved the use of an equilateral triangle and the regular pentagon and hexagon. Further scrutiny of the teacher's guide revealed that Dan followed the teacher's guide to the letter. By his admission, Dan stated that fractions were a challenge: "I am not very comfortable teaching fractions. It is not an easy topic" (Appendix H.2).

The way he used teaching and learning materials is typical of teachers lacking in content knowledge.

6.4.3 Code-switching

Language plays a huge role in teaching and learning. When the language of instruction is the learners' second language, it is bound to create problems for the learners. In two schools, the participants used both English and SiSwati during instruction, but in one school, the teacher used only English. It turned out the culture of the school had an influence on the spoken language. Dan, who was the most proficient English speaker of the three participants, stated that there was no specific reason for communicating in English other than that it was the culture of the school. Nevertheless, Dan asserted that when he taught in a rural school, he had used both English and SiSwati because English was a challenge to learners, and became a barrier to the learning process. Code switching becomes necessary when the language of instruction is the learners' second language (Setati, 2005) Dan also had to take into considerations that the mathematics books were written in English, making English the natural choice for conveying the concepts. Pam shared the same sentiment, stating that mathematics examination papers are written in English. She felt that teaching in English was crucial, otherwise, learners would find problem solving very challenging. Pam also stated that she used SiSwati for clarification if necessary: "I do use it to clarify a point, but it must not be used a lot" (Appendix H.3).

Simon's reason for using SiSwati was slightly different from that of the other two participants. Simon explained in follow-up interviews that he used SiSwati when he felt learners were not following, and then he would give examples in SiSwati. This assertion was confirmed during classroom observations when he used scenarios familiar to the learners, such as buying either a whole, half or quarter of a loaf of bread. According to Simon, learners could also ask questions in SiSwati, since forcing learners to use their second language only resulted in learners clamming up, creating difficulties for the teacher to assess learners' understanding of content and the effectiveness of the teaching method used (Cuevas, 1984). Simon himself also struggled a little with the language of instruction. Learners and teachers alike from rural schools lack proficiency in English (Lemmer, 2010), hence teachers resort to code-switching during instruction (Naidoo, 2011).

6.5 SUMMARY OF FINDINGS

The findings reveal that teachers use all the representations found in Lesh et al.'s (1987) model. Concrete representations observed in this study included counters and symmetrical objects, and diagrammatic representations included area models (rectangle and circle). None of the participants used semi-concrete representations, such as computer technology, due to a lack of computers in two schools. The study confirmed findings from studies conducted in similar contexts that the availability of computer technology is a challenge in schools, and that teachers need training in the use of computer technology.

As far as diagrammatic representations are concerned, area models and the fraction chart dominated. Data refuted claims from the literature that the circle model is the most commonly used representation for part-whole representation. Teachers in this study preferred to use the rectangular area models. The study confirmed claims from literature that real-life situations are commonly used, as teachers used metaphors selected through their long-term experience of teaching fractions. One participant used word problems to enhance his use of teaching and learning materials. The study highlighted the need for the careful selection of metaphors to avoid situations where the metaphor becomes the subject of the lesson. Findings in this study confirmed the findings of other researchers that teachers tend to favour symbolic representations and spoken language in teaching.

All participants used the various representations as scaffolds to make the fraction concept more understandable to the learners. Some participants engaged learners in activities in which they actively manipulated various representations in an effort to help learners gain a conceptual understanding of fraction addition. However, there are still teachers who use manipulatives such as concrete models and diagrams in the traditional way.

The study confirmed findings from research that rural schools lack computers, and that their availability does not guarantee their use, since teachers require training in order to use them effectively. The results confirmed findings from other studies that teachers use various representations to gain and maintain learners' interest, for conceptual understanding and to accommodate different learning styles.

6.6 RECOMMENDATIONS

This section offers suggestions on how teachers can effectively teach fractions in the intermediate phase. It also gives recommendations to the Ministry of Education and Training, pre-service institutions, and the National Curriculum Centre.

<u>For teachers:</u>

- In schools where computer technology is available, teachers should use semi-concrete manipulatives to build conceptual understanding of fractions, since they are readily available.
- Teachers should use concrete manipulatives to build a deeper understanding of the concept of fractions, instead of using them to verify rules and procedures.
- Teachers should be careful when selecting metaphors to avoid the metaphor becoming the target of the lesson.

<u>For the Ministry of Education and Training:</u>

It is recommended that the Ministry of Education and Training ensures that all schools, even those in rural areas, are equipped with computers for use by learners in learning mathematics.

<u>For pre-service teacher training institutions:</u>

Tertiary institutions should emphasize the use both concrete and semi-concrete materials such as computer technology in teaching the concept of fractions, and provide proper training for teachers.

For the National Curriculum Centre

The National Curriculum Centre (NCC) is aware that many teachers at primary school level are not qualified to teach, and some are not specialists in Mathematics. For instance, in 2012, 2391 unqualified teachers were hired by the Teaching Service Commission (Ngozo, 2012). It is therefore recommended that they include suggestions of teaching/learning strategies in their curriculum materials. Most of the exercises on fractions require the pupils to translate diagrammatic representations of fractions to symbols; however, there are no translations from symbols to diagrams or real-life situations to diagrams, or vice versa. Therefore, assessment exercises should include translations from each form of representation to each of the others.

6.7 LIMITATIONS

The presence of the researcher in the classrooms during observations could have affected the behaviour of the participants. From observations, it appeared that some learners were not that familiar with the concrete manipulatives they used, and had possibly not used them much before. Their behaviour could have been influenced by the researcher's presence, as it looked as if they were trying to impress the researcher when answering questions. Hence to get an accurate picture of the extent to which teachers use representations such as concrete models and pictures, a longitudinal study should be done. Secondly, since the sample size was small, the findings of the study cannot be generalized.

6.8 SUMMARY

The study was a multiple case design aimed at investigating the use of multiple representations by primary teachers in teaching the concept of fractions. Three teachers took part in the study. Data were collected using interviews and observations. Data were analysed and findings discussed. The researcher forwarded the recommendations to various stakeholders.

References

Adler, J. (2001). Resourcing practice and equity: Adual challenge for mathematics education. In B. Atweh, H. Forgasz & B. Nebres (Eds), *Sociocultural research on mathematics education. An international perspective*, 185-200. Mahwah, New Jersey: Lawrence Erlbaum Associates.

Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction, 16*, 183-198.

An, S., Kulm, G., & Wu, Z. (2004). The pedagogical content knowledge of middle school, mathematics teachers in China. *Journal of mathematics teacher education, 7*, 145-172.

Arcavi, A. (2003). The role of visualization in the learning of mathematics. *Educational Studies in Mathematics, 52*, 215-251.

Austin, P., Carbone , R. E., & Webb, P. (2011). Prospective primary school teachers' attempts to pose acceptable word problems on the addition of fractions: Some insights from South Africa and the United States of America. *African Journal of Research in Mathematics, Science and Technology Education, 15*(2), 168-178.

Babbie, E. F., & Babbie, H. R. (2008). *The basics of social research* (M. D. Roth Ed. 4th ed.). UK: Thomson Wadsworth.

Bal, A. (2014). The examination of representations used by classroom teacher candidates in solving mathematical problems. *Educational Sciences: Theory & Practice, 14*(6), 2349-2365. doi:DOI: 10.12738/estp.2014.6.2189

Ball, D. L. (1990a). Halves, Pieces and twoths: constructing representational contexts in teaching fractions. *National Centre for Research in Teacher Education*.

Ball, D. L. (1990b). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449-466.

Ball, D. L., & Forzani, F. M. (2010). Teaching skillful teaching. *Educational leadership, 68*(4), 40-45.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for teaching. What makes it special? *Journal of Teacher Education, 59*(5), 389-407. doi:10.1177/0022487108324554

Barmby, P., Bolden, D. S., Raine, S., & Thompson, L. (2013). Developing the use of diagrammatic representations in primary mathematics through professional development. *Educational Research, 55*(3), 263-290. doi:10.1080/00131881.2013.825164

Beckmann, S. (2004). Solving algebra and other story problems with simple diagrams: a method demonstrated in Grade 4–6 Texts used in Singapore. *The Mathematics Educator, 14*(1), 42-46.

Berthold, K., Eysink , T. H. S., & Renkl, A. (2009). Assisting self-explanation prompts are more effective than open prompts when learning with multiple representations *Instr Sci, 37*, 345-363. doi:10.1007/s11251-008-9051-z

Bolden, D. S., Barmby, P., & Harries, T. (2013). A representational approach to developing primary ITT students' confidence in their mathematics. *International Journal of Mathematical Education in Science and Technology, 44*(1), 70-83.

Brijlall, D., & Niranjan, C. (2015). Using manipulatives to support an embodied approach to learning trigonometry in a South African school: A case study. *Africa Education Review, 12*(3), 361-380. doi:10.1080/18146627.2015.1110893

Bryman, A. (2008). Social research methods. Oxford: Oxford University Press.

Bruner, J. (1966). The culture of education. Cambridge , MA. Harvard University Press.

Carpenter, T. P., & Fennema, E. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 1*(97), 413-430.

Cassell, C., & Symon, G. (2004). *Essential guide to qualitative methods in organisational research*. e-Book: SAGE publications Ltd.

Caswell, R. (2007). *Fractions from concrete to abstract using 'Playdough Mathematics''*. Retrieved from http://files.eric.ed.gov/fulltext/EJ793976.pdf

Chitiyo, G., Taukeni, S., & Chitiyo, M. (2015). The observation method. In C. Okeke & M. Van Wyk (Eds.), *Educational Research: An African approach*. South Africa: Oxford University press Southern Africa (Pty)Ltd.

Christiansen, I., & Aungamuthu, Y. (2012). Language issues, ''misconceptions'' and confusion: A qualitative analysis of KZN grade 6 learners' responses on a mathematics test. *Education as Change, 16*(1), 51-67. doi:10.1080/16823206.2012.691713

Cramer, K., Wyberg, T., & Leavitt, S. (2008). The Role of Representations in Fraction Addition and Subtraction. *Mathematics Teaching in the Middle School, 13*(8), 490-496.

