Cross Layer Hybrid ARQ 2 Cooperative Diversity

Sannesh Rabiechand Beharie

A dissertation

Presented to:

The University of KwaZulu-Natal in fulfillment of the requirements for the degree of:

MASTER OF SCIENCE IN ENGINEERING

The School of Electrical, Electronic and Computer Engineering
Durban, South Africa
June 2008

As the candidate's Supervisor I agree to the submission of this dissertation.

Date of Submission: _______________

Supervisor: _______________________

Prof. HongJun Xu

Declaration of Originality

I, Sannesh Rabiechand Beharie, declare that:

i The research reported in this dissertation, except where otherwise indicated, is my original work

ii This dissertation has not been submitted for any degree or examination at any other university.

iii This dissertation does not contain other persons' data, pictures, graphs or other information, unless specifically acknowledged as being sourced from other persons.

iv This dissertation does not contain other persons' writing, unless specifically acknowledged as being sourced from other researchers. Where other written sources have been quoted, then:

a) their words have been re-written but the general information attributed to them has been referenced;

b) where their exact words have been used, their writing has been placed inside quotation marks, and referenced.

v Where I have reproduced a publication of which I am an author, co-author or editor, I have indicated in detail which part of the publication was actually written by myself alone and have fully referenced such publications.

vi This dissertation does not contain text, graphics or tables copied and pasted from the Internet, unless specifically acknowledged, and the source being detailed in the dissertation and in the References sections.

Signed: ____________________

Sannesh R. Beharie

To my grandfather, my greatest teacher:

The Late Mr. D.K. Maraj.

and to The Omnipotent, Omnipresent and Omniscient Supreme Being who makes everything happen for an underlying reason.

Acknowledgements

Firstly I would like to acknowledge my dear mother, sisters, grandmother and the rest of my family for their continuous support, love, care and attention during the journey of my MSc. Thanks for keeping me sane at times and for providing laughter and comfort when it was needed most.

To my Supervisor, Prof. HongJun Xu for your guidance, patience, assistance, support and belief in me. He has supported me both academically and emotionally throughout my MSc which has helped me move mountains to yield fruitful research developments and contributions. In addition, I thank him, for going out his way to create an environment that was suitable for the production of high quality research within the Centre of Excellence (CoE). Thank you for sacrificing personal time to help with my research and for teaching me that academia is a way of life.

I would like to acknowledge and thank Prof. Fambirai Takawira for his guidance and support.

To Reshma Kassie and Dr. K. Maraj for proof reading my dissertation.

A special thank you to my friends from the CoE, especially Jules Mouatcho Moualeu and Narushan Pillay, for their ever so helpful discussions and advice during my research and simulation endeavours.

I wish to acknowledge my friends for their company and encouragement during my MSc, which I found to be very relaxing.

To Telkom SA for endorsing and funding my research.

Lastly and of equal importance, to my work colleagues:

1. Andrew Herd, Peter Peyper and Ronnie Havenga for their support, understanding and encouragement.
2. Dalene Janse van Rensburg for helping to print my dissertation.

Abstract

Cooperative communication allows for single users in multi user wireless network to share their antennas and achieve virtual antenna transmitters, which leads to transmit diversity. Coded Cooperation introduced channel coding into cooperative diversity over traditional pioneer cooperative diversity methods which were based on a user repeating its partner's transmitted signals in a multi-path fading channel environment in order to improve Bit Error Rate (BER) performance..

In this dissertation the Coded Cooperation is simulated and the analytical bounds are evaluated in order to understand basic cooperation principles. This is done using Rate Compatible Punctured Convolutional Codes (RCPC). Based on the understanding of these principles a new protocol called Cross Layer Hybrid Automatic Repeat reQuest (ARQ) 2 Cooperative Diversity is developed to allow for improvements in BER and throughput.

In Cross Layer Hybrid ARQ 2 Cooperation, Hybrid ARQ 2 (at the data-link layer) is combined with cooperative diversity (at the physical layer), in a cross layer design manner, to improve the BER and throughput based on feedback from the base station on the user's initial transmissions. This is done using RCPC codes which partitions a full rate code into sub code words that are transmitted as incremental packets in an effort to only transmit as much parity as is required by the base station for correct decoding of a user's information bits. This allows for cooperation to occur only when it is necessary unlike with the conventional Coded Cooperation, where bandwidth is wasted cooperating when the base station has already decoded a user's information bits.

The performance of Cross Layer Hybrid ARQ 2 Cooperation is quantised by BER and throughput. BER bounds of Cross Layer Hybrid ARQ 2 Cooperation are derived based on the Pairwise Error Probability (PEP) of the uplink channels as well as the different inter-user and base station Cyclic Redundancy Check (CRC) states. The BER is also simulated and confirmed using the derived bound. The throughput of this new scheme is also simulated and confirmed via analytical throughput bounds. This scheme maintains BER and throughput gains over the conventional Coded Cooperation even under the worst inter-user channel conditions.

Table of Contents

1 Introduction 1
1.1 Next Generation Broadband Wireless Communications 1
1.2 Multiple Input Multiple Output Systems 2
1.3 Cooperative Diversity 3
1.4 Detect and Forward Signalling 6
1.5 Amplify and Forward 8
1.6 Channel Coded Cooperation 10
1.7 Research Motivation: Cross Layer Hybrid ARQ 2 Cooperative Diversity 10
1.8 Dissertation Outline 11

2 Coded Cooperation 13
2.1 The Coded Cooperation Protocol 13
2.2 Coded Cooperation using RCPC Codes 14
2.3 The Channel Model 16
2.4 Performance Analysis of Coded Cooperation 18
2.5 No Cooperation Analysis 19
2.6 The PEP for the Uplink Channels 21
2.7 Determining the Transmission Case Probabilities 23
2.8 The End to End BER 26
2.9 Simulation Parameters and Discussion 26
2.10 Chapter Summary 32

3 Cross Layer Hybrid ARQ 2 Cooperative Diversity 33
3.1 Modelling the wireless channel 33
3.2 Cross Layer Hybrid ARQ 2 Cooperation Protocol 35
3.2.1 Case 1.1 40
3.2.2 Case 1.2 42
3.2.3 Case 1.3 42
3.2.4 Case 1.4 43
3.2.5 Case 2 44
3.2.6 Case 3 45
3.2.7 Case 4 45
3.3 No Cooperation Analysis 47
3.4 Determining the PEP on The Uplink Channels 47
3.5 Determining the Transmission Case Probabilities 48
3.6 The End to End Bit Error Probability 51
3.7 Simulation Parameters and Discussion of Results 53
3.8 Chapter Summary 58

4 Throughput Evaluation of Cross Layer HARQ 2 Cooperative Diversity 60
4.1 Throughput Improvements in Cross Layer HARQ 2 Cooperation 60
4.2 Throughput Analysis 61
4.3 Simulation Parameters and Discussion 66
4.4 Chapter Summary 69

5 Conclusions and Future Work 71
5.1 Dissertation Achievements 71
5.2 Future Work 74
5.3 Current Literature Contributions to the Research Area 75

References 76

Appendix A **81**
Derivation of the End to End BER Bounds for Cross Layer Hybrid ARQ 2 Cooperative Diversity 81
Derivation of the 50% Best Case Cooperation End to End BER Bound 81
The 25% Best Case Cooperation End to End BER Bound 97

Appendix B **98**
Derivation of throughput bounds for Cross Layer Hybrid ARQ 2 Cooperative Diversity ... 98
Derivation of Non Cooperative Direct Transmission Throughput Bound 98

List of Figures

Figure 1.1 – Cooperative Communication....4
Figure 1.2: User - Relay Relationship in Cooperative Communications....5
Figure 1.3: The Relay Channel....6
Figure 1.4: CDMA signalling protocol for detect and forward....7
Figure 1.5: Detect and forward....8
Figure 1.6: Amplify and forward....9

Figure 2.1: RCPC transmission using Coded Cooperation....15
Figure 2.2: The four cooperation transmission scenarios....16
Figure 2.3: 50% cooperation with symmetric uplinks....27
Figure 2.4: A comparison between 50% and 25% cooperation....29
Figure 2.5: Asymmetric uplink conditions....30
Figure 2.6: Independent vs. Reciprocal interuser channels for 50% cooperation....31
Figure 2.7: Independent vs. Reciprocal interuser channels for 25% cooperation....31

Figure 3.1: Parity Packet Partitioning of the total parity frame....38
Figure 3.2: Cross Layer HARQ 2 Cooperation: System Diagram....39
Figure 3.3: 50% cooperation with symmetric uplinks....54
Figure 3.4: A comparison between 50% and 25% cooperation....55
Figure 3.5: Independent vs. Reciprocal interuser channels for 25% Best Case cooperation....56
Figure 3.6: Independent vs. Reciprocal interuser channels for 50% Best Case cooperation....56
Figure 3.7: Asymmetric uplink conditions....57
Figure 3.8: Comparison between Cross Layer HARQ 2 Cooperation and Coded Cooperation 58

Figure 4.1: Throughput comparison between Cross Layer HARQ 2 Cooperation and Coded Cooperation for 50% best case cooperation....67
Figure 4.2: Throughput comparison between Cross Layer HARQ 2 Cooperation and Coded Cooperation for 25% best case cooperation....68
Figure 4.3: Theoretical throughput bound vs. throughput simulation for 50% best case cooperation....68
Figure 4.4: Theoretical throughput bound vs. throughput simulation for 25% best case cooperation....69

Figure B.1: Transmission Scenarios during Non Cooperative Direct Transmission....98

List of Tables

Table 3.1: Cross Layer Hybrid ARQ 2 Transmission Scenarios....37
Table 3.2: Summary of transmission signals for Cross Layer HARQ 2 Cooperation....46

Glossary of Acronyms

BER	Bit Error Rate
RCPC	Rate Compatible Punctured Convolutional Codes
ARQ	Automatic Repeat request
PEP	Pairwise Error Probability
CRC	Cyclic Redundancy Check
CoE	Centre of Excellence
QoS	Quality of Service
MIMO	Multiple Input Multiple Output
SISO	Single Input Single Output
MRRC	Maximal Ratio Receiver Combiner
LAN	Local Area Network
WLAN	Wireless Local Area Network
AWGN	Additive White Gaussian Noise
CDMA	Code Division Multiple Access
TDMA	Time Division Multiple Access
FDMA	Frequency Division Multiple Access
SNR	Signal to Noise Ratio
FEC	Forward Error Correction
MRC	Maximal Ratio Combining
BPSK	Binary Phase Shift Keying
CSI	Channel State Information
BLER	Block Error Rate
HARQ	Hybrid Automatic Repeat reQuest
ACK	Acknowledge
NACK	Negative Acknowledge
3G	Third Generation
LDPC	Low Density Parity Check Codes

Chapter 1

1 Introduction

1.1 Next Generation Broadband Wireless Communications

Next generation wireless networks have very demanding requirements with regard to multi rate multimedia at exceptionally high data rates. The wireless networks thus have to contain advanced protocols and algorithms to meet the Quality of Service (QoS) demands required by different media classes. Real-time voice applications, data, video and internet are some of the flavours of services that are required for the next generation network but they need to be mixed and packaged together based on consumer demands. Fixed - Mobile convergence is a huge driver for improvements on broadband wireless communications in order to deliver any kind of service to an end user based on the QoS requirements, which have to be guaranteed. In order to meet the QoS demands for the next generation network the BER and throughput of wireless systems need to be improved from that of the current generation. This is one of the biggest challenges.

Current third generation, the future fourth generation networks and beyond will not bear any similarity to the first and second generation networks at all. Tailor - made techniques in digital communications, which would suit the specific needs of a wireless network environment, are being investigated. Components such as modulation, coding and decoding are being combined and implemented to suit the specific needs of the network. These new ideas drift away from the traditional digital communication scenarios where there would be a separate source and channel encoder.

In an effort to achieve the next generation network demands, there has to be major improvements to deterrents on the wireless radio channel; which is electromagnetic in nature. One of the main problems faced on the wireless channel is multi path fading. Take a voice call as an example - mobile users experience variable signal attenuation due to multi path fading. This variability, due to signal attenuation, causes a drop in the average data rate and also degrades the instantaneous achievable data rate causing severe fluctuations in signal magnitude. The provisioning of real-time services such as voice, video and audio are affected by this severe variability of the instantaneous achievable data rate. Hence next generation wireless networks and systems have to produce algorithms and protocols to reduce the variability of the instantaneous channel throughput.

The wireless channel is modelled as a random process and supports the modulation of electromagnetic (radio) waves, with a carrier frequency of a few MHz to many GHz to transfer information between points in a given network. The output of the wireless channel can be considered to be a function of the radio propagation effects in the environment.

Multiple delayed receptions of signals transmitted to a destination occur due to reflections, scattering and diffraction of the transmitted signals of obstacles along the path which include buildings, hills, cars etc. Line of sight between a wireless transmitter and the destination is not always possible. Each path taken by each multiply delayed signal has a different attenuation, time delay and phase shift.

Since the numerous delayed signals have relative phase shifts they constructively and destructively interfere with each other. The superposition of these signals is received at the destination and results in a phenomenon called multi path fading. Rayleigh fading is a subset of Multi path fading. Rayleigh fading can be generalised to scenarios when the source and destination have various transmission paths between them.

Current generation communication systems use single antennas. Since there is a high BER and significant loss on throughput with the use of current single antenna systems today due to multipath fading, future generations will require improvements with regard to capacity and BER. This gives rise to multiple input-multiple output wireless communication systems.

1.2 Multiple Input Multiple Output Systems

With the introduction of Multiple-Input Multiple-Output (MIMO) systems multiple antennas can be deployed at the transmitter and the receiver. Since the channel statistics are very often Rayleigh in nature the ability for the receiver to determine the transmitted signal is difficult unless multiple less attenuated replicas of the same signal are transmitted and presented to the receiver to use collectively for decoding. This method of transmission is referred to as diversity.

Some of the pioneer work in the MIMO system area was looked at in [51] and [52] based on information theoretic approaches and showed that MIMO channels can improve the channel capacity upper limit far more significantly than in single antenna or Single Input Single

Output (SISO) systems. The upper limit on the capacity of a wireless channel is referred to as the Shannon limit as developed in [54]. In [51] it is showed that the capacity of a MIMO system is directly proportional to the minimum between the number of transmit antennas and the number of receive antennas.

Techniques and algorithms that exploit spatial diversity by the use of MIMO systems have evolved. In some situations the wireless channel is neither time variant nor frequency variant and hence only spatial diversity will improve the system performance. These transmission coding techniques were termed space-time codes. The first transmit diversity technique was proposed in [55]. Then [51] introduced a layered space - time architecture. Later space - time trellis coding was introduced in [56]. In [56] it is seen that space time trellis codes designed for two transmit and four receive antennas perform very well in slow fading scenarios and produced bandwidth efficiency of which is at least three times greater than that of current systems.

In [53] the production of a simple transmit diversity technique for two transmit and 1 receive antenna based on the one transmit and two receive Maximal Ratio Receiver Combiner (MRRC) was developed. The scheme was then generalized to two transmit antennas and M receive antennas.

1.3 Cooperative Diversity

Cooperative wireless communication involves work on wireless adhoc networks [24],[41]-[43], wireless sensor networks, Wireless Local Area Networks (WLAN) and cooperation in wireless cellular networks namely the current generation, Third Generation (3G), and below in which wireless agents can improve their QoS by cooperating.

The QoS improvements can be measured using performance parameters such as block error rates, bit error rates, throughput and outage probability. Note that cooperative communications involves the cooperation between users in any wireless network; however a lot of the work done in this regard is based on cellular networks, which is used as an example. In future for the purposes of brevity but without loss of generality the cellular network scenario will be considered. The same methods can be applied to ad-hoc wireless networks and wireless sensor networks alike.

Certain MIMO system applications have been used in wireless standards, one of these being a transmit diversity method called Alamouti signalling as mentioned above in [53]. From the decoder's point of view (base station or access point etc. in a wireless network) transmit diversity is beneficial in terms of increasing the performance of the system.

Size, cost and hardware limitations will cause a wireless agent not to be able to support multiple transmit antennas thus rendering transmit diversity not feasible. Handsets, for example, are limited in size in a wireless network. Nodes in a wireless sensor network are limited by size as well as power.

Cooperative communication allows for single antenna mobiles to reap some of the benefits of MIMO systems by creating a virtual MIMO system (users share their antennas and obtain transmit diversity).

By transmitting separate copies of a signal (using a virtual MIMO system) which are statistically independent from each other, diversity results and thus reduces the effects of fading. Spatial diversity, in particular, as mentioned above allows for the transmission of the same signal from different positions allowing several differently faded versions of the same signal to be available at the receiver.

Figure 1.1, taken form [16], shows two wireless nodes communicating with the base station. Since each user has only one antenna it cannot generate spatial diversity. A user may be able to detect the partner's transmitted signal and can then forward the partner's parity as well as its own data to the destination. The path between respective users and the base station are statistically independent and hence spatial diversity is created.

