CAPTURE EFFECTS IN SPREAD-ALOHA PACKET PROTOCOLS

BY

VUYOLWETHU MAXABISO WESSELS MPAKO

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ABSTRACT

Research in the field of random access protocols for narrow-band systems started as early as the 1970s with the introduction of the ALOHA protocol. From the research done in slotted narrow-band systems, it is well known that contention results in all the packets involved in the contention being unsuccessful. However, it has been shown that in the presence of unequal power levels, one of the contending packets may be successful. This is a phenomenon called capture. Packet capture has been shown to improve the performance of slotted narrow-band systems.

Recently, much work has been done in the analysis of spread-spectrum ALOHA type code-division multiple access (CDMA) protocols. The issue of designing power control techniques to improve the performance of CDMA systems by reducing multiple access interference (MAI) has been a subject of much research. It has been shown that in the presence of power control schemes, the performance of spread-ALOHA CDMA systems is improved. However, it is also widely documented that the design of power control schemes capable of the ideal of compensation of radio propagation techniques is not possible for various reasons, and hence the imperfections in power control.

None of the research known to the author has looked at capture in spread-ALOHA systems, and to a greater extent, looked at expressions for the performance of spread-ALOHA systems in the presence of capture.

In this thesis we introduce spread-ALOHA systems with capture as a manifestation of the imperfections in power control. We propose novel expressions for the computation of the performance of spread-ALOHA systems with capture.

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1 INTRODUCTION

1.1 Media Access Control Protocols in Wireless Local Area Networks

In the 21^{st} century, media access control (MAC) protocols are extensively finding use in Institute of Electrical and Electronic Engineers (IEEE) standards like wireless local area network (WLAN). The first WLAN standard, IEEE 802.11, was adopted by the IEEE in 1997. The predicted deployment of 802.11 networks for the next decade resembles that of the Internet during the early 90s. In public places such as campuses and corporations, WLAN provides convenient network connectivity and high speeds up to 11 Mbps. Similar to any 802.x protocol, the 802.11 standard defines the MAC and physical (PHY) layers for a LAN with wireless connectivity.

The IEEE 802.11 PHY layer can use any of the following modes: (1) direct-sequence spread-spectrum (DS/SS); (2) frequency-hopping spread-spectrum (FH/SS); (3) Infrared pulse modulation. The IEEE 802.11 MAC layer is responsible for a structured channel access scheme and is implemented using a *distributed coordination function* (DCF) based on the *carrier-sense multiple access with collision avoidance* (CSMA/CA) protocol. An alternative to the DCF is also provided in the form of a *point coordination function* (PCF), which is similar to a polling system for determining the user with the right to transmit.

The CSMA/CA based MAC protocol of IEEE 802.11 is designed to reduce the collisions due to multiple sources transmitting simultaneously. In these networks, each node with a packet to transmit first senses the channel to ascertain whether it is in use. If the channel is sensed to be idle for an interval greater than the *distributed inter-frame space* (DIFS), the node proceeds with its transmission. If the channel is sensed busy, the node defers transmission until the end of the ongoing transmission. The node then initializes its *backoff timer* with a randomly selected *backoff interval* and decrements this timer every time it senses the channel to be idle. The node is allowed to transmit when the backoff timer reaches zero. Since the backoff interval is chosen randomly, the probability that two or more stations will choose the same backoff value is very small.

There is another particular feature of WLANs, known as the *hidden terminal* problem, that the 802.11 MAC specification addresses. Two stations that are not within hearing distance of each other can lead to collisions at a third node which receives the transmission from both sources. To address this problem, 802.11 MAC uses a reservation-based scheme. A station with a packet to transmit sends a *ready-to-send* (RTS) packet to the receiver and receiver responds with a *clear-to-send* (CTS) packet if it is willing to accept the packet and is currently not busy. All the nodes within hearing distance of either the sender or receiver detect this RTS/CTS exchange, which also contains the timing information about the length of the ensuing transaction, or both and they defer their transmission until the current transmission is complete.

There are currently four specifications in the IEEE 802.11 family: 802.11, 802.11a, 802.11b and 802.11g. All four specifications use CSMA/CA.

- The 802.11 specification applies to WLANs and provides 1 or 2Mbps transmission in the 2.4GHz band using either FH/SS or DS/SS.
- The 802.11a specification is an extension to 802.11 that provides up to 54Mbps in the 5GHz band. 802.11a uses orthogonal frequency division multiplexing (OFDM) rather than the FH/SS or DS/SS.
- The 802.11b specification provides 11Mbps transmission (with a fallback to 5.5, 2 and 1Mbps) in the 2.4GHz band. 802.11b uses DS/SS, allowing wireless functionality comparable to Ethernet. 802.11b is also referred to as Wireless fidelity (Wi-Fi).
- The 802.11g specification offers wireless transmission over relatively short distances at 20 to 54Mbps in the 2.4GHz band. 802.11g uses OFDM.

This thesis describes an alternative MAC protocol that can be used in 4^{th} Generation networks.

1.2 Motivations for Research

From as early as the 1970's, research has been conducted in the area *random access* (RA) MAC, with the introduction of the ALOHA family of protocols for narrowband systems by Abramson [Abr70]. The pure ALOHA protocol is the most basic protocol in this family of protocols. The pure ALOHA protocol states that a newly generated packet is transmitted immediately with the hope that there is no interference by other users. Should the transmission be unsuccessful, every colliding user reschedules its transmission to a random time in the future. The slotted ALOHA variation of the ALOHA protocol introduced a slotted channel. This reduced the vulnerable period to a single slot and hence the improvement in the throughput of the system.

In the presence of unequal received power levels at the central receiving station, a single packet with the highest received power level may be successful in the presence of a phenomenon called the *capture effect*. The capture effect has been shown to improve the capacity and the performance of slotted ALOHA systems.

On the other hand, research in spread-ALOHA systems has been conducted since the 1980's. In code division multiple access (CDMA) particularly, *power control* has been proposed as a technique to mitigate the effects of *multiple access interference* (MAI). Power control ensures that all transmitting users are received with equal power levels at the central receiving station. This technique has been shown to improve the performance of CDMA type protocols.

However, ideal compensation for signal fading dips cannot be practically achieved with power control techniques, hence the imperfections in power control. It has been shown by many researchers that the imperfections in power control lead to an increase in the *bit-*

error rate (BER) and hence a reduction in the system throughput. Most researchers have suggested two approaches to calculate the BER of spread-spectrum ALOHA systems. These being accurate BER approximations, the most cited and widely used are the *standard Gaussian approximation* (SGA) and an averaged BER expression proposed by Prasad *et al* in [NMP92], [NP93] and [NWP95].

This thesis was motivated by the apparent lack of research incorporating the capture effect in spread ALOHA systems with imperfections in power control and defining the throughput of spread ALOHA protocols in terms of packet capture. In this thesis we say that in spread ALOHA protocols, power control errors manifest themselves in user's transmissions being received with unequal power levels at the central receiving station. These unequal received power levels lead to the capture effect, as is the case in slotted ALOHA. We define accurate expressions for packet capture in spread ALOHA systems to incorporate power control errors.

1.3 Thesis Outline

This thesis begins by giving the thought behind and the motivation of the research. The literature survey presented in Chapter 2 was the beginning of the research. Chapter 2 looks at MAC protocols, concentrating on the random access ALOHA protocols and the direct-sequence spread-spectrum protocols. Chapter 2 mostly concentrated on the implementation aspects of direct sequence spread-spectrum protocols like power control, soft handovers, and RAKE receivers. Some of the newer MAC protocols are also provided in this chapter for the readers' benefit.

Chapter 3 examines the SGA and several other Gaussian approximations for the BER of *direct-sequence code-division multiple access* (DS-CDMA) systems. Chapter 3 looks at using an averaged BER expression to calculate the throughput of DS-CDMA in the presence of power control errors. Chapter 3 then looks at the system capacities for the different BER approximations.

In deriving expressions for capture models, the approximation that the sum of correlated lognormal variables is a lognormal random variable is used. Chapter 4 thus examines some of the most widely used and documented lognormal sum approximations, viz. the Fenton-Wilkinson Approximation that was originally published in 1960 and the Schwartz-Yeh Approximation published in 1982. Chapter 5 presents the derivations for the capture models developed using the approximations presented in Chapter 4. Chapter 5 also presents the throughput calculations and the system capacity in the presence of a central base station capable of multiple packet reception.

Finally, this thesis concludes with a review of the significant observations and contributions of this research. The author realizes that this work has only scratched the surface of the problem of power control errors and the inclusion of packet capture due to power control errors in spread-spectrum ALOHA systems. It is thus intended for this research to merely raise awareness of the possible implications of power control errors and hope that it will prove seminal for future work on this issue.

1.4 Original Contributions

This thesis makes one major contribution: the development of an analytical model for spread-ALOHA systems that incorporates the capture effect (presented in Chapter 5).

The following publications have resulted from this work:

- V. M. W. Mpako and F. Takawira, "Effects of Power Control on the Performance of Spread ALOHA Protocols", Proceedings of COMSIG'98, Cape Town, South Africa, pp. 149 – 154, September 1998.
- V. M. W. Mpako and F. Takawira, "Capture and Imperfect Power Control in Spread-ALOHA Systems with Lognormal Shadowing", Proceedings of AFRICON'99, Cape Town, South Africa, pp. 245 – 250, October 1999.

2 OVERVIEW OF MAC PROTOCOLS

2.1 Introduction

In any means of communication, MAC protocols (also called multiple access protocols) are needed to moderate access to whatever shared medium by defining rules for the communicating devices to talk to each other in an orderly and efficient manner. The study of MAC protocols has been done extensively since the 1970s [Abr70]. Several ways have been suggested in the literature on how to divide MAC protocols into groups [Sun89], [RM90]. In our study, we classify MAC protocols into three main groups: the contentionless protocols, also often referred to as scheduled protocols; contention protocols; and a hybrid class of contentionless and contention protocols that attempts to use the best properties of the two above-mentioned classes.

The contentionless group of protocols tries to avoid situations where more than one user access the same channel at the same time, referred to as *contention*, by scheduling transmissions of the users. The contention is avoided by either permanently allocating part of the channel to a particular user, giving rise to the fixed assignment group of contentionless MAC protocols, or by scheduling users to occupy part of the channel when they have something to transmit, giving rise to the demand assignment group of contentionless MAC protocols.

The contention group of protocols traditionally has conflicts caused by the fact that users access the channel without knowing if there exist other users transmitting in the channel. Effectively, a user tries to access the channel when it has something to transmit, in a random manner, hence being called *random access protocols*. Several schemes are adopted to resolve conflicts.

The chapter is organized as follows: Section 2.2 deals with the fixed assignment contentionless MAC protocols. Section 2.3 reviews random access protocols, of which part of this dissertation is based. Performance measures of random access protocols that are pertinent to this thesis are dwelled on as well as the subject of stability of random access protocols.

In section 2.4, demand assignment protocols are reviewed. This chapter concludes with section 2.5 that discusses multiple access issues in direct sequence spread-spectrum protocols. Implementation issues of direct-sequence spread-spectrum multiple-access as used in direct-sequence code divisional multiple access protocols are also briefly reviewed, giving some useful references for the interested readers. Section 2.6 outlines the spread ALOHA protocols which are essentially a combination of narrowband random access protocols and direct sequence spread-spectrum protocols.

2.2 Fixed Assignment Protocols

Fixed assignment protocols are a form of contentionless MAC protocols. These are based on a fixed channel assignment process and are designed to share a fixed bandwidth among a certain population of terminals; essentially being circuit switched techniques. There are traditionally three examples of fixed assignment MAC protocols: *frequency division multiple access* (FDMA) where the division is done in frequency, *time division multiple access* (TDMA) where the division is done in time and *code division multiple access* (CDMA) where the division is done by allocating codes to the transmitting entities. These protocols have been dealt with extensively in the books by Martin [Mar78], Stallings [Sta85] and Kuo [Kuo81].

The FDMA protocol divides the channel bandwidth into smaller, non-overlapping frequency bands. Each transmitting user thereby has uncontested use of only one of these sub-bands. Each user is thus equipped with a transmitter and receiver for a given frequency band. The frequency bands for transmit and receiver may be different. The main advantage for FDMA is its simplicity in that traditionally it does not require any coordination or synchronization among the users since each can use its own frequency band without interference. A problem arises when different users have different loads and have uneven long-term demands. It is however possible to divide the frequency range unevenly, proportional to the demands.

The TDMA protocol divides time into consecutive frames and each frame is sub-divided into a number of non-overlapping time slots. Each user is assigned a fixed part of each frame, not overlapping with parts assigned to other users. Effectively, during the assigned slot, the entire system resources are devoted to that user. The slot assignments follow a cyclic pattern that periodically repeats itself. For proper operation of the TDMA protocol, all the users need to be synchronized so that all the users know exactly when to start and stop transmitting. The problem with this protocol is that when the communication requirements of the users in the system are unequal, the channel utilization is not efficient. In the generalized TDMA protocol, a user might be allocated more than one slot within a frame, with arbitrary distances between successive allocated slots, to get better channel utilization.

Unlike FDMA and TDMA systems, CDMA systems make division of transmission of different users by assigning each user a different code. This code is used to transform a user's signal into a wideband, spread-spectrum signal. At the receiver the multiple wideband signals received are transformed back to the original signals using the code assigned to the individual users. During this process, the desired signal power is compressed into the original signal bandwidth, while the wideband signals of the users remain wideband signals and appear as noise (interference) when compared with the desired signal.

If the number of interferers is not too large, the *signal-to-noise ratio* (SNR) of the desired signal is large enough for the desired signal to be extracted without error. CDMA thus acts as a contentionless protocol. If the number of interferers becomes too large, larger than some threshold, the interference becomes too large for the desired signal to be extracted and contention occurs. The protocol is hence interference limited. We can thus say that CDMA is contentionless until the number of users exceeds some threshold, after which it becomes a contention protocol.

CDMA protocols are classified into several variants based on the modulation scheme used. There are generally four types of CDMA protocols: direct-sequence CDMA (DS-CDMA), frequency-hopped CDMA (FH-CDMA), time-hopped CDMA (TH-CDMA) and hybrid CDMA that uses a combination of the modulation methods of the other protocols.

2.3 Random Access Protocols

In the purest form of random access protocols, a user getting ready to transmit does not have any knowledge of when it can transmit without interfering with the transmissions of other users, i.e. there is no scheduling. Users contend for the channel in a random manner. Several users that start their transmissions at the same time *may* lead to their transmissions colliding, with the effect of some or all of their data being discarded. The possibility of collisions makes the occurrence of a successful transmission a more or less random process. The random access protocol should have the ability to resolve the contention that occurs when several users transmit simultaneously. The following sub-sections review the main variants of random access protocols.

2.3.1 The p-ALOHA Protocol

The ALOHA family of protocols was first proposed by a group of researchers at the University of Hawaii [Abr70]. This is probably the richest family of MAC protocols due to its seniority as it is the first random access technique introduced; and secondly, due to its simplicity and ease of implementation.

The pure ALOHA protocol is the basic protocol in this family of MAC protocols. The system is defined as a single-hop system with an infinite population generating packets of equal length T according to a Poisson process with a packet arrival rate of λ packets/sec. If the transmission of a packet does not interfere with any other packet transmissions, the transmitted packet is received correctly; while if two or more packet transmissions overlap in time, a collision is caused and none of the colliding packets is received correctly. The users schedule the collided packets for retransmission after a random back-off delay. The randomness of the retransmission delay ensures that the same set of packets does not continue to collide indefinitely.

The assumption that the composite arrival distribution of new and retransmitted packets has a Poisson distribution is a good one in that it makes the analysis of ALOHA-type systems tractable and predicts successfully their maximal throughput. With this assumption for the arrival distribution, the probability of having m arrivals in 1 second at a packet arrival rate of λ packets/sec is given by

$$f(m) = \frac{\lambda^m}{m!} e^{-\lambda} \tag{2.1}$$

The reader is referred to the works of Kleinrock and Lam [KL75] for a complete analysis of ALOHA type protocols. In analyzing the performance of random access protocols, we define the points at which packets are scheduled for transmissions to include both the generation times of new packets and the retransmission times of previously collided packets as the scheduling points. Let the rate of scheduling points be g packets/sec, where g is referred to as the offered load of the system. If we consider a packet is scheduled for transmission at some time t, it will be successful if no other packet is scheduled for transmission in the interval$(t-T, t+T)$, with the period $2T$ being referred to as the *vulnerable period*. For a Poisson composite arrival distribution, the probability of success of a packet is given by

$$P_{suc} = e^{-2gT} \tag{2.2}$$

The *system throughput*, defined as the fraction of time that the channel carries useful information, non-colliding packets, is given by [KL75]

$$S = gTe^{-2gT} \tag{2.3}$$

The *normalized offered load* to the channel defined as the rate at which packets are transmitted in the channel is denoted by $G(= gT)$. The channel throughput is thus

$$S = Ge^{-2G} \tag{2.4}$$

The *capacity* of p-ALOHA protocols, defined as the value of G at which S takes on its maximal value, is at $G = 1/2$, at which $S = 1/(2e) \approx 0.18$.

2.3.2 The slotted-ALOHA Protocol

The slotted ALOHA protocol is a variation of the p-ALOHA with a slotted channel and a slot size of T equal to the packet transmission duration. Users transmissions are restricted to the slot boundaries, hence reducing the vulnerable period from $2T$ for p-ALOHA to T for slotted ALOHA. Using the Poisson composite arrival distribution, the throughput of slotted ALOHA protocols is given by

$$S = Ge^{-G} \tag{2.5}$$

The capacity of slotted ALOHA is thus $1/e \approx 0.36$, achieved at $G = 1$.

In order to analyze the packet delays of the system, we consider a slotted ALOHA protocol with a finite population of N users as was done in Kleinrock and Lam [KL75]. Each of the N users has a single packet buffer. All packets have the same length of T seconds, which is also the slot-duration. The users have two modes of operation – *thinking* and *backlogged*. In the thinking state, a user generates new packets with a probability p_o and does not generate a packet with probability $1 - p_o$. Packet generation is an independent geometrically distributed process with mean $1/p_o$. Newly generated packets are transmitted in the next slot. If the packet transmission is successful, the packet generation process starts again. If packet transmission is unsuccessful, the user enters the backlogged state. In the backlogged state, the user will retransmit its packet with a probability p_r and does not retransmit with probability $1-p_r$. When the packet is finally successful, it goes back to the thinking state. Once a model for the new packet generation and retransmission processes is formulated, the composite arrival distribution can be found.

Let $\widetilde{N}(t)$ denote the channel backlog, which is the number of backlogged users at the beginning of the t^{th} slot. This is also called the *state* of the system. The state-transition of the users is independent of the activities in any previous slot; hence the process is a Markov chain. The number of backlog users cannot exceed *N*, hence the chain is finite. Also a steady-state distribution exists, hence the Markov chain is ergodic. The one-step state transition probability for $\widetilde{N}(t)$ is given by

$$P_{ij} = \Pr\{\widetilde{N}(t+1) = j \,|\, \widetilde{N}(t) = i\},\ i, j = 0,1,2,\ldots, N \tag{2.6}$$

The retransmission process of every user is an independent geometric process; hence the probability that i out of j backlogged users will be scheduled for transmission in any given slot has a binomial distribution given by

$$\Pr\{i \text{ backlogged users in a slot} \mid j \text{ in backlog}\} = \binom{j}{i} p_r^i (1-p_r)^{j-i} \tag{2.7}$$

Similarly, we obtain the probability of having *i* thinking users as

$$\Pr\{i \text{ thinking users in a slot} \mid j \text{ in backlog}\} = \binom{N-j}{i} p_o^i (1-p_o)^{N-j-i} \tag{2.8}$$

Finally the matrix **P** whose elements are P_{ij} is found to be

$$P_{ij}=\begin{cases}0 & j<i-1\\ \left[ip_r(1-p_r)^{i-1}\right](1-p_o)^{N-i} & j=i-1\\ \left[1-ip_r(1-p_r)^{i-1}\right](1-p_o)^{N-i}+\left[(N-i)p_o(1-p_o)^{N-i-1}(1-p_r)^i\right] & j=i\\ \left[(N-i)p_o(1-p_o)^{N-i-1}\right]\left[1-(1-p_r)^i\right] & j=i+1\\ \binom{N-i}{j-i}p_0^{\,j-i}(1-p_0)^{N-j} & j>i+1\end{cases} \tag{2.9}$$

Since there are only a finite number of users (states) in the system, there exists a steady-state probability vector, $\overline{\Pi}=[\pi_0,\pi_1,\cdots,\pi_N]$ given by the solution to the following $N+1$ linear simultaneous equations,

$$\overline{\Pi}=\overline{\Pi}\cdot\mathrm{P} \text{ and } \sum_{i=0}^{N}\pi_i=1 \tag{2.10}$$

The throughput of the system, defined as the long-term fraction of time the channel carries useful information, is effectively the probability that only a single packet transmission occurs within a particular slot. This means that either all backlogged users remain silent and a single new user transmits, or a single backlogged user transmits while no new packet is generated. For i backlogged users, this is given by

$$\begin{aligned}P_{suc}(i)&=\Pr\{\text{successful slot} \mid i \text{ users in backlog}\}\\&=(1-p_r)^i(N-i)p_o(1-p_o)^{N-i-1}+ip_r(1-p_r)^{i-1}(1-p_o)^{N-i}\end{aligned} \tag{2.11}$$

The steady-state throughput is thus

$$S=P_{suc}=E[P_{suc}(i)]=\sum_{i=0}^{N}P_{suc}(i)\pi_i \tag{2.12}$$

We now consider a special case where the packet generation and retransmission processes have the same probabilities, that is $p_o=p_r=p$. The probability of packet success is thus given by

$$P_{suc}(i) = Np(1-p)^{N-1} \tag{2.13}$$

In this case, the normalized offered load is given by

$$G = Np \tag{2.14}$$

The system throughput is thus

$$S = E[P_{suc}(i)] = G\left(1-\frac{G}{N}\right)^{N-1} \tag{2.15}$$

The throughput for the infinite population is similarly given by

$$S = \lim_{N\to\infty} G\left(1-\frac{G}{N}\right)^{N-1} = Ge^{-G} \tag{2.16}$$

Therefore, it can be concluded that the infinite population model is the limit of the finite population model if backlogged users are not distinguished from the thinking ones and if the number of users is increased under the constraint that the total average arrival rate remains finite.

For system stability, the average rate of new packet generation must equal the average rate of packet departure from the system. When the system is in state *i*, there are $N-i$ thinking users in the system, each generating packets with a probability of p_r. Therefore, the average packet generation rate in state i is $(N-i)p_r$. The expected throughput is thus

$$S = E[(N-i)p_r] = \sum (N-i)p_r\pi_i = (N-\overline{N})p_r \tag{2.17}$$

where $\overline{N}$ is the average number of backlogged users.

If the average rate at which users join the backlog is denoted by *b*, then by Little's theorem, the average backlog delay of the system is the ratio of the average number of backlogged users to the rate at which users join the backlog population or simply $\overline{N}/b$. A fraction, $(S-b)/S$, of the total population is never backlogged; they merely suffer a delay of 1 slot. All the other users, b/S, suffer a backlog delay of $\overline{N}/b$ plus one slot in which their transmission is successful. The average delay, normalized to the packet transmission time and measured in slots, is thus

$$\hat{D} = \frac{S-b}{S} \cdot 1 + \frac{b}{S} \cdot \left(\frac{\overline{N}}{b} + 1 \right) \tag{2.18}$$

Substituting the value of $\overline{N}$ from equation 2.17 gives the desired throughput-delay relation as

$$\hat{D} = 1 - \frac{1}{p_o} + \frac{N}{S} \tag{2.19}$$

We again consider the special case where the packet generation and retransmission processes have the same probabilities, that is $p_o = p_r = p$. The average delay is thus given by

$$\hat{D} = 1 + \frac{1-(1-p)^{N-1}}{p(1-p)^{N-1}} \tag{2.20}$$

An important observation that can be made from equation 2.20 is that $\lim_{p \to 0} \hat{D} = N$. This means that for a very small p, collisions hardly ever result. However, when a collision occurs, the backlogged users remain in the backlogged state for a very long time since the average waiting time for a backlogged user is proportional to $1/p$. In other words, most packets will have a delay of unity and very few packets will have extremely large delays, giving a combined average delay of N.

2.3.3 Stability Issues of the ALOHA Protocols

Issues with the stability of ALOHA protocols were first identified by Jenq [Jen80] and Lam and Kleinrock [LK75]. When dealing with the stability of ALOHA protocols, we look at these two underlying assumptions that have been made:

- The total arrival process of new and retransmitted packets is Poisson. There is no justification for this assumption, except that it simplifies the analysis of ALOHA protocols.
- The second assumption is the stability assumption, that is, the number of backlogged users with packets waiting to be retransmitted is not steadily growing. This implies that packets are entering and departing the system at the same rate.

Both these assumptions are false. An in-depth analysis and proof that these assumptions are false is given in Rom and Sidi [RM90] and Fayolle *et al* [FLDB74]. From the research done in this subject (Rom and Sidi [RM90], Fayolle *et al* [FLDB74], Ghez *et al* [GVS87] and Hajek and van Loon [HL82]) it can be seen that an infinite population ALOHA system is unstable for a retransmission scheme that does not, in some manner, take into account the system state. The retransmission scheme discussed thus far has used a fixed retransmission policy, regardless of the system state.

Several policies for stabilizing ALOHA protocols have been suggested including one in Ghez *et al* [GVS87]. In Rom and Sidi [RM90], the following *threshold policy* was suggested: There is a threshold, β, for the number of backlogged users that can retransmit at slot t, each of them retransmitting with probability p_r, as before. This means that when the number of backlogged users does not exceed β, each of them retransmits its packet with probability p_r. If the number exceeds β, a subset of β is chosen from the backlogged users and each user in this subset retransmits with probability p_r. All other backlogged users remain silent during this slot.

The works by Hajek and van Loon [HL82] suggested a retransmission policy that is based on updating the retransmission probabilities recursively in each slot, according to what happened during that slot. The policy was that the retransmission probability of a backlogged user in slot $k+1$ is some function of the retransmission probability is the previous slot and the event that occurred in slot k. The general structure of the policy is

$$p_r(k+1) = f(p_r(k), \text{feedback of slot } k)$$

The retransmission policies of the type described in [HL82] increase the retransmission probability when an idle slot occurs and decrease it when a collision occurs. The policies of this form were found to yield maximal stable throughput of at most e^{-1}.

2.3.4 Collision Resolution Algorithms (CRA)

The previous section has looked at the stability of the ALOHA family of protocols. We have noticed that the pure ALOHA type protocols do not attempt to resolve collisions; instead, attempts to resolve collisions are deferred to the future. In this section, we introduce a family of random access protocols that attempts to resolve collisions as soon as they occur. This family is called the *collision resolution protocol*, CRP.

The basic idea of the CRP family of protocols is to use the feedback information provided from the channel so as to control the retransmission process, hence resolving collisions. Moreover, in most implementations of these protocols, new packet arrival is inhibited while the resolution of collisions is in progress. This ensures that if the rate of arrival of new packets to the system is smaller than the rate at which collisions can be resolved, then the system is stable. The term *collision resolution algorithm*, CRA, broadly refers to any protocol that handles collisions by resolving them algorithmically.