Cuevas, G. J. (1984). Mathematics learning in English as a second language. *Journal for Research in Mathematics Education, 15*(2), 134-144.

Cuoco, A. (2001). The roles of representations in school mathematics. *National Council of Teachers of Matnematics.*

Dahms , M., Geonnotti, K., Passalacqua, D., Schilk, J. N., Wetzel, A., & Zulkowsky, M. (2007, 19/12/15). The Educational Theory of Lev Vygotsky: an analysis. Google scholar.

Dakwa, F. (2015). The interview method. In C. Okeke & M. Van Wyk (Eds.), *Educational research: An African approach*. South Africa: Oxford University press Southern Africa (Pty)Ltd.

Darke, P., Shanks, G., & Broadbent, M. (1998). Successfully completing case study research: combining rigour and pragmatism. *Information Systems Journal, 8*, 273-289.

Department of Basic Education(2015): Annual Report 20115/2016), Vote no. 14.

English, L. D., & Halford, G. S. (1995). *Mathematics education: Models and processes*: Lawrence Erlbaun Associates.

Fambaza, T. (2012). *The experiences of teachers in teaching computer applications technology at the further education and training (FET) band*. (Unpublished Masters thesis), University of KwaZulu Natal, South Africa.

Gagatsis, A., & Elia , I. (2004). *The effects of different modes of representation on mathematical problem solving*. Paper presented at the 28th Conference of the International Group for the Psychology of Mathematics Education, University of Cyprus.

Gagatsis , A., & Shiakalli, M. (2004). Ability to translate from one representation of the concept of function to another and mathematical problem solving. *An international journal of experimental educational psychology, 24*(5), 645-657. doi:10.1080/0144341042000262953

Galant, J. (2013). Selecting and sequencing mathematics tasks: Seeking mathematical knowledge for teaching. *Perspectives in Education, 31*(3).

Goldin, G. A. (2002). Representations in mathematical learning and problem solving. In L. D. English (Ed.), *Handbook of International Research in Mathematics Education.*

Goldin, G. A., & Shteingold, N. (2001). Systems of representations and the development of mathematical concepts. In A. Cuoco & F. Curcio (Eds.), *Roles of representations in school mathematics*: National Council of Teachers of Mathematics.

Gouws, E., & Dicker, A.-M. (2011). Teaching mathematics that addresses learners' multiple intelligences. *Africa Education Review, 8*(3), 568-587. doi:10.1080/18146627.2011.618721

Gravemeijer, K. (2010). Preamble: From models to modelling. In K. Gravemeijer, R. Lehrer, B. Van Oers, & L. verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (Vol. 30). London: Kluwer academic publishers.

Hartely, J. (2004). Case study research. In Cassell, Catherine, symon & Gillian (Eds), essential guide to qualitative methods in organisational research. London:Sage publications Ltd, pp.323-333.

Hiebert, J. (1988). A Theory of Developing Competence with Written Mathematical Symbols. *Educational Studies in Mathematics, 19*(3), 333-355.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning*. New York: Mcmillan.

Hill, H., & Charalambous, C. Y. (2012). Knowledge, curriculum use, and quality of instruction: Unpackig a complex relationship. *Journal of curriculum studies, 44*(4), 443-446.

Johnson, T., & Lesh , R. (2003). A models and modeling perspective on technology- based representational media. In R. Lesh & H. M. Doer (Eds.), *Beyond constructivism: Models and modelling perspectives on mathematics problem solving, learning , and teaching*. (pp. 256-277): Lawrence Erlbaum associates publishers.

Khisty, L. L. (1995). Making inequality: Issues of language and meanings in mathematics teaching with Hispanic students. In W. B. Secada, E. Fennema, & L. Byrd (Eds.), *New directions for equity in mathematics education*: Cambridge University Press.

Kim, B. (2001). Social constructivism. In M. Orey (Ed), Emerging *perspectives on learning, teaching, and technology. Retrieved from* http://www.coe.uga.edu/epltt/SocialConstructivism.htm

Lakoff, G. & Nunez, R.F. (1997). The metaphorical structure of mathematics: Sketching out cognitive foundations for mind-based mathematics. In Lakoff G.(Author), *Mathematical reasoning: Analogies, metaphors, and images. Retrieved from* http://escholarship.org/uc/item/6pv6265j

Lee , S. J., Brown, R. E., & Orrill, C. H. (2011). Mathematics teachers' reasoning about fractions and decimals using drawn representations. *Mathematical thinking and http://www.coe.uga.edu/epltt/SocialConstructivism.htmlearning, 13*(3), 198-220.

Lemmer, E. (2010). Language policy and linguistic realities in South African schooling. In E. Lemmer & N. van Wyk (Eds.), *Themes in South African education*. South Africa: Routledge Publishers.

Lesh, R., Behr, M., & Post, T. (1987). Rational Number Relations and Proportions. In C. Janiver (Ed.), Problems of Representations in the Teaching and Learning of Mathematics (pp. 41-58). London: Lawrence Erlbaum.

Lesh, R., & Doer, H. M. (2003). Foundations of a model and modeling perspective on mathematics teaching, learning , and problem solving. In R. D. Lesh , H. M. (Ed.), *Beyond constructivism: models and modeling perspectives on mathematics problem solving, and teaching* (pp. 3-33). London: Lawrence Erlbaum Associates.

Lesh , R., Post, T., & Berh, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In c. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33-40). London: Lawrence Erlbaum.

Lim, K. H. (2011). Addressing the multiplication makes bigger and division makes smaller misconceptions via prediction and clickers. *International Journal of Mathematical Education in Science and Technology, 42*(8), 1081-1106.

Lourenco, O. (2012). Piaget and Vygotsky: Many resmblances and a crucial difference. *New ideas in psychology, 30*(3), 281-295.

Mahn, H. (1999). Vygotsky's methodological contribution to sociocultural theory *Remedial ans special education, 20*(6), 341.

McKinley, J. (2015). Critical argument and writer identity: Social constructivism as a theoretical framework for EFL academic writing. *Critical Inquiry in Language Studies, 12*(3), 184–207. doi:10.1080/15427587.2015.1060558.

McLeod, S. A. (2009). Jean Piaget. Retrieved from http://www.simplypsychology.org/piaget.html

McLeod, S. A. (2012). Zone of Proximal Development. Retrieved from www.simplypsychology.org/Zone-of-Proximal-Development.html

Mendiburo, M., Hasselbring, T., & Biswas, G. (2014). Teaching fractions with technology: What type of support do students need as they learn to build and interpret visual models of fractions ordering problems? *Journal of Cognitive Education and Psychology, 13*(1).

Molebale, J. J. L. (2005). *The role of practical work in learning the division of fractions by grade 7 Learners in two primary schools in Mpumalanga Ward of Hammarsdale Circuit in KwaZuluNatal.* (Master of Education), University of KwaZulu Natal.

Moyer, P. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*, 175-197.

Moodley, S. (2012). The influence of context on teachers' conceptions of professional expertise. (Masters thesis), University of Kwa Zulu Natal, South Africa.

Murphy, C., Scantlebury, K., & Milne, C. (2015). Using Vygotsky's zone of proximal development to propose and test an explanatory model for conceptualising coteaching in pre-service science teacher education *Asia-Pacific Journal of Teacher Education, 43*(4), 281-295. doi: 10.1080/1359866X.2015.1060291

Naidoo, J. (2011). *Exploring master teachers' use of visuals as tools in mathematics classrooms.* (Doctoral thesis), University of KwaZulu Natal. (205524804)

NCTM. (2000). Principles and standaerds for school mamthematics. *Veston, VA.*

Naiser, E. A., Wright, W. E., Capraro, R. M. (2003). Teaching fractions: Strategies used for teaching fractions to middle grades students. *Journal of Mathematics Teacher Education* **7:** 145–172.

Nichols, K., Stevenson, M., Heberg, J., & Gillies, A. (2015). Primary teachers' representational practices: from competency to fluency. *Cambridge journal of education*. doi:10.1080/0305764X.2015.1068741

Nickson, M. (1992). The culture of the mathematics classroom: An unknown quantity? In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning.* New York: Mcmillan.

Ngozo, S. (2012, January 23). Shocker: Over 2000 unqualified teachers hired. The Swazi Observer, p.2)

Osana, H. P., & Pitsolantis , N. (2013). Addressing the struggle to link form and understanding in fractions instruction. *British Journal of Educational Psychology, 83*, 29-56.

Pape, S., & Tchoshanov, M. (2001). The Role of Representation(s) in Developing Mathematical Understanding. *Theory into practice, 40*(2), 118-127. doi:10.1207/s15430421tip4002_6

Presmeg, N. C. (2013). Reasoning with metaphors and metonymies in mathematics learning. In L. D. English (Ed.), *Mathematical Reasoning: Analogies, metaphors, and images*: Routledge.

Rajesh, B. (2009). *The effects of using visual Literacy and visualization in the teaching and mearning of mathematics problem solving on grade 6 and grade 7.* (Master of Education), University of KwaZulu Natal.

Reiner, K., & Moyer, P. (2005). Third-graders learn about fractions using virtual manipulatives: A classroom study. *Journal computers in mathematics and science teaching, 24*(1), 5-25.

Schoenfeld, A. H., & Kilpatrick, J. (2008). *Towards a theory of profficiency in teaching mathematics*. Retrieved from https://gse.berkeley.edu/

Schumacher, S., & McMillan, J. H. (2010). *Research in education : Evidence- based enquiry* (Seventh Edition ed.): Pearson.

Setati, M. (2005). Teaching mathematics in a primary multilingual classroom. *Journal for Research in Mathematics Education, 36*(5), 447-466.

Sfard, A. (2001). There is more discourse than meets the ears: Looking at mathematics as communicating to learn more about mathematical learning. *Educational studies in mathematics, 46*, 15-37.