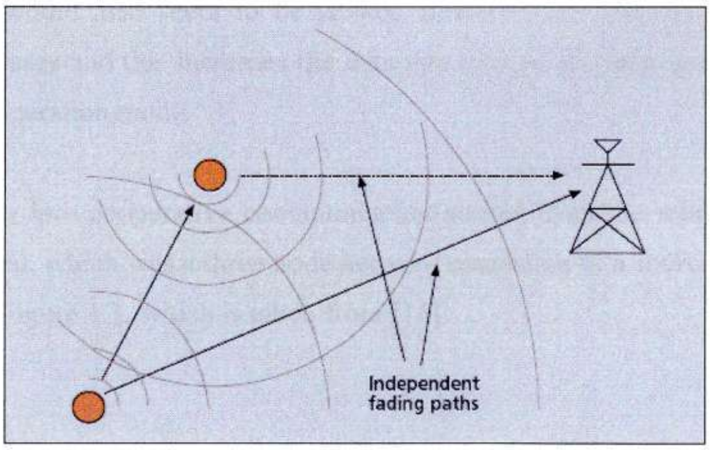

Figure 1.1 – Cooperative Communication

Some of the issues raised regarding cooperative communications are related to the loss of data rate of each user (since each user now has to send data to its partner as well as its own data to the destination) and changes to the transmit and receive powers of the wireless handsets.
In cooperative communications each wireless agent transmits and passes data to its cooperating partner as shown in Figure 1.2, which is taken from [16]. Each user then transmits data and also acts as a relay for its partner. This results in a trade-off in code rate and transmit power.

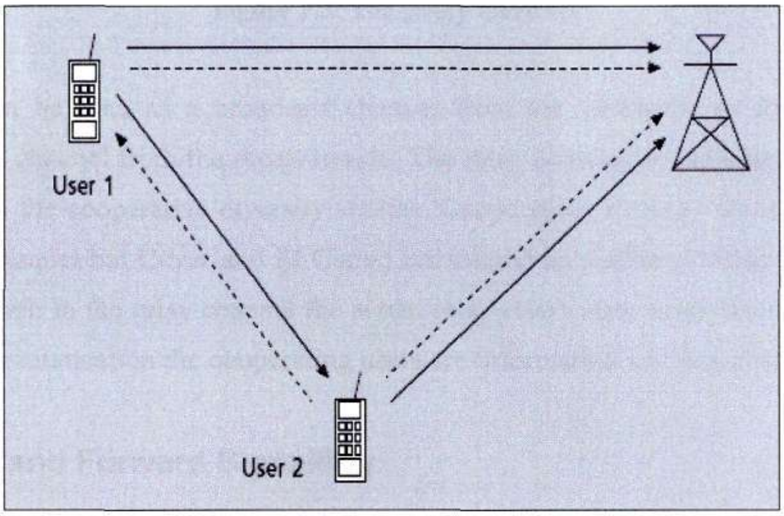

Figure 1.2: User - Relay Relationship in Cooperative Communications

Since each cooperating wireless user now transmits data for its partner and itself, it would seem that the transmit power of the mobile would have to be larger, however the baseline transmit power of each user actually reduces. The rate of data transmission from each cooperating user would also seem to be slower, however the spectral efficiency of each wireless user increases and this increases the data rate (due to increase in channel code rates) of each user in cooperation mode.

The pioneer theory into cooperative communication started from the relay channel done by Cover and El Gamal, which was a three node network consisting of a source, destination and a relay as shown in Figure 1.3, which is taken from [16].

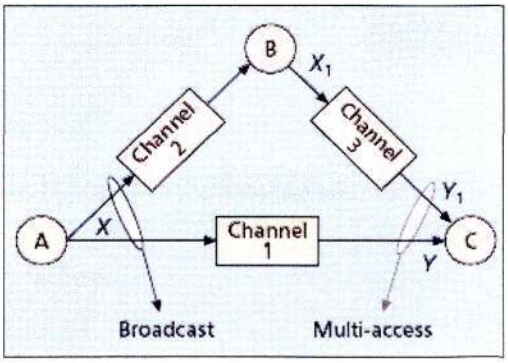

Figure 1.3: The Relay Channel

The system can be seen as a broadcast channel from the viewpoint of the source and a multiple access channel from the receiver side. The relay channel as explained in [16] differs somewhat from the cooperative diversity studies. Cooperative diversity involves the analysis of the fading channel but Cover and El Gamal considered an Additive White Gaussian Noise (AWGN) channel. In the relay channel the actual relay relays data along the main channel. In cooperative communication the cooperating users are information sources as well as relays.

1.4 Detect and Forward Signalling

The Detect and forward protocol resembles the relay channel concept and allows for a user to attempt to detect its partner's transmitted bits and then retransmit the detected bits.

Sendonaris et al. implemented detect and forward in [1]-[3] and [5]. The implementation was based on Code Division Multiple Access (CDMA) cooperation. Each user was given a unique spreading code denoted by $c_i(t)$ where $i \in \{1,2\}$ and two user cooperation was considered.

The user's data bits are given by $b_i(n)$ and n denotes the time interval for the respective transmitted information bits from the source. The index $i \in \{1,2\}$. Three bits are transmitted over each signalling period and power allocation is controlled by the definition of power control factors, of the signal amplitudes, $a_{i,j}$ and $j = (3-i)$. The signal of each user is denoted by $X_1(t)$ and $X_2(t)$ respectively and is given by:

$$X_1(t) = a_{1,1}b_1(1)c_1(t), \quad a_{1,2}b_1(2)c_1(t), \quad a_{1,3}b_1(2)c_1(t) + a_{1,4}\hat{b}_2(2)c_2(t) \tag{1.1}$$

$$X_2(t) = a_{2,1}b_2(1)c_2(t), \quad a_{2,2}b_2(2)c_2(t), \quad a_{2,3}\hat{b}_1(2)c_1(t)+a_{2,4}b_2(2)c_2(t)$$

(1.2)

Figure 1.4 below, taken from [3], shows the transmission of $X_1(t)$ and $X_2(t)$ over three bit periods.

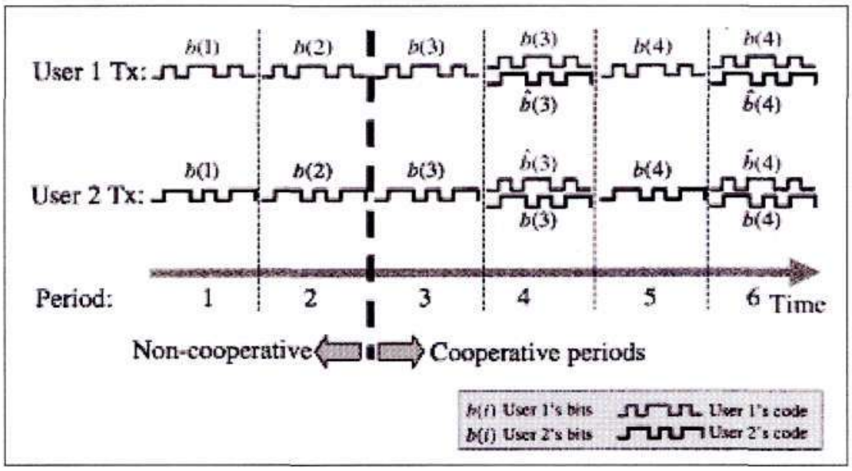

Figure 1.4: CDMA signalling protocol for detect and forward

During the first and second bit transmission intervals each user transmits its own bits to the base station and these bits are inherited by the partner due to the omni directional nature of electromagnetic waves. The partners then attempt to detect each other's second bit and during the third bit transmission interval transmit, on the uplink to the base station, a linear combination of their second bit and the partner's second bit each multiplied by the respective spreading code.

Note that the average power constraint is kept to by allocation of the factors $a_{i,j}$. Power control is obtained by varying the values of $a_{i,j}$ based on the interuser channel conditions i.e. more power is allocated to cooperation during favourable interuser conditions and less power is allocated during unfavourable cooperative conditions.

In [4] methods to improve power efficiency using power control is investigated for detect and forward. Although this protocol allows for adaptability to channel conditions there is room for error introduction when one considers the cooperating partner's incorrect detection of its partner's second transmitted bit. This results in detrimental effects on decoding at the base station.

In [1], [2], [3] and [5] information theory is used to derive bounds on outage probability, coverage analysis and achievable rate regions.

In [7] and [9] a Hybrid detect and forward method is proposed which allows for cooperation based on the fading channel instantaneous Signal to Noise Ratio (SNR). When the fading channel has a high instantaneous signal to noise ratio, the users will perform detection and forwarding of their partners' data. During opposite conditions i.e. when the channel has a low signal to noise ratio - non cooperation is adopted. This is advantageous over detect and forward in that during poor fading channel conditions in the interuser channel - no power is allocated to cooperation, which will introduce error into the system, which is done by varying the factors $a_{i,j}$ during detect and forward. Figure 1.5 below, taken from [24], illustrates the detect and forward protocol.

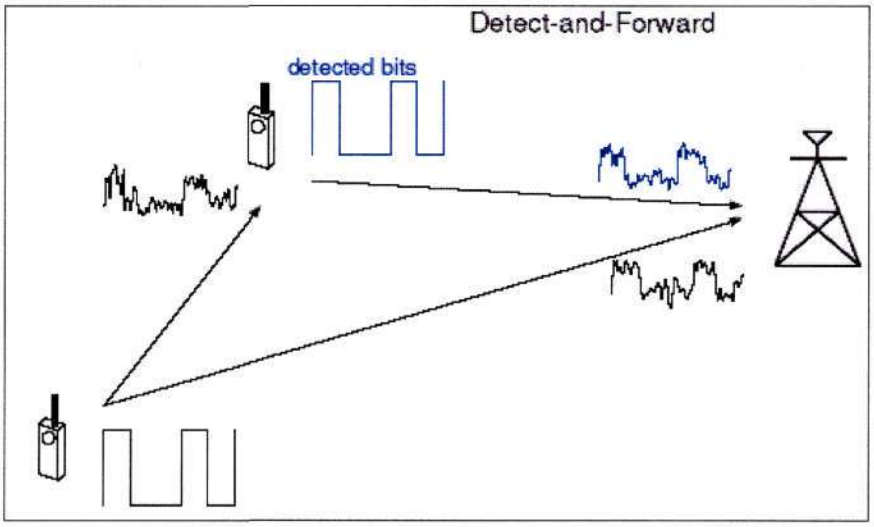

Figure 1.5: Detect and forward

1.5 Amplify and Forward

In [7] and [9] Laneman et al developed the amplify and forward protocol in which each user utilises the noisy version of the partners uplink transmission to the base station. The user then amplifies the noisy received version of the partner's transmission and forwards this to the base station.

Even though the partner amplifies noise by cooperating, the base station still has a greater chance of making a better decoding decision since it has two independently faded versions of the same signal to consider.

Amplify and forward protocol assumes that the interuser fading coefficients are known at the base station. Obtaining a method of estimating or passing this information to the base station is not a trivial exercise. In [7] and [9] it is shown that in the high SNR regime a diversity order of two can be obtained. Figure 1.6, taken from [24], gives a visual representation of the Amplify and forward protocol.

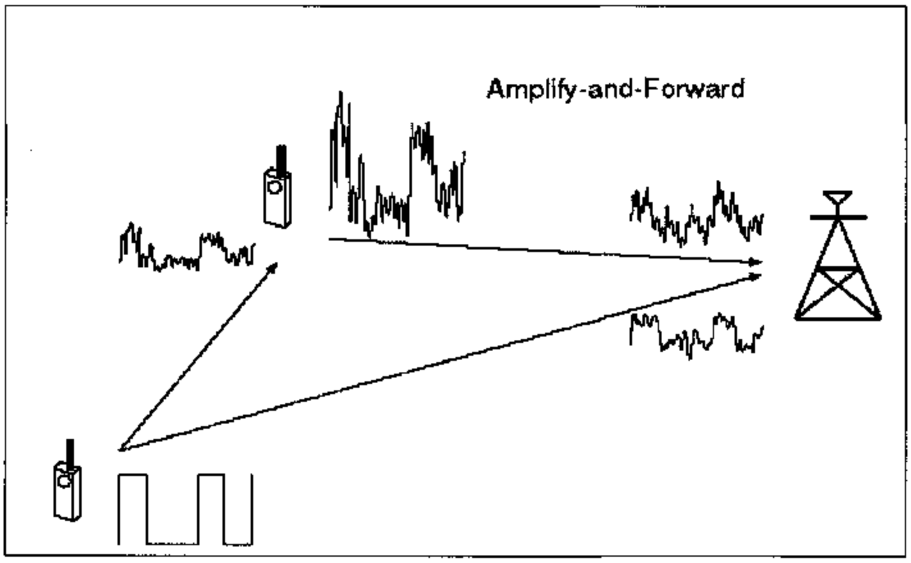

Figure 1.6: Amplify and forward

Other areas of research in Cooperative Diversity involve Space Time Cooperation in [8]. Space time transmission is considered in [26] and [27], under noisy interuser channel conditions with the assumption that each cooperating node has multiple transmit antennas. In [10] network coding gain is investigated using information theoretic concepts to derive the outage probability. In [11] and [12] modulation and demodulation of cooperative systems are considered. Cooperative routing is looked at in [13]. In [14] and [15] transmission strategies for relaying are developed which comprise of a combination of both decode and forward and amplify and forward protocols. Channel coding design and analysis is done in [28], [32] and [33] together with cooperation to achieve full diversity gain for two user cooperation. Cooperative region analysis and network geometry is investigated in [30] and [35]. Various combining methods are investigated in [29], at the base station based on cooperative information transferred from the relays to the destination. Source and channel coding is combined in [31], [36] and [38] in an effort to improve spectral efficiency and reduce distortion at the source caused by compression and channel errors.

1.6 Channel Coded Cooperation

In this cooperative signalling method [17]-[19] and [21]-[24] channel coding is combined with cooperative diversity. This is a significant improvement from amplify and forward and detect and forward where some form of repetition of the partner's information is presented to the base station. In Coded Cooperation a user attempts to decode the partner's transmitted information and if successful transmits additional parity for the partner based on a forward error correction scheme. The users employ CRC error checking in order to avoid introducing error into the system by transmitting erroneous parity for the partner. Coded Cooperation ensures that the code rate, bandwidth and transmit power are exactly the same as that of a comparable non cooperative system.

Coded Cooperation in this way addresses the concerns and risks of employing amplify and forward or detect and forward as mentioned above in [1], [3], [5], [7] and [9] in that no amplification of the partner's noise is introduced into the system or no erroneous estimates of the partner's information is forwarded to the base station (as in detect and forward).

The use of amplify and forward also bears the limitation that the base station has to have knowledge of the interuser channel fading coefficients as shown in [7] and [9], which is complex. The performance of amplify and forward and detect and forward and their associated introduction of error into the system is very much dependant on the quality of the interuser channel. During poor interuser channel conditions the system performance deteriorates rapidly due to error introduction.

1.7 Research Motivation: Cross Layer Hybrid ARQ 2 Cooperative Diversity

The conventional Coded Cooperation framework is not an efficient transmission scheme since in some cases the base station has already decoded a particular set of information bits using a user's initially transmitted parity and not all parity bits are required. The user however is not informed of this by the base station. This results in Coded Cooperation not being bandwidth efficient. This dissertation proposes an efficient transmission scheme called Cross Layer

Hybrid ARQ 2 (HARQ 2) Cooperative Diversity. By combining HARQ 2 (at the data link layer) with Cooperative Diversity (at the physical layer) feedback from the base station is available for users. Users can then transmit incremental parity packets and await feedback from the base station to decide whether further parity is required for a particular set of information bits. This is done using an incremental redundancy coding scheme. CRC is also employed for error detection. Improvements in BER and throughput are observed by the Cross Layer Design over Coded Cooperation.

1.8 Dissertation Outline

In Chapter 2 the Coded Cooperation protocol is investigated based on the work done in [24] and [21]. The scheme is analysed and the simulation parameters and certain performance issues are examined. The cooperative scheme is based on RCPC codes. The theoretical analysis of Coded Cooperation using RCPC codes is also looked at since these principles lay the foundation for further complicated theoretical analysis in Chapter 3 based on the new proposed Cross Layer Hybrid ARQ 2 protocol. A bit more detail is given with regard to Coded Cooperation theoretical analysis of the BER which is not so visible in [24] and [21] from an implementation point of view. Comparisons are made between reciprocal and independent interuser channel conditions for the BER. Simulations of BER are confirmed by theoretical analysis.

In Chapter 3 Cross Layer Hybrid ARQ 2 Cooperative diversity is proposed. Here incremental redundancy is implemented using a hybrid ARQ 2 scheme so that feedback from the base station to the users occurs. RCPC codes are employed here for the ease of incremental redundancy implementation. Users only cooperate when they need to during this scheme and improvements in BER are seen over that of Coded Cooperation. Simulation results and theoretical results are presented in this chapter for BER. The analytical bound for the BER is derived using the PEP. Comparisons of BER are made between reciprocal and independent interuser channel conditions for this scheme also to show the validity of assuming reciprocal interuser channel conditions.

Chapter 4 proposes an in depth look at throughput analysis for the Cross Layer Hybrid ARQ 2 scheme. Theoretical analysis throughput comparisons are done between the new proposed cross layer scheme and Coded Cooperation. Here concepts such as retransmission probability and packet successful rate are developed based on manipulation of the BER results. Simulations and theoretical analysis show that the throughput of Cross Layer Hybrid ARQ 2 Cooperative Diversity improved markedly over Coded Cooperation. This cross layer scheme adapts well to channel conditions and can collectively improve throughput and BER at the same time.

In chapter 5 conclusions on this dissertation are made and future avenues of work in The Cooperative Diversity research area are discussed. The contributions made in this dissertation and the summary of work done is also presented.

Chapter 2

In Chapter 1 the concept of the pioneer Coded Cooperation was presented. In this Chapter Coded Cooperation from [17]-[19] and [21]-[24], in the paradigm of RCPC codes, is investigated in detail. The analysis of Coded Cooperation with regard to BER is also looked at in an effort to create awareness of the PEP and other performance related principles that will be used extensively in Chapter 3, when further complicated performance analysis is presented. Simulations are performed and compared to the BER analytical bounds.