The basic assumptions and the underlying model of the CRP family of protocols investigated is the same as the slotted ALOHA, viz:

1. *Synchronous slotted operation*: All transmitting terminals are slot synchronized to a central broadcast channel. Furthermore, messages are split into packets of fixed size.
2. *Errorless channel*: If a given slot contains a single packet transmission, then the packet will be received correctly by the central common receiver.
3. *Destructive interference among simultaneous transmissions*: If two or more packets are transmitted in a slot, a collision occurs and the packets involved in the collision have to be retransmitted. Thus, the state of the channel in any given slot is in one of:
   i. *idle*: no packet was transmitted,
   ii. *success*: exactly one packet was transmitted (in which case it was successful), or
   iii. *collision*: at least two packets were transmitted (and none of the packets is received correctly).
4. *Immediate (errorless) binary or ternary feedback*: At the end of the current slot, the users are informed of the state of the channel, before they need to make decisions for the actions in the following slot. The feedback is *ternary*, denoted by i, s or c for idle, success and collision, respectively. For some protocols, it suffices for the users to know whether the slot contained a collision or not. This is referred to as *binary* feedback. Furthermore, it is assumed that the feedback by the central controller represents the true state of the channel.
5. *Full or limited sensing*: Full sensing implies that all users keep track of the channel from the feedback information that they receive and that no new users may join the channel while the system is in operation.

The most basic CRP is called the *binary-tree protocol* and was proposed in the later 1970s by Capetanakis [Cap79] and Tsybakov [Tsy85]. In this protocol, if a collision occurs in slot k, all the users involved wait until the collision is resolved. The colliding users are split into two groups, say, by each flipping a coin. The users in the first subset retransmit in slot $k+1$ while the users in the second subset wait until all those in the first subset successfully transmit their packets. If in slot $k+1$ the channel is either in an *idle* or *success* state, the users from the second subset retransmit in slot $k + 2$. If the channel in slot $k + 1$ is in the collision state, then the procedure is repeated, i.e., the users whose packets collided in slot $k + 1$ are split into a further two subsets by flipping a coin and operate according to the outcome of the coin flipping, and so on.

It is said that a collision is resolved when the users of the system know that all packets involved in the collision have been successfully transmitted. The time interval starting with the original collision (if any) and ending when the collision is resolved is called the *collision resolution interval*, CRI.

In describing the operation of the protocol, the time when newly generated packets are transmitted for the first time needs to be specified. Some researchers have proposed a scheme called the *obvious-access algorithm*. According to this algorithm, during the resolution of a collision no new packets, i.e. packets that did not participate in the initial collision, may join the CRI. Instead, new packets wait until the collision is resolved, at which time they are transmitted.

The performance analysis of binary-tree protocols in terms of the moments of the conditional length of the CRI among n packets, the protocol's stability and its expected packet delay is deemed to be beyond the scope of this review, but we point the reader to the works by Massey [Mas81] for further treatment of the topic.

In the binary-tree protocol, when a collision is followed by an idle slot, the outcome of the next slot is predetermined to be another collision. A modification to the standard binary-tree protocol skips this step and proceeds directly to the next level of the tree. Subsequently, this enhanced binary-tree protocol is referred to as the modified binary-tree protocol.

The binary-tree protocol uses full-sensing of the channel. There are, however, *limited sensing* protocols that improve on the fact that some users may not receive the feedback information and start transmission, hence disturbing the channel. In the limited sensing algorithms, the users monitor the feedback signals only during limited periods, preferably after having generated a packet for transmission and until the packet is transmitted successfully.

Several limited sensing protocols have been proposed, the simplest being the *free-access* protocol proposed by Mathys and Flajolet [MF83] and by Fayolle *et al* [FLDB74]. In the free-access protocol, new packets are transmitted at the beginning of the slot subsequent to their arrival time. The user then monitors the channel for feedback information and then continues operating as an "old" user. The major advantage of this protocol is its simplicity although it is not the most efficient protocol, with maximal throughputs (channel capacity) of 0.360 achieved. According to the literature surveyed, the most efficient limited sensing protocol was proposed by Humblet [Hum86] and Georgiadis and Papantoni-Kazakos [GPK87]. This protocol is documented to having achieved a channel capacity of 0.487.

The performance of the binary-tree protocol can be improved in two ways. The first is to speed up the collision resolution process by avoiding certain avoidable collisions, as is the case in the *modified binary-tree protocols*. In the binary-tree protocol, when a collision is followed by an idle slot, the outcome of the next slot is predetermined to be another collision. The *modified binary-tree protocol* skips this step and proceeds directly to the next level of the tree. The second is based on the fact that collisions among a small number of packets are resolved more efficiently than collisions among a large number of packets. Therefore, if most CRIs start with a small number of packets, the performance of the protocol is expected to improve. The *"dynamic" tree algorithm* developed by Capetanakis [Cap79] uses this observation to improve the performance of the binary-tree protocol.

2.4 Carrier Sense Protocols

One of the limiting factors of the ALOHA family of protocols is that whenever a user has a packet to transmit, it does so without consideration of the transmission of other users. This results in the user in question's packet not being successful as well as the other user's packet that was currently transmitting not being successful. There is a family of random access protocols that avoids collisions by listening to the carrier due to another user's transmission before transmitting, and refraining from transmitting if the channel is sensed busy. This family is called *carrier-sense multiple access*, CSMA. Kleinrock and Tobagi have extensively researched this family of protocols in their publications [KT75a], [KT75b], [TK77], and [Tob80]. Unlike in ALOHA, in CSMA a user that senses a channel busy can take one of several actions. A user that senses the channel busy will retransmit its packet after a randomly distributed delay. The user senses the channel again and repeats the algorithm of the protocol if the channel is still busy.

The two main types of CSMA protocols are known as the *non-persistent* CSMA and the *p-persistent* CSMA, depending on whether the retransmission by a user that finds the channel busy occurs immediately or later following the current one with probability p. Many variants and modifications of these two schemes have been proposed.

In *non-persistent CSMA*, a user generates a packet and senses the channel. If the channel is sensed idle, the user transmits the packet. If the channel is found busy, the user schedules the retransmission of its packet to some later time according to the retransmission delay distribution.

The *1-persistent CSMA* protocol attempts not to let the channel go idle if the there is some user with data to transmit. If the user senses the channel idle, it transmits the packet with a probability of 1. If the channel is sensed busy, the user waits until the channel goes idle and then immediately transmits the packet with probability 1.

There are also versions of the CSMA protocols where the time axis is slotted. There exists a slot of duration τ seconds, where τ is the maximum propagation delay. This slot, referred to as a "minislot", is not the same as the transmission time of a user's packet. All users are slot synchronized to the minislot. Users with packets that arrive during a minislot wait until the next minislot, during which they start their transmission.
It should be noted that in the 1-persistant CSMA when two or more users become ready to transmit during a packet transmission period, they wait for the end of the current packet interval and then all start their transmissions with probability 1. This means that a collision will also occur with probability 1. To counter these almost certain collisions, the starting times of transmissions can be randomized according to some parameter p, hence reducing the probability of collisions and improving the throughput.

The family of protocols that have a parameter p for the probability that a ready user transmits and the probability of the user retransmitting its packet after a delay being $1-p$ is called *p-persistent CSMA*. In this protocol a user generates a packet and senses the channel at the beginning of a minislot. If the channel is idle, the user transmits its packet with probability p. If the channel is busy, the user delays its packet transmission with probability $1\text{-}p$. If the ready user senses the channel busy again, it waits until it becomes idle and then operates as above.

There is another variation of the CSMA protocols where, on top of sensing the channel, the users detect interference among several transmissions (including their own) and abort transmissions in the event of collisions. This is achieved by having each user compare the bit stream that it has transmitted to the bit stream that it sees on the channel. If this collision detection can be done fast enough, then the duration of an unsuccessful transmission can be reduced to less than the duration of a successful transmission. This variation is called *carrier-sense multiple-access with collision detect*, CSMA/CD. This scheme was presented in [TH80]. The operation of CSMA/CD protocols is similar to the normal operation of the CSMA protocols, except that if a collision is detected during transmission, then the transmission is aborted immediately and the users schedule their transmission to some later time. The CSMA/CD protocol is used in ETHERNET local

area networks (LANs). Implementation of collision detection in LANs is relatively simple since the transmitted and the received signals are of the same order of magnitude. In radio systems, this is not the case. The received signal in radio systems is considerably weaker compared to the transmitted signal, hence the implementation of collision detection is relatively more difficult in radio systems than in LANs.
Chen and Li [CL89] proposed another scheme where a collision is broadcast to all users to ensure that the duration of the collision is long enough, hence allowing all users to become aware of the collision.

In obtaining the throughput of the CSMA schemes, we assume that all packets are of the same length of T seconds, and the maximum propagation delay in the system is τ seconds, then we define a normalized propagation time $a = \tau/T$. Using our definition of G (the normalized offered load from section 2.3.1) the throughputs, S, for the various CSMA protocols in comparison to ALOHA protocols is shown in figure 2.1 with the normalized propagation time, $a = 0.1$. The reader is again referred to the paper by Kleinrock and Tobagi [KT75a] for a more in-depth analysis of CSMA protocols.

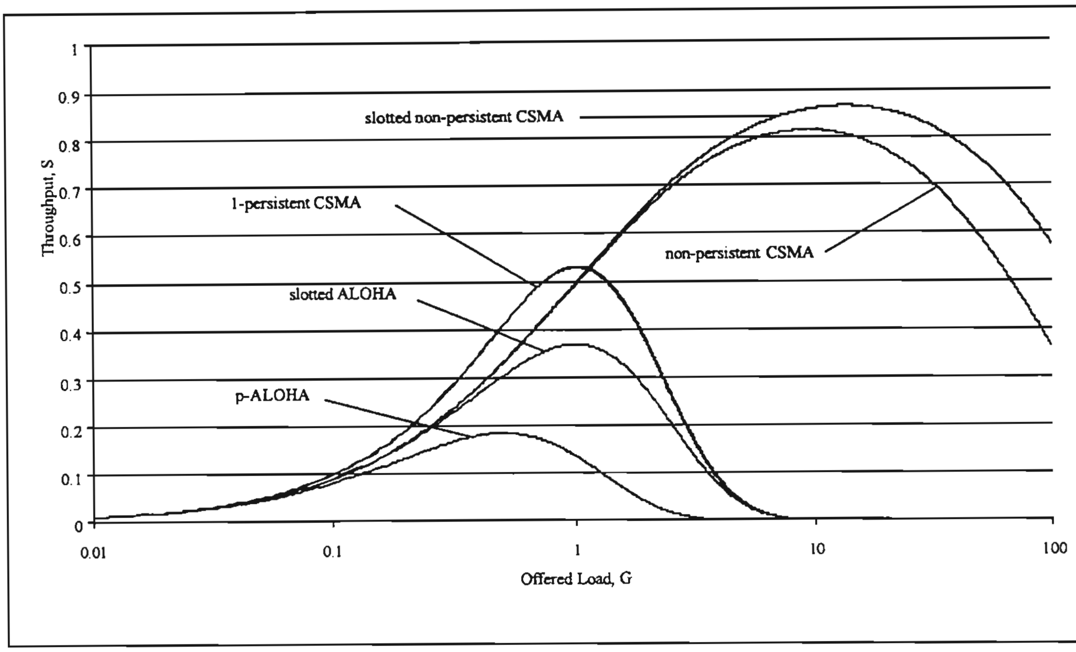

Figure 2.1 - Throughout for the various CSMA protocols in comparison to ALOHA

The performances of the various CSMA protocols were obtained based on the assumption that all users were in line-of-sight (LOS) and within range of each other. However, there are scenarios in which this is not the case. As an example, two terminals can be within range of the central controller but out-of-range of each other. Particularly in land mobile radio communications, two users can be separated by some physical obstacle that will not pass radio signals. The two users are said to be "hidden" from each other. The existence of these two hidden users greatly degrades the performance of CSMA protocols under the assumptions made above. This is referred to as the *hidden-terminal problem*.

A solution to the hidden terminal problem proposed by Tobagi and Kleinrock in [KT75b] is called the *Busy-Tone Multiple Access mode* (BTMA). In this mode, it is assumed that the central controlling station is within LOS of all users. The total available bandwidth is divided into two channels: a message channel and a busy-tone (BT) channel. As long as the central station senses a user's signal on the incoming channel, it transmits a busy-tone signal on the busy-tone channel. By sensing a carrier on the busy-tone channel, a user determines when the message channel is busy. The action taken by the user when it has sensed the busy-tone is dictated by the particular protocol being used.

In the non-persistent BTMA, which is similar to the non-persistent CSMA, whenever a user has a packet ready for transmission, it senses the busy-tone channel for t_d seconds (the detection time, which can be optimized), at the end of which the user decides whether the BT signal is present or not. If the user decides that the BT signal is absent then it transmits its packet; otherwise, it reschedules the packet for transmission at some later time incurring a random scheduling delay; at this new point in time, the user senses the BT signal again and the algorithm repeats. When a user fails to get an acknowledgement from the central stations, it means that a collision has occurred. The user then reschedules the packet for transmission at some later time, and the process repeats again. In [KT75b], the problem of detecting the busy-tone signal in the presence of noise is examined and the effect of various system parameters is characterized. This is deemed beyond the scope of this review and the reader is referred to [KT75b] for a comprehensive description of this subject.

2.5 Demand Assignment Protocols

The previous sections of this chapter have discussed two extremes in media access techniques. The first extreme, the fixed assignment protocols, have a rigid control on resources, are non-adaptive to changing resource demands, and can be resource wasteful if small delay constraints are to be met. The second extreme, the random access protocols, require no control, are simple to implement and adaptive to changing resource demands, but they can be wasteful on resources when collisions occur. Demand assignment protocols attempt to utilize both random access and fixed assignment protocols in enhancing the performance of this group of protocols. Demand assignment protocols are generally grouped into two classes that are distinguishable on the mode of control used. There are centrally controlled demand assignment protocols that are controlled by a central scheduler. There are also distributed controlled demand assignment protocols that employ a distributed algorithm executed by all users.

2.5.1 Centrally Controlled Demand Assignment Protocols

Here there are two methods: contention and polling. Both these methods require the presence of a central station that performs the control. In polling, the central station has a list that gives the order in which terminals are polled. The central controller sends polling messages to the terminals, one-by-one, asking the polled terminal to transmit. If the polled terminal has something to transmit, it goes ahead; if not, a negative reply or no reply is received by the controller, which then polls the next terminal in the order. The efficiency of polling is dependent on: small propagation delays, low overhead due to polling, and traffic that is not bursty. A performance analysis of polling that was done by Kleinrock and Tobagi in [TK76]. In [Hay78], Hayes proposed an adaptive polling scheme called *probing* that is based on a tree searching algorithm that tries to improve on the performance of polling protocols by decreasing the overhead incurred in determining which of the terminals have messages.

In a contention based demand assignment protocol, the terminal makes a request to transmit: if the channel is free, transmission goes ahead; if it is not free, the terminal must wait. The central station schedules the transmissions in a prearranged sequence according to some scheduling scheme or in the sequence in which the requests were made. However, since the radio channel is the only means of communications between the terminal and the central station, collisions will occur. The contention on the channel from the requests is exactly the same as discussed in random access protocols. In order to avoid collisions between the request packets and the actual message packets, the channel is either divided in time or in frequency. In [TK76], Kleinrock and Tobagi considered frequency division of the radio channel to form a scheme called *split-channel reservation multiple access* (SRMA). The radio bandwidth is divided into two channels: one used to transmit control information and another used for the data messages. In one mode of operation called the *request/answer-to-request/message scheme* (RAM), the bandwidth allocated for control is further divided into two channels: the request channel and the answer-to-request channel. The request channel is operated in a random access mode. When a terminal has data to transmit, it sends a request packet on the request channel with information about the message it wants to transmit. When the scheduling station correctly receives the request packet, the scheduling station computes the time at which the backlog on the message channel will empty and then transmits back to the terminal, on the answer-to-request channel, an answer packet containing the address of the answered terminal and the time at which it can start transmission. Another variation of the SRMA, called the RM schemes, has only the request channel and the message channel. When the scheduling station correctly receives the request, the terminal joins a request queue. Requests may be serviced on a "first-come first-served" basis or on any other scheduling algorithm. When the message channel is available, an answer packet containing the ID of a queued terminal scheduled for transmission is transmitted by the station on the message channel. The identified terminal starts transmitting its message on the message channel. If the terminal does not hear its ID within a certain set time interval after sending the request, the original transmission of the request packet is assumed to be unsuccessful and the request packet is retransmitted.

2.5.2 Distributed Controlled Demand Assignment Protocols

This class of demand assignment protocols offers higher reliability as the system is not dependent on the central controller. When the propagation delays are large, the performance of this class of demand assignment protocols is higher. The basic element underlying the operation of all distributed algorithms is the need to exchange control information among the users, either explicitly or implicitly. All users execute the same algorithm independently, resulting in some coordination in their actions. The system must be fully connected as all the users must receive the same information regarding the channel usage and the demand placed on the individual users. The requirement for a fully connected system exists in satellite systems. Also, the long propagation delay inherent in this class of demand assignment protocols makes satellite environments ideal for distributed controlled demand assignment protocols.

Most of the reservation-type multiple access schemes and their variants that have been researched belong to the family of distributed controlled demand assignment protocols. A class of demand assignment protocols with distributed control suitable for the satellite environment called Reservation-ALOHA (R-ALOHA) was originally proposed by Crowther *et al* in [Cro73]. In this protocol, the time axis is slotted with the slots organized into frames of equal size as in a conventional TDMA system, with the frame size greater than the satellite propagation delay. A user that has successfully accessed a slot in a frame is guaranteed access to the same slot in the succeeding frame and this slot is further reserved for this user in the coming frames, until the user stops using it. An unused slot in the current frame is a slot which was either idle or had a collision in the preceding frame. Unused slots are free to be accessed by all users in a slotted ALOHA fashion. Users only need to maintain a history of the usage of each slot for just one frame duration. This protocol is effective only if the users generate stream type traffic or long multipacket messages. The performance of this protocol degrades with single packet messages, as every time a packet is successful, the corresponding slot in the following frame is likely to remain empty.

Kleinrock and Scholl proposed a conflict-free multiple access scheme that was suitable for a small number of users in ground radio environments in [KS80] called *mini-slotted alternating priorities* (MSAP). This protocol uses the carrier sensing capabilities of each user. The time axis is slotted with the minislot size again equal to the maximum propagation delay as in the CSMA protocols. All users are again slot synchronized to the minislot. Users are considered to be ordered from 1 to M. When a packet transmission ends, the channel is assigned to the same user who transmitted the last packet if he is still busy (say user i); otherwise the channel is assigned to the next user in sequence (i.e., user (i, (mod M+1)). This is called the *alternating priorities* (AP) scheduling rule. By carrier sensing, at most one slot later, all users detect the end of transmission of user i (by the absence of the carrier). If there is no transmission, then:

1) Either, user (i mod M) + 1 starts transmission of a packet; in this case, one minislot after the beginning of his transmission, all others detect the carrier. They wait until the end of this packet's transmission and then operate as above.
2) or, user (i mod M) + 1 is idle; in this case, one minislot later, all other users detect no carrier; they then know that it is the turn of the next user in the sequence, i.e., user $[(i + 1) \bmod M] + 1$, and operate as above.

When all users are idle, the "turn" keeps changing at each minislot until it is the turn of a non-idle user.

In general, with MSAP, the M users are really passing around a token which gives them permission to transmit. The token in this case is silence. Scheduling rules other than AP are also possible, viz. round-robin (RR) or random-order (RO). MSAP exhibits the least overhead incurred in switching control between users. Mini-slotted round-robin (MSRR) is more suitable in environments with unbalanced traffic since the smaller users will be guaranteed more frequent access than with MSAP.

Packet-reservation multiple access (PRMA) was proposed by Goodman *et al* for indoor wireless communications in [GVR89]. In PRMA, time is divided into slots which are grouped into frames. The frame length is determined by the voice-packet generation rate. One voice packet is generated in a frame. A slot in a frame is either available or reserved by the voice user. Both voice and data users can contend for the available slots based on the voice-transmission probability p_t and the data-transmission probability p_r, respectively. If a voice user succeeds, this slot will be labeled as reserved and the voice user can use the corresponding slot in subsequent frames until the end of the current talkspurt. If a data user succeeds in the contention, the slot will be labeled as available, and the data user can only use this slot in the current frame and no reservation is allowed. There is no dedicated reservation packet needed in this scheme as the information packet is used for channel access.

Dynamic TDMA (D-TDMA) was first introduced in the 1970s in satellite communications [Abr70] and later proposed for third-generation wireless communications systems by Wilson *et al* in [WGJR93]. In D-TDMA, time is divided into a contiguous sequence of TDMA frames, which are subdivided into reservation slots, voice slots and data slots. The base-station keeps track of the information slots, which are either available or reserved. Two types of packets are transmitted in the channel: reservation packets and information packets. The reservation packet is used for the reservation of information slots (either voice or data slots), often including the origin and the destination addresses for data or the calling and called ID's for voice. The reservation packet is usually much shorter than the information packet.

A terminal generating a new voice talkspurt or a new data packet transmits the appropriate reservation packet in the reservation slots of the next frame, based on the voice-transmission probability p_t and the data-transmission probability p_r. If there is more than one packet transmitted in the same reservation slot, a collision occurs and all the reservation packets are destroyed. At the end of each reservation slot, the successful or unsuccessful reservation will be identified and broadcasted by the base-station. The unsuccessful user can retry in the next reservation slot with probability p_t or p_r. This reservation procedure is quite similar to PRMA.

The successful voice user will be assigned one of the available voice slots and will keep using it in subsequent frames until the end of the talkspurt. If there is no voice slot available, the voice user has to re-contend in the next frame's reservation slots. After the assignment of voice traffic, data users who transmitted successfully in reservation slots can use all the remaining slots, but cannot reserve them. If there is no slot available, the data user has to re-contend in the next frame. No reservation queue is maintained in the central controller.

2.6 Direct Sequence Spread-Spectrum Protocols

The spread-spectrum concept has emerged in answer to the unique needs of military communications. A basic description of spread-spectrum that characterizes its signal is as follows:

"Spread-spectrum is a means of transmission in which the carrier signal is an unpredictable, or pseudo-random, wide-band signal; the bandwidth of the carrier is much wider than the bandwidth of the data modulation; and reception is accomplished by cross-correlation of the received wide-band signal with a synchronously generated replica of the wide-band carrier."

Due to the nature of the signal characteristics of spread-spectrum systems, these systems have important performance attributes: low probability of interception; anti-jam capabilities; interference rejection; multiple access capability; multipath protection and immunity to selective fading; and, improved spectral efficiency.

The main parameter in spread-spectrum systems is the *processing gain* or the *spreading factor*. This is defined as the ratio of the transmission and information bandwidths. The processing gain determines the number of users that can be allowed in the system; the amount of multipath effect reduction; the difficulty to detect or jam a signal, etc. It is thus advantageous for spread-spectrum systems to have a processing gain as high as possible.

There are many different types of spread-spectrum systems. One way of categorizing them is by the modulation technique employed. The most common modulation techniques employed are the following: direct-sequence spread-spectrum (DS/SS), frequency-hopping spread-spectrum (FH/SS), time-hopping spread-spectrum (TH/SS). This thesis focuses on the DS/SS. However, a comprehensive study of these techniques is provided by Viterbi in [Vir95].

In direct-sequence spread-spectrum systems, the information bits are added in modulo-2 fashion to the pseudo-random noise (PN) sequence. The PN sequence acts as a noise-like (but deterministic) carrier used for bandwidth spreading of the signal energy. The selection of a good code is important because the type and the length of the code sets the bounds on the system capability. The PN code sequence is a pseudo-random sequence of 1's and 0's, but not a real random sequence. Some of the desirable properties of PN sequences are the following: the sequence elements should behave like independent identically distributed random variables, i.e., the sequence should be pseudo-random; it should be easy to distinguish a spreading signal from a time-shifted version of it; it should be easy to distinguish a spreading signal from other spreading signals, including time-shifted versions of them; it should be easy for the transmitter and the intended receiver to generate the spreading sequence; and, it should be difficult for any unintended receiver to acquire and regenerate the spreading sequence. The following types of PN codes have been suggested in the literature: m-sequence, Barker codes, Gold codes and Hadamard-Walsh codes. The reader is referred to the book edited by Skaung and Hjelmstad [SH85] for a more comprehensive review of PN codes.

The sum of the information bits of bit duration T_b and the PN sequence chips is then used to phase-modulate the carrier signal. The modulation of the PN sequence on the spread-spectrum carrier can be either binary phase or quadriphase. Binary phase-shift keyed (BPSK) modulation is one of the most widely used DS implementations. The received spread-spectrum signal for a single user can be represented as:

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cdot m(t) \cdot p(t) \cdot \cos(2\pi f_c t + \theta) \tag{2.21}$$

where E_b is the bit-energy, T_b is the information bit-period, $m(t)$ is the information sequence, $p(t)$ is the PN spreading sequence, f_c is the carrier frequency, θ is the carrier phase angle at $t = 0$.

The focus of our thesis is on the implementation aspects of DS/SSMA as used in DS-CDMA. The most important implementation aspects of DS-CDMA systems are: power control, RAKE receiver to combat multipath fading, the concept of soft handovers to reduce dropped calls, and multiuser detection to reduce the effects of multiple access interference. Power control forms the core of this thesis and will be as such handled in the next chapters. In this chapter, we give a brief review of the remaining concepts of DS-CDMA systems.

2.6.1 RAKE Receiver and Diversity Combining Techniques

Radio propagations in the land mobile channel are characterized by multiple reflections, diffractions and attenuation of the received signal. These are caused by natural obstacles such as buildings, hills, and so on; resulting in so-called multipath propagation. A direct sequence spread-spectrum signal waveform is well matched to the multipath faded channel. If the signals arrive more than one chip apart from each other, the receiver can resolve them. From each multipath signal's point of view, other multipath signals can be regarded as interference and they are suppressed by the processing gain. Small-scale changes, changes less than one chip, can be handled by a code-tracking loop. Code tracking loops track the time delay of each multipath signal.

Additional benefit in CDMA systems is obtained when a RAKE receiver is used to combine the resolved multipath signals. The RAKE receiver is a device that is well suited for demodulating a spread-spectrum signal in channels, where the signal bandwidth is much larger than the coherence bandwidth. The autocorrelation properties of direct-sequence spread-spectrum signals allow for using multiple correlators each of which uses a different time-shift to derive a different data estimate from the received multipath signal. By combining these estimates, a better estimate is obtained than of single correlators. A RAKE receiver consists of correlators, each receiving a multipath signal. Each correlator in a RAKE receiver is called a RAKE receiver finger.

After de-spreading by correlators, the signals from multiple antennas are combined to reduce the effects of fading and to improve the received signal strength. This is often called "spatial diversity". The base station receiver can either combine the outputs of its RAKE receiver fingers coherently, i.e., the outputs are added in voltage, or non-coherently, i.e., the outputs are added in power. The most popular diversity combining techniques are maximal-ratio combining (MRC) and equal-gain combining (EGC) [EKM96].

For coherent modulation with independent branch fading, MRC is the optimal linear combining technique. However, MRC is seldom implementable in a multipath fading channel because the receiver complexity for MRC is directly proportional to the number of resolvable paths or branches, L, that are available at the receiver.

For non-coherent combining, EGC is commonly used, whereby all available branches are equally weighted and then added incoherently. This combining technique also has the undesirable effect of having the receiver complexity dependent on L. Moreover, EGC is not optimal due the phenomenon called "non-coherent combining loss" whereby combining more signals does not necessarily enhance performance, especially for high BER's.

In the past, one simple suboptimal combing scheme that has often been mentioned in the literature is selection combining (SC), in which the branch signal with the largest amplitude (SNR) is selected for demodulation [EKM96].