Sfard, A., & McClain, K. (2002). Guest Editor's Introduction: Analyzing tools: Perspectives on the role of designed artifacts in mathematics learning. *Journal of the Learning Sciences, 11*(2), 153-161. doi:10.1080/10508406.2002.9672135

Silver, D. (2011). Using the ‘Zone’ Help Reach Every Learner. *Kappa Delta Pi Record, 47* (sup 1), 28-31. doi:10.1080/00228958.2011.10516721

Son, J., & Senk, S. L. (2010). How reform curricula in the USA and Korea present multiplication and division of fractions. *Educational Studies in Mathematics, 74*, 117-142. doi:10.1007/s10649-010-9229-6

Sriraman, B., & Lesh , R. (2007). Leaders in mathematical thinking and learning: A conversation with Zoltan P. Dienes. *Mathematical thinking and learning, 9*(1), 59-75.

Stols, G., Ono, Y., & Rogan, J. (2015). What constitutes effective mathematics teaching? Perceptions of teachers. *African Journal of Research in Mathematics, Science and Technology Education, 19*(3), 225-236. doi:10.1080/10288457.2015.1080934

Stott, D. (2016). Making Sense of the ZPD: An organising framework for mathematics education research. *African Journal of Research in Mathematics, Science and Technology Education, 20* (1), 25-34. doi:10.1080/10288457.2016.1148950

Thompson, C. J., & Davis, S. B. (2014). Classroom observation data and instruction in primary mathematics education: improving design and rigour. *mathematics Education Research Journal, 26*, 301-323. doi:10.1007/s13394-013-0099-y

Tripathi, P. (2008). Developing Mathematical understanding through multiple representation. *National Council of Teachers of Mathematics, 13*(8), 338-345.

Uttal , D. H., Scudder, K. V., & Deloache, J. S. (1997). Manipulatives as Symbols: A New Perspective on the Use of Concrete Objects to Teach Mathematics. *Journal of applied developmental psychology, 18*, 37-54.

van Compernolle., R. (2012). Promoting sociolinguistic competence in the classroom zone of proximal development. *Language teaching research, 16*(1), 39–60. doi:DOI: 10.1177/1362168811423340 van Garderen, D., Scheuermann, A., & Jackson, C. (2012). Developing representational ability in mathematics for students with learning disabilities: A content analysis of grade 6 and grade 7 textbooks. *Learning Disability Quarterly, 35*(1), 24–38. doi:DOI: 10.1177/0731948711429726

van Laren , L., & Goba, B. (2013). 'They say we are creche teachers':Experiences of re-service teachers taught through the medium of isiZulu. *Pythagoras, 31*(1), 21-28.

Van Oers, B. (2010). Informal representations an their improvements. In K. Gravemeijer, R. Lehrer, B. Van Oers, & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (Vol. 30). London: Kluwer Academic Publishers.

Vanderburg, R. M. (2006). Reviewing research on teaching writing based on vygotsky's theories: What we can learn. *Reading & Writing Quarterly, 22* (4), 375-393. doi:http://dx.doi.org/10.1080/10573560500455778

Verenikina, I. (2008). Scaffolding and learning: its role in nurturing new learners. In P. Kell, W. Vialle, D. Konza, & G. Vogl (Eds.), *Learning and the learner: exploring learning for new times*. University of Wollongong, Research Online.

Vygotsky, L. S. (1978). *Mind in society*. Cambridge: Havard University Press.

Wilkerson, T. L., & Gupta, D. (2015). Teaching and learning of fractions in elementary grades. *Curriculum and teaching dialogue*, *17*(1 and 2), 27-44.

Woolfolk, A. (2010). Educational psychology (11th ed).Columbus, OH. Pearson, Allyn & Beacon.

Wu, H. (2014). *Fractions , decimals and rational numbers.* Retrieved from https://math.berkeley.edu/~wu/NMP**fractions**.pdf

Yearley, S., & Bruce, C. (2014). A canadian effort to address fraction teaching and learning challenges. *Australian Primary Mathematics Classroom, 19*(4).

Zaretskii, V. K. (2009). The zone of proximal development. *Journal of Russian & East European Psychology, 47*(6), 70-93. doi:DOI 10.2753/RPO1061-0405470604

APPENDICES

APPENDIX A: SEMI-STRUCTURED INTERVIEW SCHEDULE WITH TEACHERS

1. Teacher Profile

a. Name:____

b. Gender:____

c. Age group:____

d. Qualification(s):____________________________________

e. Subject(s) teaching:_________________________________

f. Grade(s)teaching:___________________________________

g. Number of years teaching mathematics:_______________________

h. Number of years teaching this grade:_________________________

2. Teaching and learning materials:

a. Do you use textbooks when preparing for lessons on fractions?__________

b. If so, name them:____________________________________

c. Do you use any other sources?____________________________

d. If so, name them.____________________________________

____

e. Are the materials provided by the National Curriculum Centre adequate?____

f. If no, why?____

____

g. How do you select teaching and learning aids for your lessons?__________

____

3. **Do primary teachers value the use of multiple representations in teaching fractions?**

a. . What kind of learning aids do you usually use? (Manipulatives or visual aids). State the reason for using those representations.

____
____
____

b. Do you always use more than one representation to illustrate fractions?

____

c. What motivates you to use more than one representation when teaching fractions?

____
____
____
____
____

APPENDIX B: OBSERVATION SCHEDULE TO BE USED WITH VIDEO RECORDING

Name of school: --

Name of teacher: --

Date: --

Observation no: --

Start time: --

End time: --

1. What kinds of representations do primary school teachers use when teaching fractions in the mathematics classroom?

Representation	Not at all	Sometimes	Frequently	All the time
Concrete/virtual models Counters				
Fraction bars				
Symmetrical objects				
Computer software				
Diagrams/pictures Area models				
Fractional chart				
Number line				
Sets				
Experience-based metaphors Games				

Real-life problems				
Symbols Written				
Verbal				
Language Spoken				
Written				

2. Interactions within the classroom

Type	**Not at all**	**Sometimes**	**Frequently**	**All the time**
Whole group				
Small group				
Pairs				
Individual				

3. How do teachers use representations in teaching fractions?

____

____

____

4. Any other observations:

____

____

____

APPENDIX C: SEMI-STRUCTURED INTERVIEW SCHEDULE FOR FOLLOW-UP INTERVIEWS

School ____

Teacher's name ____

Time ____

1. Do primary teachers value the use of multiple representations in teaching fractions?

1. Do you always use more than one representation to illustrate to fraction concept?
2. What motivates you to use more than one representation when teaching fractions?
3. What can you say about your learners' response to the representations you used?
4. Do you think using multiple representation when teaching fractions can help learners understand fractions?
5. Why do you say so?
6. Has the use of multiple representations benefited your learners?
7. Have you learned anything from using multiple representation in your lesson?

2. How are representations used within different contexts?

1. Do you feel that you have used the various representations in the way you intended?
2. What preparation did you have to do in order to use these visuals in your class?
3. In what ways can the use of multiple representation help teachers improve their teaching of fractions?

3. What support do teachers need in order to use multiple representations?

1. Did you need training to use multiple representations in the classroom?
2. Do you need further support in using multiple representations? If so, what support do you need?

4. Interview questions based on lesson on fractions:

____

APPENDIX D: CONSENT FORMS

APPENDIX D. 1: FOR TEACHER

Informed Consent Form

Dear Teacher

My name is Thabisile Dlamini. I am currently doing a Master's degree at the University of KwaZulu-Natal, Edgewood Campus, in the Cluster of Mathematics and Computer Science Education, specialising in mathematics education.

I am doing an investigation into teachers' experiences of teaching fractions using multiple representations, how they translate between various representations and the reasons for those translations. The aim of the study is to identify areas where professional development for in-service teachers is needed and how pre-service training can be improved as far as the teaching and learning of fractions is concerned.

If you agree to take part in the study, you should expect the following events to take place:

- An in-depth face-to-face semi-structured interview with the researcher; duration 30 to 40 minutes.
- A maximum of two classroom observations of a 40-minute lesson on fractions.
- A short interview after each lesson to corroborate observations.
- Interviews and observations will be recorded using audio and video tape.
- When the research is complete a meeting will be convened at which the findings will be discussed.

If you have any questions about the study, please feel free to contact the following people:

- Ms Thabisile P Dlamini: (+268) 76128997 (Cell), thalite3@yahoo.com or thalite3@swazi.net (email).
- Ms Barbara Busisiwe Goba: +27 73 848 3377 (cell), gobab@ukzn.ac.za (email).

Read and sign below:

I understand the nature of the study, and I agree to participate voluntarily. I understand that I am at liberty to withdraw from the study at any point in time without any repercussions. I am aware that all interviews will be audio recorded and the observations will be video recorded.

I prefer my face to be hidden/blurred in the videotapes. Please tick **YES/NO**

I hereby agree to be audio-recorded. Please tick **YES/NO**

Participant:

Name: --

Signature: -- Date: ----------------------------------

Researcher

Name: --

Signature: -- Date: ----------------------------------

APPENDIX D. 2: FOR PARENT/GUARDIAN

UNIVERSITY OF
KWAZULU-NATAL
INYUVESI
YAKWAZULU-NATALI

Consent Form

Dear Parent/Guardian

1. My name is Thabisile Dlamini. I am currently doing a Master's degree at the University of KwaZulu-Natal, Edgewood Campus, in the Cluster of Mathematics and Computer Science Education, specialising in mathematics education. I am investigating the extent to which primary teachers use multiple representations in teaching the concept of fractions, how they use them, and their reasons for using or not using them. This letter is to request your consent for your child to participate in the above-mentioned research project.

2. This study will focus on classroom teaching of fractions in grades 4 and 5. I plan to observe lessons on fractions. The lessons will be video-taped as well as some of the learners' work. As a parent/guardian I am asking for your permission to allow your child to appear as part of the video recording and have copies of work your child might produce during the course of the lesson. Classes will continue as normal, with my presence at the back of the classroom.