2 Coded Cooperation

2.1 The Coded Cooperation Protocol

Two user cooperation is considered and each user has a K bit information source block which contains CRC concatenation. Each user uses a suitable Forward Error Correction (FEC) code to encode the K bit source block and obtain an overall rate R code. The amount of encoded bits (N) that each user possesses, from a K bit source block, can be calculated using $N = K/R$. It should be noted that since the amount of information bits is in the hundreds of bits that a few bits allocated for error detection via CRC is negligible and does not present any overhead. Note that for the simulations presented in Section 2.9, K is set to 128 bits and 16 CRC bits are concatenated into the 128 source block. This was done to conform to the simulation parameters used in [17]-[19] and [21]-[24]. In this case a 12.5 % CRC overhead is observed. This however is a simulation and in a live network scenario the maximum transfer unit would be in the order of thousands of bytes. If the IEEE 802.16 (d and e) standard is looked at one would see that the maximum transfer unit is 1500 bytes. Hence in this case the 16 bit CRC overhead is negligible.

The N bit codes that are encoded by each user are used in cooperation but its transmission is partitioned into two time periods or two frames being denoted by N_1 and N_2 respectively such that $N_1 + N_2 = N$. During first frame transmission each user transmits a weaker code of rate R_1 $(R_1 > R)$ to the base station which is inherited and attempted to be decoded by the partner to obtain the source information. During first frame transmission the amount of encoded bits transmitted by a user is given by $N_1 = K/R_1$. If a user can successfully decode the partners first frame rate R_1 code then the user can cooperate and transmits an additional N_2 encoded bits for the partner during the second frame. The user knows if it has decoded the partner's first frame correctly or not by computing the CRC syndrome for the partner's first frame. If the CRC syndrome is not equal to 0 then the user reverts to non cooperative mode and transmits N_2 parity bits for itself in the second transmission frame.

It should be noted that each user in Coded Cooperation attempts to transmit incremental redundancy for its partner and if this is not possible, due to errors in first frame transmission, the users revert to non cooperation mode. The use of CRC for error detection also allows for error propagation to be avoided.

2.2 Coded Cooperation using RCPC Codes

Coded Cooperation is a generic framework. The coding schemes, in Coded Cooperation, that can be used could be convolutional codes, block codes, product codes and even concatenated codes.

Coded Cooperation, using RCPC codes, is investigated here based on [24]. A specific RCPC code family is chosen here based on a family of codes given in [57]. The mother code chosen for the overall rate R code is $1/4$. To obtain the rate R_1 $(R_1 < R)$ code a puncturing matrix operation is performed on the mother code of rate R. The N_2 encoded bits transmitted during the second transmission frame are the remaining parity bits that were not transmitted during the first frame transmission. The following matrices are used in the Coded Cooperation framework based on the family of codes chosen from [57] and are given by, puncturing matrices, $P_{50\%}$ and $P_{25\%}$ below:

$$P_{50\%} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad P_{25\%} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

A "1" in the puncturing matrices, $P_{50\%}$ and $P_{25\%}$, denotes a bit that is transmitted in the rate R_1 code and a "0" denotes a bit that is not transmitted in the rate R_1 code but that will be transmitted in the second frame transmission with the remaining N_2 parity bits. The subscripts of 50% and 25% denote the cooperation level which is defined to be the number of parity bits that a user transmits for its partner out of the total amount of parity bits i.e. $\lambda = N_2/N$, where λ is the cooperation level. Note that each user always transmits a total of N bits during a timeslot irrespective of whether cooperation takes place or not. Figure 2.1, adapted from [24], below shows RCPC transmission during Coded Cooperation.

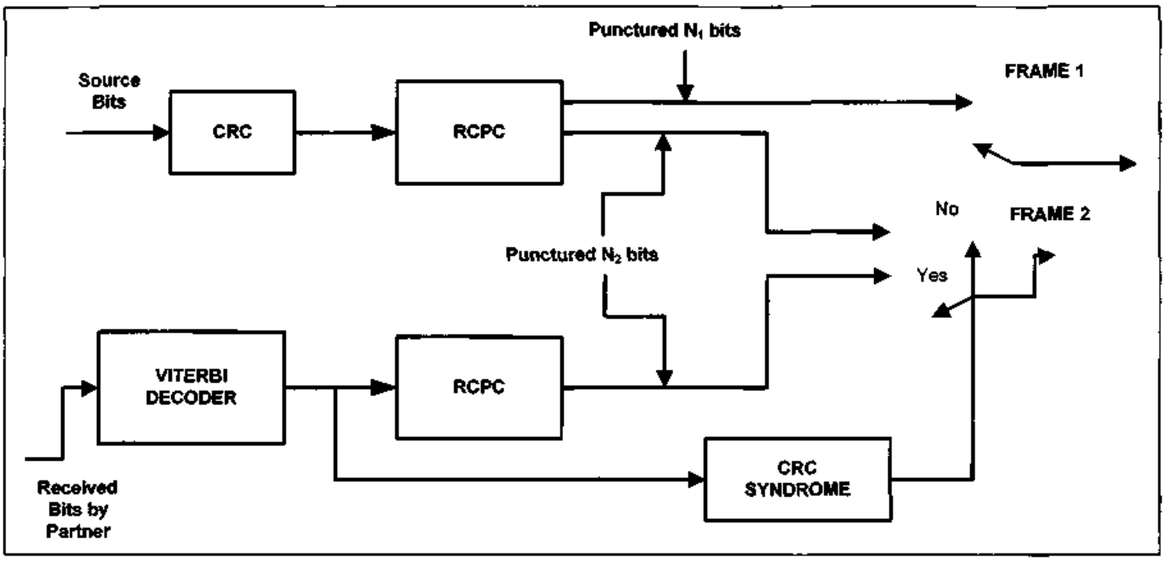

Figure 2.1: RCPC transmission using Coded Cooperation

During the second transmission frame the users act independently without knowing whether their partners have decoded the first frame transmissions correctly or not. This leads to four possible transmission scenarios.

The four transmission scenarios are named Case1, Case 2, Case 3 and Case 4 as shown in Figure 2.2. During case 1, full cooperation occurs i.e. both users decode each other's first frame transmission and hence transmit N_2 incremental parity bits for their partners in the second frame.

During case 2 both users revert to non cooperation mode since both do not decode each other's first frame transmission. This is the opposite of case 1 and is the same as that of direct transmission.

Case 3 is a partial cooperation case which is advantageous to user 1 since here user 1 cannot decode user 2's first frame, but user 2 can decode user 1's first frame. Thus user 1 and user 2 transmit user 1's parity bits and none of the users transmits parity for user 2. Since there are two independent copies of user 1's parity at the base station during the second frame transmission the base station optimally combines the copies using Maximal Ratio Combining (MRC).

Case 4 is the opposite of case 3, i.e. user 2 benefits from partial cooperation and neither user transmits parity for user 1 in the second frame. The two independent copies of parity bits transmitted for user 2 in the second frame is again optimally combined at the base station using MRC. Figure 2.2, adapted from [24], shows the four possible first frame transmission scenarios.

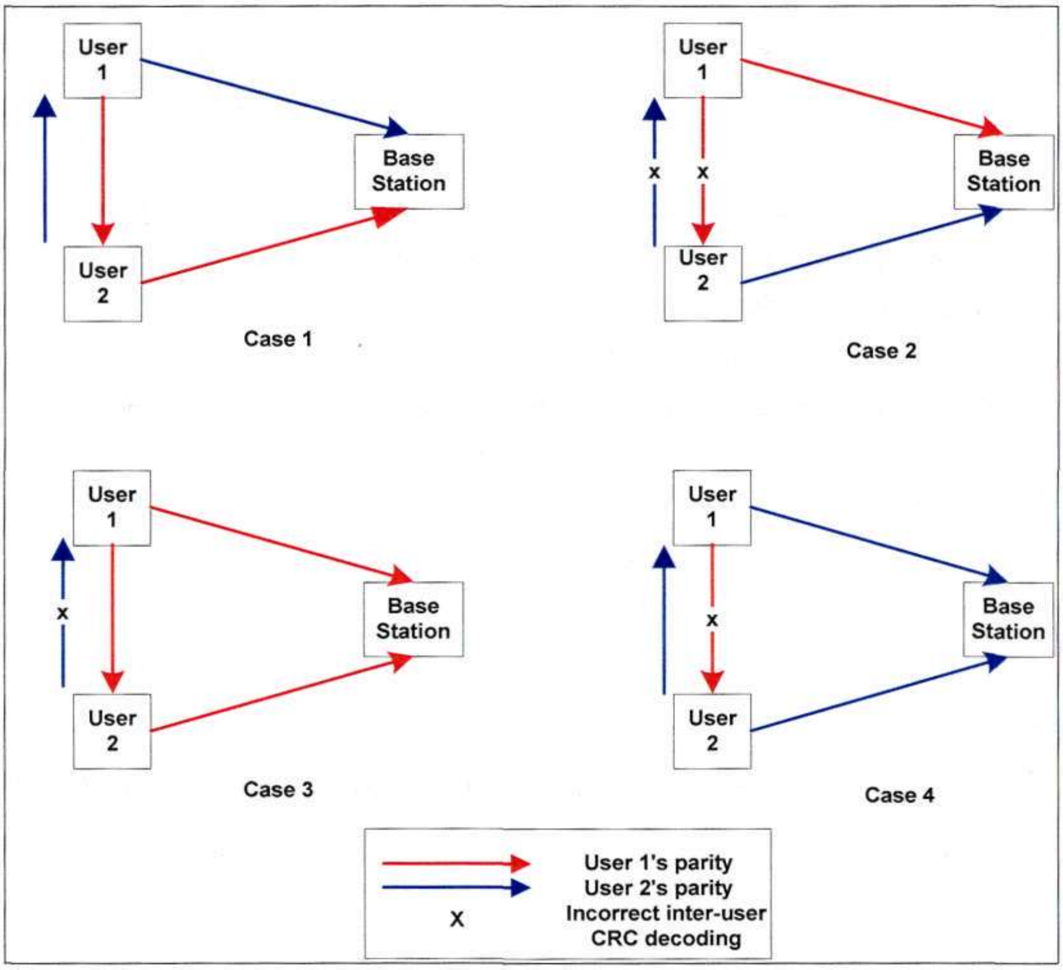

Figure 2.2: The four cooperation transmission scenarios

2.3 The Channel Model

In a two user cellular transmission scenario cooperation can take place due to the multi-user presence. The uplink channels of each user and the interuser channel are considered mutually independent of each other and experience flat Rayleigh fading. Spatial diversity is exploited during flat fading, however, the system can be implemented in a block or frequency selective scenario.

Orthogonal transmissions are undertaken by each user (using either Frequency Division Multiple Access (FDMA), CDMA or Time Division Multiple Access (TDMA)) so that each user and the base station can independently decode each other. Binary Phase Shift Keying (BPSK) modulation is used and it is assumed that all receivers have Channel State Information (CSI) and that coherent detection occurs. In this way only the magnitude of the fading needs to be taken into account and not the phase.

The users in the coded cooperative system can be defined by $i \in \{1,2\}$, being the transmitting users, and $j \in \{0,1,2\}$, being the receiving users and the base station, where $j = 0$ denotes the base station and $j \neq i$.

The channel modelling and analysis that follows is adopted from [24], however it is critical that this information is summarised and laid out here, in this chapter, so that the reader is introduced to the concepts of PEP , the Block Error Rate (BLER) for convolutional codes, the first event error probability and the case probabilities for second frame transmissions. These concepts are used as tools of foundation in the next two chapters for further complicated cross layer cooperation analysis.

The transmitted signal from each user can be described as:

$$r_{i,j}(n) = \alpha_{i,j}(n)\sqrt{E_{b,j}} \cdot b_i(n) + z_j(n) \tag{2.1}$$

Here $b_i(n) \in \{-1,1\}$ is the modulated encoded bit at time instant n, which is BPSK in nature and $E_{b,i}$ is the transmit bit energy. The term $z_j(n)$ models noise at the receiver j as samples of an independent white Gaussian noise process with a variance of $N_j/2$ and a zero mean. The fading coefficient magnitude is given by $\alpha_{i,j}(n)$. The subscript in $\alpha_{i,j}(n)$ denotes fading between user i and j and $\alpha_{i,j}(n)$ are considered as samples of an independent Rayleigh random variable. The mean square value of $\alpha_{i,j}(n)$ is denoted by:

$$\Omega_{i,j} = E_{\alpha_{i,j}}\left[\alpha_{i,j}^2(n)\right] \tag{2.2}$$

The expectation operator is given by $E_{\alpha_{i,j}}$ for the fading coefficient $\alpha_{i,j}(n)$. During slow fading conditions the fading coefficients remain constant in value over each transmitted block of N bits by each user. This implies that $\alpha_{i,j}(n)$ is the same for each instant of n over N i.e. $\alpha_{i,j}(n) = \alpha_{i,j}$. In the inter-user channel the instantaneous received SNR between user i and j can be modelled as $\gamma_{i,j}(n) = \dfrac{\alpha_{i,j}^2(n)E_{b,i}}{N_j}$ and is exponentially distributed with a mean value given by the expectation of the instantaneous SNR averaged over the fading distribution i.e.

$$\begin{aligned}\Gamma_{i,j} &= E_{\alpha_{i,j}}\left[\gamma_{i,j}(n)\right] \\ &= E_{\alpha_{i,j}}\left[\frac{\alpha_{i,j}^2(n)E_{b,i}}{N_j}\right] \\ &= \Omega_{i,j}\frac{\mathrm{E_{b,i}}}{\mathrm{N_j}}\end{aligned} \tag{2.3}$$

$\Gamma_{i,j}$ is the average received SNR and is used as a measure for SNR during all simulations and for performance analysis. Statistically similar (symmetric) and dissimilar channels (asymmetric) are considered on the uplinks i.e. cases where $\Gamma_{1,0} = \Gamma_{2,0}$ and where $\Gamma_{1,0} \neq \Gamma_{2,0}$ are considered.

In [24] using a TDMA framework with reciprocal inter-user channels are assumed. This means that $\alpha_{i,j}(n) = \alpha_{j,i}(n)$. This assumption is reasonable in TDMA and CDMA, but is not always true in FDMA since the correlation of the fading coefficients for different frequency channels is not always identical. Note that there is some correlation between the frequency channels and that they are not fully independent. In this dissertation TDMA is used and the performance loss by having independent interuser channels as opposed to reciprocal interuser channels is examined.

2.4 Performance Analysis of Coded Cooperation

In [21] the performance of Coded Cooperation is shown to be largely dependant on the PEP of the convolutional channel code. As in [21] the PEP is defined to be incorrect selection of the error code vector $e=\left[e(1),e(2),....,e(N)\right]$ when the actual transmitted code vector was $c=\left[c(1),c(2),....,c(N)\right]$. In [44] the PEP is shown to be set as a conditional probability conditioned on the instantaneous received SNR vector γ, where $\gamma=\left[\gamma(1),\gamma(2),....,\gamma(N)\right]$ and $\gamma(n)$ denotes the instantaneous SNR at code bit number n.

The conditional PEP is given by-[58]:

$$P(c\rightarrow e\mid\gamma)=Q\left(2\sum_{n\in\eta}\gamma(n)\right) \tag{2.4}$$

where $Q(x)$ represents the Gaussian Q-function. η is the set denoting the positions in which $c(n)$ and $e(n)$ differ, i.e. $c(n)\neq e(n)$. If the hamming distance between vector c and e is computed (d) then the cardinality of η is obtained. Without loss of generality the all zero sequence is assumed to be input to the decoder during this performance analysis. The reason for this is that for a convolutional code a tree diagram can be drawn and if a comparison is made (in the hamming sense) between sequences of codes generated in a tree diagram up to a point to the all zero sequence this would yield the same result as a comparison between sequences of codes generated up to a point in the tree diagram and any other code sequence. Thus for simplicity the all zero sequence is assumed to be transmitted. Note that the PEP depends on d and not on c and e.

2.5 No Cooperation Analysis

During non-cooperation (direct transmission) a user transmits all the parity bits, using the mother code rate, to the base station on its uplink. This implies that $d_1=d$ and $d_2=0$. Note that d_1 and d_2 are the individual hamming distance for the first and second frames transmitted through user 1 and user 2's uplink channel respectively.

Since we only take into account the fading magnitude, the PEP is dependant on a scalar value of the instantaneous received SNR γ. Hence we have the conditional PEP given by:

$$P(d\mid\gamma)=Q\left(\sqrt{2d\gamma}\right) \tag{2.5}$$

To obtain the unconditional PEP the conditional PEP is averaged over the instantaneous SNR range i.e.

$$P(d)=\int_0^\infty P(d\mid\gamma)p_\gamma(\gamma)d\gamma \tag{2.6}$$

Using the alternate form of the Q function as outlined in [59], i.e.

$$Q(x)=\frac{1}{\pi}\int_0^{\pi/2}\exp\left(\frac{-x^2}{2\sin^2\theta}\right)d\theta \tag{2.7}$$

Where $x>0$. $P(d)$ can be calculated as:

$$P(d)=\frac{1}{\pi}\int_0^{\pi/2}\left[\int_0^\infty \exp\left(-\frac{d\gamma}{\sin^2\theta}\right)p_\gamma(\gamma)d\gamma\right]d\theta \tag{2.8}$$

In (2.8) the inner integral within square brackets has a form that fits the description of a moment generating function, which is very similar to the Laplace transform, which has the opposite sign in the exponent.