2.6.2 Multi-user Detection

With the use of RAKE receivers, CDMA receivers only consider other users' signals as interference. This means that the capacity of DS-CDMA systems using only a RAKE receiver is interference limited. As new users, or interferers, enter the system, other users' quality of service degrades.

Multi-user detection (MUD), also referred to as *interference cancellation* in the literature, provides a means for reducing the intracell interference and thus increasing the system capacity. The concept of MUD was first proposed by Schneider in [Sch79]. The actual capacity increase depends on the efficiency of the algorithm, the radio environment, and the system load.

Verdú's work [Ver86] published in 1986, proposed and analyzed the optimal multiuser detector, or the maximum likelihood sequence estimator. Unfortunately, this detector is much too complex for practical DS-CDMA systems. The optimal detector provides huge gains in performance and capacity over the conventional detectors; it also minimizes the need for power control. Due to its complexity, a number of suboptimal multiuser and interference cancellation receivers have been developed in the late 1980s.

The suboptimum detectors can be divided into two main categories: the linear detectors and subtractive interference cancellation detectors. Linear detectors apply a linear mapping (transformation) into the soft outputs of the conventional detector to reduce the multiple access interference seen by each user. The two most popular of the linear detectors are decorrelator and linear minimum mean square error (LMMSE) detectors. In subtractive interference cancellation detection, the basic principle is the creation at the receiver of separate estimates of the multiple access interference contributed by each user in order to subtract out some or all the multiple access interference seen by each user. Such detectors are often implemented with multiple stages, where the expectation is that the decisions will improve at the output of successive stages. Examples of interference cancellation are parallel interference cancellation and successive (serial) interference cancellation. The reader is referred to the work by Moshavi [Mos96] for a more comprehensive study of multiuser detection.

2.7 Spread-spectrum ALOHA Protocols

In this chapter it has been shown that the main limiting factor of narrowband random access ALOHA type protocols is contention. That is, the ALOHA channel guarantees errorless access for only a single user. More than one user accessing the channel results in all the users' packets being corrupt i.e., the packet success rate is zero, assuming that there is no capture effect.

Fixed CDMA systems offer the advantage of allowing more than one user to access the channel, but with some error rate. The CDMA channel is almost contentionless in that collisions are not associated with a 100 percent packet error probability, but with an error probability that is a function of the number of contending users. The biggest drawback of CDMA systems is the increased bandwidth utilization.

Spread-spectrum ALOHA systems combine the concepts of CDMA with the concepts of narrowband random access protocols. In spread ALOHA systems, the error probability is dependent on the number of users. Users' transmissions are not successful due to *multiple-access interference* (MAI). As the number of contending users increases from zero to infinity, the packet error probability increases from almost zero towards unity.
Over the years, several authors have studied spread ALOHA protocols. In [Ray81], Raychaudhuri presented a comprehensive analysis of a slotted spread ALOHA with packet lengths equal to the slot size for both an infinite population model with a Poisson arrival process and a finite population model with a Markov arrival process. In the analysis of the finite population case for a p-persistent CRA, Raychaudhuri showed the system to be unstable for fixed values of p which were too large. His results showed that SS/ALOHA is no different to narrowband ALOHA systems with regards to system stability and channel efficiency. The shape of the throughput-delay curves characteristic of the ALOHA was the same as that of SS/ALOHA with the difference being that SS/ALOHA was a scaled up version of the narrowband case with respect to the average throughput per slot. The work showed that if the throughput of spread ALOHA systems, S, is divided by the spreading gain or the bandwidth expansion factor, N, then the normalized efficiency (S/N) of spread ALOHA is worse than that of slotted ALOHA systems. Effectively, the SS/ALOHA improves upon the narrowband throughput of 0.184 for p-ALOHA or 0.368 for slotted ALOHA, at the expense of the increased bandwidth due to spreading.

For this literature review, we presented a simplified analysis of a slotted SS/ALOHA system, not including the effects of fading, capture and additive white Gaussian noise (AWGN). The population model is assumed to be infinite (to simplify the analysis) and the combined arrival process is Poisson with a mean arrival rate of λ (packets per second). This assumption allows for easy modeling of the traffic arrival statistics and for determining the probability of having m users attempting to transmit during a time period τ (seconds), given that the mean arrival rate from all the users in the network is λ (packets per second). This probability of having m users attempting to transmit during a time period τ is given by

$$P(m,\tau) = \frac{e^{-\lambda\tau}(\lambda\tau)^m}{m!} \tag{2.22}$$

The SS/ALOHA system under consideration is time slotted with a slot size of t_s. The packet sizes have a fixed length of L bits and are transmitted in exactly one slot. It is assumed that the level of MAI is also constant during a slot as the number of users is also constant. It is also assumed that no *forward error-correction* (FEC) is used, hence the probability of success of a reference packet during k other spread-spectrum packet transmissions, $P_{PS}(k)$, is conditioned on the fact that all L bits in the packet are successful, i.e.,

$$P_{PS}(k) = [1 - P_{BE}(k)]^L \tag{2.23}$$

where $P_{BE}(k)$ is the bit-error probability (this may be approximated by the SGA provided in Chapter 3). The overall probability of packet success must include all possible values of k. If the load offered to the slotted-SS/ALOHA system is $G = \lambda t_s$, then the throughput S is given by

$$\begin{aligned} S &= \sum_{k=1}^{\infty} k \cdot P(m = k, \tau = t_s) \cdot P_{PS}(k) \\ &= Ge^{-G} \sum_{k=0}^{\infty} \frac{G^k}{k!} \cdot P_{PS}(k+1) \end{aligned} \tag{2.24}$$

where $P(m = k, \tau = t_s)$ is the Poisson arrival process given by equation 2.22. The throughput, S, for slotted SS/ALOHA are plotted in figure 2.2 for a bit duration, L, of

1024 bits and various processing gains, along with the throughput of a narrowband slotted ALOHA system.

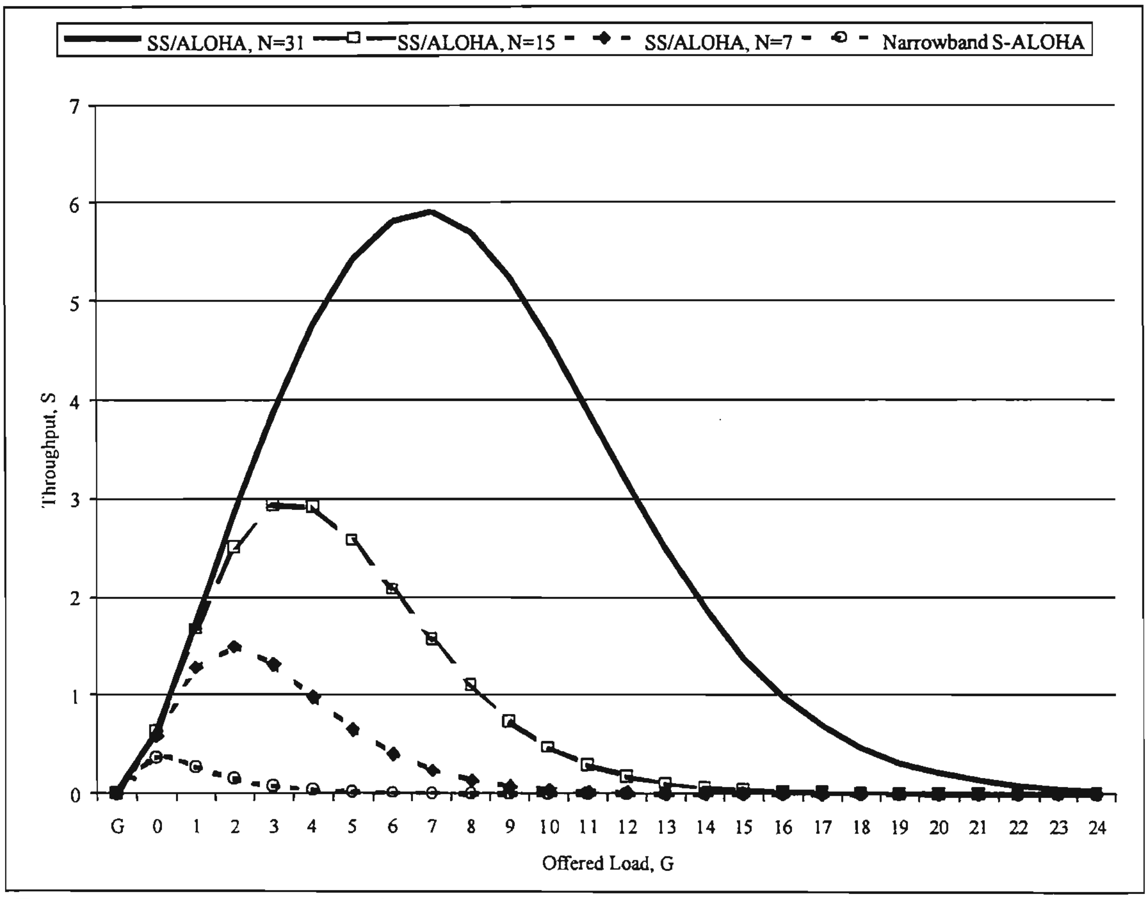

Figure 2.2 - Throughout of SS/ALOHA for the various processing gains in comparison to narrowband ALOHA

Storey and Tobagi [ST89] analyzed a spread ALOHA system with variable packet lengths with an exponential distribution in an infinite population model. They showed that spread ALOHA systems with longer packet lengths give better performance at high loads. This is somewhat intuitive since narrowband ALOHA only supports one packet at a time, and longer packets have a longer period of vulnerability. In [JR93], Joseph and Raychaudhuri extended Storey and Tobagi's work to analyze the effects of variable packet lengths in a spread ALOHA system with a finite population. They analyzed the packet arrival process from idle and backlogged users and showed that longer packets have lower success probabilities than shorter packets due to the longer vulnerability periods.

2.8 Summary

This chapter gave a literature survey of MAC protocols that are commonly used in applications. The MAC protocols were classified into fixed assignment (TDMA and FDMA), random access (the ALOHA protocols and the CSMA protocols), demand assignment (the centrally controlled and distributed controlled protocols) and then spread-spectrum multiple access protocols. The newer types of demand assignment protocols like PRMA and D-TDMA were also reviewed. More attention was given to the ALOHA type and the direct-sequence spread-spectrum protocols as they form a basis for this thesis. Performance measures and the applications of the protocols were also briefly discussed, in particular the applications of the protocols in land radio mobile communications. This chapter concluded by describing a combination of narrowband random access protocols and spread-spectrum protocols called spread ALOHA protocols.

3 PERFORMANCE OF SPREAD-SPECTRUM S-ALOHA SYSTEMS

3.1 Introduction

Power control is one of the most important requirements for DS-CDMA cellular systems. In the uplink direction (from the mobile station to the base-station), the need for power control arises due to the multi-user environment with multiple access interference. All the users transmit messages by using the same bandwidth at the same time. Users' transmissions thus interfere with each others' transmission, causing collisions. Moreover, when a non-reference user (interfering user) is much closer to the base-station than the intended reference user, it may happen that the interference caused by this non-reference user (however suppressed) has more power than the reference user. Now only the non-reference user will be received by the receiver at the base-station. This scenario is called the *near-far effect*. This has been shown to degrade the performance of DS-CDMA systems. To alleviate the near-far effect, all signals, irrespective of their distance from the base-station, should arrive at the base-station with the same power level. Power control attempts to achieve this constant received power level for each user. Due to power control, users closer to the base-station are not required to transmit as much power as users further away from the base-station. The battery life of mobile stations in cellular systems is thus prolonged. The implementation of power control also reduces interference in neighboring cells. Moreover, power control compensates for fading dips.

In the recent literature, several schemes have been proposed for implementing power control in DS-CDMA. An adaptive power control (APC) scheme is desirable to combat the effects of fading, shadowing and path losses. Such a scheme attempts to maintain a constant average performance among the users, minimize the required transmit power at each mobile station, and reduces multiple-access interference effects. For Universal

Mobile Telecommunications Systems (UMTS), two forms of APC have been proposed, viz. open-loop and closed-loop APC. The open-loop power control scheme is used prior to the mobile station initiating transmission on the random access channel (RACH) [MM95]. In such a scheme, the channel state on the downlink (base to mobile station direction) is estimated by the mobile by measuring the carrier received signal strength at the mobile, and this estimate is used as a measure of the channel state on the uplink. This technique assumes that the downlink and uplink paths are perfectly correlated. The mobile then calculates the required mean output power level required to achieve the access requirement of the cell it wishes to connect to. The mobile will now send its random access burst at this calculated value. If no positive or negative acknowledgement is received, the mobile will increase its power by some defined power-ramping factor and send a second random access burst. This process is repeated until an acknowledgement is received or the set maximum number of retries is reached. It should be noted that the implementation of this scheme is based on the assumption that the downlink and uplink channels are correlated. However, due to multipath fading, this is not necessarily the case.

An alternative to the open-loop power control scheme is closed-loop power control [AC93 & CMDR98]. In closed-loop implementations, the uplink channel is estimated by the base-station, which then transmits this information back to the mobile on the downlink for use in controlling the mobile's transmit power. In a terrestrial cellular environment, such a scheme is feasible under most conditions. However, the closed-loop scheme is typically not effective in land mobile satellite systems due to the sensitivity of the scheme to round trip delays. In UMTS, two closed-loop power control schemes are defined, viz. closed-loop power control inner loop and closed-loop power control outer loop. In the inner-loop closed-loop power control scheme, the aim is to adjust the mobile's transmitted power such that the received uplink signal-to-interference ratio (SIR) is kept at a given SIR target (SIR_{target}). A similar process is used in the downlink, to control the relative power weighting to be applied to each downlink dedicated channel.

While the inner-loop closed-loop power control is used to maintain a target SIR, the outer-loop power control adjusts the SIR target in the base station according to the needs of the individual radio link and aims at a constant quality, usually defined as a certain target bit-error rate (BER) or frame-erasure rate (FER). The SIR target is adjusted to compensate for variations in mobile speeds and multipath profiles when actually mobile.

Most of the work published on CDMA has assumed perfect power control, e.g. the work by Foschini and Miljanic [FM95]. It is known, however, that power control mechanisms that have been used cannot achieve ideal compensation for fading, resulting in imperfections in power control. The papers by Cameron and Woerner [CW96] and by van Nee *et al* [NWP95] have shown that imperfections in power control lead to an increased BER and hence a reduction in the system throughput.

Our area of interest is in giving quantitative investigation of the power control errors and the effects thereof on the performance of spread-spectrum ALOHA systems. The aim of this chapter is to provide a comparison of different approximation techniques for the evaluation of interference, and the consequent errors on the transmitted data. These approximations are used to define the SIR and BER which are used to estimate the performances of spread-spectrum ALOHA systems on uplink channels in the presence of power control errors.

Two approaches have been suggested. The first approach uses accurate BER approximations, the most cited and widely used being the so-called standard Gaussian approximation (SGA) that have been studied by Pursley in [Pur77], Morrow and Lehnert in [Mor98] and [ML89] and later by Sunay and McLane [SM96]. The second approach uses an averaged BER expression that was proposed by van Nee, Prasad *et al* in [NMP92], [NP93] and [NWP95].

This chapter is organized as follows. In Section 3.2, we introduce the system and the channel models. The SGA and several other Gaussian approximations for the BER are examined in Section 3.3. Prasad's averaged BER expressions are presented and analyzed in Section 3.4. Numerical results and discussions are provided in Section 3.5. A summary of the topics discussed in this chapter is provided in Section 3.6.

3.2 System Model

3.2.1 Network Model

The network under consideration is a packet single-hop star-network in which mobile stations communicate with each other, and with a backbone network, via a base-station using a shared wireless CDMA channel as in the papers by Mpako and Takawira in [MT99] and by Judge and Takawira in [JT00]. The forward link (downlink) and the reverse link (uplink) are assumed to be on separate frequency bands. Only the reverse link will be considered in this thesis as it is random access in nature and is inherently less efficient than the forward link.

Time is divided into fixed length slots of duration t_s, and it is assumed that all terminals in the cell are exactly slot synchronized with the base-station. Data messages may comprise several contiguous packets. Each packet has a fixed length packet size containing a fixed number of b bits such that each packet is transmitted in exactly one time slot. Each transmitted bit is spread using an N chip direct sequence PN spreading code. The base-station allocates a unique PN code to each mobile in its cell and is aware of the presence of each mobile user in the cell, and the codes allocated to them. Packets that are corrupted due to MAI are regrouped and retransmitted as "new" messages.

It is assumed that the cell size is small enough such that the round trip propagation delay (mobile station to base-station and back again) is always less than the slot size t_s.

3.2.2 Channel Model

The propagation is described by three effects: the attenuation due to the distance, r, proportional to $r^{-\eta}$, where η, the power loss exponent assumes a value of ≈ 4 in land mobile radio environments; the shadowing, described by means of a lognormal random variable; and the fast Rayleigh fading. The received power from a mobile at distance r can be expressed as

$$P_r = R^2 S r^{-\eta} P_T \tag{3.1}$$

where R is a Rayleigh distributed random variable with unit power; $S\ \left(=e^{\xi}\right)$ accounts for the lognormal shadowing $(\xi \sim N(0,\sigma))$ with a probability distribution function (pdf) given by

$$f(S) = \frac{1}{\sqrt{2\pi}\sigma S}\exp\left\{\frac{-(\log S-\mu)^2}{2\sigma^2}\right\} \tag{3.2}$$

where μ and σ are the mean and the standard deviation due to shadowing, respectively; and η is the deterministic path loss attenuation. It is assumed that the effects of Rayleigh fading are mitigated by the use of a RAKE receiver, coding and interleaving.

We use the same assumption that has been made in the literature, that power control schemes are unable to achieve ideal compensation for fading, hence giving rise to imperfections in power control. We further assume that the power control errors are lognormal distributed as is the case in the papers by Cameron and Woerner [CW96] and Viterbi *et al* [VVZ93]. The pdf of the received power is thus given by equation 3.2 where μ and σ are the mean and the standard deviations of the received powers.

3.2.3 Signal Model

Using the notations for the transmitted signal used in [ML89], the k^{th} user's transmitted signal is

$$s_k(t) = \sqrt{2P_k}\,b_k(t-\tau'_k)a_k(t-\tau'_k)\cos(\omega_c t+\theta_k) \tag{3.3}$$

where P_k is the power of the transmitted signal, ω_c is the carrier frequency, θ_k is the phase offset, $b_k(t)$, the data signal, is a sequence of unit amplitude rectangular pulses of duration T_b, $a_k(t)$, the spectral-spreading signal, is a sequence of unit amplitude rectangular pulses (chips) of duration T_c and τ'_k incorporates the differences in propagation and message start times as measured from a reference signal. The ratio of the bit duration to the chip duration is the main parameter for spread-spectrum systems called the processing gain or the spreading factor $(N = T_b/T_c)$. In the previous chapter, it was mentioned that the processing gain determines the number of users that can be allowed in the system.

The signals from each mobile station are sent on the wireless channel and the total received signal from the desired user and the $K-1$ undesired users is given by

$$r(t) = \sum_{k=0}^{K-1} \sum_{m_k=0}^{M_k-1} \sqrt{2P_k}\,\alpha_{k,m_k} b_k\left(t-\tau_{k,m_k}\right) a_k\left(t-\tau_{k,m_k}\right) \cos\left(\omega_c t + \phi_{k,m_k}\right) + n(t) \tag{3.4}$$

where $n(t)$ is the AWGN with power spectral density equal $N_0/2$; α_{k,m_k} is the path gain component which is equal to one for the additive white Gaussian channel and is a Rayleigh distributed random variable for the frequency non-selective fading channel; the variable ϕ_{k,m_k} includes the value of offset transmitted carrier, θ_k and the variable τ_{k,m_k} includes the channels delays τ'_k and τ''_{k,m_k}. This is the most general expression for the received signal because it considers both multipath and multi-service system.

The receiver tries to decode the received signal and to recognize a desired user with correlators and filters. Without loss of generality, the user zero, *0*, is the desired user while the other $K-1$ users are interference. The receiver takes back the signal to baseband and then multiplies it by the spreading signal of the desired user $(a_0(t))$ and integrates on the bit-period. The receiver is supposed to be synchronized, both in phase and in time, with the multipath zero. The decision maker, the m^{th} bit, can be described as in [SM96] as

$$
\begin{aligned}
Z_0(m) &= \int_{mT_b}^{(m+1)T_b} r(t) a_0(t-\tau_{0,0}) \cos(\omega_c t) dt \\
&= b_0(m) \alpha_{0,0} \sqrt{P_0/2} T_b + \sum_{k=0}^{K-1} \sum_{\substack{m_k=0 \\ m_0 \neq 0}}^{M_k-1} I_{k,m_k} + v
\end{aligned}
\tag{3.5}
$$

where $b_0(m)$ is the m^{th} bit from user *0*.

The noise and interference terms can be expressed as

$$
v = \int_{mT_b}^{(m+1)T_b} n(t) a_0(t-\tau_{0,0}) \cos(\omega_c t) dt \tag{3.6}
$$

$$
\begin{aligned}
I &= \sum_{k=0}^{K-1} \sum_{\substack{m_k=0 \\ m_0 \neq 0}}^{M_k-1} I_{k,m_k} \\
&= \sum_{k=0}^{K-1} \sum_{\substack{m_k=0 \\ m_0 \neq 0}}^{M_k-1} \int_{mT_b}^{(m+1)T_b} \alpha_{k,m_k} s(t-\tau_{k,m_k}) e^{j\phi_{k,mk}} a_0(t-\tau_{0,0}) \cos(\omega_c t) dt
\end{aligned}
\tag{3.7}
$$

The noise term is a zero mean Gaussian random variable with variance $\sigma_v^2 = N_0 T_b / 4$.

It is worth mentioning that the MAI term includes:

- All the multipath components relative to the desired user; $I_{0,1}, \ldots, I_{0,M_0-1}$, while the (0,0) component is the direct ray;
- All the direct and multipath components relative to the interfering users: $I_{k,0}, \ldots, I_{k,M_k-1}$ for all $k = 1,2,\ldots,K-1$.

Thus, the decision statistic defined in equation 3.5 can be re-written as

$$
Z_0(m) = D_0(m) + I + v \tag{3.8}
$$

where $D_0(m)$ is the desired signal component, the first term of equation 3.5, I is the MAI term given in equation 3.7 and v is the AWGN term of equation 3.6.

In multiple access systems, one of the fundamental design parameters is the SIR at the receiver. The SIR measures the ratio between the useful power and the amount of interference generated by all the other sources sharing the same communication channel. From the above equations, it is easy to express the SIR as

$$SIR = \frac{E\left[D_0(m)^2\right]}{E\left[(I+v)^2\right]} = \frac{E\left[D_0(m)^2\right]}{E\left[I^2\right]+E\left[v^2\right]} \tag{3.9}$$

The statistical averages in equation 3.9 can be calculated as follows: The variance for the desired useful term is given by [ML89] as

$$E\left[D_0(m)^2\right] = \frac{P_0 T_b^2}{2} \tag{3.10}$$

where P_0 is the transmitted power of the desired user and T_b is the bit duration.

For the noise term, the variance is given by

$$E\left[v^2\right] = \sigma_v^2 = \frac{N_0 T_b}{4} \tag{3.11}$$

where $N_0/2$ is the power spectral density of the AWGN, $n(t)$.

Expressions for the interference terms can be found in the works by Holtzman in [Hol92], Vojcic *et al* in [VPM94], Morrow and Lehnert in [ML89] and Sunay and McLane in [SM96]. All these expressions exploit the Gaussian approximation for the summations of the interfering terms and operate subsequent statistical averages over the environmental parameters, thus obtaining the variance of the interference term in purely AWGN channels to be

$$E\left[I^2\right] = \sigma_I^2 = \frac{N T_b^2}{6}\sum_{k=1}^{K-1} P_k \tag{3.12}$$

where N is the processing gain, T_b is the bit duration and P_k is the transmitted power of the k^{th} user.

3.2.4 Traffic Model

The traffic model considered in this thesis is similar to the one considered in the papers by Mpako and Takawira in [MT99] and by Judge and Takawira in [JT00]. The model assumes that the K users have two modes of operation – "idle mode" and "backlog mode". In the idle mode, there is no backlogged packet to be retransmitted, and new packets are transmitted in any given slot with probability p_0. The distribution of new message lengths, $L_0(l)$, is assumed to be geometric with parameter μ_0, and hence, mean length $\overline{L}_0 = \mu_0^{-1}$. Users enter the backlogged mode when an attempt to transmit a new packet fails due to MAI. In this mode, the retransmission of the backlogged packet occurs in any given slot with probability p_r. The distribution of retransmitted messages, $L_r(l)$, is more complicated to compute and depends on several factors such as the offered load and the partial message success rates. Once a model for the new packet generation process and the retransmission process is formulated, the required composite arrival distribution, $f_M(m)$, can be found. However, in our traffic model, we assume that message retransmission probability is the same as the new message generation probability (i.e. $p = p_0 = p_r$), hence the composite arrival distribution is Binomial and is given by

$$f_M(m) = \binom{K}{m} p^m (1-p)^{K-m} \tag{3.13}$$

If the combined distribution of the new packets and the retransmitted packets is Binomial with a mean arrival rate of λ packets per second and the slot size is t_s, then the *average offered load* G (packets per second) is given by

$$G = \lambda t_s \tag{3.14}$$

As already cited, the success of a packet of b bits depends on the packet being received at the central base station. That is, the probability of packet success can be given by

$$P_{Succ}^{Pkt}(k) = (1 - BER)^b \tag{3.15}$$

The number of successful packets per slot is called the packet throughput, S. This is found by averaging over all possible packet arrivals as

$$S = \sum_{k=1}^{K} k \cdot f_M(k) \cdot P_{Succ}^{Pkt}(k) \tag{3.16}$$

Approximate expressions for the BER of a spread-spectrum ALOHA system are provided in the next section.

3.3 BER Approximations

In the literature reviewed, power control errors have been shown to increase the BER and thus reduce the throughput. In our quest to give quantitative methods for describing power control errors, we present some of the well known techniques for approximating BER in spread-spectrum ALOHA systems. These are the standard Gaussian approximation and the improved Gaussian approximation that are based on the assumption that the sum of a large number of the independent identically distributed random variables is a Gaussian variable. The other technique uses an averaged expression for BER that was proposed by van Nee, Prasad *et al* in [NMP92], [NP93] and [NWP95].

3.3.1 The Standard Gaussian Approximation

The standard Gaussian approximation is based on the Central Limit Theorem (CLT). The assumption that is made is that the sum of a large number of independent and identically distributed (iid) variables is a Gaussian variable. That is, the bit decision statistic Z_0 may be modeled as a Gaussian random variable. This is, however, still an approximation, but the advantage is that it offers easy and fast computations.

Equation 3.8 gave the expression of the decision statistic of the transmitted bit as

$$Z_0(m) = D_0(m) + I + v \tag{3.17}$$

where $D_0(m)$ is the desired signal component and represents a deterministic variable given the transmitted bit, while I and v are independent identically distributed (iid) zero mean Gaussian random variables with variances given in equations 3.12 and 3.11, respectively. Z_0 is thus a Gaussian random variable with mean D_0 and variance given by $E[I^2] + E[v^2]$.