3. All data collected will be used purely for research purposes. All data collected will be archived and securely stored at the University of Kwa Zulu Natal for a duration of five years, after which it will be destroyed.

Please note that if permission is not granted I will respect your decision, and your child will not appear in video recordings and his/her work will not be reproduced.

Please complete the form below and return it to the class teacher. Thank you for your support.

Yours Faithfully

Thabisile Dlamini

. Cell; (+268) 76128997

Read and sign below

I____ (please print full name) parent/guardian of _______________________________________, give consent to the following:

Video-taping of mathematics lessons on fractions in which my child might appear as part of the video text. Please tick **YES/NO**

Copies made of classwork, homework or assessment that my child might produce as part of these lessons. Please tick **YES/NO**.

APPENDIX E: CLASSROOM OBSERVATIONS

APPENDIX E. 1: SIMON

Observation 1

T: What operation were we talking about?

L: Solving problems.

L: Solving problems involving what?

P: Eight and nine.

T: What were we doing with 8 and 9?

L: Making sets.

T: When making sets, what operation did we use?

L: Making group sets.

T: Making group sets is one thing.

L: Dividing.

T: We were dividing, very good. Now let us revise division. Eight divided by 4.

L: Two.

T: We are going to have two sets of four.

16 ÷ 4

The next one is 16 ÷ 4.

L: Four (Khumalo.)

T: You all agree?

L: Yes.

T: Sixteen members shared into four sets, each set will have?

L: Four members.

T: Very good; 24 divided by 12?

L: Two. (Dlamini.)

T: We are going to have two sets of twelve. Now I want six people; three boys and three girls and form a group.

(Learners stand in front of chalk board).

T: *Yakhani sibaya* (form a kraal shape). Yes, this is a what?

L: This is a set of pupils.

T: Very good. How many members do I have in the set?

L: Six members.

T: We are having six members in the set. Now I want to make two groups, so I'm going to divide my set. What are we supposed to do now? Each member goes into one part until they are finished. (Children move into new sets)

T: How many members does each set have?

L: Three members.

T: Very good. Each set has three members. What have we done with six?

L: Divided six.

T: We have divided six into two equal sets. Each set, this set of three girls is what of the whole set?

L: Three-thirds.

T: Do you agree? Are there any three equal groups here?

L: No.

T: No, there are not. It is not true that there are three thirds. There are two groups. Each set is call what of the whole group?

L: One third.

L: One set of three.

T: One set of three. How many sets are there? This set has been divided into two equal groups. Now we have this group. How many groups do I have?

L: One set of 2. 1 set of 2= ½

T: What kind of number is this?

L: A fraction.

T: Now today we are going to deal with fractions. What is a fraction? You can share your thoughts with your partners. Discuss with somebody next to you. How did we define a fraction? (Discussion in process, teacher moves around).

T: Let me remind you what a fraction is. You will pay me for reminding you what you were taught (jokingly). A fraction is part of a whole thing. (Writes a second definition on the board).

T: We said we have a group of six pupils in a group. What did we do with this group?

L: We divided it into two sets.

T: Six pupils in a group divided into two sets. Three pupils were in one set. The group that was here is a whole. We divided in into two sets. We said each is what of a set of the whole group?

L: Fraction. (Dlamini.)

T: Each set is a fraction of the whole group. Let's have a group of eight pupils. This is a group. How many groups do we have?

L: One group.

T: This one group has how many members?

L: Eight members.

T: So one group is formed out of eight members. I want to find a fraction of the eight members. So I'm going to make sets. Now we have how many sets?

L: Two sets.

T: Now we have two sets. This set is done what here?

L: Divided into two sets.

T: We say this group here is what of the whole group?

L: Fraction of the group.

T: Now what fraction is this group of the whole set? What fraction of the whole group is each set?

L: Four-fourths.

T: How many equal sets do we have here?

L: Two.

T: One group is what of the whole group?

L: Fraction.

T: What fraction, I want the name of the fraction. (Learners confused. Teacher takes them back to original example of fraction,) Each group is half of eight members. How many members are in each group?

L: Four.

T: Four is half of eight. (Teacher draws a circle on the floor and divides it into four equal parts. Each pupil goes into each set until there are equal numbers of pupils. In each set under the watchful eye of the teacher, ensuring that there is no gender bias.)

T: Now this group is divided into?

L: Four equal parts.

T: Now this group is divided into four equal parts. (Emphasis on four equal parts). Each set is a what? (Writes of the board: “4”).

L: Fraction.

T: What fraction of the whole group is each set? (No response).

T: This is one set not two. Out of how many sets?

L: Four.

T: Out of four sets. What fraction is this?

L: One fourth. (Teacher writes ¼ on the board.)

T: This set is one fourth. What we have done is finding a fourth of eight. How many members are in each set?

L: Two members.

T: Any problems? Now let’s go into our groups. (Learners sit in groups of four and wait for teachers instructions. Teacher distributes counters (stones) – twelve per group. Learners count as he puts them on the table and piece of chalk.

T: Now I have given each group how many stones?

L: Twelve stones.

T: Let’s take the stones as a group. We are going to make sets. Put them in one set and use the chalk to make boundary around the stones. Draw boundary on the desk. (Leaners follow instructions.)

T: Now this is a group of twelve. Now divide the group into two parts. Now how many sets do we have?

L: Two sets.

T: Each set is what of the whole group?

L: It is a fraction.

T: What fraction is each set of the whole group?

L: (No response.)

T: The problem is the English.

L: Two halves.

T: Each set is what of the whole group?

L: Half.

T: Half of twelve is what?

L: Six.

T: Now divide the group into four sets. (Teacher moves around ensuring that all learners are actively participating.) How many sets do we have? Each set is what of the whole group?

L: One fourth.

T: Each set is a fourth of the whole group. (Selects a pupil to write a fourth on the board. Learner writes the following.)

½ of 8 = 4 ¼ of 8 = 2

½ of 12 = 6¼ of 12 = 3

T: One fourth of twelve is how many members?

L: Three.

T: (Selects pupils at random to write on the board. Distributes pieces of paper. Instructs pupils to draw a circle on the piece of paper and draw six members inside.)

T: Now divide the set you have drawn into two parts. Now each set is what?

L: Fraction.

T: What fraction of the whole group is each set?

L: One half.

T: Next to the set write, “½ of 6”. (This was a struggle for most of the learners).

T: Write in numerical form.

Observation 2

T: Which are the four operators you have learned of?

L: Divide.

T: The value of dividing.

L: Times.

T: Times equals multiplication.

L: Minus.

T: Minus equals subtraction.

L: Plus.

T: Plus is called what?

L: Add.

T: Addition. It is called addition. (Teacher reviews addition of whole numbers. Asks learners the following 3+4, 2+5, 1+0, 0+2. Learners respond orally.)

T: You still remember how to add numbers. Now, what were we learning about yesterday?

L: Fractions.

T: Today we are going to operate fractions. We are going to operate fractions using fractions. (Writes on the board: “Adding fractions”). Let us remind ourselves, what is a fraction?

L: (Responds softly.)

T: Speak aloud so that everyone can hear you.

L: A fraction is a part of a whole thing.

T: A fraction is part of a whole thing, you still remember. A fraction has two parts. Which are the two parts of a fraction? Are they two or three? I can’t remember.

L: Half.

T: Let us look at half. Can someone write half, in numerical form on the board?

L: Learner writes half on the board: “½”.

T: Is that half?

Class: Yes.

T: Let’s give her a round of applause. (Class claps hands.) This is half (pointing to the fraction symbol ½ on the board). This fraction has different parts. How many parts? How many different things do you see?

L: Three.

T: There are three things. (Points to the numerator and dominator and the line separating the two.) These things do not represent the same thing, they are different. One is called what?

L: Numerator.

T: Is called a numerator, very good, you still remember. (Writes numerator next to 1). The two is called a what?

L: Denominator.

T: Then what is this? (Pointing to the line separating numerator and denominator.)

L: Divide.

T: It stands for the word divide. (Writes divide next to the line.)

T: Let us read what is on the board.

T & L: Numerator divide by denominator.

T: What is a denominator?

L & T: Denominator is the number, number of equal parts in a whole.

T: A whole. Think of something that we normally buy and like. What do we like? Bread. Our whole is a loaf of bread. Nowadays when you buy a loaf of bread you find it already sliced. When you buy it in the plastic, we call it a whole loaf of bread. Then we buy what, if it's not a loaf of bread? What do we do?

L: Half a loaf.

T: Half of a loaf of bread. What else do we buy?

L: Quarter of a loaf of bread.

T: A quarter of a loaf of bread. Do you understand?

L: Yes.

T: Today we are going to learn about adding fractions. The word denominator is a very important word. Could you please lend me a desk? (Two learners leave their desk and the teacher pushes it to the front.)

T: Now we have a desk. What do we have here? There is a seat here. How many pupils can we sit here?

L: Four pupils.

T: So only four can sit here, we can't add anymore?

L: Yes.

T: Okay. (Points at pupils at random to come and sit on the desk, until he could not fit anyone any more.) How many pupils are seated on this desk?

L: Four pupils.

T: The four could be called what of a fraction? Numerator or denominator?

L: Denominator.

T: Denominator. Since we are adding, I will need another desk. (Moves another desk to the front.) Are the desks the same size? Let us see if the same number of people will sit on this desk. (Four pupils sit on the second desk.) Are they the same?

L: Yes.

T: They are equal in number on the desks. We have four in each desk. (Teacher removes two students from one desk.)

T: How many are remaining?

L: Two.

T: The remaining two are called what?

L: Numerator.

T: That is the numerator. (He writes "$\frac{2}{4}$".) Let us read.

L & T: Two fourths.

T: Two fourths, not two over four. Or you can say two out of four. You have taken out two out of four. What else can you say? Two of the four, not two over four. That means it is finished or overflowing. Let me write another fraction. (He completes the fraction, putting 1 in the numerator ¼. Then he puts an addition sign between the fractions 2/4 + ¼.) Now let me make an example of a kombi (mini-bus). A kombi can carry how many people?