The moment generating function of a random variable x is given by:

$$M_x(s)=\int_0^\infty e^{sx}p(x)dx \tag{2.9}$$

Applying (2.9) to the random variable γ the following solution is obtained for $M_\gamma(-s)$:

$$M_\gamma(-s)=\frac{1}{1+s\Gamma} \tag{2.10}$$

where $s>0$

Hence substituting (2.10) into (2.8) it implies that,

$$P(d)=\frac{1}{\pi}\int_0^{\pi/2}M_\gamma\left(\frac{-d}{\sin^2\theta}\right)d\theta \tag{2.11}$$

Using (2.10) in (2.11) it implies that:

$$P(d)=\frac{1}{\pi}\int_{0}^{\pi/2}\frac{1}{\left(1+\frac{d\Gamma}{\sin^2\theta}\right)}d\theta \tag{2.12a}$$

The PEP can be upper bounded by using the maximum value of $\sin^2\theta$ which is 1. This implies that:

$$P(d)\leq\frac{1}{\pi}\int_{0}^{\pi/2}\frac{1}{(1+d\Gamma)}d\theta \tag{2.13b}$$

$$\leq\frac{1}{\pi}\left[\frac{1}{1+d\Gamma}\right] \tag{2.14}$$

The PEP is thus given by (2.13):

$$P(d)\leq\frac{1}{\pi}\left[\frac{1}{1+d\Gamma}\right] \tag{2.15}$$

2.6 The PEP for the Uplink Channels

During slow fading conditions $\alpha_{i,j}(n)=\alpha_{i,j}$ i.e. the fading vector reduces to a scalar during the transmission of the entire parity block of N bits.

The same principle applies to the received instantaneous SNR at all receivers i.e. $\gamma_{i,0}(n)=\gamma_{i,0}$, where $i\in\{1,2\}$.

During case 1 when full cooperation occurs each user's second frame transmission involves transmitting N_2 incremental parity bits for its partner. A user's total N parity bits are spread over both uplink channels.

The PEP for case 1 on the uplink of user 1 is given by:

$$P(d\mid\gamma_{1,0},\gamma_{2,0})=Q\left(\sqrt{2d_1\gamma_{1,0}+2d_2\gamma_{2,0}}\right) \tag{2.16}$$

Note that $d_1+d_2=d$. Also d_1 denotes the distance between c_1 and e_1 for the first frame transmission for user 1 and d_2 denotes the distance between c_2 and e_2 for user 2.

Again to obtain the unconditional PEP for case 1, (2.14) is averaged over the SNR range i.e.

$$P(d)=\int_0^\infty\int_0^\infty P(d\mid\gamma_{1,0},\gamma_{2,0})p(\gamma_{1,0})p(\gamma_{2,0})d\gamma_{1,0}d\gamma_{2,0} \tag{2.17}$$

Using the alternative form of the Q function in (2.7) and applying techniques used in [60] the unconditional PEP for user 1 during case 1 can be derived to be:

$$P(d)=\frac{1}{\pi}\int_0^{\pi/2}\left[\int_0^\infty \exp\left(\frac{-d_1\gamma_{1,0}}{\sin^2\theta}\right)p(\gamma_{1,0})d\gamma_{1,0}\right]\cdot\left[\int_0^\infty \exp\left(\frac{-d_2\gamma_{2,0}}{\sin^2\theta}\right)p(\gamma_{2,0})d\gamma_{2,0}\right]d\theta \tag{2.18}$$

Again using (2.9) and applying the moment generating function approach $P(d)$ is simplified to be:

$$P(d)=\frac{1}{\pi}\int_0^{\pi/2} M_{\gamma_{1,0}}\left(\frac{-d_1\Gamma_{1,0}}{\sin^2\theta}\right)\cdot M_{\gamma_{2,0}}\left(\frac{-d_2\Gamma_{2,0}}{\sin^2\theta}\right)d\theta \tag{2.19}$$

Looking at (2.17) again an upper bound for $P(d)$ can be obtained by again noting that the maximum value of $\sin^2\theta$ in the integrand is 1.

The upper bound on the PEP for case 1 is thus given by (2.18) which is derived below:

$$P(d)=\frac{1}{\pi}\int_0^{\pi/2} M_{\gamma_{1,0}}(-d_1\Gamma_{1,0})\cdot M_{\gamma_{2,0}}(-d_2\Gamma_{2,0})d\theta \tag{2.20 a}$$

$$\Rightarrow P(d)\le\frac{1}{\pi}\int_0^{\pi/2}\left[\frac{1}{1+d_1\Gamma_{1,0}}\right]\cdot\left[\frac{1}{1+d_2\Gamma_{2,0}}\right]d\theta \tag{2.17b}$$

$$P(d)\le\frac{1}{\pi}\left[\frac{1}{1+d_1\Gamma_{1,0}}\right]\cdot\left[\frac{1}{1+d_2\Gamma_{2,0}}\right] \tag{2.21}$$

Equation (2.18) shows that a diversity order of two is obtained for user 1 and 2 in the high SNR regime. During case 3 when partial cooperation occurs, which benefits user 1, both users transmit incremental parity for user 1 during the second frame. This is where MRC is used to optimally combine the second frame transmissions for user 1 at the base station. The conditional PEP for user 1 is given by:

$$P(d\mid\gamma_{1,0},\gamma_{2,0})=Q\left(\sqrt{2d_1\gamma_{1,0}+2d_2(\gamma_{1,0}+\gamma_{2,0})}\right) \tag{2.22}$$

$$= Q\left(\sqrt{2d\gamma_{1,0} + 2d_2\gamma_{2,0}}\right) \tag{2.23}$$

Applying (2.15) to (2.20) the unconditional PEP for user 1 during case 3 is given by:

$$P(d) = \frac{1}{\pi}\int_0^{\pi/2} M_{\gamma_{1,0}}\left(\frac{-d\Gamma_{1,0}}{\sin^2\theta}\right)\cdot M_{\gamma_{2,0}}\left(\frac{-d_2\Gamma_{2,0}}{\sin^2\theta}\right)d\theta \tag{2.24}$$

$$\leq \frac{1}{\pi}\left[\frac{1}{1+d\Gamma_{1,0}}\right]\cdot\left[\frac{1}{1+d_2\Gamma_{2,0}}\right] \tag{2.25}$$

Equation (2.22) shows that user 1 receives a diversity order of two.

2.7 Determining the Transmission Case Probabilities

As explained in Section 2.2 four possible transmission scenarios are envisaged based on the first frame transmission decoding at the respective user. The BLER of the first frame transmissions can be used to compute the cooperative transmission case probability.

For a convolutional code the BLER is given by [46-(12)] or [24-(3.26)]:

$$P_{block}(\gamma) \leq 1 - \left(1 - P_E(\gamma)\right)^B \leq B \cdot P_E(\gamma) \tag{2.26}$$

$P_E(\gamma)$ is the first event error probability, which is dependant on γ, which is a vector containing the instantaneous SNR values for channel. The number B denotes the number of branches in the trellis for the transmitted code. The first error event probability is upper bounded by-[61]:

$$P_E(\gamma) \leq \sum_{d=d_f}^{\infty} a(d)P(d\,|\,\gamma) \tag{2.27}$$

Here d_f is the free distance of the convolutional code and $a(d)$ is the multiplicity of the error events with hamming weight d.

As in [24] the four cases are assigned to a set called θ, where $\theta \in \{1,2,3,4\}$. Since each case is dependant on the channel, it has to be conditioned on the channel SNR. The conditional case probability for case 1 is given by:

$$\begin{aligned} P(\theta = 1\,|\,\gamma_{1,2},\gamma_{2,1}) &= \left(1 - P_{block,1}(\gamma_{1,2})\right)\left(1 - P_{block,2}(\gamma_{2,1})\right) \\ &\geq \left(1 - P_{E,1}(\gamma_{1,2})\right)^B \left(1 - P_{E,2}(\gamma_{2,1})\right)^B \end{aligned} \tag{2.28}$$

In (2.25) the factor $\left(1-P_{block,2}\left(\gamma_{2,1}\right)\right)$ denotes correct decoding by user 1 of user 2's first frame transmission. The opposite holds true for the factor $\left(1-P_{block,1}\left(\gamma_{1,2}\right)\right)$, i.e. this factor denotes correct decoding by user 2 for user 1's first frame transmission. The product of these two terms hence provide for case 1 of full cooperation.

Using similar logic the other conditional case probabilities can be derived. In order to remove the condition of the transmission case probability on the instantaneous received channel SNR, the conditional case probability has to be averaged over the instantaneous SNR range. This means that:

$$P(\theta)=\int_{\gamma_{2,1}}\int_{\gamma_{1,2}}P(\theta\mid\gamma_{1,2},\gamma_{2,1})p(\gamma_{1,2})p(\gamma_{2,1})d\gamma_{1,2}d\gamma_{2,1} \tag{2.29}$$

During reciprocal interuser channel conditions, $\gamma_{1,2}=\gamma_{2,1}$, we have (2.26) reducing to a single integral given by:

$$P(\theta)=\int_{\gamma_{1,2}}P(\theta\mid\gamma_{1,2})p(\gamma_{1,2})d\gamma_{1,2} \tag{2.30}$$

Note that $\gamma_{1,2}=\gamma_{2,1}$ and they are scalars since slow fading is considered. During independent interuser channel conditions $\gamma_{1,2}\neq\gamma_{2,1}$ even though $\gamma_{1,2},\gamma_{2,1}$ are still scalars. The case probability for case 1 during cooperative transmission under independent interuser channels is given below:

$$P(\theta=1)=\left(1-P_{block,1}\right)\cdot\left(1-P_{block,2}\right) \tag{2.31}$$

In an effort to obtain tight theoretical bounds the techniques employed in [46] called limiting the bound before averaging are used to evaluate (2.28) and (2.29) together with the correct PEP. Take case 1 as an example, the unconditional case probability is given by:

$$P(\theta=1)\geq\int_0^\infty\left(1-\min\left[1,\sum_{d=d_{free}}^{\infty}a(d)p(d\mid\gamma_{1,2})\right]\right)^B\left(1-\min\left[1,\sum_{d=d_{free}}^{\infty}a(d)p(d\mid\gamma_{1,2})\right]\right)^B p(\gamma_{1,2})d\gamma_{1,2} \tag{2.32}$$

$$\geq\int_0^\infty\left(1-\min\left[1,\sum_{d=d_{free}}^{\infty}a(d)p(d\mid\gamma_{2,1})\right]\right)^{2B} p(\gamma_{1,2})d\gamma_{1,2} \tag{2.33}$$

Where:

$$p(d \mid \gamma_{1,2}) = \mathrm{Q}\left(\sqrt{2\mathrm{d}\gamma_{1,2}}\right) \tag{2.34}$$

During independent interuser channel conditions the BLER for a user i is given by:

$$P_{block,i} \leq 1 - \int_{0}^{\infty} \left[1 - \min\left[1, \sum_{d=d_{free}}^{\infty} a(d) p(d \mid \gamma_{i,j}) \right] \right]^{B} p(\gamma_{i,j}) d\gamma_{i,j} \tag{2.35}$$

Equation (2.32) can be used in (2.28) to obtain the probability of case 1 occurring during independent interuser channel conditions. In all of the BLER equations the PEP used is the conditional PEP.

During case 2 i.e. non cooperative transmission, the conditional PEP used during computation of (2.30) or (2.32) is given by (2.5).

Since the limit before average technique is performed on the bounds a minimization operation is performed which means that (2.30) or (2.32) have to be computed via a numerical method.

2.8 The End to End BER

The end to end BER is a combination of the unconditional bit error rate for each case averaged over the four transmission scenarios. Since the case probability is a discrete variable the evaluation of the end to end BER will involve a summation for the averaging of the four transmission cases.

The end to end BER is given by:

$$P_b = \sum_{i=1}^{4} P_b(\theta)P(\theta = i) \tag{2.36}$$

Where $P_b(\theta)$ is given by:

$$P_b(\theta) \le \int_0^\infty\int_0^\infty \min\left[\frac{1}{2}, \frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)p(d \mid \gamma_{1,0}, \gamma_{2,0}, \theta)\right] p(\gamma_{1,0})p(\gamma_{2,0})d\gamma_{1,0}d\gamma_{2,0} \tag{2.37}$$

$P_b(\theta)$ is the unconditional bit error rate for a specific instance of θ. The variable $c(d)$ is the multiplicity of information bit error events with Hamming weight d.

The variable k_c is the amount of input information bits in each branch of the trellis. Note that the conditional BER is given by [24]:

$$P_b(\gamma, \theta) \le \frac{1}{k_c}\sum_{d=d_f}^{\infty} c(d)P(d \mid \gamma, \theta) \tag{2.38}$$

2.9 Simulation Parameters and Discussion

The BER performance of Coded Cooperation is examined using RCPC codes as outlined in Section 2.2. The puncturing period of the code is $P = 8$, the memory length is $M = 4$ and the mother code rate is $R = 1/4$ which is taken from [57]. The distance spectra i.e. $a(d), c(d)$ and the separation of d into d_1 and d_2 is computed via enumeration. In order to take into account error detection a 16 bit CRC code is used which has a generator polynomial given by $g_{crc}(x) = 15935$ in hexadecimal. Figures 2.3 to 2.7 show the simulated and theoretical analysis results for Coded Cooperation. The amount of information bits in the source packet is kept at 128 $(K = 128)$.

All graphs shown below plot channel SNR or average received SNR versus BER. Plotting BER versus information bit SNR will yield the same results but with a $10\log R$ db shift on the abscissa.

Symmetrical uplink channel conditions are considered in Figure 2.3 below.

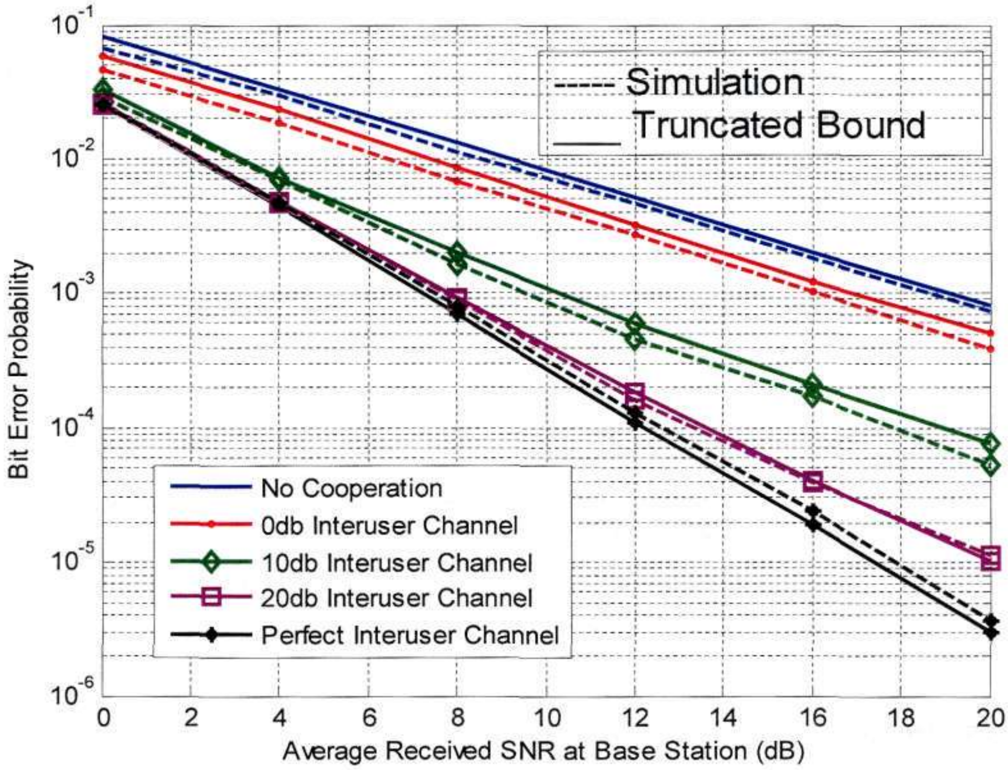

Figure 2.3: 50% cooperation with symmetric uplinks

This means that $\Gamma_{1,0} = \Gamma_{2,0}$. The cooperation level here is kept at 50% and reciprocal interuser channels are considered i.e. $\gamma_{1,2} = \gamma_{2,1}$. Coded Cooperation shows massive improvements over non cooperation as the interuser channel quality improves. Even under 0dB interuser channel conditions Coded Cooperation maintains a 1.8-3dB improvement in performance over non cooperation.

Note that the result of having a BER gain under 0dB interuser channel conditions for a 0dB uplink channel SNR is counter intuitive, because this scenario seems like a case of complete non cooperation. Note however that with cooperation an interuser channel is introduced. User's now try to exploit the omni directional nature of electromagnetic waves and attempt to decode the parity that a partner is trying to transmit to the base station. Since the interuser channel is modelled as Rayleigh, the statistical distribution of the fading coefficients will produce fading coefficients in some cases that do not fade the modulated BPSK much and this will result in case 1 being observed (full cooperation) in the interuser channel or even case 3 and 4 (partial cooperation where one user decodes its partner but the partner does not decode the other user). During these cases a BER gain will be observed, for a user over non cooperation, because two independently faded signals are transmitted for one user to the base station. The BER of a specific user thus improves due to better signal quality at the base station, where the base station uses an MRC combining scheme to combine one users received signal from two different fading paths. Also note that the uplink channels are i.i.d and therefore these aid in creating spatial diversity which provides BER improvement for a user.

Note, however, with the above mentioned points of having case 1,3 and 4 occuring – case 2 also occurs and is more probable. Hence the BER improvement in performance is small at an uplink SNR of 0dB for a 0dB interuser channel (as high as 1.8 dB) and improves as the interuser SNR improves.