We define a Gaussian random variable ξ such that $\xi = I + v$. The variance is then given by

$$\sigma_\xi^2 = \sigma_I^2 + \sigma_v^2 = \frac{NT_b^2}{6}\sum_{k=1}^{K-1} P_k + \frac{N_0 T_b}{4} \tag{3.18}$$

Since the noise and interference terms are Gaussian, the probability of error over the channel can be given by the Q function of the SIR (given in equation 3.9) as

$$BER^{SGA} = Q\left(\sqrt{SIR}\right) = Q\left(\sqrt{\frac{D_0}{\sigma_\xi^2}}\right) = Q\left(\sqrt{\frac{P_0 T_b^2}{2\sigma_\xi^2}}\right) \tag{3.19}$$

where $Q(\cdot)$ is the standard Q function given by

$$Q(x) = \frac{1}{2} erfc\left(\frac{x}{\sqrt{2}}\right) \tag{3.20}$$

and the SIR from equation 3.9 is

$$SIR^{SGA} = \frac{D_0}{\sigma_\xi^2} = \frac{P_0 T_b^2}{2\sigma_\xi^2} \tag{3.9}$$

However, mobile radio environments are interference-limited and not noise-limited, hence the effects of the AWGN can be neglected (by setting the term $\sigma_v^2 = 0$ equation 3.11) and the BER thus given by

$$BER^{SGA} = Q\left(\sqrt{\frac{3N}{\sum_{k=1}^{K-1} \frac{P_k}{P_0}}}\right) \tag{3.21}$$

It should be noted that the above BER expression assumes some knowledge of the received powers, $\{P_k\}$. However, CDMA systems normally implement some form of the power control for alleviating the near-far effect and to ensure that all signals arrive at the receiver with the same powers, i.e. $P_k = P_0, \forall k$. Hence in the presence of perfect power control, the BER is given by

$$BER^{SGA} = Q\left(\sqrt{\frac{3N}{K-1}}\right) \tag{3.22}$$

Equation 3.22 thus gives the BER expression in the interference-limited case with perfect power control and is called the SGA approximation for the probability of bit errors.

3.3.2 Other Gaussian Approximations

The work by Pursley, Sarwate and Stark in [PSS82] developed upper and lower bounds for the probability of bit error that are accurate but computationally difficult. When these upper and lower bounds were compared to the SGA, it was shown that the SGA gives accurate results when the number of users, K, is large. Furthermore, even when K is large, in the presence of imperfections in power control, the MAI is not accurately modeled as a Gaussian random variable. This makes the use of the SGA inappropriate.

In the previous sections, the interference term, I, was a zero-mean Gaussian random variable with a variance given by

$$E\left[I^2\right] = \sigma_I^2 = \frac{NT_b^2}{6}\sum_{k=1}^{K-1} P_k \qquad (3.12)$$

This was obtained by subsequently averaging the summation in equation 3.7 over the distributions of the parameters defining the radio propagation pathϕ_k, τ_k, B. In essence, the expressions for the SIR and BER for a single user channel are evaluated as a deterministic measure of the radio propagation environment.

A different approach to the calculation of the bit error rate is given by Morrow and Lehnert in [ML89], which is based upon the presupposition that the MAI converges to a Gaussian random variable as the processing gain, N, becomes large for any number of simultaneous users, K, when the chip delays, S, and the carrier phases, ϕ, are fixed. Therefore, this approximation also uses the CLT, but now with the MAI being a Gaussian variable conditioned on the users states that are defined by the delay and the phase. When this is done, ψ is defined as the conditioned variance of the MAI for specific operating conditions. That is

$$\psi = \mathrm{var}\left(I \middle| \varphi_k, \tau_k, P_k, B\right) \qquad (3.23)$$

Hence the conditional variance of the MAI is itself a random variable. The SIR therefore becomes

$$SIR^{IGA}\Big|_{\psi} = \frac{P_0 T_b^2}{2\psi} \tag{3.24}$$

This expression is similar to the one presented in equation 3.9 with the fundamental difference being that equation 3.9 evaluated the SIR with the assumption that the average value of the variance of the MAI term is known. If the distribution $f(\psi)$ of ψ is known, then the SIR and the BER can be found by averaging over all possible values of ψ. This is shown in equations 3.25 and 3.26.

$$SIR^{IGA} = \int_0^{\infty} \frac{P_0 T_b^2}{2\psi} f(\psi) d\psi \tag{3.25}$$

$$BER^{IGA} = \int_0^{\infty} Q\left(\sqrt{\frac{P_0 T_b^2}{2\psi}}\right) f(\psi) d\psi \tag{3.26}$$

When the distribution of ψ is found, this Gaussian approximation, called the Improved Gaussian Approximation (IGA), is extremely accurate as measured against the bounds of the probability of bit error given by Pursley *et al* in [PSS82]. Furthermore, in the case of perfect power control, the technique given in equations 3.25 and 3.26 yields accurate results for a very small number of interfering users. In [ML92], Morrow and Lehnert provide a complete study of the computational complexity of the IGA. This thesis will not dwell on the topic, but just bring to light some of the approximations that have been proposed in the literature.

In [Hol92], Holtzman proposed another approximation for the probability of bit errors that does not require the distribution of ψ, but only requires the mean and the variance of ψ to be determined based upon the expansion of differences methods. This new approximation is referred to as the simplified improved Gaussian approximation (SIGA).

The simplified SIR and BER expressions are based on the fact that if $g(x)$ is a continuous function and x is a random variable with a mean value of μ_x and variance σ_x^2, then the average value $E[g(x)]$ can be expressed by making use of the Taylor's expression as

$$E[g(x)] = g(\mu_x + x) + \frac{1}{2}\sigma_x^2 g''(\mu_x) + \ldots \tag{3.27}$$

Further computational savings can be obtained by expanding in differences rather than derivatives, so that

$$E[g(x)] \cong g(\mu_x) + \frac{\sigma_x^2}{2}\frac{g(\mu_x + h) - 2g(\mu_x) + g(\mu_x - h)}{h^2} \tag{3.28}$$

Choosing $h = \sqrt{3}\sigma_x$, equation 3.28 then reduces to

$$E[g(x)] \cong \frac{2}{3}g(\mu_x) + \frac{1}{6}g\left(\mu_x + \sqrt{3}\sigma_x\right) + \frac{1}{6}g\left(\mu_x - \sqrt{3}\sigma_x\right) \tag{3.29}$$

Using equation 3.29, the expressions for the SIR and BER are thus given by

$$SIR^{SIGA} = \frac{2}{3}a + \frac{1}{6}b + \frac{1}{6}c \tag{3.30}$$

and

$$BER^{SIGA} = \frac{2}{3}Q\left(\sqrt{a}\right) + \frac{1}{6}Q\left(\sqrt{b}\right) + \frac{1}{6}Q\left(\sqrt{c}\right) \tag{3.31}$$

where

$$a = \frac{N^2}{\mu_x} \tag{3.32}$$

$$b = \frac{N^2}{\mu_x + \sqrt{3}\sigma_x} \tag{3.33}$$

and

$$c = \frac{N^2}{\mu_x - \sqrt{3}\sigma_x} \tag{3.34}$$

The values for μ_x and σ_x^2 are given in [Hol92] as

$$\mu_x = (K-1)\frac{N}{3} \tag{3.35}$$

and, for any $i \neq j$,

$$\sigma_x^2 = (K-1)\left[N^2\frac{23}{360} + N\left(\frac{1}{20}+\frac{K-2}{36}\right) - \frac{1}{20} - \frac{K-2}{36}\right] \tag{3.36}$$

The SIGA SIR and BER given in equations 3.30 and 3.31 consist of a weighted average of three positive quantities: a, which is the SGA; b, which is greater than or equal to a; and c, which is less than or equal to a. If the weights in equation 3.28 and the coefficient of σ_x are selected appropriately, the SIGA reduces to the IGA with less computational difficulties. Since μ_x is proportional to K and σ_x is proportional to $\sqrt{K}$, the SIGA converges to the SGA as K increases.

One shortcoming of the SIGA is that $\sqrt{3}\sigma$ can be greater than μ_x under some conditions for smaller values of K. This leads to the evaluation of c in equation 3.34 being negative and hence an imaginary Q function argument in equation 3.31.

3.4 The Averaged BER Model

In this section we present an expression for calculating the probability of bit error for BPSK modulated spread-spectrum ALOHA systems that was used by Prasad *et al*, in [NMP92] and [NP93], and by the author of this thesis in [MT99]. The signal model is identical to the one defined in Section 3.2 by equations 3.3 to 3.8. The probability of bit error, conditioned on the received powers, $P_{r,i}$, is given by

$$BER^{Ave}\Big|_{P_{r,i}} = Q\left(\frac{P_{r,i}}{\sqrt{2\sigma_\xi^2}}\right) \tag{3.37}$$

where σ_ξ^2 is the sum of the noise and the interference powers similar to equation 3.18 given by

$$\sigma_\xi^2 = \sigma_I^2 + N_0 T_b / 4 \tag{3.38}$$

In [NP93] it was shown that a closed-form expression can be derived for the variance of the K -1 interfering terms. This approach normalizes the sum of noise and interference powers given in equation 3.18 in Section 3.3 to give the normalized equation for the variance of the interfering terms as

$$\sigma_I^2 = \frac{2(K-1)P_{r,i}}{3N} \tag{3.39}$$

where $P_{r,i}$ is the received power of the i^{th} user, and is different for each user for the imperfect power control case.

The probability of bit error is obtained by averaging equation 3.37 over all possible values of $P_{r,i}$ using the expression for the pdf of $P_{r,i}$ given in equation 3.2. The average probability of bit error in the presence of imperfect power control is thus

$$BER^{Ave} = \int_0^\infty Q\left(\frac{P_{r,i}}{\sqrt{2\sigma^2}}\right) \cdot f(P_{r,i}) dP_{r,i} \tag{3.40}$$

where $f(P_{r,i})$ is the lognormal pdf of the received powers given in equation 3.1.

It should be noted that when we have perfect power control, $P_{r,i}$ is deterministic and the average probability of bit error found by substituting the variance from equation 3.39 reduces to the SGA given in equation 3.22.

3.5 Results

In this section the results of a set of numerical tests obtained through the implementation of the equations presented in the preceding sections of this chapter are presented. We consider the system model presented in section 3.2 in the implementations of the equations.

In the first scenario, we consider a system with perfect power control where the channel is modeled with fading. In the case of the SGA we use equations 3.9 and 3.21 for the SIR and the BER, respectively. In the case of the SIGA we use equations 3.29 and 3.30 for the SIR and the BER, respectively.

Figure 3.1 and figure 3.2 show a comparison of the SIR and BER for the SGA and SIGA approximations for different values of processing gains. In figure 3.2, it is shown that the SGA and the SIGA models achieve very similar BER values when the number of simultaneous users is large. However, much lower BER's are obtained with the SGA when the number of simultaneous users is small. These BER values have been shown to be very pessimistic in [PSS82], where accurate upper and lower bounds for the probability of bit error where obtained.

Figure 3.3 shows a comparison of the SGA BER and the BER from the Averaged BER model ($\mu = 0$, $\sigma = 1dB$). The Averaged BER model attempts to calculate the BER in the presence of power control errors by averaging the BER values across all possible values of the received power levels. This is achieved by integrating the BER expression with respect to the lognormally distributed received power levels.

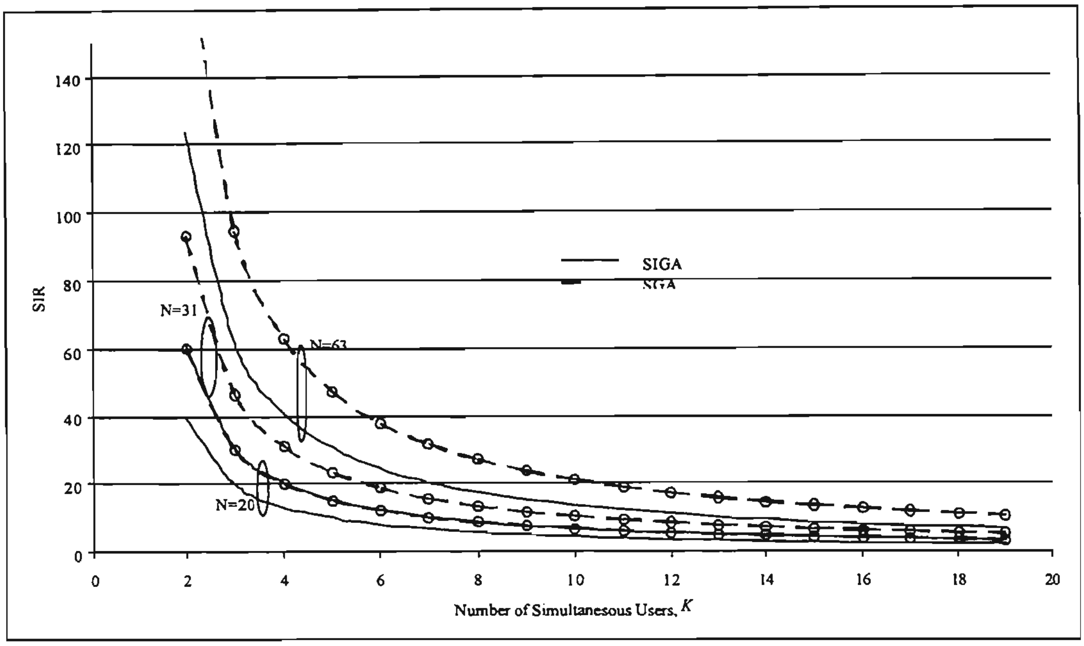

Figure 3.1 - SIR for the SGA and SIGA for different processing gains, N

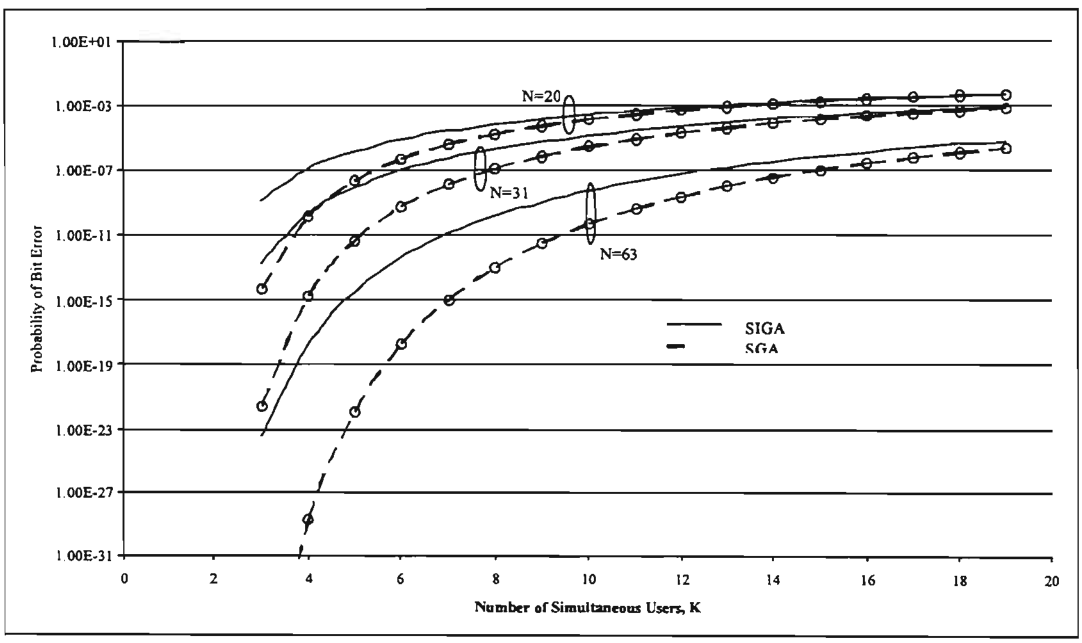

Figure 3.2 - Comparison of the BER for the SGA and SIGA for different values of the processing gain, N

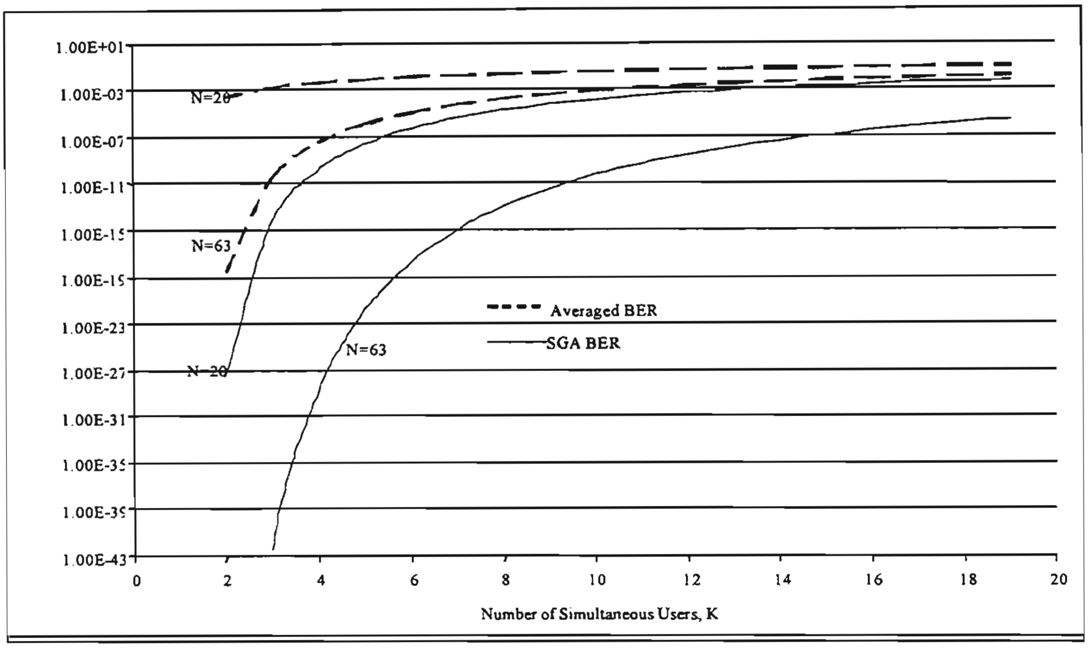

Figure 3.3 - BER for the SGA and the Averaged BER Models for different processng gains, N (μ = 0dB, σ = 1dB)

The BER approximations used in this chapter has assumed that the radio channel was only interference limited and not noise limited and hence ignored AWGN. In the presence of AWGN, the SNR is given by

$$SNR = \left\{ \frac{N_0}{2E_b} + \frac{1}{SIR} \right\}^{-1} \qquad (3.41)$$

where E_b is the energy per data bit, E_b/N_0 is the energy per data bit to the noise ratio that accounts for the AWGN, and SIR accounts for the signal-to-interference ratio given by $\frac{3N}{K-1}$ for the SGA approximation and by equation 3.30 for the SIGA approximation.

Tables 3.1 and 3.2 give a comparison of the SGA BER, SIGA BER with the lower and upper bounds for the BER given by Pursley *et al* in [PSS82]. In table 3.1 the upper and lower bounds for the BER are compared to the SGA BER and the SIGA BER for $K = 4$

and $N = 31$. In table 3.2 the upper and lower bounds for the BER are compared to the SGA BER and the SIGA BER for $K = 3$ and $N = 127$.

E_b/N_0	BER^L	BER^U	BER^{SGA}	BER^{SIGA}
4	1.87E-02	1.99E-02	1.88E-02	1.66E-02
6	5.85E-03	6.67E-03	5.92E-03	4.56E-03
8	1.36E-03	1.74E-03	1.37E-03	8.16E-04
10	2.44E-04	3.74E-04	2.45E-04	9.23E-05
12	3.74E-05	7.29E-05	3.77E-05	7.10E-06
14	5.37E-06	1.42E-05	5.98E-06	4.48E-07

Table 3-1: BER Bounds compared to SGA & SIGA BER (K=4, N=31)

E_b/N_0	BER^L	BER^U	BER^{SGA}	BER^{SIGA}
4	1.35E-02	1.36E-02	1.35E-02	1.28E-02
6	2.86E-03	2.91E-03	2.85E-03	2.51E-03
8	2.95E-04	3.07E-04	2.91E-04	2.16E-04
10	1.11E-05	1.22E-05	1.05E-05	5.23E-06
12	1.17E-07	1.41E-07	9.29E-08	1.87E-08
14	3.03E-10	4.35E-10	1.44E-10	3.99E-12

Table 3-2: BER Bounds compared to SGA & SIGA BER (K=3, N=127)

From tables 3.1 and 3.2, it can be seen that the SGA BER model is more accurate than the SIGA BER model, when the upper and lower bounds of the BER are used. Moreover, the SIGA BER model gives more optimistic BER results for higher E_b/N_0 values.

Figure 3.4 shows a comparison of the SGA and SIGA BER analytical models to simulations of a spread-spectrum slotted ALOHA system where the success of a packet in a slot is determined by tossing a coin. In figure 3.4 the user population is 50 users with a processing gain of 32 and a packet size of 100 bits. For the simulation, the average packet length was set at 4 packets, with an average retransmission period of 2 packets.

From figure 3.4, it is evident that the simulation results give a higher throughput at high loads. This is largely influenced by the packet retransmission period. Generally, shorter retransmission periods will lead to more frequent collisions and hence a reduction in the packet throughput at high traffic loads.

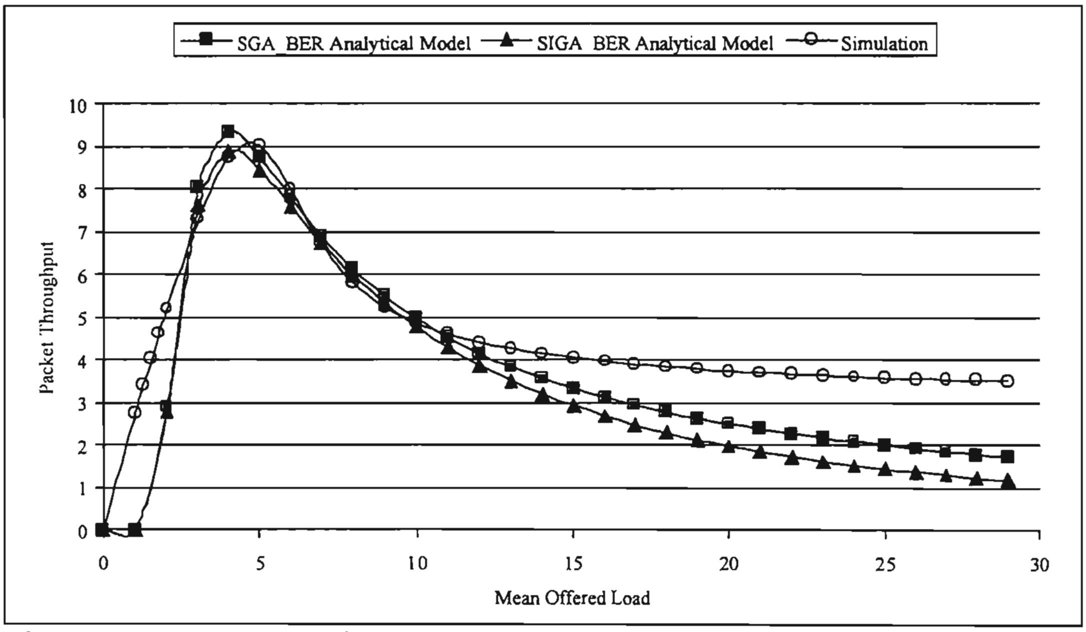

Figure 3.4 - Comparison of the SGA & SIGA BER Models to simulation results. (K = 50 users, b = 100, μ = 0dB, σ = 2dB, N = 32)

Figures 3.5 to 3.7 show the system throughput S in packets per slot given by equation 3.16 against the offered load G measured as the number of arriving packets from both idle and retransmitting users. Figure 3.5 shows the effect of the packet size on the system throughput. With a packet size of 50 bits, the capacity achieved with the Averaged BER Model is 14.8 packets per slot. At much higher packet sizes of 1024 bits, the capacity of the Averaged BER Model is reduced to 3.8 packets per slot. This is due to the fact that the Averaged BER Model has much higher BER probabilities and these get more pronounced as the number of bits within the packets is increased.

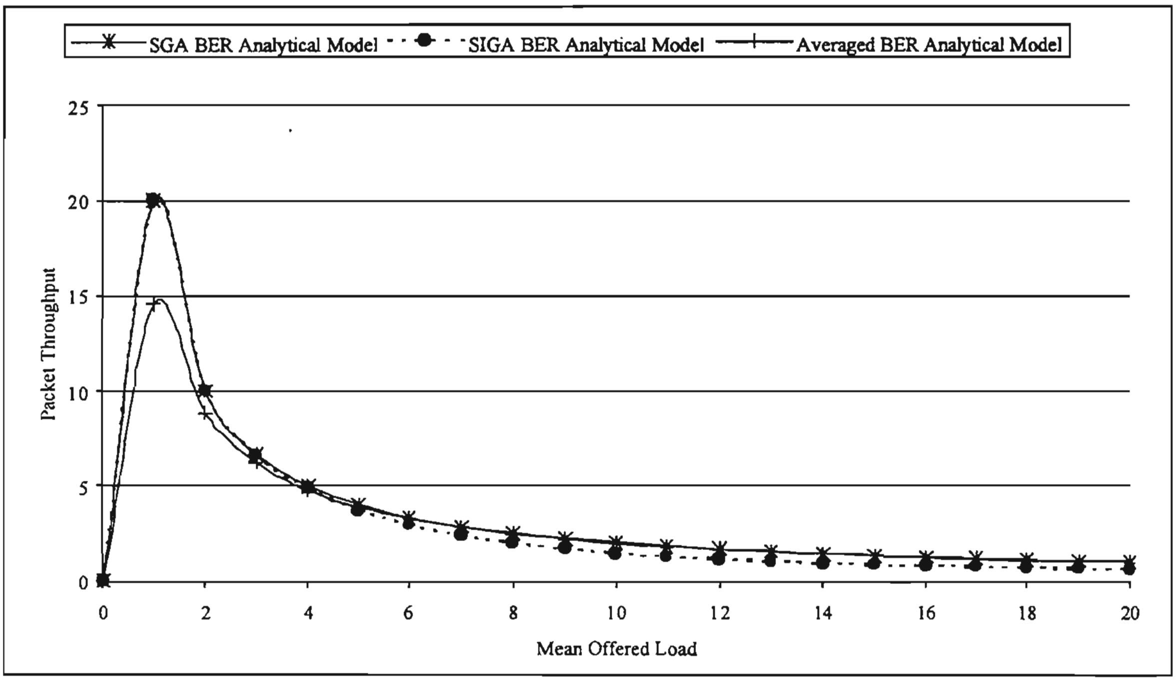

Figure 3.5 - Effect of the packet size on the throughput of the SGA BER, SIGA BER and the Averaged BER Analytical models ($K = 20$ users, $b = 50$, $\mu = 0dB$, $\sigma = 2dB$, $N = 128$)

Figure 3.6 shows the effect of the processing gain, N, on the system throughput with the packet size, b, fixed at 100 bits per packet. In this figure, what is again noticeable is the fact that an increase in N improves the capacity of the system with the Averaged BER Model. There is still no change in the throughputs of the system using the SGA BER and SIGA BER Models. However, the SGA BER Model has marginally higher packet throughputs at much higher traffic loads.

Figures 3.5 and 3.6 show that the Averaged BER model is very susceptible to changes in the channel characteristics, and hence not a good model to use to quantify the effects of power control errors in the performance of spread-spectrum slotted ALOHA systems. The effects of the processing gain and the bit durations on the performance of the SGA BER and SIGA BER models are more clearly shown in figures 3.7 and 3.8, respectively. In general, the higher the processing gain, the higher is the system capacity. However, figures 3.7 and 3.8 show that the longer the bit durations the lower is the system throughput. This is due to the fact that longer packets will have a higher probability of bit errors.