L: Fifteen people.

T: Fifteen people. A desk can carry how many people?

L: Four.

T: Is it possible for a kombi to carry thirty people?

L: No.

T: Is it possible for a desk to carry eight people?

L: No.

T: Always carries four. Now we are putting together or combining. Let us combine the people in the two desks. So how are we going to do this? You on the second desk move to the first desk.

T: We have added two fractions, what is the denominator?

L: Four.

T: How many are they now?

L: Three 2/4 + ¼ = ¾.

T: Three is the numerator. If we say two fourths plus one fourth the answer is three quarters. Do you understand? If you don't we will play this game again. You are going to tell the people to sit on the desks. (He writes a problem on the board: ¼ + ¼. Instructs pupils to do the demonstration using the desks. One pupil stands up to give instructions to other pupils. Instructs a pupil to sit at one of the desks, representing ¼. When the pupil instructs another pupil to sit on the same desk as the first learner the teacher interrupts. Tells learners to think carefully about what they are doing.)

T: Is he doing the right thing? We are adding the two fractions and there are two desks. So what is he supposed to do?

L: One pupil in each desk.

T: Now we have one pupil in each desk representing on quarter. What is the next step?

L: Combine. (A learner stands up and instructs the learner from the second desk to move to the second desk.)

T: So what do we get? Ask them what is the denominator?

L: What is the denominator?

Class: Four.

T: Write on the board. What remains now is numerator?

L: Numerator (Learner writes ¼ + ¼ = 2/4.)

T: The answer?

Class: Two fourths as a numeral.

T: Which means one fourth and one fourth …?

L: Two fourths.

T: The sum of the two fractions is two fourths. Thank you, you may sit down.

T: We have been using ourselves, now we are going to use something else. Now we are going to use this area model. (Shows pupils a rectangle which has been subdivided into equal parts.) Into how many parts has this been divided?

L: Four equal parts.

T: (Draws a rectangle on the board and portions it into parts). Are the parts on the board equal?

L: No.

T: (Emphasises that the parts should be equal). The parts should be equal, not almost equal. The parts in this piece of paper are all equal. Now what can this piece of paper stand for? Something we have already done.

L: It can stand for the desk.

T: Four people were seated on this desk and we said they are all the same size. (Takes another piece of paper, also divided into four equal parts, to represent the second desk. Teacher sticks the two pieces on the board and instructs pupils to represent 2/4 + ¼ = ¾.)

T: How are we going to represent these fractions using these pieces of paper?

L: Shading.

T: We will shade two parts of the four. (Learner stands up to shade using marker.) Did she do the right thing? Choose one pupil to shade the other fraction. (Learner stands up to shade one part out of the four.)

T: Now we want the answer. Now let me use this square on the board to make it easy. Now we want to find the sum of the two fractions. (He pastes the two area models on the square board. To find the answer he uses the unit squares. Teacher instructs pupils to move into groups. Gives then plain paper to use, a ruler and pencil to draw equal parts.

Instructs learners to represent 1/5 + 3/5, by drawing rectangles that are 5cm long. Each square will be one square centimetre in size. He moves around the class attending to each group, guiding pupils through probing and asking questions.)

APPENDIX E.2 : DAN

Observation 1

T: The subject is maths. The topic is adding fractions with the same denominator. Okay, if I say denominator or … Somebody give me a fraction, any fraction that you know.

L: Five. (Teacher writes five on the board.)

T: You say five is a fraction, how many say five is a fraction? (No show of hands.) How many say five is not a fraction? (Several students raise their hands.) Do you even know why five is not a fraction?

L: (Not audible.)

T: Alright. Is this a fraction? (Pointing at five.)

L: No.

T: Can somebody give me a fraction?

L: Five over ten. (Teacher writes 5/10 on the board.)

T: Any other fraction that you know?

L: One over two. (Writes ½ on board.)

T: One over two. Okay, these are fractions (pointing at 5/10 and 1/2). But this one is not a fraction (pointing at five). This is what? (Points at five.)

L: It is a number.

T: This is a number. (Points at the fraction. Learners laugh). This is a number which is a fraction, but what do we call such a number (pointing at five); how is it different from this? (Pointing at the fraction.)

L: It is a single number.

T: Such numbers we call them whole numbers (writes whole number next to five) and these are fractions (pointing at ½ and 5/10). When you count 1, 2, 3, 4, 5 until you reach billion, you are counting whole numbers. Right, they are whole numbers. Why do we say it's a whole number? It means not a part of it has been taken away.

T+L: Not a part of it has been taken away.

T: If I give you five rand, you are one and you have five rand. Even if I give you five apples and you are one. Are you sharing the five apples?

L: No.

T: You have a whole number, five, you are not sharing. But if somebody were to come and sit next to you and say may we share the five apples, and for some reason you become so kind, like some people are so generous, they find it so easy to give. Can you think of anyone who is generous in this class? If you pass by Zena and she has five oranges, I'm sure she would give some because she is a generous person. Okay now let me remind ourselves what a fraction is. (Looks at the board.) Which part of a fraction is a denominator and which one is not? There is 1 and 2, there is 5 and 10.

L: Two.

T: Two is a denominator and here? (Pointing at 5/10.)

L: Ten.

T: If the topic says "adding fractions with the same denominator", it means the denominator have to be what? The same. Before we go any further let's look at this story (written on the white board with red). Unfortunately, my red is a bit fuzzy for other people, but I will narrate the story. It says, "Mr Thwala has five sweets. He gives three sweets to Thabo and two sweets to Gugu." This implies that, initially Mr Thwala has five sweets. Is this a whole number or a fraction?

L: Whole number.

T: It is a whole number. Five is a whole number and it belongs to Mr Thwala. But now Mr Thwala becomes generous. He decides to give three sweets to Thabo and Gugu two sweets. The question is what fraction of the set of sweets has been given to Thabo and what fraction has been given to Gugu? Remember I said it starts with a whole number, but once you start sharing then you have fractions. You understand?

L: Yes.

T: Fractions come from a whole. If you have one big cake and I decide to cut it into (uses gestures). From one, if I decide to share it with four people in equal parts, then we will have?

L: A quarter.

T: In this case (pointing at the problem on the board) they are saying Mr Thwala has to share the five sweets between Thabo and Gugu. He gave Thabo three and Gugu two. What fraction of the set of sweets has been given to Gugu? Class, what fraction?

L: Two over five.

T: Very good. Two over five. Why are we saying two over five? From the big circle which has … (He draws a circle and counts the five small circles. Then puts a circle around the two small circles.) We say its two sweets out of how many?

L: Five.

T: If I may ask, how many sweets have been given to Thabo?

L: Three over five.

T: Three over five (writes 3/5 on the board), that’s Thabo. If I may show this in a diagram form (using a ruler, draws a rectangle on the board). We want to show Thabo’s fraction. If I may ask into how many parts should I divide this?

L: Five parts.

T: Five parts, very good. We try to divide, even though they won’t be equal. (Divides rectangle into five parts.) How many parts do we have?

T+L: (Count “1, 2, 3, 4, 5” parts.)

T: This represents the number of sweets that we had originally. But we want to show by shading this, 2/5 which was received by Gugu. So how many parts should we shade?

L: Two.

T: Yes, two. Let me shade using red. How many parts remain unshaded?

L: Three.

T: These were the parts that were given to?

L: Thabo.

T: So this diagram is basically showing us what fractions? The shaded is for Gugu (2/5) and the unshaded Thabo (3/5). Let me show you other fractions, so I can see that you still remember. (Sticks a sheet of paper with an area model of a circle on the board.) Can you all see?

L: Yes.

T: Those at the back?

L: Yes.

T: What fraction is this? First of all how many parts can you see?

L: Four.

T: There are four parts and we said we know the number of parts, what does it say to you? It means you know the denominator. Which means all the fractions here (pointing at area model) are over what?

L: Over four.

T: What fraction of the whole shape is the shaded part showing?

L: Three over four.

T: Three over four (writes ¾ next to area model.) There are three shaded parts out of 1, 2, 3, 4. So three over four. Let me put another one here. (Sticks another area model, this time a rectangle.) How many parts are there?

L: Five.

T: Five, How many parts are shaded?

L: One part.

T: So what fraction is that? The shaded fraction?

L: One over five. (Teacher writes 1/5 next to area model.)

T: The fraction is one over five. If you can see this then you are able to tell any. I can shade 3, or 4 or 5, so long as you can count into how many parts has the diagram been divided. That gives the denominator already, and then you count how many are shaded and that gives you the numerator. (Pointing at the area model, counts number of parts into which rectangle has been divided – denominator, then counts shaded parts – numerator. Writes 1/5.) But the topic says we are adding fractions. Therefore, we are still moving towards adding fractions. (Takes another area model, sticks it on the board.) Now let us look at this one. Into how many parts has this shape been divided?

L: Four.

T: How many parts are shaded?

L: Two.

T: What fraction is this?

L: Two over four.

T: The fraction is two over four. Now let me take this one. (Sticks another sheet of paper on the board with a rectangle divided into ten equal parts.) Into how many parts has the shape been divided?

L: Ten parts.

T: You notice that the parts have been shaded in different colours.

L: Yes.

T: So which means you are going to have different fractions. You cannot just say six over because you have different colours. So what is the fraction represented by colour green?

L: Three.

T: Three what?

L: Three over ten.

T: (Writes 3/10 next to the area model). So what is the other one?

L: Three over ten.

T: It is another three over ten. (Writes 3/10 on the board.). If maybe you had taken away 3/10 and now you want it back. If we are adding 3/10 which was taken away: 3/10 + 3/10. Remember what did and said about this? How do we add fractions with the same denominator?

T: What do we do?

T+L: Add the numerators.

T: So in this case what are we going to get?

L: Six over ten.

T: Six over … We do not add the denominators; that is the rule because look here (referring to diagram). If we were to add ten and ten it will give you six over twenty. Is this true?

L: No.

T: This diagram is over ten. Let us work on another example. We are just playing around. (Sticks another diagram on the board.)