At 10 dB interuser channel conditions Coded Cooperation shows a gain of between 9-10 dB over non cooperation at a BER of 10^{-3}. As the interuser channel quality improves so does the BER of Coded Cooperation. Coded Cooperation always has a superior performance over non cooperation even under the worst interuser channel conditions.

A comparison between 50% and 25% cooperation is made in Figure 2.4 for 10dB and perfect interuser channels. At perfect interuser channel conditions case 1 dominates and full cooperation is observed with the performance 50% cooperation exceeding the performance of 25% cooperation due to the maximisation of d_1 and d_2, during 50% cooperation, as shown in the PEP in (2.18). During 10dB interuser channel conditions, at higher uplink SNR's, 25% cooperation has a better performance than that of the performance of 50% cooperation. The gains by which the 25% cooperation performance exceeds the 50% cooperation performance, at 10dB interuser channel conditions, is as high as 2 dB since the transmission of a stronger code in the first frame is vital to the overall performance of a user during poor interuser channel conditions.

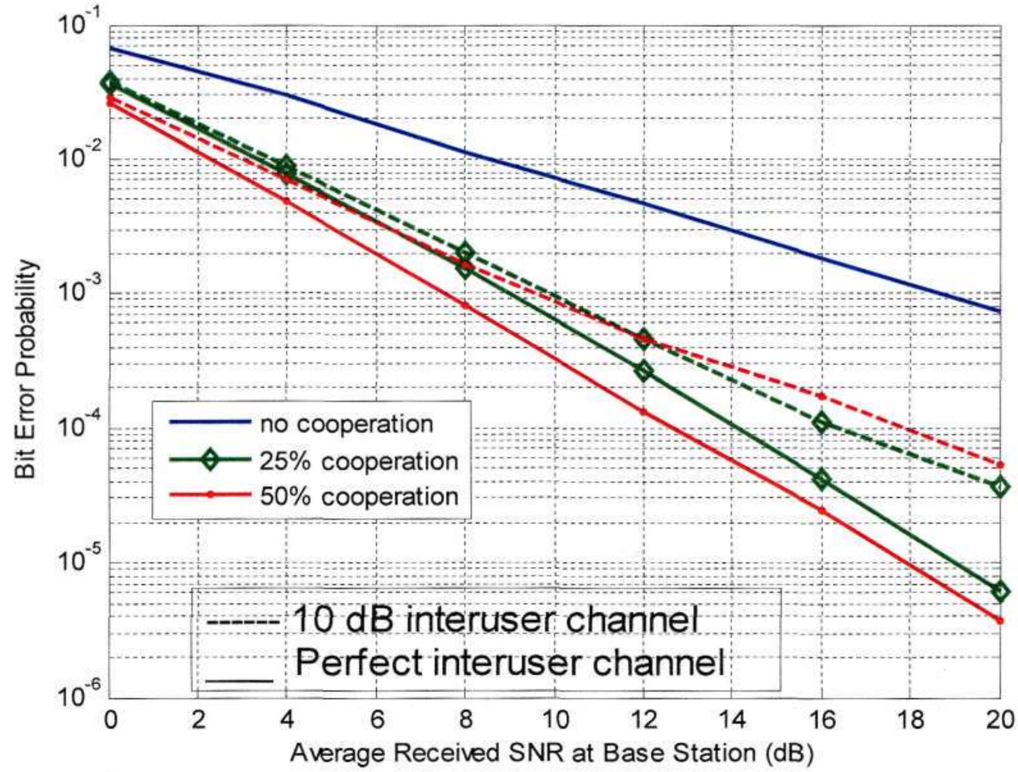

Figure 2.4: A comparison between 50% and 25% cooperation

Asymmetric uplink channels are considered in Figure 2.5. User 1's uplink SNR is fixed at 20dB and user 2's uplink SNR changes from 0dB-20dB. The interuser average received channel SNR is set to 10dB. The results show that user 1 with a high uplink channel SNR has dramatic performance improvements by cooperating. This significant performance improvement over non cooperation is very high even when user 2's uplink SNR is at 0dB for user 1. User 2's BER performance also improves by a margin of 11-12 dB over non cooperation.

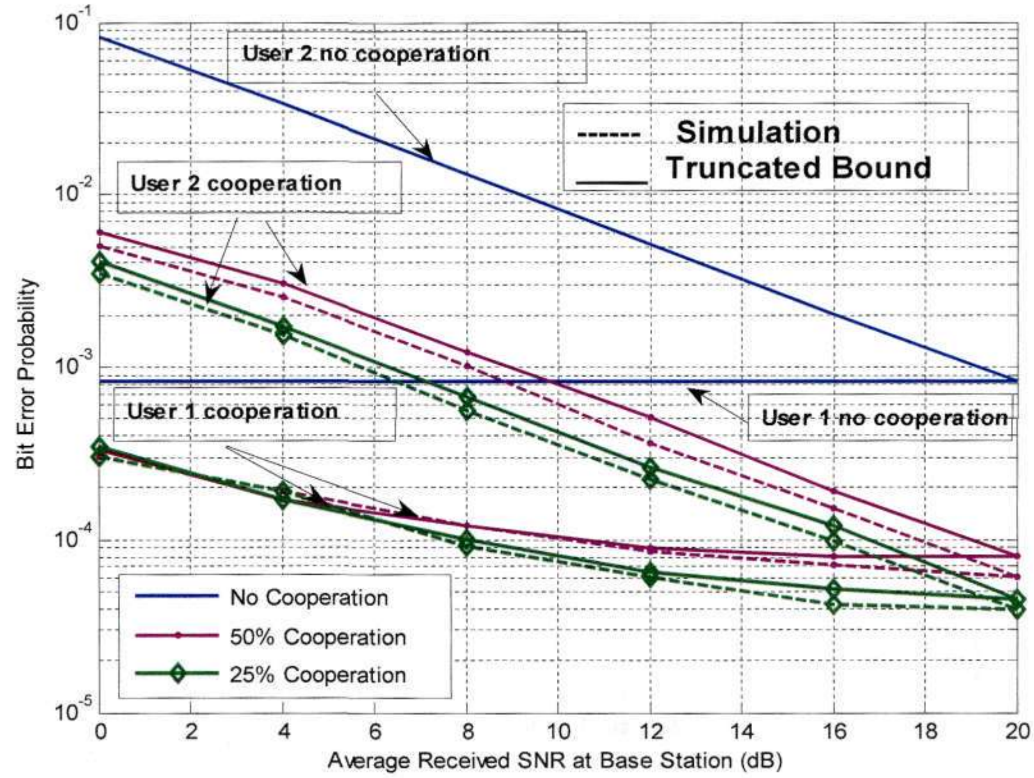

Figure 2.5: Asymmetric uplink conditions

Figure 2.6 and Figure 2.7 show comparisons between reciprocal and independent interuser channels for 50% and 25% cooperation respectively. During 50% cooperation the loss in performance due to independent interuser channel conditions can go as high as 2.5dB depending on the interuser channel quality. During 25% cooperation the performance loss when independent interuser channels are used is less than 0.8 dB.

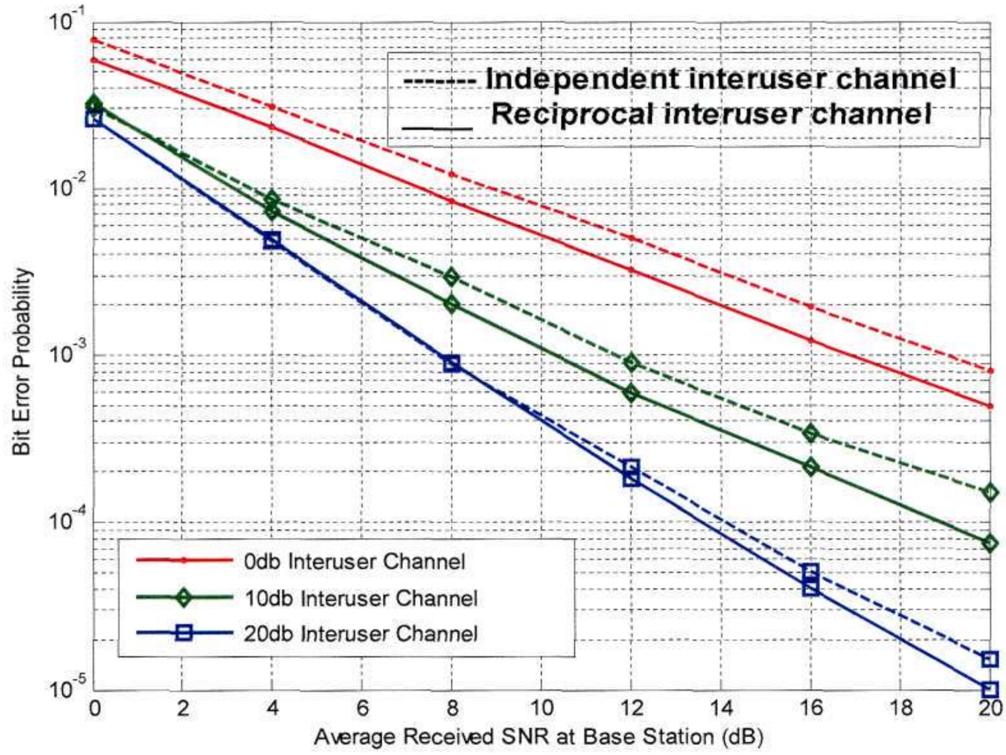

Figure 2.6: Independent vs. Reciprocal interuser channels for 50% cooperation

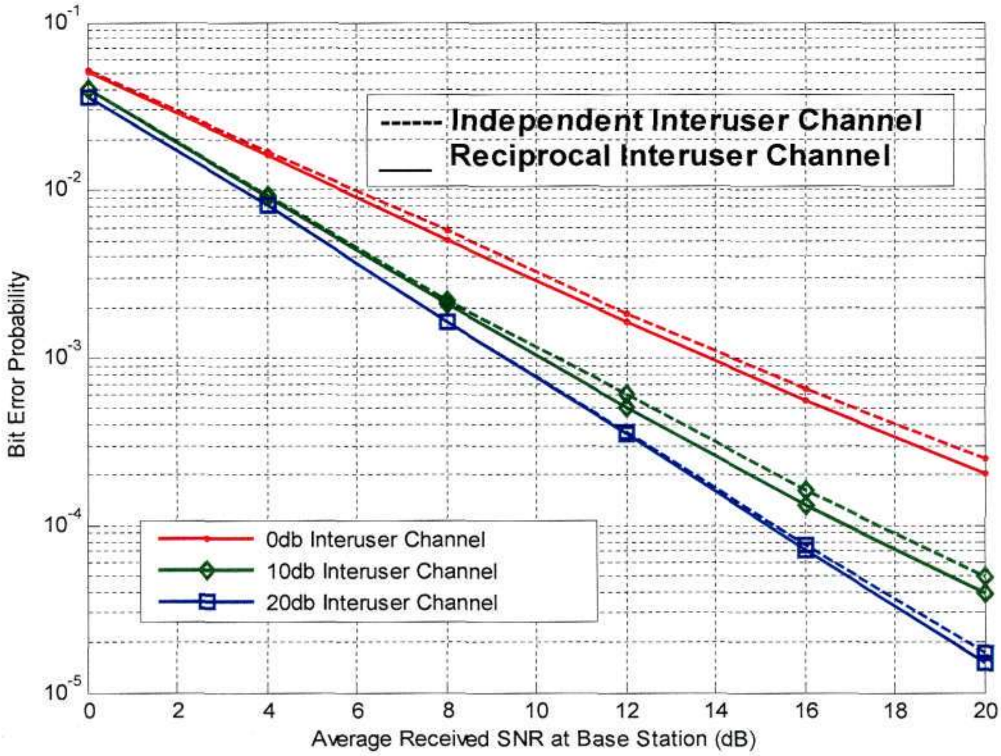

Figure 2.7: Independent vs. Reciprocal interuser channels for 25% cooperation

2.10 Chapter Summary

An RCPC implementation in a Coded Cooperation framework was considered in this chapter based on the work done in [24]. Two user cooperation is enabled by the partitioning of code words into two frames of size N_1 and N_2. A user will always receive N_1 bits from the partner and will, based upon successful decoding of the partner's first transmitted frame, transmit N_2 bits of parity for the partner. If the decoding of the N_1 bits from the partner is unsuccessful the user will transmit N_2 parity bits for itself. The base station will receive N_1 and N_2 bits, for each user, via two independently faded paths.

The BER performance analysis is considered based on the four possible transmission scenarios and PEP for each transmission scenario. Coded Cooperation maintains impressive BER gains over non cooperation. Even under poor interuser channel conditions Coded Cooperation maintains a BER performance gain of 1.8-3 that of a non cooperative mode.

The code rate, transmit power and bandwidth of each user during Coded Cooperation is the same as that of a user transmitting during a non cooperative transmission. The analytical analysis concepts for the BER of Coded Cooperation, explained in this chapter, sets up a foundation for more complicated Cross Layer Cooperation BER analysis in the next chapter.

Chapter 3

In Chapter 2 Coded Cooperation was investigated in detail. Coded Cooperation is however not one of the most efficient transmission schemes, since in some cases the base station has already decoded a particular set of information bits based on a user's initially transmitted parity. The user is not informed of this, however, and Coded Cooperation allows for a user to transmit parity for information bits that have already been decoded. This is bandwidth inefficient. This chapter introduces a new framework called Cross Layer HARQ 2 Cooperative Diversity which combines Cooperative Diversity at the physical layer with HARQ 2 at the data–link layer in an effort to improve BER over Coded Cooperation. The use of HARQ 2 allows for the user to be informed from the base station on whether further parity is required for a particular set of information bits or not. The BER of Cross Layer Hybrid ARQ 2 Cooperative Diversity is examined via simulations and by the computation of theoretical bounds based on the PEP and the individual transmission case probabilities. The BER of Cross Layer Hybrid ARQ 2 Cooperation is compared to that of Coded Cooperation showing BER improvements over Coded Cooperation. Cross Layer HARQ 2 Cooperation results in efficient use of bandwidth for users during cooperation by selectively cooperating, only when necessary, based on feedback from the Base Station.

3 Cross Layer Hybrid ARQ 2 Cooperative Diversity

3.1 Modelling the wireless channel

Consider two wireless users transmitting data to a base station. The interuser channel and the uplink channels for each user are mutually independent and follow a block fading model.

The multiple access scheme used is TDMA. TDMA allows for a cooperating user and the destination (the base station) to decode each other's parity.

Perfect CSI is assumed to be available at all receivers (cooperating users and the base station). This is possible through the use of some pilot symbol channel estimation technique which will not be pursued in this work.

Coherent detection is employed at all receivers also, so that the phase components of fading coefficients can be ignored and only the magnitude of the fading coefficients are taken into account. BPSK modulation is employed and the received sample for a particular fading block "l" can be modelled as:

$$y_{i,j,l}(n) = \alpha_{i,j,l}(n)\sqrt{E_{b,i}}\,x_{i,j,l}(n) + n_j(n) \tag{3.1}$$

In (3.1), i $(i \in \{1,2\})$ denotes the transmitting user, for that transmission, j $(j \in \{0,1,2\})$ the receiving user and the index l $(l \in \{1,2,3,4\})$ being the fading block index for the RCPC codes used. Note also that $i \neq j$.The block fading model resembles that of [46].The variable $y_{i,j,l}$ is a received sample of user i's transmission at user j during fading block l. $E_{b,i}$ is the energy of a transmitted bit, $x_{i,j,l}$ is a BPSK modulated symbol i.e. $x_{i,j,l} \in \{-1,1\}$ at time instant n $(n \in \{1,2,....,K\})$ and $n_j(n)$ accounts for noise and other additive interference effects at the receiver j and is modelled as samples of a zero mean Gaussian random variable with variance of $N_j/2$.

$\alpha_{i,j,l}(n)$ are modelled as independent samples of a Rayleigh random variable for a particular block l. Reciprocity of interuser channels are assumed i.e. $\alpha_{i,j,l}(n) = \alpha_{j,i,l}(n)$ based on [21] - [24] and [17]-[19]. The mean square value of $\alpha_{i,j,l}(n)$ is given by:

$$\Omega_{i,j,l} = E_{\alpha_{i,j,l}}\left[\alpha^2_{i,j,l}(n)\right] \tag{3.2}$$

Based on the perfect CSI assumption at the receiver the instantaneous signal to noise ratio at receiver j being transmitted from user i is given by:

$$\gamma_{i,j,l}(n) = \frac{\alpha^2_{i,j,l}(n)E_{b,i}}{N_j} \tag{3.3}$$

$\gamma_{i,j,l}(n)$ is exponentially distributed with a mean value of:

$$\Gamma_{i,j,l} = E_{\alpha_{i,j,l}}\left[\gamma_{i,j,l}(n)\right] = E_{\alpha_{i,j,l}}\left[\frac{\alpha^2_{i,j,l}(n)E_{b,i}}{N_j}\right] = \Omega_{i,j,l}\frac{E_{b,i}}{N_j} \tag{3.4}$$

In (3.3) and (3.4) $\Omega_{i,j,l}$ and $\Gamma_{i,j,l}$ are constant over the range of n.

3.2 Cross Layer Hybrid ARQ 2 Cooperation Protocol

Two users i and j have information bits comprising of K bit blocks including c concatenated CRC bits. Each K bit information block is encoded into N data bits by a $R = K/N = 1/4$ rate mother convolutional code. The ratio of the augmented number of CRC bits to the information block size i.e. c/K is almost zero and hence adds negligible overhead.