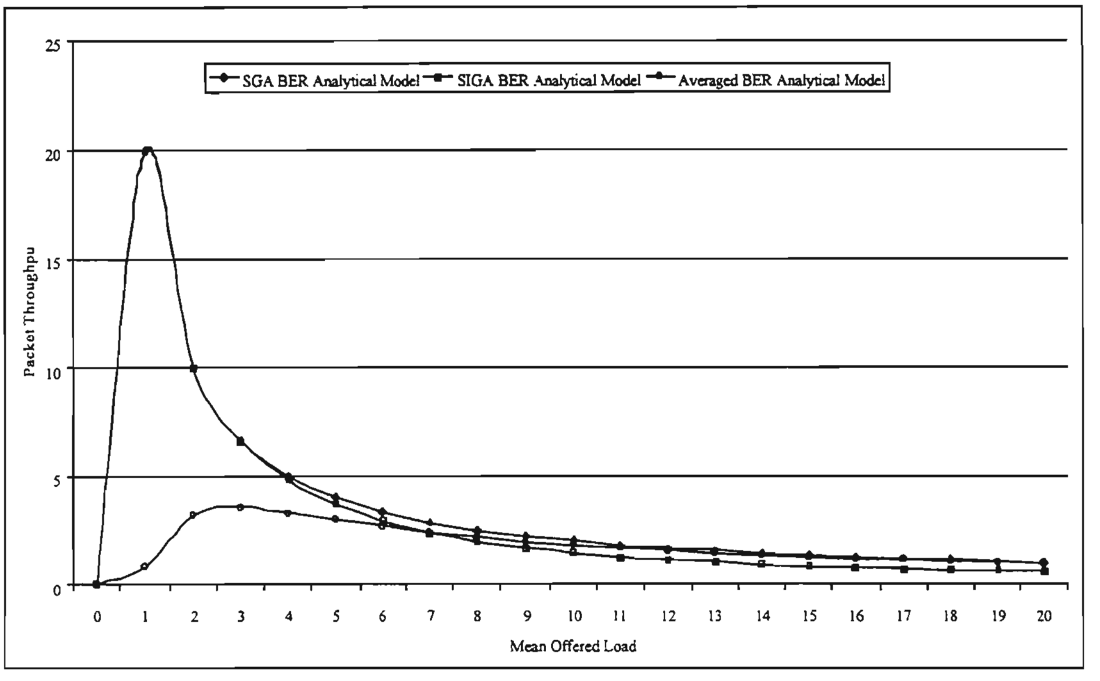

Figure 3.6 - Throughputs of the SGA BER, SIGA BER and the Averaged BER Analytical Models. (K = 20 users, b = 500, μ = 0dB, σ = 2dB, N = 128).

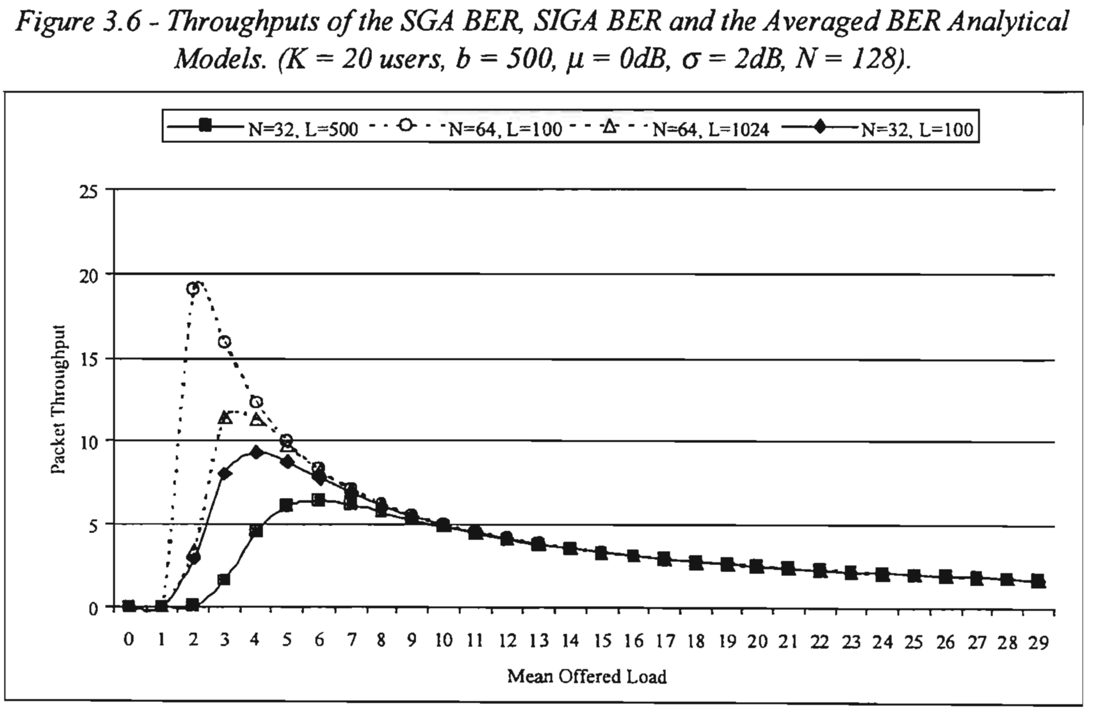

Figure 3.7 - Packet throughputs of the SGA BER Model for various values of the processing gain and the number of bits per packet. (K = 50 users).

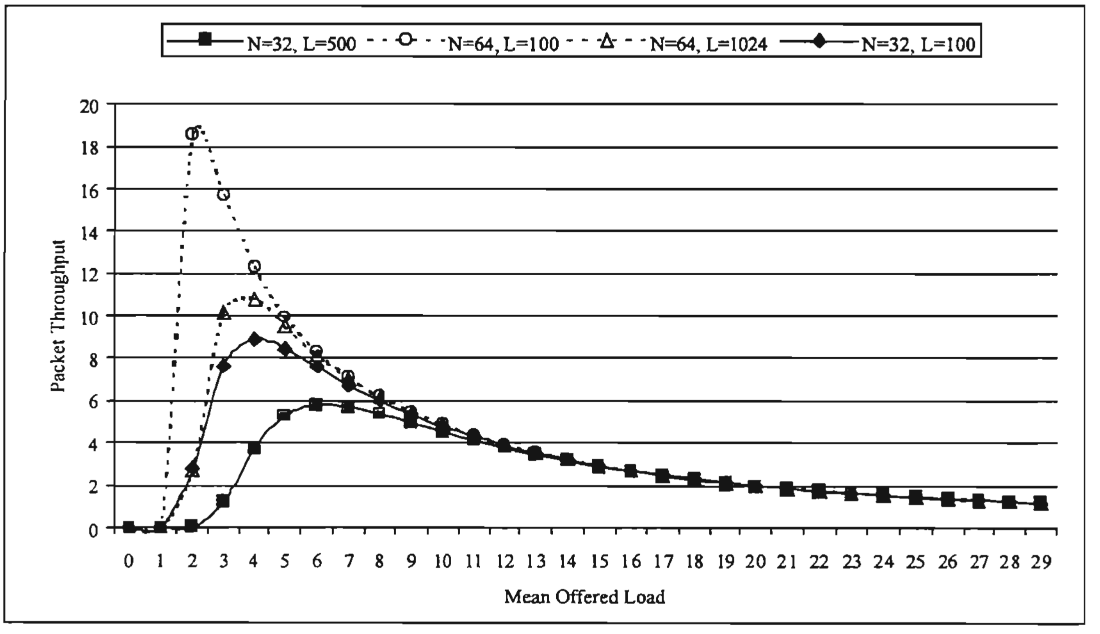

Figure 3.8 - Packet throughputs of the SIGA BER Model for various values of the processing gain and the number of bits per packet. (K = 50 users).

3.6 Summary

This chapter has attempted to tackle the well-known problem of describing the interference generated by a number of co-channel independent users in DS-CDMA systems. The Central Limit Theorem has been applied in the assumptions for the Gaussian approximation that has been used to derive analytical solutions of the problem at hand, in terms of SIR and BER.

Several methods have been suggested to improve on the SGA where the number of users is small. These improvements have come in the form of the IGA and SIGA. The problem with the IGA has been shown to be the one of finding the distribution function of the variance term used. The SIGA has been shown to have some numerical constraints. This work has shown that the SIGA gives optimistic BER values for higher E_b/N_0 than the

SGA as benchmarked against the upper and lower bounds of the BER given by Pursely *et al*.

Another method that was developed by Prasad *et al* has also been investigated. The Averaged BER Model includes power control errors in that the BER expression is averaged by the distribution of the received power levels. This model has been shown to give very pessimistic BER values. The pessimistic BER values obtained with this model can be attributed to the averaging used. It has also been shown that in the case of perfect power control, the received power levels are all equal and the expression for the averaged BER model reduces to the SGA model.

The performance of these MAI approximation techniques has further been investigated in terms of the packet throughputs of the system using a finite population traffic model with the combined arrival of new and retransmitted packets being binomial. The results have shown that the maximum capacity is achieved with the SGA BER model and that to obtain the highest throughput one needs to design with a high processing gain and low bit durations.

4 A STUDY OF LOGNORMAL SUM APPROXIMATIONS

4.1 Introduction

In the computation of the Standard Gaussian Approximation (SGA) in the preceding chapter, the Central Limit Theorem (CLT) was used in assuming that the sum of a large number of independent and identically distributed (iid) variables is Gaussian. This was an acceptable assumption as it offered simple and fast computations. However, in mobile radio systems with correlated and uncorrelated shadowing components, it is often desired to express the mean and the variance of the resultant sum in terms of the individual means and variances of the lognormal components. In these scenarios, the power sum of a finite number of lognormal (correlated and uncorrelated) random variables can be approximated by another lognormal random variable [Fen60], [SY82]. In the following chapters, an exact expression for the means and variances of the sum of lognormal components will be desired; hence the subject is treated in advance in this chapter.

This chapter presents a study of the most widely used and documented lognormal sum approximations, viz. the Fenton-Wilkinson Approximation that was originally published in 1960 in [Fen60] and the Schwartz-Yeh Approximation published in [SY82]. These approximations were first derived for the uncorrelated signals and later extended to correlated signals by Safak in [Saf93], Beaulieu *et al* in [AB94] and Ho in [Ho95]. This chapter will also present the effect of correlation and the variability of the component means and variances to the sum distribution.

4.2 Lognormal Sum Approximations

From the literature researched on the subject, there is no known exact result for the distribution of the sum of several lognormal random components. However, many approximations have been used to approximate this sum, and they have modeled the sum distribution as a lognormal random variable too. That is, the sum of t lognormal random variables can be represented by the expression

$$L = e^{Y_1} + e^{Y_2} + \cdots + e^{Y_t} \cong e^{Z} \tag{4.1}$$

where the terms Y_i and Z are Gaussian random variables.

The correlation coefficient of Y_i and Y_j is defined by

$$\rho = \rho_{ij} = \frac{E\left[\left(Y_i - m_{Y_i}\right)\left(Y_j - m_{Y_j}\right)\right]}{\sigma_{Y_i}\sigma_{Y_j}} \tag{4.2}$$

where m and σ represent the mean and the standard deviation of the indexed random variables.

4.2.1 Fenton-Wilkinson Approximation

In the approximation for the sum of lognormal random variables by Fenton and Wilkinson [Fen60], the mean m_z and the standard deviation σ_z of Z are derived by matching the first two moments of the both sides of equation 4.1. If the first moment of $(L_1 + L_2 + \cdots + L_N)$ is denoted by u_1, it can be derived to be

$$u_1 = E[L] = E\left[e^{Z}\right] = \sum_{i=1}^{N} e^{m_{Y_i} + \sigma_{Y_i}^2/2} \tag{4.3}$$

By matching the second moment, u_2, we obtain

$$u_2 = E\left[L^2\right] = E\left[e^{2Z}\right] = \sum_{i=1}^{N} e^{2m_{Y_i} + 2\sigma_{Y_i}^2} + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left\{e^{m_{Y_i} + m_{Y_j} + \frac{1}{2}\left(\sigma_{Y_i}^2 + \sigma_{Y_j}^2 + 2\rho\sigma_{Y_i}\sigma_{Y_j}\right)}\right\} \tag{4.4}$$

Solving equations 4.3 and 4.4 simultaneously yields the mean and the standard deviation of Z to be

$$m_Z = 2\ln u_1 - \frac{1}{2}\ln u_2 \tag{4.5}$$

and

$$\sigma_Z = \ln u_2 - 2\ln u_1 \tag{4.6}$$

In [SY82], the Fenton-Wilkinson approximation was found to be more applicable when the standard deviations of the lognormal components are lower than 4dB for uncorrelated signal components. For higher values of standard deviation, the Fenton-Wilkinson tends to underestimate the mean and overestimate the variance of the sum distribution. When the lognormal components are correlated, it was found that the Fenton-Wilkinson approximation was adequately accurate at values of standard deviation higher than 12dB.

4.2.2 Schwartz-Yeh Approximation

The Schwartz-Yeh approximation is also based on the assumption that the sum is lognormal as in equation 4.1. The first and second moments of the Gaussian random variable, Z, however, are not obtained by using the same assumption (moments matching technique). Rather, exact expressions for the first two moments of the sum of two lognormal random variables are derived. By again assuming that the sum of two lognormal random variables is a lognormal random variable, nesting and recursive techniques are then used to extend the approach to a larger number of cumulative random variables ($t > 2$).

The calculations required in the computation of the moments using the Schwartz-Yeh approximation are quite complex and they require a lot of computational capacity before the results converge to the required accuracy in digits. The computations require rounding off that introduces errors and complicated integration using the trapezoidal rule and appropriate transformations in the integration range. An extension to this approximation provided by Ho in [Ho95], circumvents the difficulties provided in the computation of the moments using the Schwartz-Yeh approximation. In the literature, Ho's extension to this approximation is referred to as the Ho's Schwartz-Yeh approximation.

We first present the general case of two independent random variables with unequal means and variances. We then extend this approximation to the case where the two random variables are correlated as well as having different means and variances. Starting from equation 4.1, for the two components, we have

$$e^{Z_2} = L = L_1 + L_2 = e^{Y_1} + e^{Y_2} \tag{4.7}$$

or

$$Z_2 = \ln\left(e^{Y_1} + e^{Y_2}\right) \tag{4.8}$$

We define a Gaussian random variable

$$w = Y_2 - Y_1 \tag{4.9}$$

with

$$m_w = \overline{w} = m_{Y_2} - m_{Y_1} \tag{4.10}$$

and

$$\sigma_w^2 = \sigma_{Y_2}^2 + \sigma_{Y_1}^2 \tag{4.11}$$

The expressions required for the mean and the standard deviation calculations (sum of two lognormal components) using Ho's Schwartz-Yeh method are presented in equations 4.12 to 4.21.

$$m_{Z_2} = m_{Y_1} + G_1 \tag{4.12}$$

$$\sigma_{Z_2}^2 = \sigma_{Y_1}^2 - G_1^2 - 2\sigma_{Y_1}^2(I_2 + I_0) + G_2 \tag{4.13}$$

$$G_1 = E\left[\ln\left(1 + e^w\right)\right] = A_0 + I_1 \tag{4.14}$$

$$G_2 = E\left[\ln^2\left(1 + e^w\right)\right] = I_3 + 2I_4 + \sigma_w^2 I_0 + m_w A_0 \tag{4.15}$$

$$G_3 = E\left[(w - m_w)\ln\left(1 + e^w\right)\right] = \sigma_w^2(I_2 + I_0) \tag{4.16}$$

$$I_4 = \sigma_w^2\left[f_w(0)\ln 2 - I_5\right] + m_w I_6 \tag{4.17}$$

$$A_0 = \frac{\sigma_w}{\sqrt{2\pi}} e^{-m_w^2 / 2\sigma_w^2} + m_w I_0 \tag{4.18}$$

$$I_i = \int_0^1 h_i(v) v^{-1} dv \tag{4.19}$$

$$h_i(v)=\begin{cases}\dfrac{1}{\sqrt{2\pi}}\exp\left[\dfrac{-\left(\ln v+{}^{m_w}\!/_{\sigma_w}\right)^2}{2}\right], & i=0\\ \left[f_w(\ln v)+f_w(-\ln v)\right](\ln v+1), & i=1\\ \left[f_w(\ln v)-f_w(-\ln v)\right]\left(1+v^{-1}\right)^{-1}, & i=2\\ \left[f_w(\ln v)+f_w(-\ln v)\right]\ln^2(1+v), & i=3\\ -f_w(-\ln v)\ln v\ln(1+v), & i=4\\ f_w(-\ln v)\left(1+v^{-1}\right)^{-1}, & i=5\\ f_w(-\ln v)\ln(1+v), & i=6\end{cases} \tag{4.20}$$

$$f_w(w)=\frac{1}{\sqrt{2\pi\sigma_w^2}}\exp\left[-\frac{(w-m_w)^2}{2\sigma_w^2}\right] \tag{4.21}$$

The results provided above can account for correlation with a simple modification to equation 4.11 such that

$$\sigma_w^2=\sigma_{Y_2}^2+\sigma_{Y_1}^2-2\rho\sigma_{Y_1}\sigma_{Y_2} \tag{4.22}$$

Furthermore, using nesting and recursion, the Schwartz-Yeh approximation can be extended to more than two lognormal components, with correlation. That is, for $t\geq 3$, the mean and variance of $Z_t=\ln\left(e^{Z_{t-1}}+L_t\right)$ are given by

$$m_{Z_t}=m_{Z_{t-1}}+G_1, \tag{4.23}$$

$$\sigma_{Z_t}^2=\sigma_{Z_{t-1}}^2-G_1^2+G_2-2\cdot\left(\rho\sigma_{Y_t}-\sigma_{Z_{t-1}}\right)\cdot\sigma_{Z_{t-1}}\cdot\frac{G_3}{\sigma_{w_t}^2} \tag{4.24}$$

with

$$w_t=Y_t-Z_{t-1} \tag{4.25}$$

$$m_{w_t}=\overline{w}=m_{Y_t}-m_{Z_{t-1}} \tag{4.26}$$

and

$$\sigma_{w_t}^2=\sigma_{Y_t}^2+\sigma_{Z_{t-1}}^2-2\rho\sigma_{Z_{t-1}}\sigma_{Y_t} \tag{4.27}$$

4.3 Results

Figure 4.1 and figure 4.2 show the mean and the standard deviation of the sum of lognormal random variables, respectively, with a varying number of identically distributed components for the Fenton-Wilkinson Approximation, the Schwartz-Yeh approximation and from simulations with the mean of the lognormal components, μ is 3dB, the standard deviation of the lognormal components, σ is 2dB and the correlation coefficient of the lognormal components, ? is 0.7.

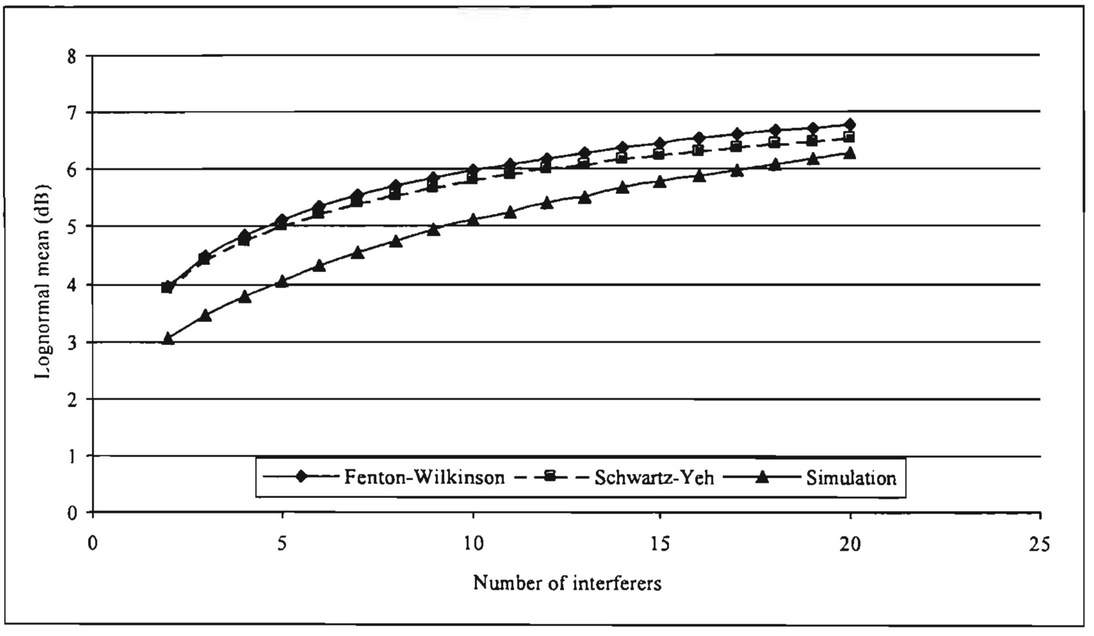

Figure 4.1 - Mean of the lognormal components for the Fenton-Wilkinson, Schwartz-Yeh approximations and simulations for the mean of the lognormal components against the number of interferers (μ = 3dB, σ = 2dB, ? = 0.7)

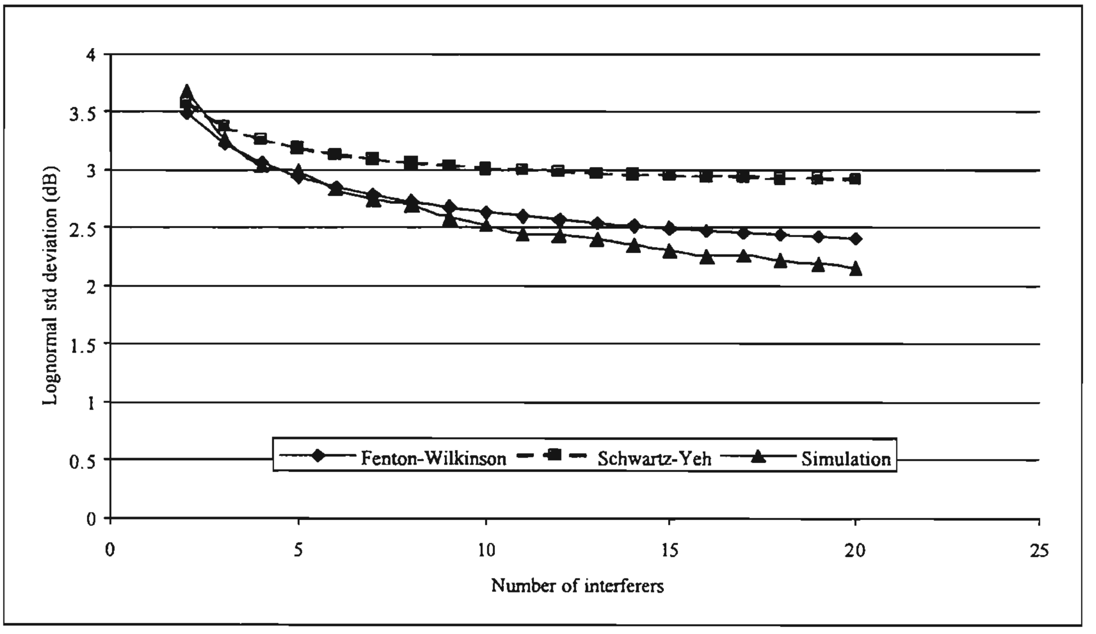

Figure 4.2 – Standard deviation of the lognormal components for the Fenton-Wilkinson, Schwartz-Yeh approximations and simulations for the standard deviation of the lognormal components against the number of interferers (μ = 3dB, σ = 2dB, ? = 0.7)

Figure 4.3 and figure 4.4 show a comparison of the two approximations with simulation results where the mean of the lognormal components, μ, is 1dB, the standard deviation of the lognormal components, σ, is 2dB and the correlation coefficient of the lognormal components, ?, is 0.6.

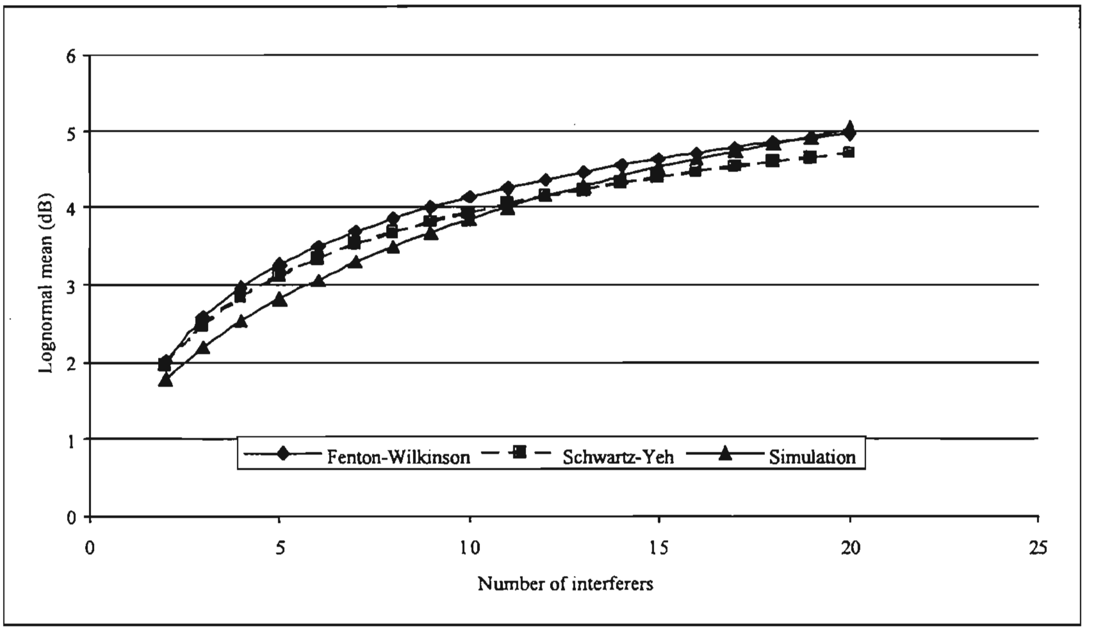

Figure 4.3 - Mean of the lognormal components for the Fenton-Wilkinson, Schwartz-Yeh approximations and simulations for the mean of the lognormal components against the number of interferers (μ = 1dB, σ = 2dB, ? = 0.6)

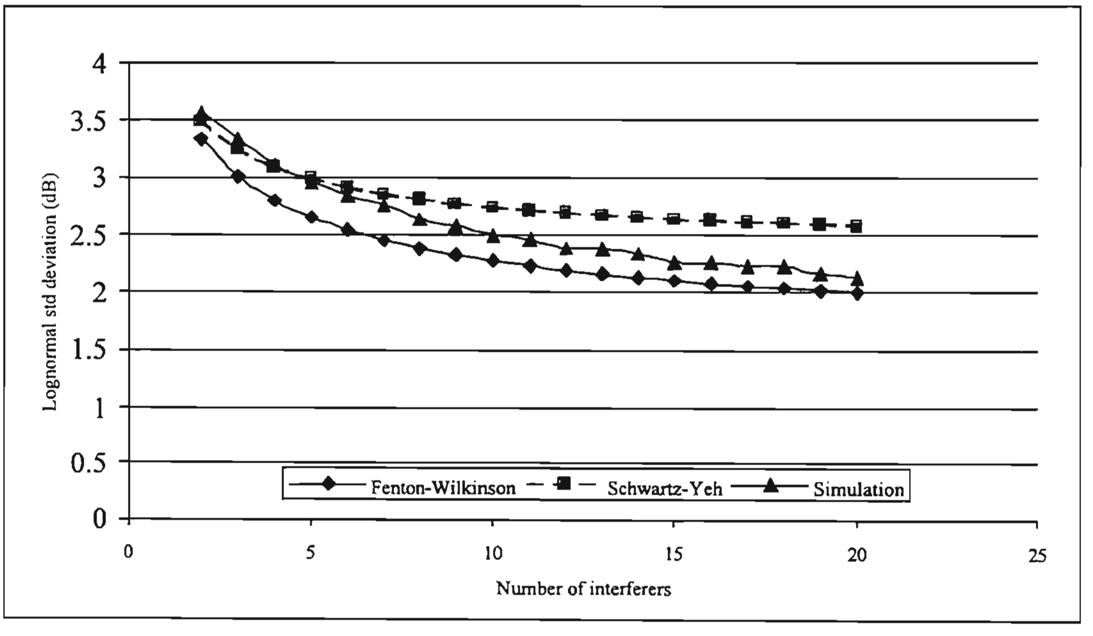

Figure 4.4 - Standard deviation of the lognormal components for the Fenton-Wilkinson, Schwartz-Yeh approximations and simulations for the standard deviation of the lognormal components against the number of interferers (μ = 1dB, σ = 2dB, ? = 0.6)

From figures 4.2 and 4.4, it can be seen that the Fenton-Wilkinson approximation is closest to the simulated results, with the error increasing with the number of interfering components.