T: How many parts do we have here?

L: Five parts.

T: Five parts which means every fraction we will talk about concerning this diagram will be over what?

L: Five.

T: Can you identify two fractions from this diagram?

L: One over five, three over five. (Writes answer next to diagram.)

T: So 1/5 + 3/5 is what?

L: No response (from three pupils).

L Four over five. (Teacher uses diagram to explain how the other pupil arrived at four over five. Even uses the previous example to clarify.)

T: Now let us look at this chart. (Sticks a fraction chart on the board.) So that we can get the concept of whole from whole. This is a whole, like a complete bar of chocolate, it has not been shared by anyone, but for some reason if the whole can be divided for two people, what will each person get?

L: One over two.

T: One over two and we call these halves. If now the same bar is divided among three people. We will now have three parts. What do we call these?

L: Thirds.

T: If the same whole (one bar of chocolate) can be shared among four people. What does it create?

L: Four parts.

T: Remember it is equally shared (emphasising that all pieces are equal). Any questions? If the whole is shared amongst six people equally. These are sixths. The same applies if we have eight people, we get eighths. We can divide based on the number of people. There are so many fractions depending on how many parts you want to divide the whole. They all come from a whole. Now suppose we were adding quarters, one-quarter plus one-quarter (pointing at chart).

L: Two quarters.

T: Two quarters based on how many are shaded or unshaded (linking fraction chart and area model). Any questions? Turn to page 54. (Reads question one, pupils answer questions orally. Learners had to translate from area model to symbols (addition of fractions)). I would like you to do number 2. (As he moves around marking, he notices that pupils are using a slanted line to separate numerator from denominator. He discourages them from that and encourages them to use a horizontal line.)

T: As much as it is not wrong, but at this level we do not allow children to do this (drawing a slanted line on the board). Let’s concentrate on straight lines. (Draws a horizontal line.)

Observation 2

T: What did we learn yesterday?

L: Adding fractions.

T: Yes, adding fractions. What kind of fractions were we adding?

L: Fractions with the same denominator.

T: Fractions with the same denominator. Anyone who wants to add to that? (No response.) So that is what we learned yesterday. Today we are taking that further, how can we subtract fractions with the same denominator? What is the difference between adding and subtracting?

L: When you are adding you are increasing.

T: Yes, if you can say let us add weight on you (pointing to one of the pupils), you will never look the same, and maybe by the time you are done you would be as big as this house. But what about subtracting? When we are subtracting what are we doing?

L: We are adding.

L: We are decreasing.

T: When we are subtracting we are decreasing. Others are getting confused by the word decrease. Let us use a simple term "taking away". When subtracting you are taking away. If for example you have twenty sweets and I come and take ten. Are you going to have more or less sweets?

L: Less sweets.

T: Less sweets than you had before. These are operations we use. (Writes on the board "+ adding (increase more) – subtracting (decrease less)".) Now let us remind ourselves about fractions. (Writes words for the fractions:

Halves – ½

Thirds – 1/3

Quarters – ¼

Eights – 1/8. Instructs pupils to write the numerals next to the words.)

T: (Sticks an area model of a circle divided into four equal parts.) How many parts are shaded?

L: Three.

T: If I remove one of the shaded parts, how many parts would remain?

L: Two over four.

T: Can someone come forward to write a mathematical sentence for what we just did?

L: Learner writes ¾ - ¼ = 2/4.

T: (Sticks another area model on the board.) What fraction of the whole shape is shaded?

L: Three over four.

T: (Writes ¾ under diagram.) Now if I decide to cross these two … (Puts cross on the two parts.) What fraction of the whole shape are crossed parts?

L: Two over four.

T: Yes. (Writes 2/4 on board.) Now same process, if I decide to remove the crossed parts, what will remain?

L: One over four.

T: Now who can write a mathematical sentence to illustrate what we have just done?

L: (Learner gives incorrect answer. Teacher uses diagram to help learner understand. Learner is given another chance to write mathematical sentence and this time he succeeds: ¾ - 2/4 = ¼.)

T: Finally he gets it. Anyone lost? Are we still together?

L: Yes.

T: (Sticks eight pens on the board, three blue, five red.) If I decide to remove these (removes all blue pens) what do I have?

L: Five.

T: What fraction is that?

L: Five over eight.

T: I'm remaining with 5/8. What happened for me to remain with 5/8. What did I take out?

L: Learner writes 8/8 – 3/8 = 5/8.

T: Let me show you another way. (Draws a number line on the board showing fifths.) These are fifths. If you want 5/5 – 2/5 = 3/5 ... (He uses a number line to illustrate how one arrives at the same answer using the number line.) You move how many steps?

L: Two steps.

T: Work out two more problems. You can use the number line to show subtraction, even addition though we did not do it. We will do it tomorrow. Any questions?

L: Let's do the first one again.

T: (Teacher demonstrates using number line. Instructs two more pupils to show subtraction on the number line until he is satisfied that everybody understands. Instructs learners to do class work, shown below.)

<u>Question 1</u>

Write mathematical sentences, given area models.

<u>Question 2</u>

Work out subtraction problems using number line. Number line drawn, e.g.: ¾ - 2/4 = ?

<u>Question 3</u>

Interpret number line by writing mathematical sentence.

APPENDIX E.3 : PAM

Observation 1

T: What are factors?

L: (Three pupils give incorrect responses.)

T: Any numbers that you multiply to get the product. (Lists factors of six on the board.)

T: What are multiples?

L: (Gives incorrect response.)

T: We will go outside and play and short game that will help us generate multiples of different numbers; “There is a fire on the mountain”. Then we will come back and add common fractions with different denominators. In Grade 4, you added fractions with the same denominator. In Grade 4, it was as good as adding the numerators, denominator remain the same. (Gives an example, writing in on the board. Instructs pupils go outside.)

T: Let us form a big circle. There is a fire on the mountain!

L: Run, run, run. (Learners run around the big circle.)

T: In threes! (Learners stand in groups of three.) At the end of the game you are going to tell me the number whose multiples we were accumulating. Are you all in threes?

T: 3, 6, 9, 12, 15, 18, … 36. Multiples of 3. (They continue playing for five minutes, generating multiples of different numbers and then return to the classroom.)

T: Now let us look at these fractions; one fifth plus one half. (Writes 1/5 + ½.) These two fractions have different denominators. For us to be able to add fractions the denominators must be the same. So how can we make these denominators of these fractions the same? We have to find what we call the lowest common multiple. How do we find the lowest common multiple? You first find multiples of five and multiples of two, then find multiples that are common, that is, that appear in five and multiples that appear in two. You accumulate these numbers, and then identify the lowest one which is called the lowest common multiple.

You are fresh from the game; multiples of five?

L: 5, 10, 15, 20, 25, 30, 35, 40 …

T: Multiples of two?

L: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 …

T: Common multiples; multiples that appear in five and in two. (Underlines “10” from both lists, writes, “Lowest Common Multiple is 10”.) By lowest we mean the smallest of them all. This means we make our denominator ten. How do we do that? We want to make denominator from 5 to 10. What do we do to 5 to get 10?

L: Multiply by two.

T: Multiply our five by two. If you are not sure you go to your list of multiples and count, you find ten in the second position which means you are going to multiply 5 by 2 to get 10. What you to the denominator, you also do to the?

L: Numerator.

T: You multiply the numerator by two. Now we have this denominator two (pointing at 1/2). Now you look at the multiples of two and find ten. Then count from the left to ten, to find the number you need to multiply two to get ten. So you multiply two by …?

L: Five.

T: Multiply 2 by 5 to get 10. Then turn to the numerator, 1 multiply by 2.

L: Two.

T: (Writes 2/10 + 5/10.) Now our fractions are having the same denominator. It is now simple for us to add the fractions, we add the numerators. Do you understand?

L: Yes.

T: Now we have taken Grade 4 stuff to Grade 6. (Adds two fractions, 2/10 + 5/10 = 7/10.) Seven out of ten. It is so simple to find multiples, you just think of a game you have been playing outside to accumulate multiples, find common multiples, then find the lowest common multiple. You can also add fractions with different denominators by using a fraction chart. (Sticks fraction chart on board.) Please study the fraction chart. Look at the divisions. We have one whole (pointing at the chart bar labelled, "whole") and we also having one half (pointing at the second bar which is divided into two parts). What does that mean? It means our whole is divided into two parts. All these divisions are from the whole (pointing at the other bar). So how do we add fractions using this fractional chart? We are going to add one fifth plus one half. What did we get when we added 1/5 and ½?

L: Seven out of ten.

T: Now, how do we use the fractional chart? We take a strip that is one fifth and another one that is one half. (Sticks both strips on the fraction chart.) We said our LCM is ten. We will look at the fraction with denominator ten. Then we take the strips and stick them adjacent to each other. Then we will compare the answer we get with the first answer 7/10. (Sticks strips on chart.) What is the answer?

L: Seven out of ten.

L: Three out of ten.

T: How did you get the answer seven out of ten?

L: (Learner demonstrates on the chart.)

T: How did you get the three out of ten?

L: (Learner demonstrates, counting the part not covered by strip.)

T: The correct answer is seven out of ten. You count the part covered by the strip, where it ends is your answer. The part that is not covered by the strip is not your answer. Last one before I give you an exercise. I need a volunteer to add 1/3 + ½ on the fraction chart.

L: (Learner lists multiples of three and two and identifies the LCM of 3 and 2 as 6, multiplies them and gets 2/6 + 3/6 = 5/6.)

T: Last volunteer to show this one on the fraction chart.

L: (Selects two strips, one representing one third and the other one half, sticks them on chart. After brief guidance from teacher, another learner stands up. Removes strips and sticks on the whole representing sixths, counts part covered by strips.)

T: (Satisfied that every pupil understands addition of fractions. Instructs pupils to do three problems.)