Users i and j cooperate in a selective manner i.e. instead of using the interuser CRC syndrome as an indication of whether to cooperate or not, the users look at the base station CRC syndrome as well as the interuser CRC syndrome as an indication of whether to cooperate. The base station CRC syndrome is the HARQ 2 feedback signal from the base station, on whether extra incremental parity is required for a particular user's transmitted parity or not. The best case cooperation level $\lambda_{i,j}$ is defined to be the ratio of the maximum number of cooperation bits user i transmits for user j, divided by mother code encoded length i.e. $\lambda_{i,j} = N_{i,j(MAX)}/N$ where the notation i,j refers to user i's cooperation with user j.

Each user's N encoded bits are partitioned into four packets i.e. $B_{i,1}, B_{i,2}, B_{i,3}, B_{i,4}$ of equal size $N_{i,1}, N_{i,2}, N_{i,3}, N_{i,4}$ such that $lenght(N) = \sum_{r=1}^{4} lenght(N_r)$. $\lambda_{i,j}$ is discretized by two values for best case cooperation of 0.25 and 0.5, which denote 25% and 50% best case cooperation.

Based on the a priori value of $\lambda_{i,j}$, cooperation is combined with Hybrid ARQ 2 (feedback from the base station) to allow for cooperative transmission only when the base station needs additional incremental parity from the partner in order to decode a user's parity correctly. User i will transmit an initial amount of parity packets to the base station. The base station will attempt to decode this parity. The partner, user j, will also receive this parity transmitted by user i (by listening to user i's transmission) and will attempt to decode it, using TDMA and RCPC decoding i.e. Viterbi decoding.

If $\lambda_{i,j}$ is set at 0.5 then user i or j's initial transmissions to the base station are $B_{i/j,1}$ and $B_{i/j,2}$, (which is also picked up by "listening partner i or j" who attempts to decode it) i.e.

$$B_{i/j}(t) = B_{i/j,1} + B_{i/j,2} = \sum_{l=1}^{N_1} \left|B_{i/j,1}\right| \delta(t-l) + \sum_{l=N_1+1}^{N_2} \left|B_{i/j,2}\right| \delta(t-l) \qquad \textbf{(3.5)}$$

Using a puncturing matrix specified in [58] only $B_{i/j,1} + B_{i/j,2}$ are transmitted out of the total N available parity bits. The notation i/j denotes either user i or j and $\delta(.)$ denotes the unit impulse function.

If $\lambda_{i,j}$ is set at 0.25 then user i or j's initial transmissions to the base station are $B_{i/j,1} + B_{i/j,2} + B_{i/j,3}$ (which is also picked up by "listening partner i or j" who attempts to decode it) such that only $B_{i/j,1} + B_{i/j,2} + B_{i/j,3}$ are transmitted out of the total of N available parity bits i.e.

$$B_{i/j}(t) = B_{i/j,1} + B_{i/j,2} + B_{i/j,3} = \sum_{l=1}^{N_1} \left|B_{i/j,1}\right| \delta(t-l) + \sum_{l=N_1+1}^{N_2} \left|B_{i/j,2}\right| \delta(t-l) + \sum_{l=N_2+1}^{N_3} \left|B_{i/j,3}\right| \delta(t-l)$$
$$\textbf{(3.6)}$$

Selective cooperation by a user i or j is based on the base station's decoding of the initially transmitted packets (using Viterbi decoding and computing the CRC parity check sum or the syndrome calculation of the initially transmitted packets) of the cooperating user, the cooperation level $\lambda_{i,j}$ and the inter-user CRC syndrome calculations.

Note that the base station combines the initial parity packets together and then decodes them. At least two parity packets have to be sent for there to be sufficient parity available at the base station for decoding i.e. for a $1/4$ mother code rate, R, the minimum punctured rate chosen-based on the family of codes present in [58] is 0.5. It is for this reason that 50% is chosen for one of the best case cooperation levels. If the base station can decode the transmitted data in the initial transmissions of user i or j then the base station responds with an Acknowledge (ACK) and user i/j transmits parity for the next K bit information block.

If the base station cannot decode the combined parity packets a Negative Acknowledge (NACK) signal is fed back to the transmitting user i and is also picked up by the cooperating user j. User i and j then cooperate to transmit parity for user i incrementally (in response to incremental feedback from the base station) until the total N parity bits of user i is completed i.e. user i / j transmits either $B_{i/j,3} + B_{i/j,4}$ or $B_{i/j,3}$ on their own uplink channels. After the completion of N parity bits if a NACK is still received then the parity frame (comprising of incremental packets) is considered erroneous and a new packet transmission is started for a new N bit parity frame.

Based on the interuser CRC states i.e. crc1 and crc2 and the decoder CRC states i.e. dcrc1 and dcrc2, 16 possible transmission scenarios for initial transmissions could occur. These are tabulated in Table 3.1 below where a "0" denotes a NACK and a "1" an ACK.

Table 3.1: Cross Layer Hybrid ARQ 2 Transmission Scenarios

Case	$dcrc_1$	$dcrc_2$	crc_1	crc_2
1.1	0	0	0	0
1.2	0	0	0	1
1.3	0	0	1	0
1.4	0	0	1	1
2.1	0	1	0	0
2.2	0	1	0	1
2.3	0	1	1	0
2.4	0	1	1	1
3.1	1	0	0	0
3.2	1	0	0	1
3.3	1	0	1	0
3.4	1	0	1	1
4.1	1	1	0	0
4.2	1	1	0	1
4.3	1	1	1	0
4.4	1	1	1	1

Note that the total parity frame of each user consists of N parity bits for a K bit information block. The N bit frame is partitioned into sub code words i.e. $B_{i,l}$, $i \in \{1,2\}$ and $l \in \{1,2,3,4\}$, which are obtained by applying a puncturing matrix to the entire N bit frame. The puncturing matrices chosen are based on the $1/4$ rate mother code and the family of codes present in [57].

Figure 3.1 below shows the partitioning of a N bit parity frame into sub code words or parity packets. This shows the total amount of parity packets a user will posses for a K bit information block.

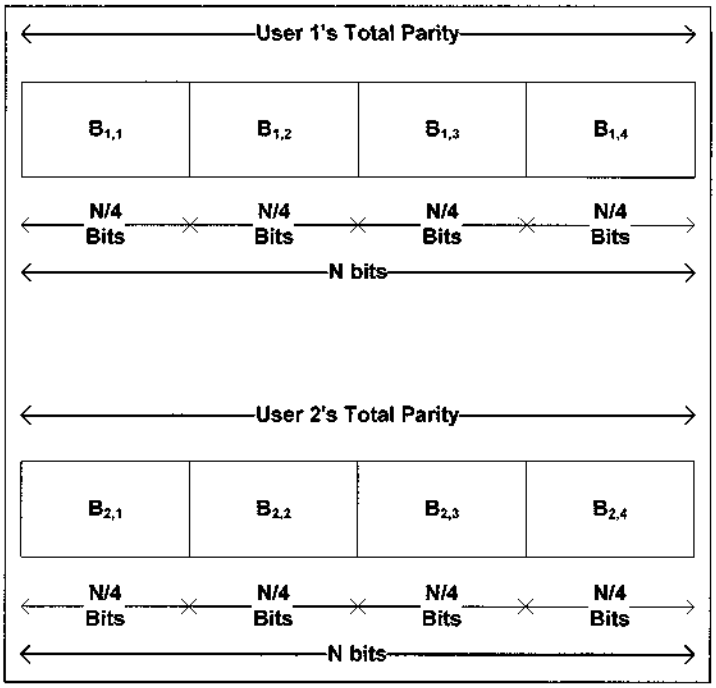

Figure 3.1: Parity Packet Partitioning of the total parity frame

Note that the notation for the transmission cases is written with a main case and its four sub cases i.e. case 1 which corresponds to $dcrc_1$ and $dcrc_2$ (base station CRC flags) equal to 0 and 0 (i.e. $dcrc_1$=NACK and $dcrc_2$=NACK) respectively has four sub cases, based on the interuser CRC decoding i.e. the values of crc_1 and crc_2, which are case 1.1, case 1.2, case 1.3 and case 1.4. In this way 16 possible transmission cases can be derived from the base station CRC flags given by $dcrc_1$ and $dcrc_2$.

The subsequent cooperation and transmission of the remaining user and cooperating partner's parity packets is explained in each sub case. Figure 3.2 shows a system diagram of Cross Layer Hybrid ARQ 2 Cooperation.

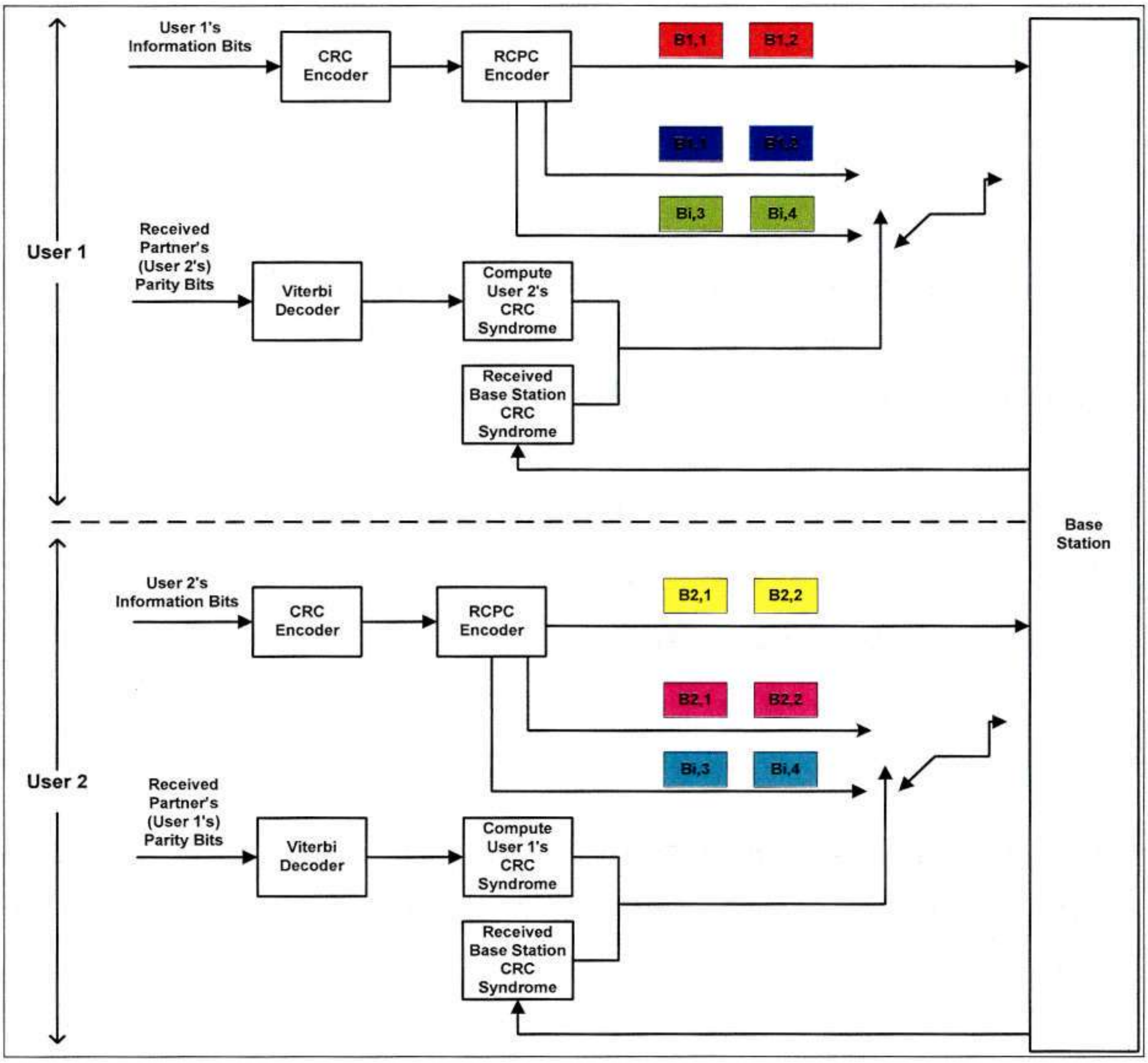

Figure 3.2: Cross Layer HARQ 2 Cooperation: System Diagram

Initially user 1 transmits $B_{1,1}$ and $B_{1,2}$ to the base station, shown by the red packets in Figure 3.2. The base station and the partner receive these two red packets and decode them using a Viterbi decoder and then compute the CRC syndrome. The base station sends either an ACK or a NACK to user 1, on the downlink, which is inherited by user 2. Based on the received CRC signal from the base station and the interuser CRC the users decide to cooperate or not. If for example user 1's red packets were decoded correctly, the first time, by the base station and user 1 is not cooperating then user 1 will transmit the two dark blue packets which is new parity ($B_{1,1}$ and $B_{1,2}$) for a new information packet for himself. On the other hand if the red packets were decoded incorrectly then user 1 will incrementally transmit the green packets for himself which would be parity ($B_{1,3}$ and $B_{1,4}$ if needed). Note that the subscript "i" is used in the green packets for $B_{i,3}$ and $B_{i,4}$ because user 1 could also be transmitting incremental cooperation packets for user 2 (user 1 is cooperating) since $i \in \{1,2\}$ in this context.

Similarly user 2 transmits $B_{2,1}$ and $B_{2,2}$ (yellow packets) for itself initially. Based on the feedback from the base station to user 2, which is inherited by user 1, and the interuser CRC states user 2 will either cooperate with user 1 by transmitting the light blue packets for the partner ($B_{1,3}$ and $B_{1,4}$ if needed) or user 2 could also not have to cooperate and could have its own initial parity decoded correctly by the base station, in which case user 2 will transmit the purple packets for himself (which is new parity for a new set of information bits) i.e. $B_{2,1}$ and $B_{2,2}$. The subscript "i" is used in the light blue packets for $B_{i,3}$ and $B_{i,4}$ because user 2 could also be transmitting incremental parity packets for itself since $i \in \{1,2\}$ in this context (during non cooperation). Note that in Figure 3.2 the transmissions of user 1 and 2 are symmetrical about the dashed line.

3.2.1 Case 1.1

During case 1.1 user 1 and 2 have incorrect interuser CRC. The base station signals NACK for both user 1's initial transmission of $B_{1,1} + B_{1,2}$ and users 2's initial transmission of $B_{2,1} + B_{2,2}$ which are the half rate punctured subcode words from the 0.25 rate mother code. During 50% best case cooperation user 1 will transmit $B_{1,3}$ and user 2 will transmit $B_{2,3}$ i.e. each user transmits parity for themselves incrementally. If after transmission of $B_{1,3}$ and $B_{2,3}$ by user 1 and user 2 respectively, the base station still signals NACK's for the decoding of $B_{1,1} + B_{1,2} + B_{1,3}$ and $B_{2,1} + B_{2,2} + B_{2,3}$ - then user 1 will transmit an additional parity packet $B_{1,4}$ and user 2 will transmit an additional parity packet $B_{2,4}$.

Note that if an ACK is reached at any time for $B_{i,3}$ or $B_{i,4}$, the transmission of parity for that set of information bits for user i is stopped and the next set of parity bits is transmitted for the next set of information bits for each user.

During 25% best case cooperation user 1 will transmit $B_{1,4}$ and user 2 will transmit $B_{2,4}$ i.e. each user transmits incremental parity for themselves. If after the transmission of

$B_{1,1}+B_{1,2}+B_{1,3}+B_{1,4}$ by user 1 and $B_{2,1}+B_{2,2}+B_{2,3}+B_{2,4}$ by user 2, the base station still signals a NACK for user 1's parity and a NACK for user 2's parity i.e. $dcrc_1$=0 and $dcrc_2$=0 then the data packets transmitted by user 1 and 2 are considered erroneous and add to the BER. The next set of parity, for new information packets, is transmitted by the users.

As an example if $B_{2,3}$ is given an ACK from the base station then user 2 does not have to transmit $B_{2,4}$ and user 1 can continue to transmit $B_{1,4}$ if $B_{1,3}$ was received incorrectly. This could lead to some "disynchronisation" between sets of N bits for each user i.e. user 1 could be transmitting the parity packet $B_{1,4}$ for the previous set of information bits whilst user 2 will be transmitting parity packets $B_{2,1}$ and $B_{2,2}$ for the next of its information bits. HARQ 2 allows for the transmission of incremental parity packets to a receiver and for feedback signals from the receiver (ACK or NACK) on whether more incremental parity packets are required. HARQ 2 can be implemented in any multiple access scheme, TDMA, CDMA and FDMA. These multiple access schemes cannot solve the problem of packet synchronisation issues so that cooperation opportunities are not missed in some cases.

One method of solving the above problem is to have user 1 and 2 transmit $B_{1,4}$ (this happens in the case when $B_{2,3}$ leads to correct decoding of user 2's parity but $B_{1,3}$ is received incorrectly for user 1's parity) for user 1 and have MRC at the base station to combine user ones received signal that is transmitted form both users. This would result in further BER gains. Cross Layer Hybrid ARQ 2 Cooperative Diversity tries to improve throughput also and therefore has difficulty in using the approach of both users transmitting $B_{1,4}$ for user 1. Instead the protocol allows for the case of "disynchronisation" to occur (user 1 could be transmitting the parity packet $B_{1,4}$ for the previous set of information bits whilst user 2 will be transmitting parity packets $B_{2,1}$ and $B_{2,2}$ for the next of its information bits) with the following reasoning below.

Cross Layer Hybrid ARQ 2 Cooperative Diversity is a cross layer design between the physical and data-link layer. In order to also improve throughput, the protocol assumes that there are higher layer (network layer) scheduling algorithms that will take QoS constraints into account for specific types of traffic so that desynchronized cooperation packets will be buffered and scheduled at the correct time in order to facilitate synchronized cooperation and still meet QoS demands.