Figure 4.5 and figure 4.6 show a comparison of the two approximations with simulation results where the mean of the lognormal components, μ, is 1dB, the standard deviation of the lognormal components, σ, is 4dB and the correlation coefficient of the lognormal components, ?, is 0.6.

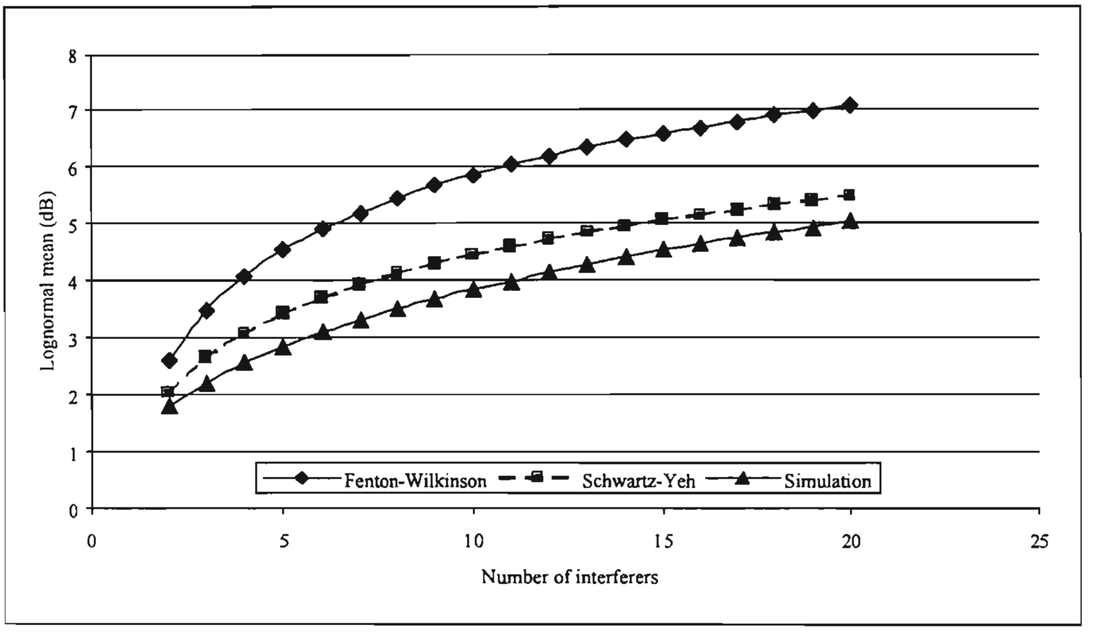

Figure 4.5- Mean of the lognormal components for the Fenton-Wilkinson, Schwartz-Yeh approximations and simulations for the mean of the lognormal components against the number of interferers (μ = 1dB, σ = 4dB, ? = 0.6)

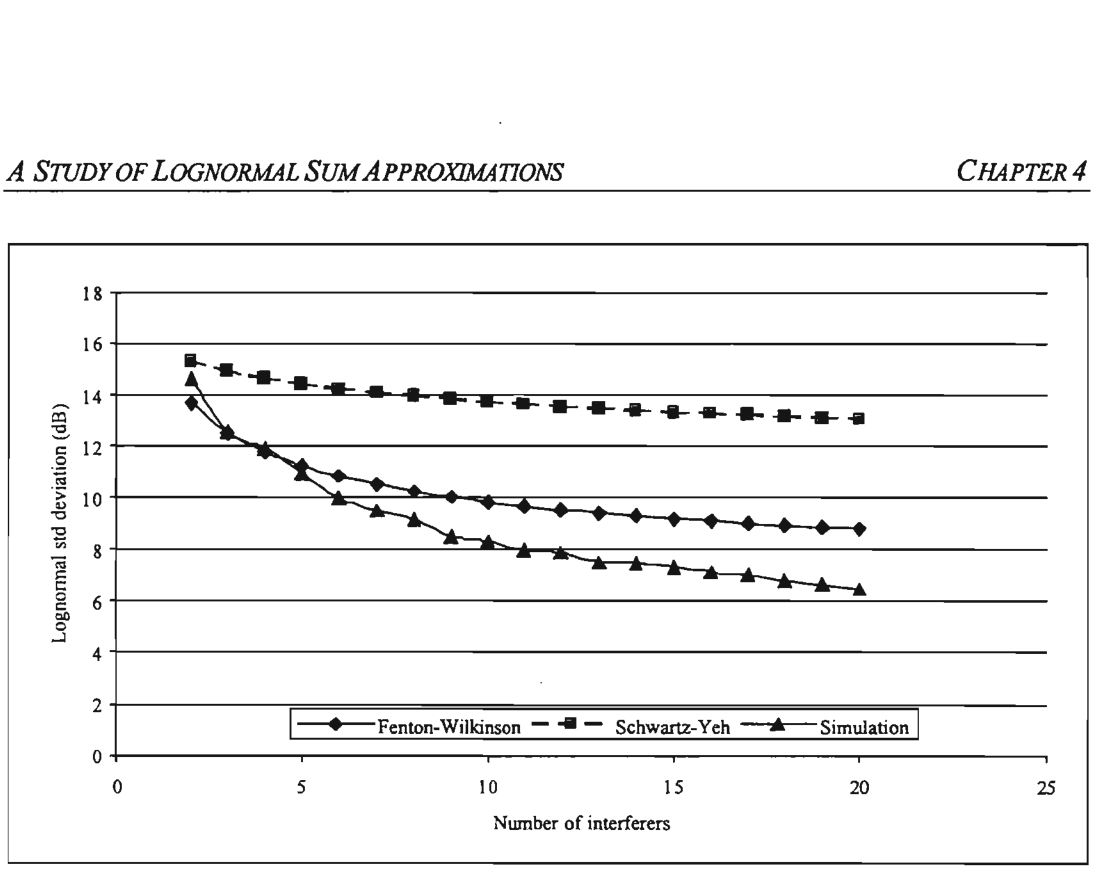

Figure 4.6 - Standard deviation of the lognormal components for the Fenton-Wilkinson, Schwartz-Yeh approximations and simulations for the standard deviation of the lognormal components against the number of interferers(μ = 1dB, σ = 4dB, ? = 0.6)

According to the literature ([AB94] & [Ho95]), the Fenton-Wilkinson approximation is best applied for the range of standard deviations of $4\text{dB} \leq \sigma \leq 12\text{dB}$. The results obtained from the above approximations and simulations confirm that the Fenton-Wilkinson approximation is indeed the better approximation to use for the standard deviations of 4dB and greater are used. The Schwartz-Yeh approximation tends to give more optimistic standard deviation results.

4.4 Summary

This chapter has described the two widely used approximations for the sum of lognormal random variates. The work presented in this chapter forms a strong basis for the following chapter. Chapter 5 makes reference to the expressions for the mean and standard deviation calculations presented here.

The correlation of the lognormal components was not studied in the earlier papers on this subject. We have included some of the findings of the researchers that have included correlation. This chapter has also compared the two approximations with simulated results, including the effects of the correlated lognormal components.

The works by Ho [Ho95] and Beaulieu *et al* [AB94] have done simulations to verify the results of the Fenton-Wilkinson and Schwartz-Yeh approximations described in this paper. The general result is that when the standard deviation is less than 4dB, it is better to use the Schwartz-Yeh approximation. However, when the standard deviation is greater than 4dB, then the Fenton-Wilkinson approximation is more accurate with more realistic approximations for the mean and standard deviation of the sums of the components.

5 CAPTURE MODELS FOR SPREAD-SPECTRUM ALOHA SYSTEMS

5.1 Introduction

Chapter 2 has provided a brief and yet concise overview of random access protocols, in particular slotted ALOHA. The issues around contention in slotted ALOHA systems and the eventual collisions where packets become unsuccessful have been highlighted. These unsuccessful packets may be retransmitted after some random delay, the assumption here being that whenever two or more packets overlap at the receiver (creating contention), all of them getting lost in the process, is very pessimistic. Other researchers have considered situations in which multiple mobiles send packets to a central base station using slotted ALOHA. The power of the received signals from the different mobiles will vary due to the differences in the length of the transmission path and due to multipath fading effects. These power variations have been observed to actually improve the throughput of random access protocols like slotted ALOHA (the works by Roberts [Rob75], Zorzi and Rao [ZR94] and Krishna and LaMaire [KL94] have researched this). This phenomenon, known as *capture*, occurs when differences in received power levels result in successful reception of a packet in the presence of typically weaker contending transmissions. Some other works (Davis and Gronemeyer [DG80]) studied the effects of delay capture in slotted ALOHA, where the receiver captures a packet because the packet arrives a short time before any other packet transmitted during the same slot. The capture phenomenon can also be considered as erroneous operation of the system, corresponding to the ability to receive a packet successfully although more than one packet is transmitted at the same time.

The research that has been done in this subject has considered single packet capture scenarios. These works have defined some capture ratio, β, such that a packet will be captured by the base station's receiver if the ratio of the received signal power of the packet in question to the received signal powers of the interfering packets is greater than this capture ratio [ZR94]. In this chapter, we consider a spread-spectrum slotted ALOHA system similar to the one described in Chapter 3, where the base station is capable of "capturing" multiple packets, simultaneously. We consider the effects of a radio propagation model that is characterized by lognormal shadowing and the signal at the base station's receiver is attenuated due to path losses.

This chapter is organized as follows: Section 5.2 presents the system model that compirses the network model and the channel model describing the radio propagation model, which are very similar to the ones used in Chapter 3. The capture model that is formulated in this thesis is presented in Sections 5.3 and 5.4. In the formulation of the capture models described in Sections 5.3 and 5.4, we use the approximations for the sums of lognormal components that were presented in Chapter 4. Some numerical results from the analysis presented in this chapter are provided in Section 5.5.

5.2 System Model

The traffic model considered is identical to the one described in Chapter 3. The reader is referred to Chapter 3 for more details.

5.2.1 Network Model

The network model used in this chapter is based on the one used in Chapter 3, with some differences. The network architecture under consideration is a single-hop star-network cell architecture in which mobile data terminals communicate with each other, or with a global backbone network, via a centralized base station using a shared wireless DS-CDMA channel as in [MT99] and [JT00]. As before, only the reverse link will be considered.

The time axis is slotted with a slot size equal to the packet transmission period. All mobile terminals in the base station's coverage area are slot synchronized with the base station. Data messages may comprise several contiguous packets. Each packet has a fixed length packet size containing a fixed number of b bits such that each packet is transmitted in exactly one time slot. Each transmitted bit is spread using an N chip direct sequence PN spreading code. The base station allocates a unique PN code to each terminal in its cell and is aware of the presence of each user in the cell, and the codes allocated to them. The only difference between the models described in this chapter to the one described in Chapter 3 is that in this model, successful packets are those that are captured by the base station. Two models for capture are presented in the following sections of this chapter. Packets that are not captured (successful) are again regrouped and retransmitted as "new" messages.

5.2.2 Channel Model

As in Chapter 3, the propagation is described by three effects: the attenuation due to the distance, r, proportional to $r^{-\eta}$, where η, the power loss exponent, assumes a value of ≈ 4 in land mobile radio environments; the shadowing, described by means of a lognormal random variable; and the fast Rayleigh fading, which causes the instantaneous envelope of the received signal to be Rayleigh distributed, and its power to be an exponentially distributed random variable. The received power from a mobile at distance r can be expressed as

$$P_r = R^2 S r^{-\eta} P_T \tag{5.1}$$

where R is a Rayleigh distributed random variable with unit power due to multipath fading; $S \left(= e^{\xi}\right)$ accounts for the lognormal shadowing with a probability distribution function (pdf) given by

$$f(S) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{\frac{-(\log S - \mu)^2}{2\sigma^2}\right\} \tag{5.2}$$

where μ and σ are the mean and the standard deviation due to shadowing, respectively; and ξ is the dB attenuation due to shadowing that is Gaussian with zero mean and a standard deviation of σ; and η is the deterministic path loss attenuation. The effects of Rayleigh fading are mitigated by the use of a RAKE receiver, with the use of coding and interleaving.

A mobile station at distance r from the base station thus has received power proportional to the attenuation given by

$$P_r \propto r^{-\eta} e^{\xi} \tag{5.3}$$

The dB losses can also be expressed as

$$\log \alpha(r,\xi) = -\eta \log r + \xi \tag{5.4}$$

The radio propagation model used is developed in analogy to the one developed by Viterbi in [VVGZ94]. Since the dB losses due to shadowing are Gaussian, we assume a joint Gaussian probability distribution function for losses from two different mobile stations. We take into consideration that the lognormal shadowing is a sum of a component in the near-field of the mobile station which is independent from one mobile station to another; and a component in the far-field of the mobile station which is common to all mobiles being served by a central base station. We thus express the random component of the dB loss from the i^{th} mobile station ($i = 1,2,\cdots,N$)

$$\xi_i = a\zeta + b\zeta_i \text{ where } a^2 + b^2 = 1 \tag{5.5}$$

with

$$\xi_i, \zeta, \zeta_i \sim N(0,\sigma) \text{ for all } i,$$

$$E[\zeta\zeta_i] = 0 \text{ for all } i,$$

and

$$E[\zeta_i\zeta_j] = 0 \text{ for all } i \text{ and } j,\ i \neq j.$$

The normalized covariance (correlation coefficient) of the losses from the two mobile stations, i and j, is

$$\rho_{ij} = \frac{E[\xi_i \xi_j]}{\sigma^2} \tag{5.6}$$

We assume constant correlation between the two mobiles, i and $j, i \neq j$. Equation 5.5 then reduces to

$$\xi_i = \sqrt{\rho}\zeta + \sqrt{1-\rho}\zeta_i \tag{5.7}$$

Using the propagation model described above, we express the instantaneous SNR for user i as

$$SNR = \frac{P_{r,i}}{W + \sum_{\substack{j=1 \\ j \neq i}}^{K} P_{r,j}} = \frac{r_i^{-\eta} e^{\xi_i}}{W + \sum_{\substack{j=1 \\ j \neq i}}^{K} r_j^{-\eta} e^{\xi_j}} \tag{5.8}$$

where W is the background AWGN and K is the total number of users in the system. We assume that the cell shape is approximated by a circle of radius r. We further assume that the locations of different mobiles are statistically independent, with the mobiles uniformly distributed in the base station's coverage area. Thus the probability density function of the distance between a mobile and the base station is

$$g(r) = \frac{2(r - R_o)}{(R - R_o)^2}, \quad R_o \leq r \leq R \tag{5.9}$$

Capture is defined as the event that the SNR exceeds a predetermined threshold, often referred to as the *capture ratio*. We assume that when a user's packet is captured, it is correctly received, otherwise its packet is corrupt and then retransmitted after some random backoff delay. Therefore, the probability of a packet success is defined as

$$P_s = \Pr\{SNR > \beta\} \tag{5.10}$$

where β is the capture ratio which depends on the coding scheme and modulation techniques used. The capture ratio was analytically evaluated by Lau and Leung in [LL92] in the absence of fading for a bell-shaped traffic model. For narrowband systems, the capture ratio β typically ranges from 4 to 10dB, but values of 0 to 1dB are possible for CDMA systems ([ZR94] and [KL94]).

5.3 Capture Model #1

In defining Capture Model #1, we use the radio propagation model described above. The probability of capture is thus

$$P_{cap} = \Pr\left\{ \frac{r_i^{-\eta} e^{\xi_i}}{W + \sum_{\substack{j=1 \\ j \neq i}}^{K} r_j^{-\eta} e^{\xi_j}} > \beta \right\} \tag{5.11}$$

In order to concentrate on the effects of multiple access interference, the effect of AWGN is neglected (i.e. $W = 0$). We assume that the product of the uniformly distributed random variable, r, and the lognormal component due to shadowing, e^{ξ} given in equation 5.11 is also lognormal. We denote this lognormal random variable as e^{Y} where Y is a Gaussian random variable. We derive the moments of the lognormal random variable using the moments matching technique as detailed below.

$$L = r^{-\eta} e^{\xi} = e^{Y} \tag{5.12}$$

The mean is given by

$$E\{r^{-\eta} e^{\xi}\} = r^{-\eta} E\{e^{\xi}\} = E\{e^{Y}\}$$

$$r^{-\eta} e^{m_x + \frac{1}{2}\sigma_x^2} = e^{m_Y + \frac{1}{2}\sigma_Y^2} \tag{5.13}$$

The variance is given by

$$r^{-2\eta} e^{2m_x + 2\sigma_x^2} = e^{2m_Y + 2\sigma_Y^2} \tag{5.14}$$

Taking the logarithms for equations 5.13 and 5.14, we obtain equations 5.15 and 5.16, respectively.

$$-\eta \log r + m_x + \frac{1}{2}\sigma_x^2 = m_Y + \frac{1}{2}\sigma_Y^2 \tag{5.15}$$

$$-2\eta \log r + 2m_x + 2\sigma_x^2 = 2m_Y + 2\sigma_Y^2 \tag{5.16}$$

Solving equations 5.15 and 5.16 for m_Y and σ_Y^2 simultaneously yields

$$\sigma_Y^2 = \sigma_x^2 \tag{5.17}$$

and,

$$m_Y = m_x - \eta \log r \tag{5.18}$$

We now write equation 5.11 as

$$P_{cap} = \Pr\left\{ \frac{e^{Y_i}}{\sum_{\substack{j=1 \\ j \neq i}}^{K} e^{Y_j}} > \beta \right\} \tag{5.19}$$

In finding an analytical solution for equation 5.19, we use the findings made by Fenton and Wilkinson and Schwartz and Yeh that were presented in Chapter 4, that the sum of lognormal random variables is also lognormal. We thus define a Gaussian random variable Z such that

$$L = e^{Y_1} + e^{Y_2} + \cdots + e^{Y_K} \cong e^{Z} \tag{5.20}$$

The capture probability defined in equation 5.19 can now be written as

$$P_{cap} = \Pr\left\{e^{Y_i - Z} > \beta\right\} = \Pr\left\{Y_i - Z > \ln \beta\right\} \tag{5.21}$$

The moments of the Gaussian random $Y_i - Z$ are given by

$$E[Y_i - Z] = m_i - m_Z \tag{5.22}$$

and

$$E\left[(Y_i - Z)^2\right] = \sigma_i^2 + \sigma_Z^2 - 2\rho\sigma_i\sigma_Z \tag{5.23}$$

In obtaining the solution for equation 5.21, we use the well known Gaussian cumulative distribution function (cdf) that has been used in many papers in computing the outage probabilities in the presence of jointly correlated Gaussian random variables. In [PSGR00], Pratesi *et al* defined a probability of outage as

$$P_{out} = \Pr\{SIR > \ln(\gamma)\} \tag{5.24}$$

where γ is a power reduction ratio and the SIR is given by

$$SIR = \ln\left(e^{y}\right) \tag{5.25}$$

and y is a Gaussian random variable with a mean of η_y and a standard deviation of σ_y. Using the Gaussian cdf, the outage probability in [PSGR000] is thus given by

$$P_{out} = Q\left(\frac{\ln(\gamma)-\eta_y}{\sigma_y}\right) \tag{5.26}$$

where $Q(\cdot)$ is the standard Q function given by

$$Q(x) = \frac{1}{2}erfc\left(\frac{x}{\sqrt{2}}\right) \tag{5.27}$$

We use results similar to the ones from [PSGR00] presented in equation 5.26 in calculating the capture probability of equation 5.21. We include the moments of the Gaussian random $Y_i - Z$ given in equations 5.22 and 5.23 to obtain the capture ratio as

$$P_{cap} = \int_0^R \left[Q\left(\frac{\ln\beta - E\{Y_i - Z\} - \sqrt{\rho E\{(Y_i - Z)^2\}}}{\sqrt{(1-\rho)E\{(Y_i - Z)^2\}}}\right)\right]\cdot g(r)dr_i \tag{5.28}$$

where $g(r)$ is the probability density function of the distance between a mobile and the base station given in equation 5.9. By substituting the moments given in equations 5.22 and 5.23, the probability of capture for Capture Model #1 is thus given by

$$P_{cap} = \int_0^R \left[Q\left(\frac{\ln\beta - (m_i - m_Z) - \sqrt{\rho\left(\sigma_i^2 + \sigma_Z^2 - 2\rho\sigma_i\sigma_Z\right)}}{\sqrt{(1-\rho)\left(\sigma_i^2 + \sigma_Z^2 - 2\rho\sigma_i\sigma_Z\right)}}\right)\right]\cdot g(r)dr_i \tag{5.29}$$

What should be noted from the above analysis is that in the presence of perfect power control, all the signals from the different users are received with the same received power levels; hence there is no capture. Power control alleviates the near-far effect; hence the distance of the users from the base station is compensated for. However, it has been shown that power control schemes that mitigate the effects of the lognormal shadowing are not practically possible; hence the received power levels are still lognormally distributed. The analysis for Capture Model #1 in the presence of perfect power control thus follows in similar fashion to section 5.2.2, with the instantaneous signal-to-noise ratio for the i^{th} user given by

$$P_{cap} = \Pr\left\{ \frac{P_{r,i}}{\sum_{\substack{j=1 \\ j \neq i}}^{K} P_{r,j}} > \beta \right\} \tag{5.19}$$

where the received power levels from the i^{th} user, $P_{r,i}$, have lognormally distributed power control errors such that $P_{r,i} = e^{Y_i}$ where Y is a Gaussian random variable that accounts for the power control errors with a mean of m_i and a standard deviation of σ_i.

The analysis that follows is similar to the one performed from equation 5.19 to equation 5.29 with the exception that we don't integrate equation 5.29 with respect to the i^{th} user's distance from the central base station, since this is compensated for by power control. The capture probability for Capture Model #1 in the presence of power control errors is thus given by

$$P_{cap} = Q\left(\frac{\ln\beta - (m_i - m_Z) - \sqrt{\rho(\sigma_i^2 + \sigma_Z^2 - 2\rho\sigma_i\sigma_Z)}}{\sqrt{(1-\rho)(\sigma_i^2 + \sigma_Z^2 - 2\rho\sigma_i\sigma_Z)}} \right) \tag{5.30}$$

5.4 Capture Model #2

The definition of this model for capture is instigated from the multipacket reception capabilities of spread-spectrum ALOHA systems. We consider a base station's receiver capable of capturing l packets simultaneously from the population of n $(n \leq N)$ contending packets. Our aim is to find

$P_{cap}(l|n) = \Pr\{l$ pkts are captured and $n-l$ pkts are not captured, given n contending pkts$\}$

This capture probability can be written as

$$P_{cap}(l|n) = \Pr\left\{ \begin{array}{l} \dfrac{P_1}{\sum\limits_{j\neq 1}^{K} P_j} > \beta, \dfrac{P_2}{\sum\limits_{j\neq 2}^{K} P_j} > \beta, \cdots, \dfrac{P_{l_K}}{\sum\limits_{j\neq l_K}^{K} P_j} > \beta, \\ \dfrac{P_{l_K+1}}{\sum\limits_{j\neq l_K+1}^{K} P_j} < \beta, \dfrac{P_{l_K+2}}{\sum\limits_{j\neq l_K+2}^{K} P_j} < \beta, \cdots, \dfrac{P_K}{\sum\limits_{j\neq K}^{K} P_j} < \beta \end{array} \right\} \tag{5.31}$$

Using the propagation model presented in this chapter (described by the path attenuation due to distances from the base station and the lognormal shadowing), and using the assumption made in the preceding section, that the product of the uniformly distributed random variable, r, and the lognormal component due to shadowing, e^{ξ} given in equation 5.11 is also lognormal and that we ignore AWGN, we can rewrite equation 5.31 as

$$P_{cap}(l|n) = \Pr\left\{ \begin{array}{l} \dfrac{e^{Y_1}}{\sum\limits_{j\neq 1}^{K} e^{Y_j}} > \beta, \dfrac{e^{Y_2}}{\sum\limits_{j\neq 2}^{K} e^{Y_j}} > \beta, \cdots, \dfrac{e^{Y_{l_K}}}{\sum\limits_{j\neq l_K}^{K} e^{Y_j}} > \beta, \\ \dfrac{e^{Y_{l_K+1}}}{\sum\limits_{j\neq l_K+1}^{K} e^{Y_j}} < \beta, \dfrac{e^{Y_{l_K+2}}}{\sum\limits_{j\neq l_K+2}^{K} e^{Y_j}} < \beta, \cdots, \dfrac{e^{Y_K}}{\sum\limits_{j\neq K}^{K} e^{Y_j}} < \beta \end{array} \right\} \tag{5.32}$$

Taking logarithms on both sides of equation 5.32, we get

$$P_{cap}(l|n) = \Pr\left\{ \begin{array}{l} Y_1 - Z > \ln\beta,\ Y_2 - Z > \ln\beta,\ \cdots,\ Y_{l_K} - Z > \ln\beta, \\ Y_{l_K+1} - Z < \ln\beta,\ Y_{l_K+2} - Z < \ln\beta,\ \cdots,\ Y_K - Z < \ln\beta \end{array} \right\} \tag{5.33}$$

where Y_i and Z are Gaussian random variables as described in the earlier sections of this chapter. By rearranging the signs from equation 5.33, we get

$$P_{cap}(l|n) = \Pr\left\{ \begin{array}{l} 2\beta - (Y_1 - Z) < \ln\beta,\ 2\beta - (Y_2 - Z) < \ln\beta, \\ \cdots, 2\beta - (Y_{l_K} - Z) < \ln\beta, Y_{l_K+1} - Z < \ln\beta, \\ Y_{l_K+2} - Z < \ln\beta,\ \cdots,\ Y_K - Z < \ln\beta \end{array} \right\} \tag{5.34}$$

As in the derivation of the capture probability for Capture Model #1, in obtaining an analytical expression for Capture Model #2 we again use the derivations from Gupta [Gup63] and Zhang and Aalo [ZA01] in our development of packet capture, thus getting

$$P_{cap}(l|n)=\int_0^{R_1}\int_0^{R_2}\cdots\int_0^{R_K}\left[\int_{-\infty}^{\infty}\left[\prod_{i=1}^{l_K}Q\left(\frac{\ln\beta+\hat{\mu}_i+\sqrt{\rho}\hat{\sigma}_i x}{\sqrt{1-\rho\hat{\sigma}_i}}\right)\cdot\prod_{j=l_N+1}^{K}Q\left(\frac{\ln\beta+\hat{v}_j+\sqrt{\rho}\hat{\omega}_j x}{\sqrt{1-\rho\hat{\omega}_j}}\right)\cdot f(x)\right]dx\right]g(r)d^K r \tag{5.35}$$

where the means are given by

$$\hat{\mu}_i=E[2\beta-(Y_i-Z)]=2\beta-m_i+m_Z \tag{5.36}$$

$$\hat{v}_i=E[Y_i-Z]=m_i-m_Z \tag{5.37}$$

with the variances being

$$\hat{\sigma}_i^2=Var[2\beta-(Y_i-Z)]=Var[Y_i-Z]=\sigma_i^2+\sigma_Z^2-2\rho\sigma_i\sigma_Z \tag{5.38}$$

$$\hat{\omega}_i^2=Var[Y_i-Z]=\sigma_i^2+\sigma_Z^2-2\rho\sigma_i\sigma_Z \tag{5.39}$$

and m_Z and σ_Z are the mean and the standard deviation of the sum of lognormal components that can be obtained from the Fenton-Wilkinson and the Schwartz-Yeh approximations that have been discussed previously. The derivation of equation 5.35 is shown in Appendix 1.