Observation 2

T: Today we are adding mixed numbers. But before that I want clever heads and clever rabbits. (A learner comes up and hands a plate to the teacher. Teacher places plate on the table.) Everybody sit up straight and close your exercise books. (She takes out pieces of papers from a bag with problems on addition of whole numbers written on them, promises to reward the first correct answer with money.)

T: 13+25?

L: 36.

T: Wrong.

L: 36.

T: No second chance because you are correcting her answer. (This carries on until prize is claimed.)

T: We were adding what types of numbers, now?

L: Whole numbers.

T: Last time we were adding fractions. In Grade 4 we added fractions with the same denominator. (Writes 1/3 + 2/3 =.) We said you add the numerators.

L: (Add numerators and the teacher writes “3/3”.)

T: Do we leave a fraction like this?

L: No.

T: You have to simplify the fraction. (She writes “1”.)

T: In Grade 5 and 6 you add fractions with different denominators. (Writes 1/6 + ½.) So how do we add fractions with different denominators? Do we just leave them as they are and say, “Ah these fractions have different denominators so we cannot add them”? What so we do?

L: You find multiples of two and six. (Teacher writes the multiples of two and six on the board.)

T: Step two?

L: Find the lowest common multiple.

T: What do we mean by LCM?

L: The smallest number.

T: 6, 12 and 18 are all common but we want the smallest of the all, which is?

L: Six.

T: (Writes LCM = 6.) So what does this mean? It means the denominators 6 and 2, when we change them, they must all be out of 6. (Writes.)

T: Do we leave a fraction like this?

L: (Most say yes and one says no.)

T: What should we do?

L: Simplify the fraction. (Gives the answer 2/3.)

T: Today we are adding mixed numbers. A mixed number is a number that is made up of a whole number and a fraction. (Writes.) Be careful how you write a mixed number. If you write … This is not a mixed number, the whole number must be same size as the fraction. Do you understand?

L: Yes.

T: Before we continue, we change mixed numbers to improper fractions. Do you remember that?

L: Yes, denominator × whole number + numerator, divided by denominator.

T: (Writes a top-heavy or improper fraction.) This is called a top-heavy fraction. Why is it called a top-heavy fraction or improper fraction?

L: Numerator is greater than the denominator.

T: (Calls two volunteers. Puts five apples on a plate and gives two to one pupil and cuts one apple into four equal parts. Demonstrates how to add mixed numbers using apples as her concrete materials. She gives one learner two and a quarter apples and the other, two and two quarters of an apple. Then she instructed one learner to add the two mixed numbers. The learner first takes the whole apples from each learner, then the "fractions". Then he counts the number of wholes and writes this on the board, then counts the quarters and write the fraction, 3(3/4).)

T: The volunteer first took the whole numbers. Why is it so easy? It is because the fractions have the same denominator.

T: Let me have two volunteers to add 1(2/8) + 1(1/8). It means our apples will be divided into how many parts?

L: Eight parts.

T: (Cuts apple into eight parts and distributes it according to the fractions.) Here is another method. You change the mixed number to a top heavy fraction. Now these two fractions have the same denominator, so we add the numerators. Are the answers not the same?

L: They are the same.

T: Now let us add mixed numbers which have fractions with different denominators. (Writes on the board.) The first step is the same, changing mixed numbers to top-heavy fractions. This takes us back where we started, fractions with different denominators. We start by accumulating multiples of the denominators. Remember, “Fire on the mountain, fire on the mountain”? You accumulate multiples of the denominators. (Lists multiples of three and two, lists common multiples, and identifies the LCM as six.)

T: This means our denominator will be?

L: Six.

T: (Works out problem with learners.)

APPENDIX F: DATA FROM PRE-OBSERVATION INTERVIEWS

Teaching and learning materials

1. **Do you use textbooks when preparing for lessons on fractions? If so, name them.**

Simon: I use Modern Basic Mathematics from South Africa, which is good for oral and short written exercises. It can define some mathematics words very well, If one is not mathematically inclined can use that book to get the meaning of each word.

Dan: I use the pupils' book and teachers' guide.

Pam: I. do. I use the pupils' book and the teacher's guide.

2. Do you use any other sources? If so, name them.

Simon: Like I said, I use a book called Modern Basic Algebra from South Africa.

Dan: No, I don't.

Pam: Yes, I do. Sometimes you find other books that simplify the concept clearer. For example, you find that other books break down the concept for Grade 6, They start the concept in Grade 1 to where you are in Grade 6. We get books from World Vision, which are donated to the school

3. Are the materials provided by the National Curriculum Centre adequate? If not, why?

Simon: I think they are adequate, but I may be biased as I am involved in writing those materials.

Dan: I believe so.

Pam: No. Sometimes you find other books that simplify the concept clearer. For example, you find that other books break down the concept for Grade 6, they start the concept in Grade 1 to where you are in Grade 6.

1. How do you select teaching and learning aids for your lessons?

Simon: I think of my learners and their level of understanding.

Dan: Interesting. What I usually do is, my wife is a mathematician, so I just bounce some thoughts with her, we just share. So when I'm not too sure about a lesson I usually bounce ideas with her.

Pam: I use the teaching materials and my experience of teaching fractions. What works and what doesn't work.

Do primary teachers value the use of multiple representation in teaching fractions?

1. What kind of learning aids do you usually use? (Manipulatives or visual aids.) State the reason for using those representations?

Dan: I usually combine them, diagrams and concrete materials to cater for all learning styles, to ensure that no child is left out.

Pam: I use diagrams and pictures but sometimes they are a bit confusing. But when you bring the concrete they enjoy the manipulation, they understand more clearly.

2. Do you always use more than one representation to illustrate fractions?

Simon: Yes I do.

Dan:Yes I do.

Pam: Yes.

3. What motivates you to use more than one representation when teaching fractions?

Simon: It is just motivating the pupils to get the real concept. Using one thing you may say they are able to do, but have few examples that you have put for them. But if you have many examples, some examples they have experienced in their lives. So it is better to make more so that one can make a relationship to what he or she has experienced before.

Dan: It's mainly understanding that children have different levels of understanding, you can't just concentrate on one method of teaching or representation because you find that others would benefit. But you will find that you have lost a number of them, so you have to try to do as much as you can to try and accommodate all of them.

Pam: When you have used more than one representation you can tell when it comes to evaluation, everybody wants to show the teacher that they have understood.

4. How do learners respond to the use of multiple representations?

Simon: Learners become excited. But it depends on how they are introduced. If you introduce the chart first it creates confusion. It should be real life first.

Dan: I find that it is actually interesting to them, they enjoy seeing things. It works for me because the learners tend to be interested then, than when you keep talking and talking.

Pam: They are so excited. They all want to volunteer yet they cannot. They are so motivated. Once you put the visual aids on the table everybody wants to volunteer, then the teacher has to choose, otherwise there will be confusion because everyone wants to participate.

G.DATA FROM FOLLOW-UP INTERVIEWS

1. Do teachers value the use of multiple representation in teaching fractions?

Simon: Learners become excited. But it depends on how they are introduced. If you introduce the chart first it creates confusion. It should be real life first.

Dan: I find that it is actually interesting to them, they enjoy seeing things. It works for me because the learners tend to be interested then, than when you keep talking and talking.

Pam: They are so excited. They all want to volunteer yet they cannot. They are so motivated. Once you put the visual aids on the table everybody wants to volunteer, then the teacher has to choose otherwise there will be confusion because everyone wants to participate.

2. Do you think using multiple representation when teaching fractions can help learners understand fractions?

Dan: Yes, I believe so, so far it has worked for me because like I said you find that you lose some but you are to be able to help them when you try something else.

Pam: The visual and concrete aids go very well with fractions. You seem to flow once they have manipulated. The fraction concept is too abstract for the learners.

3. Has the use of multiple representation benefited your learners?

Simon: Yes, a lot. When you assess them you can tell by the way they respond to oral or written questions during the lesson or at the end of the lesson.

Dan: I believe so, because when I mark for them even those who are slow, you have won a big percentage of the students.

Pam: Yes, they follow the lesson, no one seems lost. When you have used more than one representation you can tell when it comes to evaluation, everybody wants to show the teacher that they have understood.

4. Have you learned anything from using multiple representation in your lesson?

Simon: As you have seen, I started with an activity involving the learners themselves. If you introduce the visual aids first, it creates confusion. It I had started with the chart and paper strips it would have been difficult for them to understand the meaning of a whole. So you have to start with real life first. That is why I used the desk and taxi as examples in real life.

Dan: Yes, you know we learn every day, as you try this and that. You must have noticed that at the beginning, there are some students that I kept calling on at the back, but today as we kept trying this and that, they were able to concentrate and grasp. You learn that one should not stick to one teaching style, try to diversify your teaching. You find that one method will win even that child whom most teachers had written off.

Pam: Yes. They help a lot, they help the learners' understanding. But you have to prepare a lot, sometimes use your own money to buy apples. You have to think, what visual aids you need, and how can I use them to benefit the learners?

A. How are representations used within different contexts?

1. Do you feel that you have used the various representations the way you intended?

Simon: I used them the way I wanted to use them. In some of them I have to change along the way, catching on the understanding of the pupils, then you have

Dan: Yes, I believe so.

Pam: Yes, I did.

2. What preparation did you have to do in order to use these visuals in your class?

Simon: I sit down think of my lesson, think of my learners, their level of understanding because I have to cater for the different capabilities.

Dan: Interesting. What I usually do is my wife is a mathematician, so I just bounce some thoughts with her, we just share. So when I'm not too sure about a lesson I usually bounce ideas with her.

Pam: I had to ask another teacher. I consult other teachers in the school.

3. In what ways can the use of multiple representation help teachers improve their teaching?

Simon: It depends on the teachers understanding of the content. If you are lacking in content knowledge, it becomes difficult to think about the content. If you for instance you know from your own experience where you struggled, hence you can think of ways of teaching that will benefit learners. How can I teach the content which I struggled to understand while in school? So the more teaching aids you have, the more effective the teaching will be.