3.2.2 Case 1.2

During case 1.2 user 2's interuser CRC flag is correct i.e. user 2 can decode user 1's initial transmission of $B_{1,1} + B_{1,2}$ but user 1's interuser CRC flag is incorrect i.e. user 1 cannot decode user 2's $B_{2,1} + B_{2,2}$ initial transmission. The base station has also not been able to decode $B_{1,1} + B_{1,2}$ transmitted by user 1 and $B_{2,1} + B_{2,2}$ transmitted by user 2 and signals NACK for dcrc1 and dcrc2. During 50% best case cooperation both users transmit parity for user 1 i.e. both users transmit $B_{1,3}$. The base station employs MRC to combine the same parity signals from the both users for user 1. If the base station is still not able to decode $B_{1,1} + B_{1,2} + B_{1,3}{}^{MRC}$ then user 1 and 2 transmit $B_{1,4}$ for user 1. Again the base station employs MRC to combine both the $B_{1,4}$ - same parity signals coming from both users.

Note that if an ACK is reached at any time during the transmission of $B_{1,3}$ or $B_{1,4}$ (from both users) then the transmission of parity, for that set of information bits for user 1, is stopped and the next set of parity bits is transmitted for a new set of information bits.

During 25% best case cooperation user 1 and 2 will transmit $B_{1,4}$ for user 1. The base station will use MRC to combine the same parity transmitted for user 1 by both users.

If after the transmission of $B_{1,4}{}^{MRC}$ by user 1 and 2, the base station still signals a NACK for user 1's parity i.e. $dcrc_1$=0 then the parity packets transmitted for user 1 is considered erroneous and adds to the BER. The next set of parity for new information packets is transmitted by the users.

3.2.3 Case 1.3

During case 1.3 user 2's interuser CRC flag is incorrect i.e. user 2 cannot decode user 1's initial transmission of $B_{1,1} + B_{1,2}$ but user 1's interuser CRC flag is correct i.e. user 1 can decode user 2's $B_{2,1} + B_{2,2}$ initial transmission. The base station has also not been able to decode $B_{1,1} + B_{1,2}$ for user 1 and $B_{2,1} + B_{2,2}$ for user 2 and signals NACK for $dcrc_1$ and $dcrc_2$. During 50% best case cooperation both users transmit parity for user 2 i.e. both users transmit $B_{2,3}$. The base station employs MRC to combine the same parity signals from the both users for user 2. If the base station still is not able to decode $B_{2,1}+B_{2,2}+B_{2,3}{}^{MRC}$ then user 1 and 2 transmit $B_{2,4}$ for user 2. Again the base station employs MRC to combine the same parity $B_{2,4}$ signals coming from both users.

Note that if an ACK is reached at any time during the transmission of $B_{2,3}$ or $B_{2,4}$ the transmission of parity for that set of information bits for user 2 is stopped and the next set of parity bits are transmitted for a next set of information bits for each user. During 25% best case cooperation user 1 and 2 will transmit $B_{2,4}$ for user 2. The base station will use MRC to combine the same parity transmitted for user 2 by both users.

After the transmission of $B_{2,4}{}^{MRC}$ by user 1 and 2, the base station still signals a NACK for user 2's parity i.e. $dcrc_2$=0 then the parity packets transmitted for user 2 is considered erroneous and adds to the BER. The next set of parity for new information packets is transmitted by the users.

3.2.4 Case 1.4

During case 1.4 user 1 and 2 have correct interuser CRC. The base station signals NACK for both user 1's initial transmission of $B_{1,1}+B_{1,2}$ and users 2's initial transmission of $B_{2,1}+B_{2,2}$ which are the half rate punctured sub code words from the 0.25 rate mother code. During 50% best case cooperation user 1 will transmit $B_{2,3}$ and user 2 will transmit $B_{1,3}$ i.e. each user transmits parity for its partner incrementally. If after transmission of $B_{2,3}$ and $B_{1,3}$ by user 1 and 2 respectively, the base station still signals NACKs for the decoding of $B_{2,1}+B_{2,2}+B_{2,3}$ and $B_{1,1}+B_{1,2}+B_{1,3}$ - then user 1 will transmit an additional parity packet of $B_{2,4}$ and user 2 will transmit an additional parity packet of $B_{1,4}$.

Note that if an ACK is reached at any time during the transmission of $B_{i,3}$ or $B_{i,4}$ then the transmission of parity for that set of information bits for user i is stopped and the next set of parity bits is transmitted for the next set of information bits - for each user.

During 25% best case cooperation user 1 will transmit $B_{2,4}$ and user 2 will transmit $B_{1,4}$ i.e. each user transmits incremental parity for its partner. After transmission of $B_{2,4}$ and $B_{1,4}$ by user 1 and 2 respectively the base station signals NACK for the decoding of

$B_{1,1}+B_{1,2}+B_{1,3}+B_{1,4}$ and $B_{2,1}+B_{2,2}+B_{2,3}+B_{2,4}$ then the packet is considered an error packet and a new packet transmission is started.

After the transmission of $B_{1,1}+B_{1,2}+B_{1,3}{}^{user2}+B_{1,4}{}^{user2}$ and $B_{2,1}+B_{2,2}+B_{2,3}{}^{user1}+B_{2,4}{}^{user1}$ by user 1 and user 2, if the base station still signals a NACK for user 1's parity and a NACK for user 2's parity i.e. $dcrc_1=0$ and $dcrc_2=0$ then the parity packets transmitted by user 1 and 2 are considered erroneous and add to the BER for user 1 and 2. The next set of parity for the new information packets is transmitted by the users. The subscript in the notation $B_{2,3}{}^{user1}$ means user 1 transmits the parity for user 2 during the third incremental parity packet transmission of $B_{2,1}+B_{2,2}+B_{2,3}{}^{user1}+B_{2,4}{}^{user1}$.

3.2.5 Case 2

During this case $dcrc_1=0$ and $dcrc_2=1$ i.e. the base station has only signalled a NACK for user 1's parity transmission of $B_{1,1}+B_{1,2}$. Hence parity has to be transmitted for user 1 only in this case. Based on whether or not user 2 has decoded user 1's initial transmission of $B_{1,1}+B_{1,2}$ i.e. if crc_2 is equal to 1 or 0, user 2 will either transmit incremental parity for user 1 or not. During case 2.1 and 2.3 user 2 will have $crc_2=0$ and will not be able to incrementally transmit $B_{1,3}$ and $B_{1,4}$ (if needed) for user 1 and user 1 will transmit $B_{1,3}$ and $B_{1,4}$ (if needed) for itself only.

In cases 2.2 and 2.4 user 2 has $crc_2=1$ and thus user 2 can transmit $B_{1,3}$ and $B_{1,4}$ (if needed) for user 1. The base station then uses MRC to combine the two spatially diversely received same parity signals i.e. $B_{1,3}{}^{user1}, B_{1,4}{}^{user1}, B_{1,3}{}^{user2}\, and\ B_{1,4}{}^{user2}$ (if needed) from user 1 and 2 for user 1. The "if needed" words in brackets mean that in the case where user 1 and 2 transmitted $B_{1,3}$ for user 1 and MRC was performed at the base station and the base station could still not decode the transmitted parity for user 1, then user 1 and 2 would transmit $B_{1,4}$ for user 1 because this parity is now needed by the base station. The base station again will perform MRC for the received $B_{1,4}$ signal from user 1 and 2 for user 1's parity.

3.2.6 Case 3

This case is the same as case 2 with the roles of user 1 and 2 swapped.

3.2.7 Case 4

During the first frame transmissions by user 1 and 2, $B_{1,1}+B_{1,2}$ and $B_{2,1}+B_{2,2}$ are transmitted correctly and the base station signals an ACK for both of them i.e. $dcrc_1$=1 and $dcrc_2$=1. The users do not need to transmit any more parity to the base station, for that set of information bits, and hence continue transmitting parity for the next set of information bits. Table 3.2 on the next page summarises all the possible transmissions by the users based on the scenario.

Table 3.2: Summary of transmission signals for Cross Layer HARQ 2 Cooperation

Case	$dcrc_1$	$dcrc_2$	crc_1	crc_2	User 1's Transmission	User 2's Transmission	MRC Combining Employed
1.1	0	0	0	0	$B_{1,3}$ and $B_{1,4}$ (if needed)	$B_{2,3}$ and $B_{2,4}$ (if needed)	No
1.2	0	0	0	1	$B_{1,3}$ and $B_{1,4}$ (if needed)	$B_{1,3}$ and $B_{1,4}$ (if needed)	Yes
1.3	0	0	1	0	$B_{2,3}$ and $B_{2,4}$ (if needed)	$B_{2,3}$ and $B_{2,4}$ (if needed)	Yes
1.4	0	0	1	1	$B_{2,3}$ and $B_{2,4}$ (if needed)	$B_{1,3}$ and $B_{1,4}$ (if needed)	No
2.1	0	1	0	0	$B_{1,3}$ and $B_{1,4}$ (if needed)	$B_{2,1}+B_{2,2}$ for next frame	No
2.2	0	1	0	1	$B_{1,3}$ and $B_{1,4}$ (if needed)	$B_{1,3}$ and $B_{1,4}$ (if needed)	Yes
2.3	0	1	1	0	$B_{1,3}$ and $B_{1,4}$ (if needed)	$B_{2,1}+B_{2,2}$ for next frame	No
2.4	0	1	1	1	$B_{1,3}$ and $B_{1,4}$ (if needed)	$B_{1,3}$ and $B_{1,4}$ (if needed)	Yes
3.1	1	0	0	0	$B_{1,1}+B_{1,2}$ for next frame	$B_{2,3}$ and $B_{2,4}$ (if needed)	No
3.2	1	0	0	1	$B_{1,1}+B_{1,2}$ for next frame	$B_{2,3}$ and $B_{2,4}$ (if needed)	No
3.3	1	0	1	0	$B_{2,3}$ and $B_{2,4}$ (if needed)	$B_{2,3}$ and $B_{2,4}$ (if needed)	Yes
3.4	1	0	1	1	$B_{2,3}$ and $B_{2,4}$ (if needed)	$B_{2,3}$ and $B_{2,4}$ (if needed)	Yes
4.1	1	1	0	0	$B_{1,1}+B_{1,2}$ for next frame	$B_{2,1}+B_{2,2}$ for next frame	No
4.2	1	1	0	1	$B_{1,1}+B_{1,2}$ for next frame	$B_{2,1}+B_{2,2}$ for next frame	No
4.3	1	1	1	0	$B_{1,1}+B_{1,2}$ for next frame	$B_{2,1}+B_{2,2}$ for next frame	No
4.4	1	1	1	1	$B_{1,1}+B_{1,2}$ for next frame	$B_{2,1}+B_{2,2}$ for next frame	No

3.3 No Cooperation Analysis

During no cooperation a user, using a $R = \frac{1}{4}$ rate, transmits incremental packets up to the full mother code to the base station, incrementally. The conditional PEP is dependant on the instantaneous received SNR given by (3.3) and is given by:

$$p\left(d \mid \gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4}\right) = Q\left(\sqrt{2\sum_{i=1}^{4} d_{1,i}\gamma_{1,i}}\right) \tag{3.7}$$

where $\gamma_{1,1},\gamma_{1,2},\gamma_{1,3}$ and $\gamma_{1,4}$ are mutually independent of each other. To obtain the unconditional PEP the conditional PEP is averaged over the range of gamma i.e.

$$P(d) = \int_{\alpha} P(d \mid \gamma)\, p_{\gamma}(\gamma)\, d\gamma \tag{3.8}$$

Since there are four fading blocks four fold integration is performed to compute a lower bound for the unconditional PEP Shown below.

$$P(d) \le \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} p\left(d \mid \gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4}\right)\cdot \prod_{i=1}^{4} p\left(\gamma_{1,i}\right) d\gamma_{1,1}\, d\gamma_{1,2}\, d\gamma_{1,3}\, d\gamma_{1,4} \tag{3.9}$$

Using the moment generating function and the alternate form of the Q function as shown in (2.9) and (2.7) respectively, the unconditional PEP is simplified to be:

$$P(d) = \frac{1}{\pi}\int_0^{\pi/2} M_{\gamma_{1,1}}\left(\frac{-d_1\Gamma_{1,1}}{\sin^2\theta}\right).M_{\gamma_{1,2}}\left(\frac{-d_2\Gamma_{1,2}}{\sin^2\theta}\right).M_{\gamma_{1,3}}\left(\frac{-d_3\Gamma_{1,3}}{\sin^2\theta}\right).M_{\gamma_{1,4}}\left(\frac{-d_4\Gamma_{1,4}}{\sin^2\theta}\right)d\theta$$

$$= \frac{1}{\pi}\left[\frac{1}{1+d_1\Gamma_{1,1}}\right].\left[\frac{1}{1+d_2\Gamma_{1,2}}\right].\left[\frac{1}{1+d_3\Gamma_{1,3}}\right].\left[\frac{1}{1+d_4\Gamma_{1,4}}\right] \tag{3.10}$$

3.4 Determining the PEP on The Uplink Channels

During block fading conditions $\alpha_{i,j,l}(n) = \alpha_{j,i,l}(n)$ over the range of N. The fading block vector, $\alpha_{i,j,l}(n)$, thus reduces to a scalar.

Based on the interuser and base station CRC states, the conditional PEP can be determined. The CRC states are explained in Table 3.1.

During case 1.1 no cooperation occurs and so each user transmits parity for itself. The PEP for case 1.1 for each user is given by:

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}\right) = Q\left(\sqrt{2\sum_{i=1}^{4} d_{1,i}\gamma_{1,i}}\right) \tag{3.11}$$

Note that $d = \sum_{i=1}^{4} d_{1,i}$ where $d_{1,i}$ are the distances (Hamming distance) between c_u and e_u where $u \in \{1,2,3,4\}$ or portions of the error event for each sub code word or incremental packet. During case 1.4 we have full cooperation (since the interuser CRC states $crc_1 = 1$ and $crc_2 = 1$ and the base station CRC states i.e. $dcrc_1$ and $dcrc_2$ are equal to 0) with each user transmitting parity for the other and the conditional PEP for each user is given by:

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{2,3}, \gamma_{2,4}\right) = Q\left(\sqrt{\left(2\sum_{i=1}^{2} d_{1,i}\gamma_{1,i} + 2\sum_{j=3}^{4} d_{2,j}\gamma_{2,j}\right)}\right) \tag{3.12}$$

During case 1.2 partial cooperation occurs i.e. we have MRC combining at the base station since both users transmit incremental parity for user 1. The conditional PEP for user 1 is given by:

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}, \gamma_{2,3}, \gamma_{2,4}\right) = Q\left(\sqrt{\left(2\sum_{i=1}^{4} d_{1,i}\gamma_{1,i} + 2\sum_{j=3}^{4} d_{2,j}\gamma_{2,j}\right)}\right) \tag{3.13}$$

3.5 Determining the Transmission Case Probabilities

Cross layer hybrid ARQ 2 cooperation has 16 possible transmission scenarios which are determined based on the BLER during transmission of the first 2 or 3 incremental packets depending on the best case cooperation level $\lambda_{i,j}$.

Using (2.23) the BLER for a convolutional code is bounded by:

$$P_{block}(\gamma) \leq 1 - \left(1 - P_E(\gamma)\right)^B \leq B \cdot P_E(\gamma) \tag{3.14}$$

Where the first event error probability $P_E(\gamma)$ is given by:

$$P_E(\gamma) \leq \sum_{d=d_f}^{\infty} a(d) P(d \mid \gamma) \tag{3.15}$$

The probability of case 1.1 can be calculated, under reciprocal interuser channel conditions, using:

$$\begin{aligned} &p\left(\theta = 1.1 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{2,1}, \gamma_{2,2}\right)_{2Trans} = \\ &\left[\prod_{i=1}^{2} p_{block,1}\left(\gamma_{1,2,i}\right)\right]^2 \cdot \prod_{j=1}^{2} p_{block,1}\left(\gamma_{1,j}\right) \cdot \prod_{k=1}^{2} p_{block,2}\left(\gamma_{2,k}\right) \end{aligned} \tag{3.16}$$

Where the factor $p_{block,1}\left(\gamma_{1,2,i}\right)$ denotes the interuser BLER under reciprocal interuser channel conditions. The factor $p_{block,1}\left(\gamma_{1,j}\right)$ denotes the BLER for an incremental packet on user 1's uplink channel and the factor $p_{block,2}\left(\gamma_{2,k}\right)$ denotes the BLER for an incremental packet on user 2's uplink channel. The "2Trans" subscript in (3.16) means two incremental packets are transmitted first i.e. 50% best case cooperation is employed. During 25% best case cooperation (3.16) is modified to:

$$\begin{aligned} &p\left(\theta = 1.1 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,2,3}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}\right)_{3Trans} = \\ &\left[\prod_{i=1}^{3} p_{block,1}\left(\gamma_{1,2,i}\right)\right]^2 \cdot \prod_{j=1}^{3} p_{block,1}\left(\gamma_{1,j}\right) \cdot \prod_{k=1}^{3} p_{block,2}\left(\gamma_{2,k}\right) \end{aligned} \tag{3.17}$$

Where the subscript "3Trans" means three initial incremental packets are transmitted i.e. 25% best case cooperation is employed.