The functions $f(x)$ and $g(r)$ represent the standard Gaussian pdf and the pdf of the distance between the i^{th} mobile and the central base station given in equations 5.2 and 5.9, respectively. $Q(x)$ is the standard Gaussian cdf given in equation 5.27.

We then rewrite the capture probability given in equation 5.35 as

$$P_{cap}(l|n)=\int_0^{R_1}\int_0^{R_2}\cdots\int_0^{R_K}\left[\int_{-\infty}^{\infty}\left[\prod_{i=1}^{l_K}Q\left(\frac{\ln\beta+2\beta-m_i+m_Z+\sqrt{\rho}\hat{\sigma}_i x}{\sqrt{1-\rho\hat{\sigma}_i}}\right)\cdot\prod_{j=l_N+1}^{K}Q\left(\frac{\ln\beta+m_i-m_Z+\sqrt{\rho}\hat{\omega}_j x}{\sqrt{1-\rho\hat{\omega}_j}}\right)\right]\cdot f(x)dx\right]\cdot g(r)d^K r \tag{5.40}$$

In similar fashion to the previous section, in the presence of power control, there is no capture. Imperfections in power control again result in lognormally distributed received powers. Similar to the analysis performed in equations 5.31 to 5.34, the capture probability in the presence of power control errors is given by

$$P_{cap}(l|n)=\int_{-\infty}^{\infty}\left[\begin{array}{l}\prod_{i=1}^{l_K} Q\left(\dfrac{\ln\beta+2\beta-m_i+m_Z+\sqrt{\rho}\hat{\sigma}_i x}{\sqrt{1-\rho\hat{\sigma}_i}}\right)\cdot\\ \prod_{j=l_N+1}^{K} Q\left(\dfrac{\ln\beta+m_i-m_Z+\sqrt{\rho}\hat{\omega}_j x}{\sqrt{1-\rho\hat{\omega}_j}}\right)\end{array}\right]\cdot f(x)dx \qquad (5.41)$$

where $f(x)$ is the Gaussian pdf that accounts for the lognormally distributed power control errors.

5.5 Results

In this section we present the results for packet capture probability versus the capture ratio obtained from the expressions of the capture models obtained in sections 5.3 and 5.4. We assume that the mobiles have a uniform spatial distribution in the base station's coverage area with a normalized maximum MS to BS distance of 1.

In figures 5.1 and 5.2, the capture probability for Capture Model #1, as given by equation 5.29, is plotted against the capture ratio for various values of the correlation coefficient ρ. The lognormal mean is assumed to be 0dB with a standard deviation of 2dB with a data population of 3 users in figure 5.1 and 5 users in figure 5.2. The lognormal sum approximations used are the Fenton-Wilkinson approximation (using equations 4.5 and 4.6) and the Schwartz-Yeh approximation (using equations 4.23 and 4.24) discussed in Chapter 4.

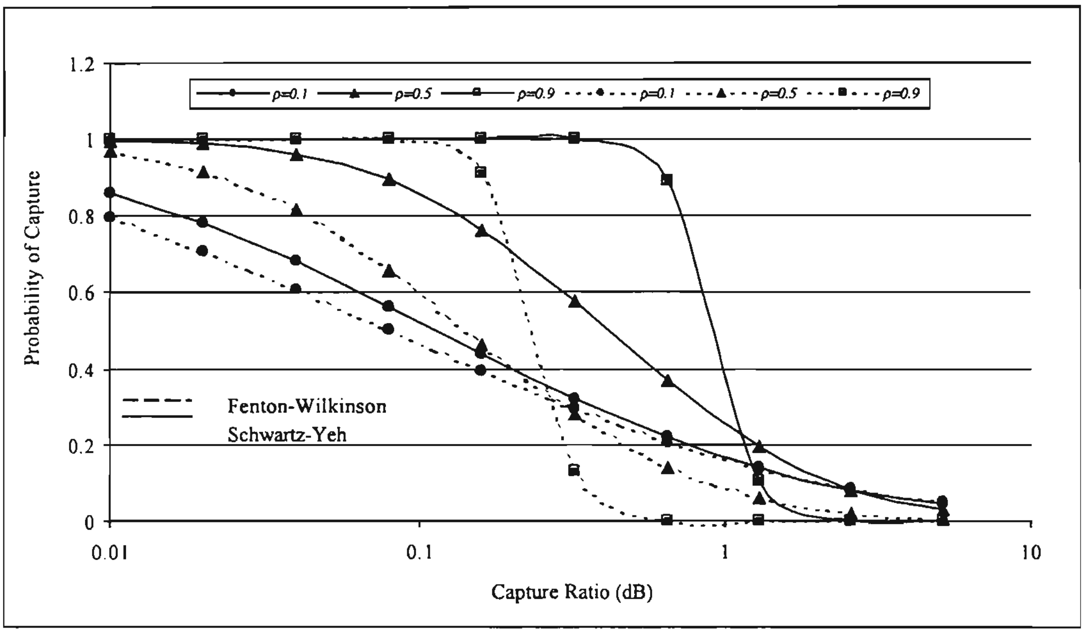

Figure 5.1 - Capture Probability versus the Capture Ratio for the Fenton-Wilkinson (dotted lines) and Schwartz-Yeh (solid lines) Approximations. (K=3, μ=0dB, σ=2dB, ?=0.1, 0.5 & 0.9)

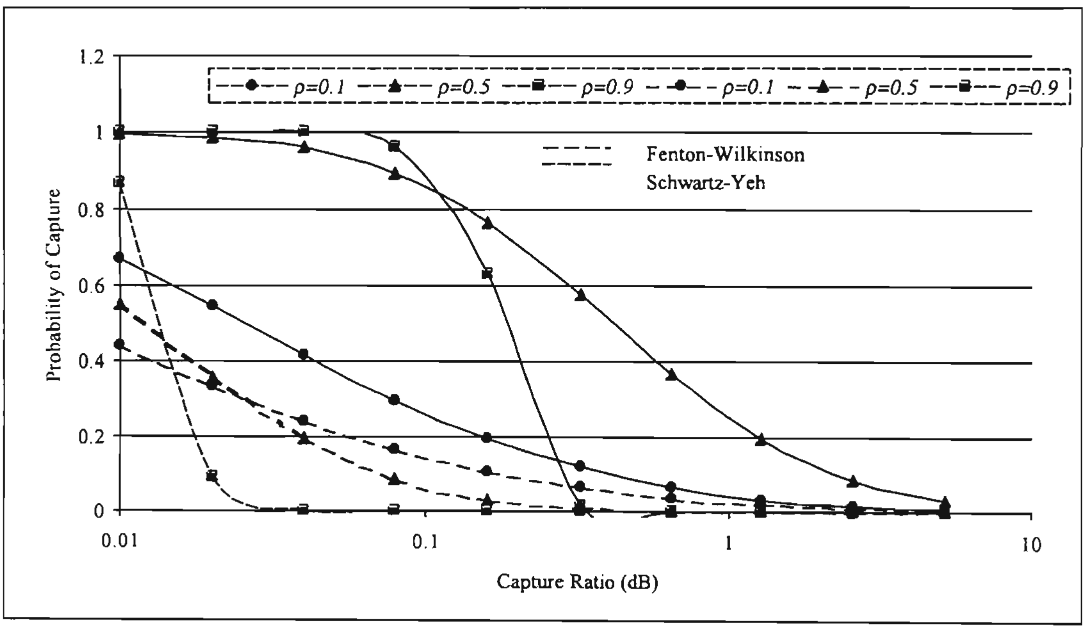

Figure 5.2 - Capture Probability versus the Capture Ratio for the Fenton-Wilkinson (dotted lines) and Schwartz-Yeh (solid lines) Approximations. (K=5, μ=0dB, σ=2dB, ?=0.1, 0.5 & 0.9)

It can be observed that the Fenton-Wilkinson approximation yields lower capture probabilities as compared to the Schwartz-Yeh approximation. This observation can be tied in with the discussions from Chapter 4, where it was shown that the Fenton-Wilkinson approximation yields very pessimistic results, especially for high values of the standard deviation. Another important observation that can be made from the figures 5.1 and 5.2 is the effect of correlated lognormal shadowing on the performance of spread-spectrum ALOHA systems. As the correlation coefficient, ?, increases, the system performance improves. For example, to achieve a packet success (capture) probability of 0.6 a margin of 3.6dB is required for the system with correlated shadowing with a correlation of ? = 0.1 to provide the same performance as that for the system with correlated shadowing with a correlation of ? = 0.5. Alternatively, when the correlation coefficient ? is 0.1 and the capture ratio β is 0.1dB, the capture probability is 0.522; a correlation coefficient of 0.5 achieves a capture probability of 0.859; and a correlation coefficient of 0.9 achieves a capture probability of 0.998.

In figures 5.3 and 5.4 we plot the capture probability for Capture Model #1 against the capture ratio for different values of the standard deviation for a fixed correlation coefficient (0.5 and 0.9, respectively) and a fixed data population of 3 users using the Schwartz-Yeh approximation. The aim of figures 5.3 and 5.4 is to illustrate the effect of the standard deviation on the capture probability.

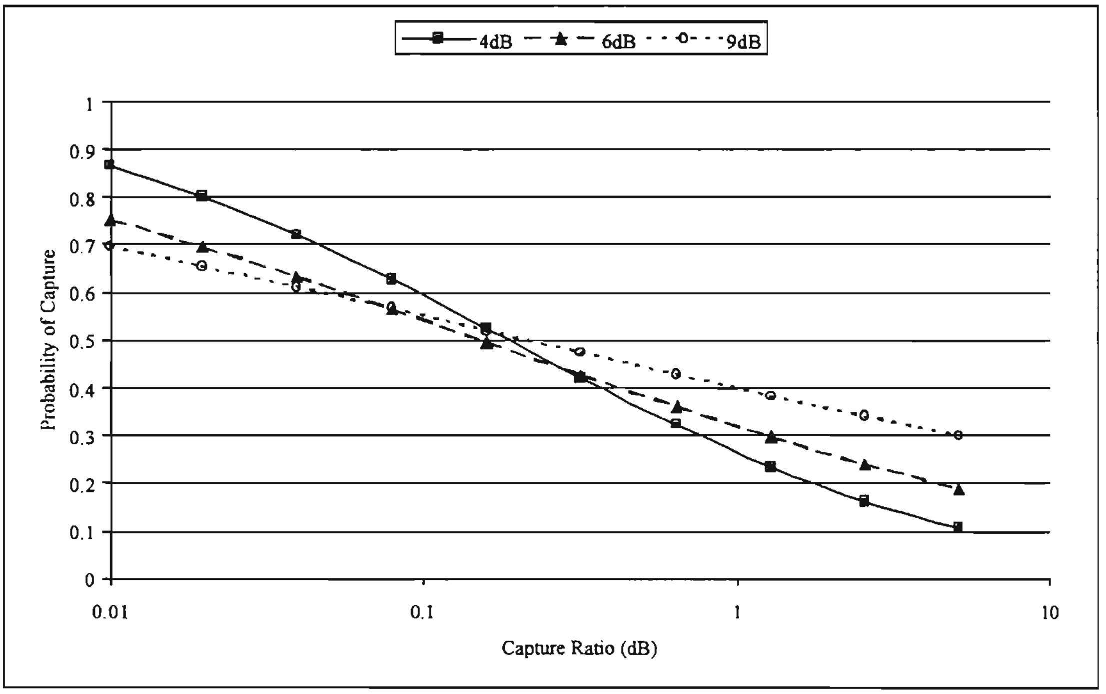

Figure 5.3 - Capture Probability against the Capture Ratio for the Schwartz-Yeh approximation showing the effect of the lognormal variance on the capture probability. (K=3, μ=0dB, σ=4dB, 6dB & 9dB, ?=0.5)

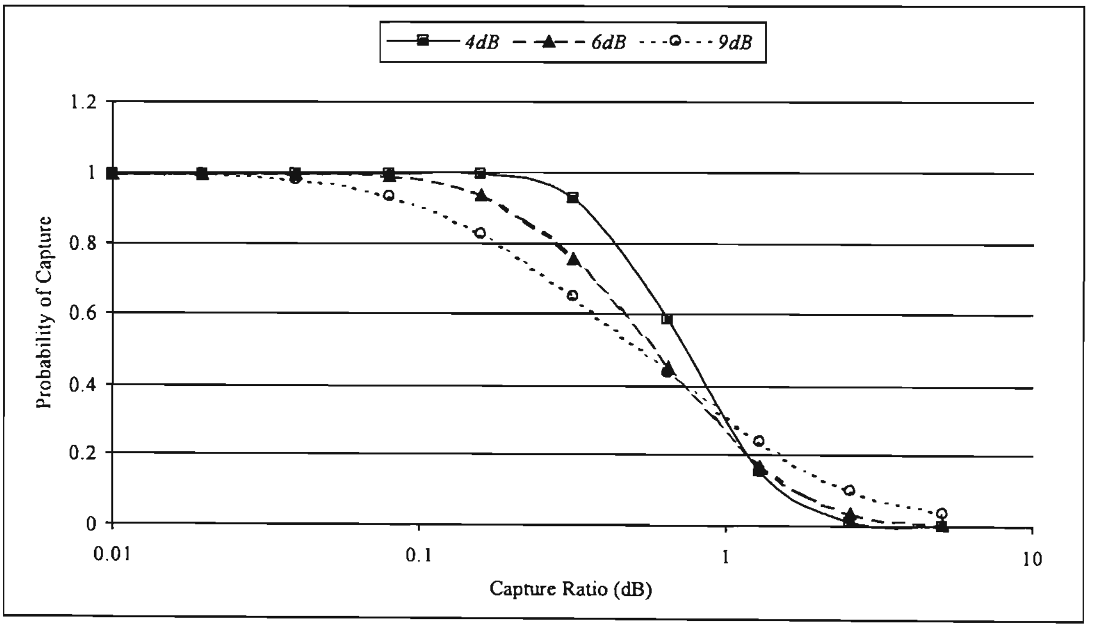

Figure 5.4 - Capture Probability against the Capture Ratio for the Schwartz-Yeh approximation showing the effect of the lognormal variance on the capture probability. (K=3, μ=0dB, σ=4dB, 6dB & 9dB, ?=0.9)

It can be observed that a low variance between the lognormal components achieves a higher capture probability for low values of the capture ratio. However, when the capture ratio approaches and exceeds 1dB (narrow-band systems), a low variance between the lognormal components yields lower capture probability. This result can be generalized to saying that for wideband spread-spectrum ALOHA type systems, a low variance between the lognormal components yields a high capture probability. For narrowband spread-spectrum ALOHA type systems, a low variance between the lognormal components yields a low capture probability.

In the computations of the packet capture probability for Capture Model #2, the multi-dimensional integrals in equation 5.40 pose some computational difficulties. Due to this difficulty, the probability was computed using semi-analytical techniques. The function $g(r)$ was computed using simulations and then multiplied by the integral of the inner function, $f(x)$. The simulation was performed by generating the positions of the users by generating a pseudorandom number u that is uniformly distributed in the $[0, 1]$. The user i's position, r_i, is deduced according to equation 5.9 using the percentile transformation method [Pap91, page 226] by

$$r_i = R_0 + (R - R_0)\sqrt{u_i} \tag{5.41}$$

Repeating the process 20000 times, we can estimate the value of the users' positions by taking the average of all the observations of the users' positions. For ease of computations, we normalize the MS to BS distances to $R_0 = 0$ and $R = 1$.

Figure 5.5 and figure 5.6 (data populations of 3 users and 5 users, respectively) depict the effect of the correlation coefficient, ρ, on the capture probability of Capture Model #2. As already seen with Capture Model #1, the higher correlation between the received power levels of the MSs at the BS improves the probability of capture of the mobile's packets at the base station.

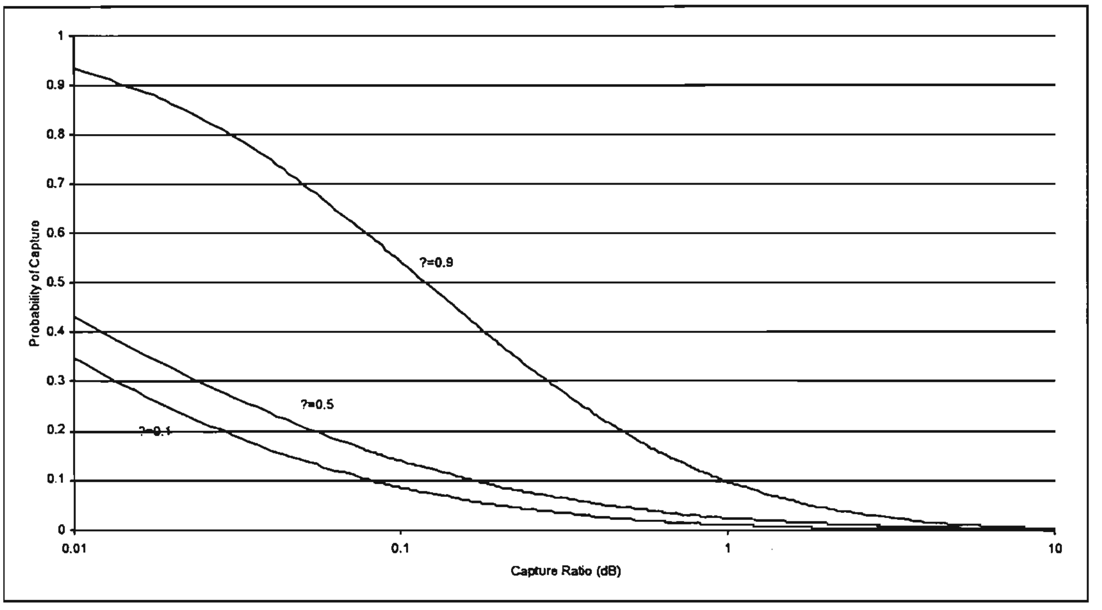

Figure 5.5 - Capture Probability against the Capture Ratio for Capture Model #2 showing the effect of correlation coefficient, ?, on the capture probability. (K=3, μ=0dB, σ=2dB, ?=0.1, 0.5 & 0.9, R_0 = 0, R = 1)

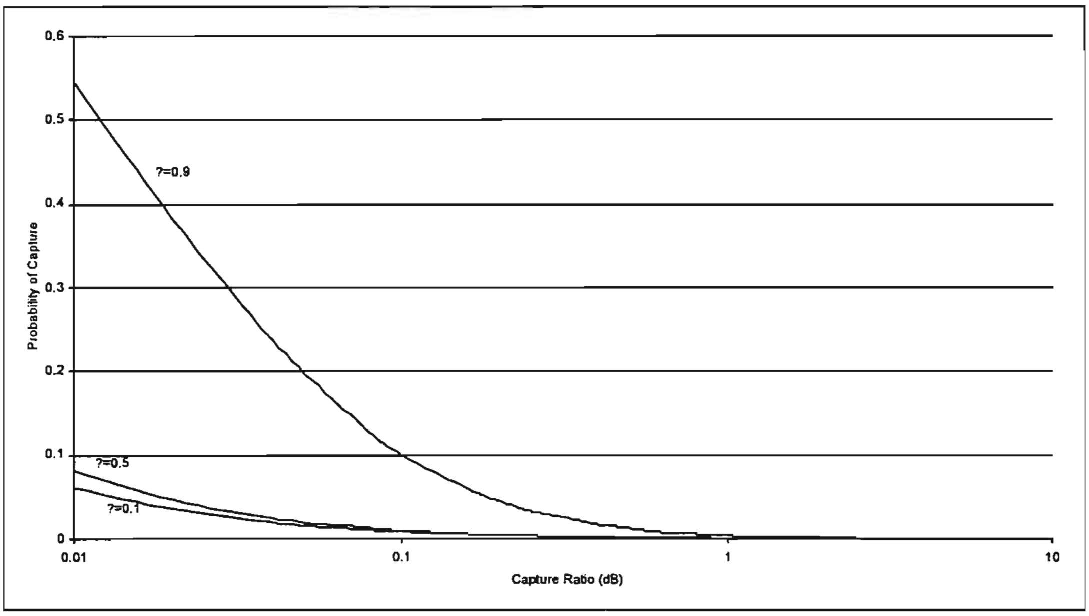

Figure 5.6 - Capture Probability against the Capture Ratio for Capture Model #2 showing the effect of correlation coefficient, ?, on the capture probability. (K=5, m=0dB, s=2dB, ?=0.1, 0.5 & 0.9, R_0 = 0, R = 1)

Figure 5.7 shows plots of the capture probability versus the capture ratio for various values of the lognormal variance. It can be noted from this plot that as the lognormal variance of the received power levels is increased, there is a decrease in the capture probability. However, as the capture ratio goes higher than 1dB, i.e., the system is now narrowband, the higher lognormal variances gives rise to higher capture probabilities. The same observation was made for Capture Model #1. The choice of the expected lognormal variance between the received power levels would thus be a very important factor in the design of wideband packet CDMA systems with the capture effect.

Figure 5.8 and figure 5.9 show the effect of the cell size on the capture probability. In figure 5.8, the minimum distance between the MSs and the BS (R_0) is varied for plots of capture probability versus the capture ratio while the maximum distance between the MSs and the BS is fixed at R. This would represent a macro-cellular system in which the BS is spilling or overshooting. In figure 5.9, the maximum distance between the MSs and the BS (R) is varied while R_0 is fixed. This would represent a micro-cellular system. The important observation from the two plots is that R_0 and R do not have separable effects on the capture probability, but, that capture probability really depends on the cell size (R - R_0). That is, to maximize the capture probability, one needs to design for maximum cell size.

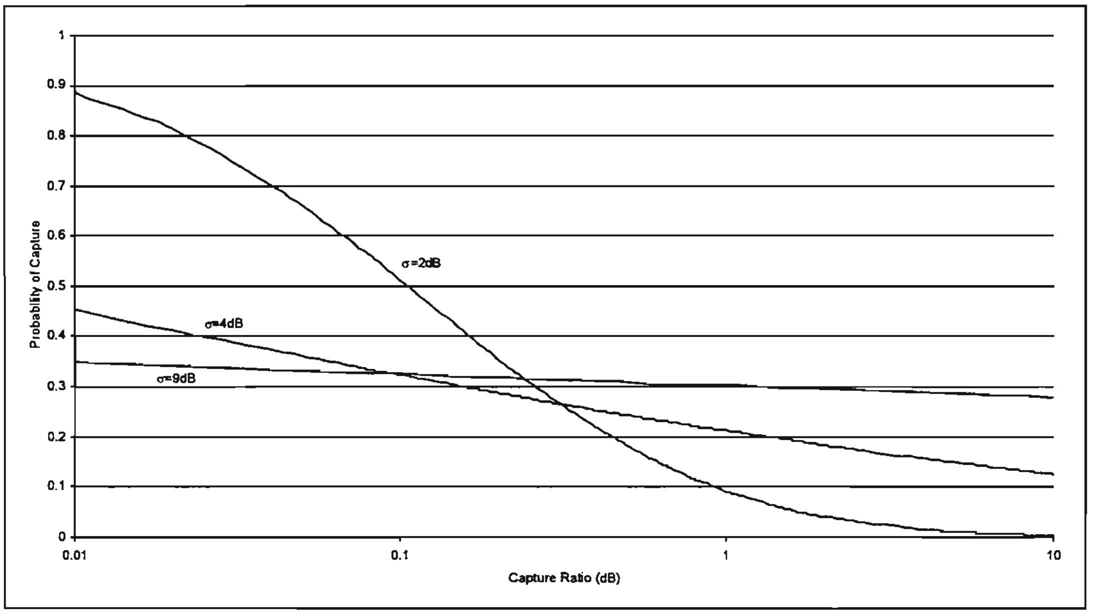

Figure 5.7 - Capture Probability for Capture Model #2 showing the effect of the lognormal variance, s, on the capture probability. (K=3, μ=0dB, σ=2dB, 4dB & 9dB, ?=0.9, R_0 = 0, R = 1)

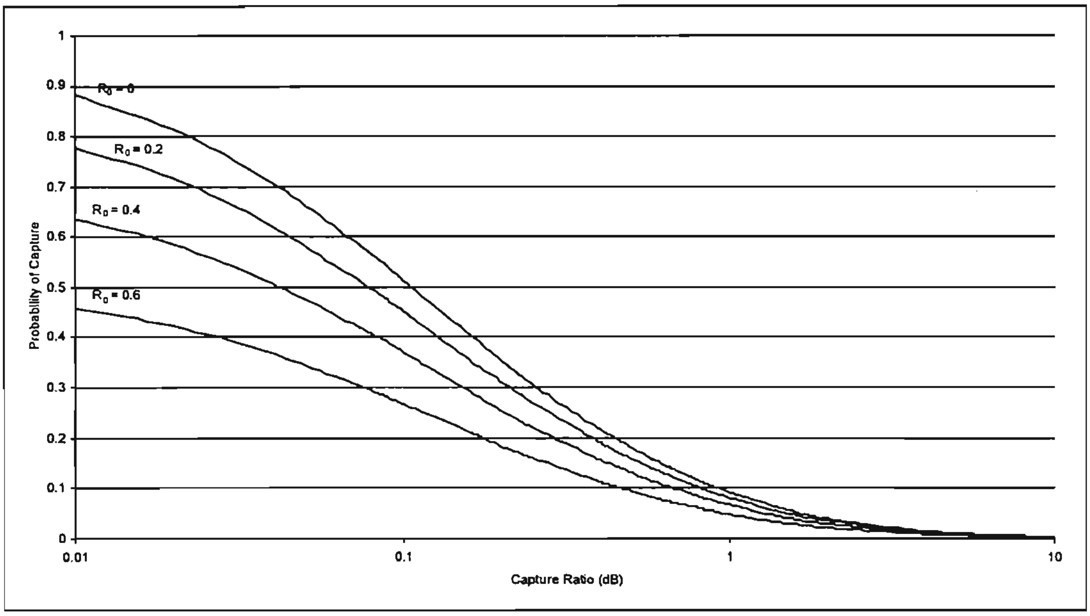

Figure 5.8 - Capture Probability for Capture Model #2 showing the effect of the minimum distance between the MSs and the BS, R_0, on the capture probability. (K=3, μ=0dB, σ=2dB, ?=0.9, R_0 = 0, 0.2, 0.4 & 0.6, R = 1)

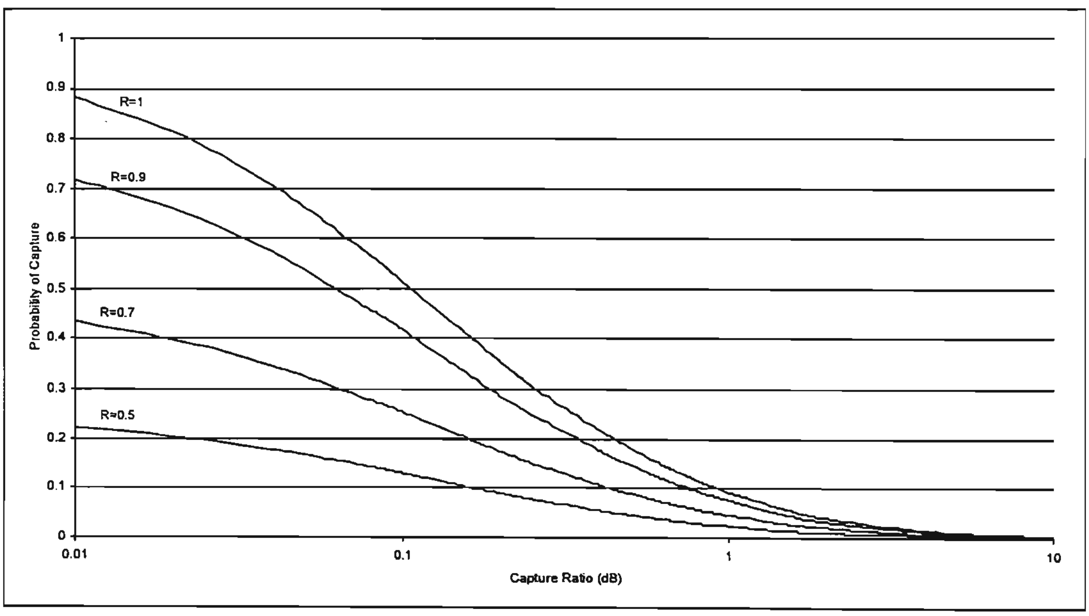

Figure 5.9 - Capture Probability against the Capture Ratio for Capture Model #2 showing the effect of the maximum distance between the MSs and the BS, R, on the capture probability. (K=3, μ=0dB, σ=2dB, ?=0.9, R_0 = 0, R = 1, 0.9, 0.7 & 0.5)

In figures 5.10 to 5.11 we look at the effect of the capture ratio, β, on the capacity of spread-spectrum ALOHA systems in the presence of the capture models described in Section 5.4 and 5.5 as a function of the mean offered load. The throughput equation and the Binomial traffic model given in Chapter 3 are used with the packet success rate being effectively the same as the capture probability.