B. What support do teachers need to use multiple representation?

1. Did you need training to use multiple representation in the classroom?

Simon: We all go through pre-service training, so I believe we learn something from college. But nowadays people go to college to get a certificate and earn money, not to teach people. I say this because most of these teachers cannot even differentiate between a factor and a multiple.

Dan: I believe so because I don't think I am at the point where I can say I am doing my best. I think I can really appreciate something that would boost what I already have.

Pam: Yes, because you can have visual aids and be not able to use them. Like I said, researching is important and consulting other teachers. Other teachers can help you in identifying representations to use and how to use them to clarify a concept. Because you can have a lot of concrete and visual and not be able to use them.

2. If you had computers in the school, would you use them for teaching mathematics?

Simon: I think if you have computers you can use them as long as you know how to operate a computer. Teaching with computers can mean less work for teachers because in a computer you prepare you lesson and put in the visual aids you want to use. Even if you have a challenge in drawing accurate diagrams, using computer technology you can draw perfect diagrams. Computers are also helpful in terms of storing information, I can easily retrieve stored information.

Dan I believe so because I don't think I am at the point where I can say I am doing my best. I think I can really appreciate something that would help. What I already have. Like sitting down and understanding what is appealing to children now because you may find that things have evolved. Right now you don't know what is really working, you are still stuck. Like today, the way the children responded you may think it's working but you may find that there is something even more that can work even better and faster.

Pam: Yes we do, with technology, computers can help us a lot. It is easier to manipulate fractions on the screen.

APPENDIX H: INDIVIDUAL FOLLOW-UP INTERVIEWS

1. SIMON

Q: All your lessons begin with an activity? Why is that?

R: The method of teaching that we use must be child-centred.

Q: Why do you always begin each lesson with what you call “human activity”? And how does it help the learners?

R: It is because of the way of learning that I have learned from my panel, that the best concrete object is the person by himself which means the human resource. Then you can come to other things, but the human resource is the best because it is always available. It is very beneficial to learners because they can remember well if they have taken part.

Q: You introduced the lesson by asking questions on division of whole numbers. Why did you do that?

R: The word divide is used to emphasise that when you are dealing with fractions you are dealing with division. As you saw in today’s lesson, a fraction was interpreted as the numerator divided by the denominator.

Q; You kept on emphasising that they shouldn’t say “over”. Why?

R: It’s a misconception to say “over” because in English when you say over you mean something that as gone past or overflowing; it’s more than what is necessary.

Q: You used a desk and kombi as examples of addition of fractions. Can you explain?

R: The desk means full capacity of the objects, as it was only able to hold four and a kombi can carry only fifteen and that means it is one whole. If I have less than fifteen, it means I have a fraction of the kombi.

Q: What if the pupils were big and only two could be seated at the desk?

R: Well that is just capacity, the amount that it can carry to its brim. It is not exact number.

Q: You used the learners, stones and diagrams as your visual representations. How does it help the learners?

R: The first one, using human resources, represents real life, followed by the counters representing concrete, then diagrams for semi-concrete.

Q: Are you saying it makes it easier for the learners to understand?

R: Yes, without starting with these concrete objects involving themselves, it's hard for them to understand.

Q: Today you used learners, desk and diagrams, then gave them a symbolic problem to solve using diagrams.

R: I used the diagram to emphasise the importance of the equal-ness of divisions because in most cases pupils cannot understand a fraction, because some teachers make the parts unequal, but in a fraction all parts must be equal.

Q: You used a rectangle; why did you choose this particular shape? Why not a circle?

R: Drawing a circle requires a lot of mathematics, so it could have taken me a long time to draw. I would need a protractor to ensure that all angles are equal. If you do not consider that when dividing your circle, you are creating misconceptions in the learners.

Q: You gave your learners a symbolic problem which they had to translate to an area model to find solution.

R: That was interpretation, symbolic to diagram and vice versa. If you cannot use numerals, use diagrams.

Q: Which language do you use?

R: I use both languages. If I use SiSwati it is only because I can see that a learner does not understand. I make an example in SiSwati.

Q: Are they allowed to respond in SiSwati?

R: Yes, we do allow them in order to get what they have. If I can say in English they will be quiet, no response, so you will not be able to evaluate yourself if you are teaching effectively or not.

2. DAN

Q: When writing fractions, you emphasised that learners should not use /, but — ?

R: I wouldn’t say there is a really good reason. I’m looking at their level, like I was saying, it is not wrong, but at this level if you can get the proper thing right, then you can play around later. If you can get the basics right then later on you can. I was just saying let’s do the same thing now then later on we can play around.

Q: What is your experience of teaching fractions?

R: Sometimes I feel like I’m not reaching all the students, like the previous topic on fractions, I had to consult my colleague, Mrs T. I’m not very comfortable teaching fractions. It is not an easy topic.

Q: You have computers in the school. Are you allowed to use them for teaching mathematics?

R: That is an idea we have never explored. But when I was teaching in a private school we used computers for teaching.

Q: I noticed that you always use English in your lessons. Is there a reason for that?
R: This school is actually known as an English-medium school. When I came here I found that it is a norm, that is how things are done, but as a teacher you are flexible, based on your learners. When I came here I discovered that these learners, most of them understand English easily. And coming from a private school where the language of instruction has always been English, so I just got used to teaching in English, not to alternate the languages. But when I was teaching in one of the rural schools, there I used both languages.
Q: What was your reason for using both languages?
R: I used both languages because I could see that the English language was a challenge for most of the student, so I used to come down to their level, try and … to English at the same time. Because at the same time you know that mathematics is in English, so you have to teach in English.
Q: You have other nationalities besides Swazis in your classroom?
R: Yes, I have Zimbabweans and Indians who do understand SiSwati and to accommodate those you have to stick to English.
Q: When teaching addition and subtraction of fractions you used both circle and rectangle area models, why?

R: These are the shapes they are familiar with. More so, using the circle is to accommodate the divisions, when you go up to eight. It is not easy with the triangle. There is no specific reason, just to show them that you can use different shapes.
Q: You came with shapes already drawn, any reason for that?
R: That was part of the guidance from the teachers' guide, it says you must come with them already drawn and demonstrate.
Q: I observed that when you are teaching you always use the whole group discussions and demonstrations. Why is that?
R: First of all, I believe that children should first see it from the teacher. You can talk about it, then you have to demonstrate it, then you use questions where they can actually give you feedback. Grouping could also work if you have enough material or if whatever you want to discuss demands that they work in groups. I felt that the lesson did need that much (grouping) to group them, but just to interact with them, that they understand what you have demonstrated. It was not like that was the only perfect method. If I had noted that, that the method did not work, maybe I would have tried something else. If the feedback says you not going together then you can try something else.

3. <u>PAM</u>

Q: Which types of visual aids do you use?

R: I use diagrams and pictures but sometimes they are a bit confusing. But when you bring the concrete they enjoy the manipulation, they understand more clearly.

Q: So what you are saying is before you can introduce diagrammatic representation, you start with concrete objects?

R: Using the concrete first helps them to draw the diagrams easily because they are able to relate the diagram to the concrete. Each time the pupil draws a diagram he will remember the concrete objects he has manipulated.

Q: You mentioned earlier that it is easier for pupils to represent fractions on a number line after manipulating concrete objects.

R: Yes, because when you are representing a number sentence on fractions on a number line, from the concrete it will help the child to divide the number line accordingly. For example, when multiplying three by a fourth, when using apples, each apple will be divided into four equal parts. Then when you go to the number line, mark it from zero. From zero the child will divide equal spaces equivalent to the division of the whole, in this case, four. From zero to one there will be four divisions.

Q: Where do you get the supplementary materials?

R: We get books from World Vision, which are donated to the school. As you go through the boxes sometimes you get good book. Not only mathematics, even other subjects like English.

Q: Which representations do you avoid? I noticed you do not use real-life problems.

R: Word problems are a challenge to learners. Pupils fail to understand the problem.

Q: I noticed that you used English throughout the lesson.

R: We are trying. In lower grades (Grades 1 to 4) they use both languages for clarification. In upper grades we have to use English because the examination paper is written in English.

Q: Do you ever use SiSwati?

R: Yes, I do, to clarify a point, but it must not be used a lot.

APPENDIX I: TURNITIN REPORT

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APPENDIX J: ETHICAL CLEARANCE

UNIVERSITY OF KWAZULU-NATAL
INYUVESI YAKWAZULU-NATALI

04 April 2016

Ms Thabisile P Dlamini 214583611
School of Education
Edgewood Campus

Dear Ms Dlamini

Protocol reference number: HSS/0491/015M
Project title: An investigation into the use of multiple representations in the teaching and learning of fractions at primary level in Swaziland: A case of Manzini Region.

Expedited Approval
In response to your application dated 18 May 2015, the Humanities & Social Sciences Research Ethics Committee has considered the abovementioned application and the protocol have been granted **FULL APPROVAL.**

Any alteration/s to the approved research protocol i.e. Questionnaire/Interview Schedule, Informed Consent Form, Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through the amendment/modification prior to its implementation. In case you have further queries, please quote the above reference number.

Please note: Research data should be securely stored in the discipline/department for a period of 5 years.

The ethical clearance certificate is only valid for a period of 3 years from the date of issue. Thereafter Recertification must be applied for on an annual basis.

I take this opportunity of wishing you everything of the best with your study.

Yours faithfully

Dr Shenuka Singh (Chair)

/px

cc Supervisor: Mrs Barbara B Goba
cc Academic Leader Research: Professor Pholoho Morojele
cc School Administrator: Ms Tyzer Khumalo

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APPENDIX K; EDITOR'S REPORT

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Language Specialist

Editing, copywriting, indexing, formatting, translation

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23 December 2016

To whom it may concern

This is to certify that I, Alexa Kirsten Barnby, ID no. 5106090097080, a full-time language practitioner employed by the University of South Africa and accredited by the South African Translators' Institute, have edited the master's dissertation "An investigation of the use of multiple representation in teaching fractions at primary school level in Swaziland" by Thabisile Priscilla Dlamini.

The onus is, however, on the author to make the changes suggested and to address the comments.

AK Barnby