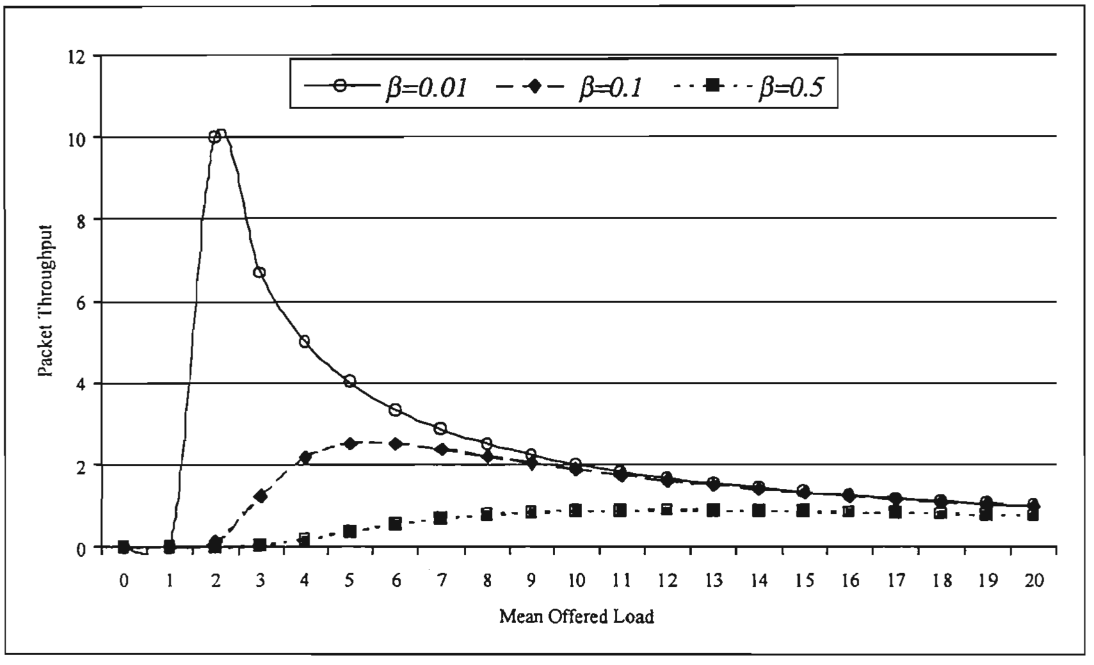

Figure 5.10 - Effect of the capture ratio, b, on the system throughput for Capture model #1 (K=20, μ=0dB, σ=2dB, ρ=0.9, R_0 = 0 & R = 1)

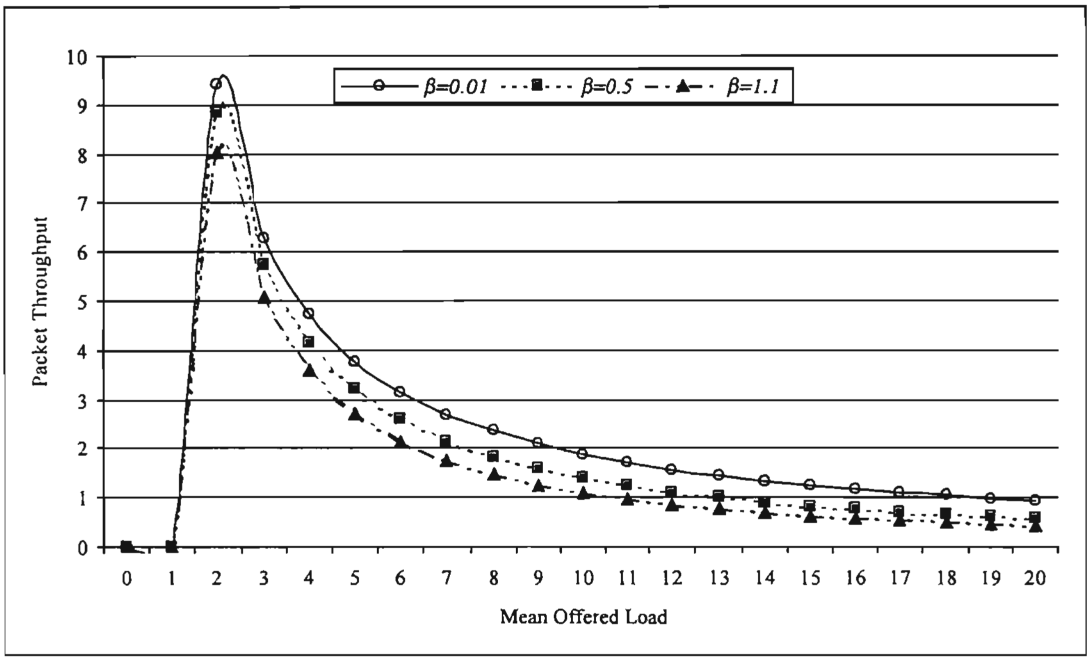

Figure 5.11 - Effect of the capture ratio, b, on the system throughput for Capture model #2 (K=20, μ=0dB, σ=2dB, ρ=0.9, R_0 = 0 & R = 1)

It can be noted that the capture ratio has a more pronounced effect on capture model #1 than on capture model #2. When the capture ratio approaches 1, when we say the system is narrowband, the throughput of the system is greatly reduced.

Figures 5.12 and 5.13 show the effect of the correlation of the received power levels on the system throughput when the capture ratio is set conservatively at 0.1. It can be noted that a high correlation of the received power levels gives a higher throughput for both capture models. However, capture model #2 is more resilient to the effects of the correlation as compared to capture model #1.

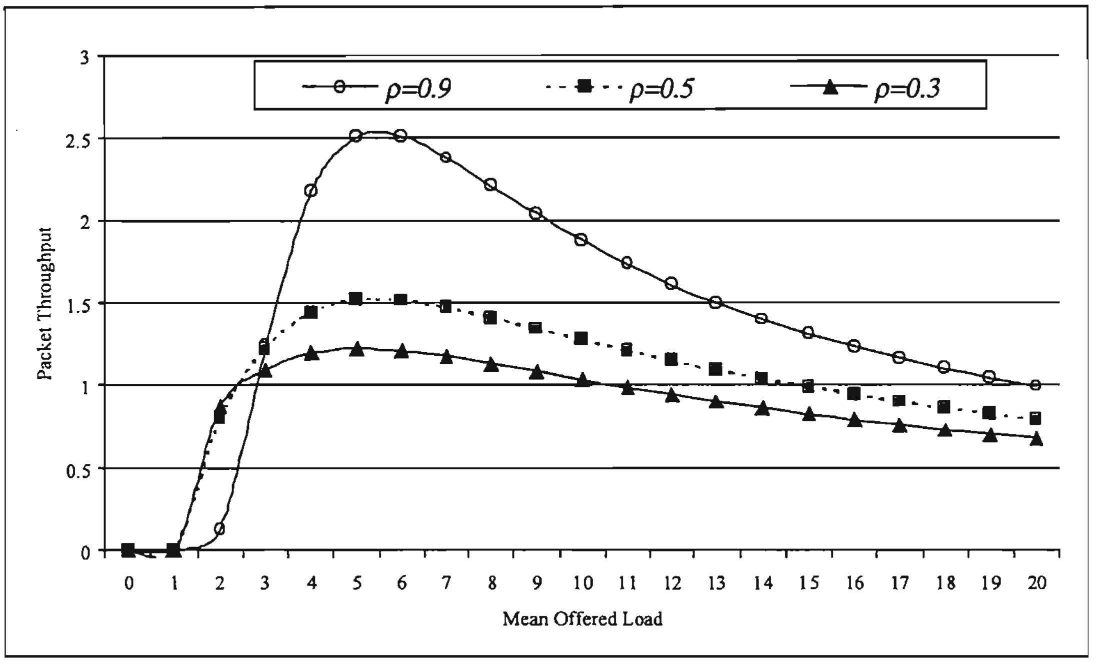

Figure 5.12 - Effect of the correlation coefficient, ρ, on the system throughput for Capture model #1 (K=20, μ=0dB, σ=2dB, β=0.1, R_0 = 0 & R = 1)

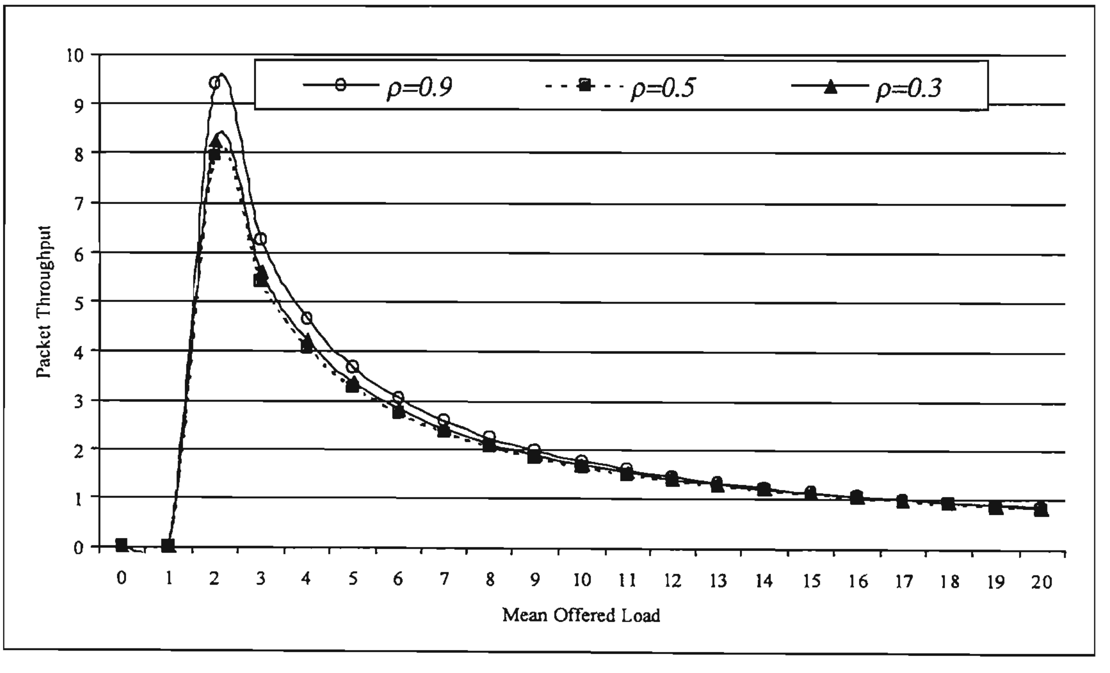

Figure 5.13 - Effect of the correlation coefficient, ρ, on the system throughput for Capture model #2 (K=20, μ=0dB, σ=2dB, β=0.1, R_0 = 0 & R = 1)

In figures 5.10 and 5.11 it was shown that the smaller capture ratios give higher system capacities. Figures 5.14 and 5.15 compare the throughputs of the two capture models for a constant correlation coefficient of 0.9 with capture ratios of 0.001 and 0.1, respectively. From the two figures, it is shown that the capacity of capture model #1 is higher than that of capture model #2. However, capture model #2 is more resilient to the effects of the radio propagation channel, making capture model #2 a better model for approximating the effects of power control errors that result in the capture effect.

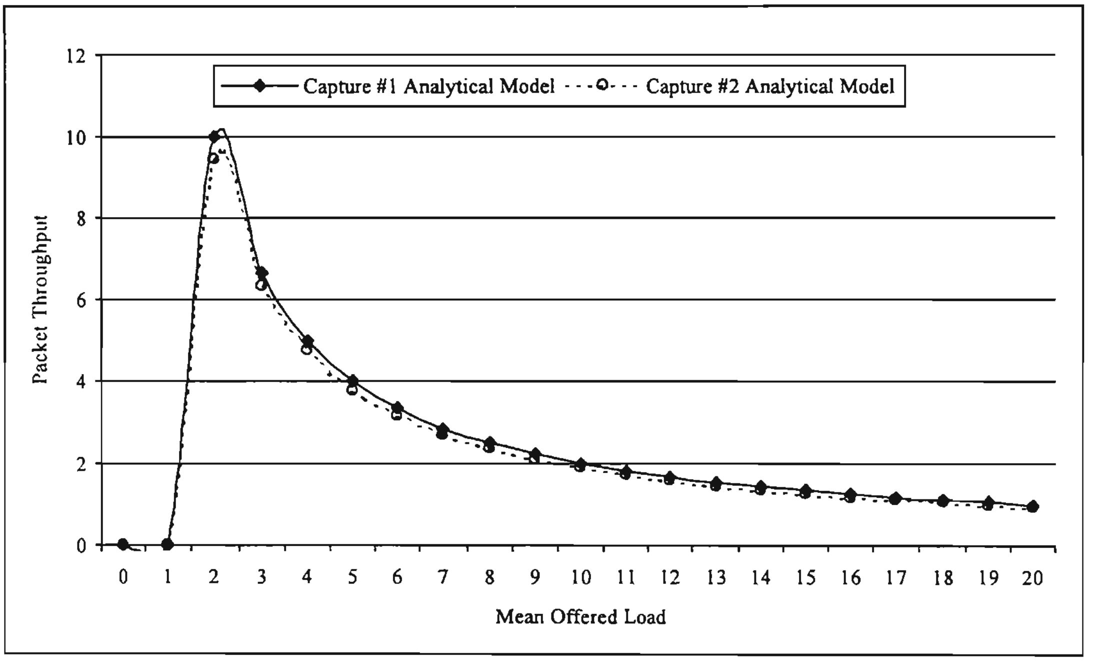

Figure 5.14 - Comparison of the capacities of Capture Model #1 and Capture Model #2 (K=20, μ=0dB, σ=2dB, β=0.001, ρ=0.9, R_0 = 0 & R = 1)

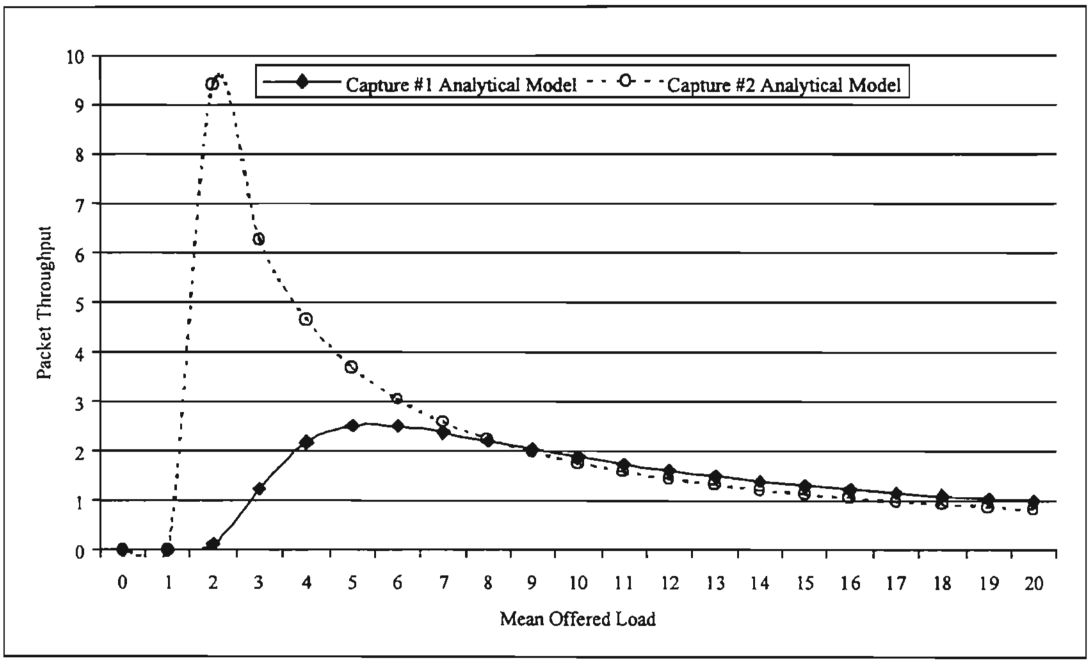

Figure 5.15 - Comparison of the capacities of Capture Model #1 and Capture Model #2 ($K=20$, $\mu=0dB$, $\sigma=2dB$, $\beta=0.1$, $\rho=0.9$, $R_0 = 0$ & $R = 1$)

When the throughputs for the capture models shown in figures 5.10 to figure 5.15 are compared to the SGA BER model and SIGA BER model throughputs shown in figures 3.4 to 3.8, it is evident that the SGA and SIGA BER models given have higher throughputs than the capture models described in this chapter. However, the high throughputs obtained for the SGA and SIGA BER models are obtained at the expense of higher processing gains and thus much higher bandwidths.

5.6 Summary

This chapter has considered a spread-spectrum slotted ALOHA system in which the base station is capable of capturing a single packet and another model in which the base station is capable of capturing multiple packets. Both capture models have been described analytically. We have seen the effect of system parameters on the capture ratios and their effect on the capacity of systems described by both capture models. For both capture models as the correlation coefficient, ?, increases, the system capacity improves. This chapter has also shown that for wideband spread-spectrum ALOHA type systems; an increase in variance between the lognormal components yields a high capture probability. For narrowband spread-spectrum ALOHA type systems, an increase in the variance between the lognormal components yields a low capture probability. It has also been shown that maximizing the cell size maximizes the system capacity.

Another important observation made in this chapter is that a system in which the base station is capable of multiple packet capture, achieves higher capacity for narrow-band systems. Moreover, the multiple packet capture system has been shown to be more resilient to radio channel and system changes than the single packet capture model.

6 CONCLUSIONS

This thesis has provided a survey of the literature on MAC protocols, concentrating on the random access ALOHA and the direct sequence spread-spectrum protocols. Important direct sequence spread-spectrum implementation aspects have been presented. Some of the more recent MAC protocols have also been presented in the literature survey provided by this thesis.

In chapter 3, we have provided the various methods that have been suggested by previous researchers for computing the BER and the throughput of spread-spectrum systems. We have shown that the averaged BER model proposed by Prasad *et al* in [NMP92], [NP93] and [NWP95] gives very pessimistic BER results, due to the averaging. We have shown that the SGA BER model gives a higher capacity than the SIGA BER model at higher traffic loads. Another approximation that has been suggested is the IGA. We have shown that this approximation can only be used if the characteristic function of the MAI is known and that the IGA approximation is computationally complex. We have further shown that the averaged BER model reduces to the SGA BER model in the presence of perfect power control.

The main contribution of this research has been the inclusion of the well-known capture effect from narrow-band ALOHA systems to the spread-spectrum slotted ALOHA systems. We have shown that in spread-spectrum slotted ALOHA systems, imperfections in power control lead to signals from different terminals being received at the central base station with unequal power levels. We have said that these unequal power levels at the central base station lead to capture. Our main area of concern has been in giving quantitative results of the effects of capture, as a manifestation of power control errors, in spread-spectrum slotted ALOHA systems. We have given two expressions for computing the probability that one or several packets will be received and decoded correctly in the presence of contending packets.

In evaluating the capture expressions, we have used the assumption made by Foschini *et al* in [FM95] and Woerner *et al* in [CW96], that power control errors are lognormally distributed; hence the received power levels have a lognormal distribution. We have said that power control alleviates the near-far effect, but that practical implementations of power control cannot track the lognormal shadowing; hence the lognormal power control errors.

In evaluating the sum of interfering users at the central receiver with lognormally distributed received power levels, we have used the assumption made by Fenton and Wilkinson in [Fen60] and by Schwartz and Yeh in [SY82], that the sum of lognormally distributed random variables is also lognormal. These approximations have been used to approximate the moments of the sums of lognormal random variables.

This thesis has then given expressions for capture in the case when there is no power control and when there are power control errors. When there is perfect power control, there is no capture. Two capture expressions have been given. The first one has assumed single packet capture. The second expression has assumed that the central base station is capable of receiving several packets simultaneously, as is the case in spread-spectrum systems. This thesis has shown that the second capture expression offers resilience to radio characteristics and system characteristics; hence its superiority. We have also shown that as mentioned in other works, the value of the capture ratio dictates the type of system. In general, capture ratios of greater than 1 result in narrow-band systems and hence the reduction in capacity.

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Appendix 1 Supportive Work for Chapter 5

Derivation of the capture probability for Capture Model #2, the probability that l packets are captured and $n-l$ packets are not captured, given n contending packets:

In [Gup63, page 800] Gupta has shown that if

$$F_n(h_1,h_2,\cdots,h_n;\{\rho_{ij}\})=\int_{-\infty}^{h_1}\int_{-\infty}^{h_2}\cdots\int_{-\infty}f(x_1,x_2,\cdots,x_n;\{\rho_{ij}\})dx_1\cdots dx_n \tag{A.1}$$

and the correlation matrix $\{\rho_{ij}\}$ of the x_i's has the structure $\rho_{ij}=\alpha_i\alpha_j(i\neq j)$ where $-1\leq\alpha_i\leq+1$, then these variates X_i can be generated from $n+1$ independent standard normal variates $Z_1,Z_2,\cdots,Z_n;\ Y$ by the transformation

$$X_i=(1-\alpha_i)^{\frac{1}{2}}Z_i+\alpha_iY \tag{A.2}$$

It then follows that

$$F_n(h_1,h_2,\cdots,h_n;\{\rho_{ij}\})=\int_{-\infty}^{\infty}\left[\prod_{i=1}^{n}F\left(\frac{h_i-\alpha_iy}{(1-\alpha_i^2)^{\frac{1}{2}}}\right)\right]f(y)dy \tag{A.3}$$

If $\rho_{ij}=\rho$ for all i,j, then equation A.3 reduces to

$$F_n(h,h,\cdots,h;\{\rho\})=\int_{-\infty}^{\infty}F^n\left[\frac{\left(h+\rho^{\frac{1}{2}}y\right)}{(1-\rho)^{\frac{1}{2}}}\right]f(y)dy \tag{A.4}$$

Adopting equation A.4 to the equation 5.32, with the case where all the variables of equation 5.32 had the same sign yields

$$\begin{aligned}P_{cap}(l\mid n)&=\Pr\{Y_1-Z>\ln\beta,Y_2-Z>\ln\beta,\cdots,Y_n-Z>\ln\beta\}\\&=\int_{-\infty}^{\infty}\prod_{i=1}^{n}F\left(\frac{\ln\beta+E[Y_i-Z]+\sqrt{\rho E[Y_i-Z]^2}\,x}{\sqrt{(1-\rho)E[Y_i-Z]^2}}\right)\cdot f(x)dx\end{aligned} \tag{A.5}$$

where Y_i and Z are Gaussian random variables; then $F(\bullet)$ is the well-known standard Gaussian cumulative distribution function (Q function) given in equation 5.27 and $f(x)$ is the standard Gaussian probability distribution function given in equation 5.2. However, the variables of equation 5.32 do not have the same sign, hence equation 5.32 yields

$$P_{cap}(l \mid n) = \Pr\{h_1 > \ln\beta, h_2 > \ln\beta, \cdots, h_l > \ln\beta, k_{l+1} < \ln\beta, k_{l+2} < \ln\beta, \cdots, k_n < \ln\beta\}$$

$$= \int_{-\infty}^{\infty}\prod_{i=1}^{l} F\left(\frac{\ln\beta + E[h_i] + \sqrt{\rho E[h_i]^2}}{\sqrt{(1-\rho)E[h_i]^2}}\right)\cdot\prod_{j=l+1}^{n} G\left(\frac{\ln\beta - E[k_j] - \sqrt{\rho E[k_j]^2}}{\sqrt{(1-\rho)E[k_j]^2}}\right)\cdot f(x)dx$$

(A.6)

where h_i and k_j are both Gaussian random variables, hence $F(\cdot)$ and $G(\cdot)$ are given by the standard Gaussian cumulative distribution function (Q function) given by equation 5.27.

The Gaussian random variable Y_i was obtained from equation 5.12 where

$$L = r^{-\eta}e^{\xi} = e^{Y} \tag{5.12}$$

From equation 5.12, the effect of the distance, r, of the mobile from the base station can be included into equation A.6 by the multiple integration used in 5.40 using the probability distribution function of the distance of the mobile from the base station given in equation 5.9, such that

$$P_{cap}(l|n) = \int_0^{R_1}\int_0^{R_2}\cdots\int_0^{R_K}\left[\int_{-\infty}^{\infty}\left[\begin{array}{c}\prod_{i=1}^{l_K} Q\left(\frac{\ln\beta + 2\beta - m_i + m_Z + \sqrt{\rho}\hat{\sigma}_i x}{\sqrt{1-\rho\hat{\sigma}_i}}\right)\cdot \\ \prod_{j=l_N+1}^{K} Q\left(\frac{\ln\beta + m_i - m_Z + \sqrt{\rho}\hat{\omega}_j x}{\sqrt{1-\rho\hat{\omega}_j}}\right)\end{array}\right]\cdot f(x)dx\right]\cdot g(r)d^K r \tag{5.40}$$