Equation (3.11) and (3.15) are used in (3.14) together with the limit before averaging techniques as outlined in [47] to obtain a tight bound for the case 1.1 conditional probability given on the next page in (3.18).

$$p\left(\theta = 1.1 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{2,1}, \gamma_{2,2}\right)_{2Trans} =$$

$$\left[\prod_{i=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)p(d \mid \gamma_{1,2,i})\right)\right)^{B}\right)\right]^{2} \cdot \prod_{j=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)p(d \mid \gamma_{1,j})\right)\right)^{B}\right) \cdots$$

$$\prod_{k=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)p(d \mid \gamma_{2,k})\right)\right)^{B}\right)$$

(3.18)

To obtain the unconditional case probability the conditional case probability is averaged over the instantaneous SNR range i.e.

$$P(\theta) = \int_{\gamma} P(\theta \mid \gamma) p(\gamma) d\gamma \qquad \textbf{(3.19)}$$

Where γ is the fading vector representing the channel state. Using (3.18) the case probability for case 1.1 is derived to be:

$$p(\theta = 1.1)_{2Trans} = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} p\left(\theta = 1.1 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{2,1}, \gamma_{2,2}\right) \cdot \prod_{i=1}^{2} p\left(\gamma_{1,2,i}\right) \cdot \prod_{j=1}^{2} p\left(\gamma_{1,j}\right) \cdots$$

$$\prod_{k=1}^{2} p\left(\gamma_{2,k}\right) \cdot \prod_{i=1}^{2} d\gamma_{1,2,i} \cdot \prod_{j=1}^{2} d\gamma_{1,j} \cdot \prod_{k=1}^{2} d\gamma_{2,k}$$

(3.20)

This happens during 50% best case cooperation. Note that in (3.20) the unconditional case probability is not averaged over $\gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{2,1,1}, \gamma_{2,1,2}$ but only over $\gamma_{1,2,1}, \gamma_{1,2,2}$ since the channel is assumed to be reciprocal.

During 25% best case cooperation, the case probability for case 1.1 can be calculated to be:

$$p(\theta = 1.1)_{3Trans} = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} p\left(\theta = 1.1 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,2,3}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}\right) \cdots$$

$$\prod_{i=1}^{3} p\left(\gamma_{1,2,i}\right) \cdot \prod_{j=1}^{3} p\left(\gamma_{1,j}\right) \cdot \prod_{k=1}^{3} p\left(\gamma_{2,k}\right) \cdot \prod_{i=1}^{3} d\gamma_{1,2,i} \cdot \prod_{j=1}^{3} d\gamma_{1,j} \cdot \prod_{k=1}^{3} d\gamma_{2,k}$$

(3.21)

Similarly the case probabilities for the other transmission cases can be calculated based on the interuser and base station CRC states. Equation (3.18) is thus integrated over the entire instantaneous SNR range in (3.20) to obtain $p(\theta = 1.1)_{2Trans}$.

Case 1.4 for example has the following conditional case probability for 50% best case cooperation:

$$p\left(\theta=1.4 \mid \gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,2}\right)_{2Trans} = \left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right)\right]^{2} \cdot \prod_{j=1}^{2} p_{block,1}\left(\gamma_{1,j}\right)\prod_{k=1}^{2} p_{block,2}\left(\gamma_{2,k}\right) \quad \textbf{(3.22)}$$

In this case full cooperation occurs and each user transmits parity to its partner incrementally.

During independent interuser channel conditions the conditional case probability has to be modified with regard to the interuser BLER factor for the transmission case probability. As an example if case 1.4 had occurred under independent interuser channel conditions then (3.22) has to be modified to:

$$p\left(\theta=1.4 \mid \gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,1}\right)_{2Trans} = \left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right)\right] \cdot \left[\prod_{j=1}^{2}\left(1-p_{block,1}\left(\gamma_{2,1,j}\right)\right)\right] \cdot \prod_{k=1}^{2} p_{block,1}\left(\gamma_{1,k}\right)\prod_{l=1}^{2} p_{block,2}\left(\gamma_{2,l}\right) \quad \textbf{(3.23)}$$

Where the factor $\left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right)\right] \cdot \left[\prod_{j=1}^{2}\left(1-p_{block,1}\left(\gamma_{2,1,j}\right)\right)\right]$ represents the interuser CRC states. Here $\gamma_{1,2,i}$ is not equal to $\gamma_{2,1,j}$.

Equation (3.22) has the factor $\left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right)\right]^{2}$ for the interuser states, since $\gamma_{1,2,i}$ is equal to $\gamma_{2,1,j}$ - the interuser bit error probability for each incremental packet is equal, which results in the square of a user's product of bit error rates for its transmission. This is shown by the square factor in this paragraph above.

3.6 The End to End Bit Error Probability

The BER for Cross layer Hybrid ARQ 2 Cooperative Diversity is given by the BER for each transmission case weighted by the case probability and takes into account correct decoding before the maximum amount of packets is transmitted. Appendix A contains detailed derivations of the end to end BER bounds. In this chapter the end to end theoretical BER bound equations are presented and explained. This is done by introducing the concepts of the individual transmission case probability and the individual BER for each transmission case probability.

When cooperation level lies between 0-50% (i.e. 50% best case cooperation) the end to end BER is given by:

$$p_{BER} = \sum_{l=1}^{2}\sum_{r=1}^{4}\sum_{j=2}^{4} p_b\left(\theta = l,r\right)_j \, p\left(\theta = l,r\right)_j \tag{3.24}$$

The factor $p_b\left(\theta = l,r\right)_j$ denotes the BER for a specific case on the uplink channel whilst the factor $p\left(\theta = l,r\right)_j$ denotes the case probability.

Since $p_b\left(\theta = l,r\right)_j$ is the BER at a specific instance of θ it can be generalized as:

$$p_b\left(\theta = l,r\right)_j \leq \int_{\gamma} \min\left[\frac{1}{2}, \frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)\, p(d \mid \gamma)\right] p(\gamma)\, d\gamma \tag{3.25}$$

Since γ is the vector state of the channel, for a specific transmission scenario, (3.25) has to be expanded based on the individual case. The PEP i.e. $p(d \mid \gamma)$ has to also be determined for each case.

During case 1.4 the BER for 50% best case cooperation is given by:

$$P_b\left(\theta = 1.4\right)_{2trans} \leq \int_0^{\infty}\int_0^{\infty} \min\left[\frac{1}{2}, \frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)\, p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \theta\right)\right] \cdot \prod_{i=1}^{2} p\left(\gamma_{1,i}\right) d\gamma_{1,i} \tag{3.26}$$

Where:

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}\right) = Q\left(\sqrt{\left(2\sum_{i=1}^{2} d_{1,i}\gamma_{1,i}\right)}\right) \tag{3.27}$$

During 25% best case cooperation the end to end BER is given by:

$$p_{BER} = \sum_{l=1}^{2}\sum_{r=1}^{4}\sum_{j=3}^{4} p_b\left(\theta = l,r\right)_j \, p\left(\theta = l,r\right)_j \tag{3.28}$$

The only difference between this equation and Equation (3.24) is that the 25% cooperation level is taken into account in this equation by changing the lower limit of the subscript j i.e. 3 initial parity packets are transmitted.

3.7 Simulation Parameters and Discussion of Results

The performance analysis of Cross Layer HARQ 2 Cooperation is analysed here using the BER performance metric. The mother code is still kept at $R = 1/4$, the memory length, $M = 4$ and the family of RCPC codes is again used from [57]. The amount of information bits in the source packet is kept at $K = 128$ information bits. The distance spectra $a(d)$ and $c(d)$ are computed via computer enumeration as well as the separation of the minimum distance d into $d_{1,v}$ or $d_{2,v}$ where $v \in \{1,2,3,4\}$.

Error detection is handled via 16 bit CRC (augmented into the K bit source packet) with the generator polynomial equal to: $g_{crc}(x) = 15935$ in hexadecimal notation. Figures 3.3 to 3.8 show comparisons between simulated and theoretical BER for Cross Layer HARQ 2 Cooperation.

Figure 3.3 shows the performance of BER under symmetrical uplink ($\Gamma_{1,0} = \Gamma_{2,0}$) channels for 50% best case cooperation, with reciprocal interuser channel conditions i.e. $\gamma_{1,2,l} = \gamma_{2,1,l}$ where $l \in \{1,2\}$. Cross layer HARQ 2 Cooperation shows massive improvements in performance over non cooperation, for the block fading channel, as the interuser channel quality improves. Even under 0dB interuser channel conditions Cross layer HARQ 2 shows a 0.6 dB improvement in performance over non cooperation at a BER of 10^{-4}. As the interuser channel quality improves the BER of Cross Layer HARQ 2 cooperation also improves.

At 10dB interuser channel conditions Cross Layer HARQ 2 Cooperation maintains a 2.1 dB gain over non cooperation at a BER of 10^{-4}. Note that since this is a block fading channel the SNR regime is smaller over the range of BER.

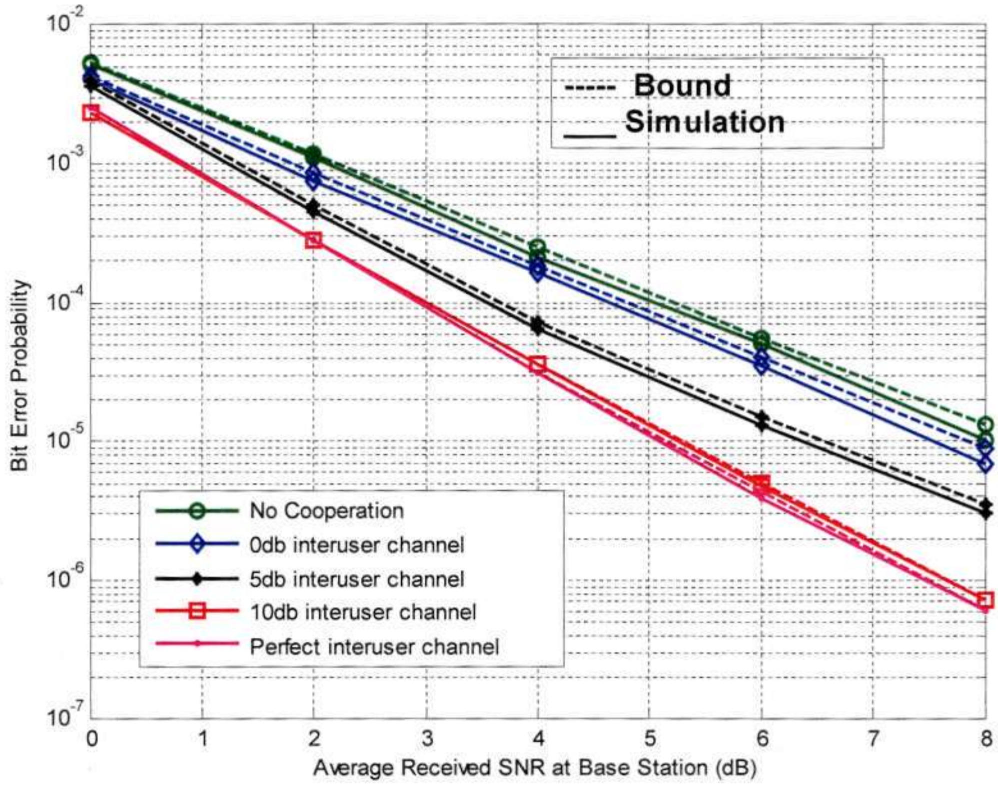

Figure 3.3: 50% cooperation with symmetric uplinks

Figure 3.4 shows a comparison between 50% and 25% best case cooperation under 5dB and perfect interuser channel conditions. During 5dB interuser channel conditions, indicated by a dotted line in the legend, 25% best case cooperation is better than 50% best case cooperation under higher SNR conditions, since more parity is needed by the base station during the initial transmission by the user to decode the user's parity correctly. Hence a stronger code in the initial transmission by a user is of more importance to a user's performance.

During perfect interuser channel conditions the case $x.4$ is observed, where $x \in \{1,2,3,4\}$. At lower uplink SNR the probability of $x=1$ i.e. case 1.4 occurring is the highest and as the uplink SNR reaches a maximum of 8dB the probability of $x=4$ is the highest i.e. case 4.4 occurring. In the intermediate SNR note the transmission case scenarios will fluctuate between case 1.4 to case 4.4 with case 2.4 and case 3.4 occurring more often as partial cooperation. Note that during case 1.4 full cooperation is observed and during case 4.4 since correct transmissions are received at the base station no cooperation is required and the users transmit their next set of parity for their next set of information bits.

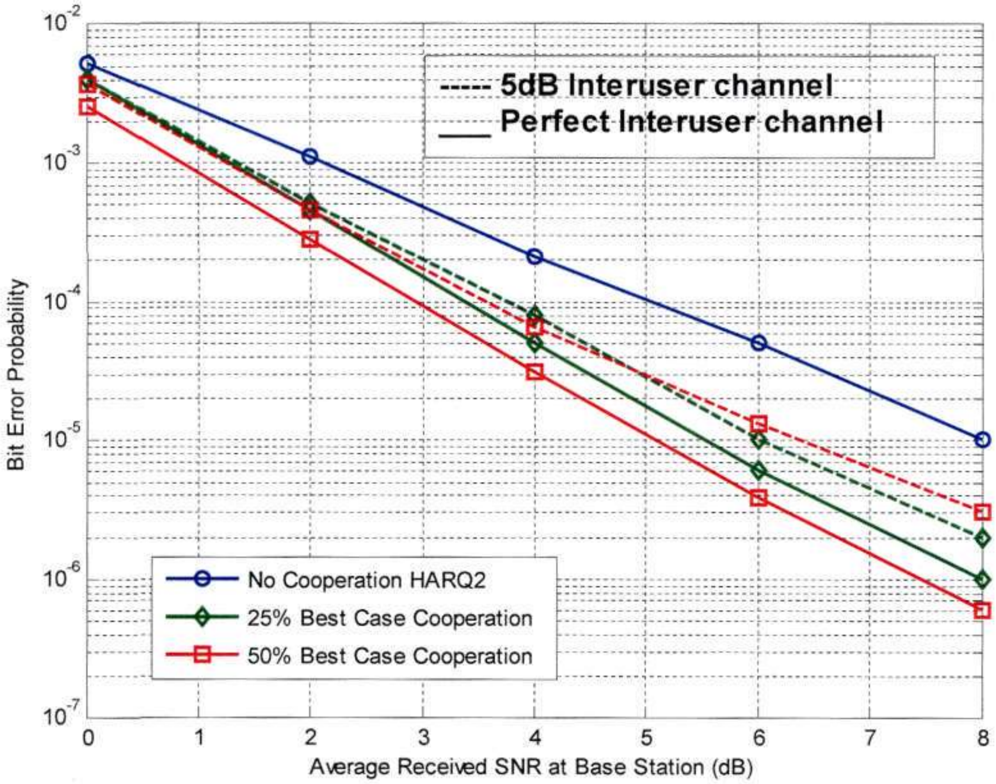

Figure 3.4: A comparison between 50% and 25% cooperation

Figure 3.5 and 3.6 shows a comparison between reciprocal and independent interuser channel conditions for both 50% and 25% best case cooperation. The loss in BER performance between Coded Cooperation and Cross Layer HARQ 2 cooperation is always within 1 dB.

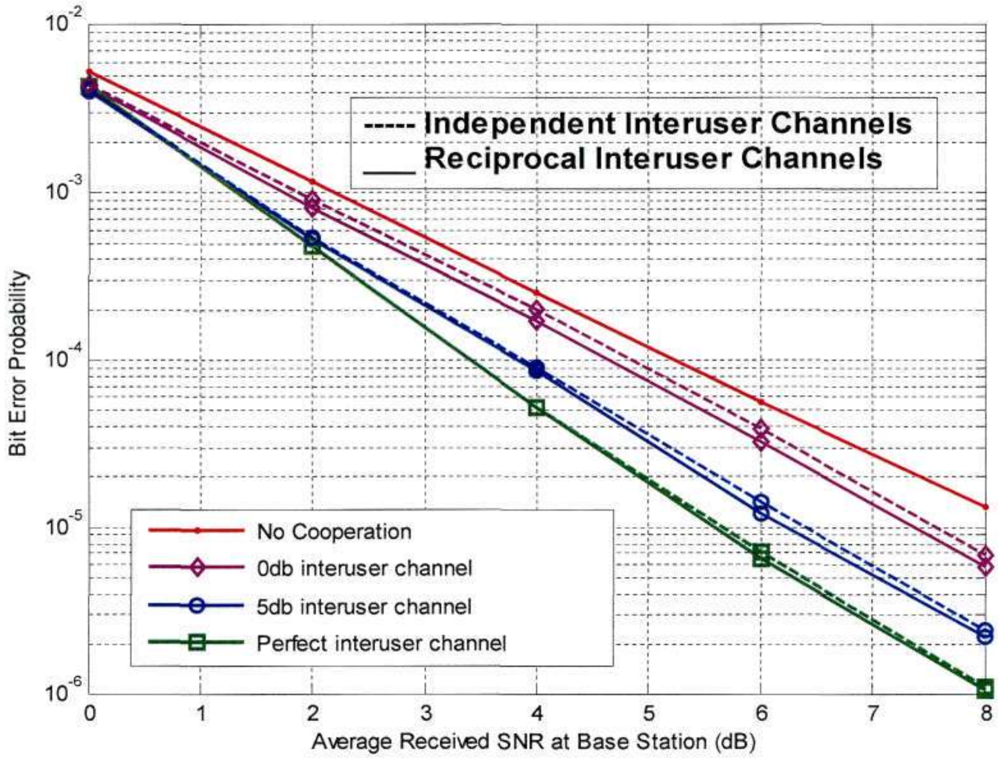

Figure 3.5: Independent vs. Reciprocal interuser channels for 25% Best Case cooperation

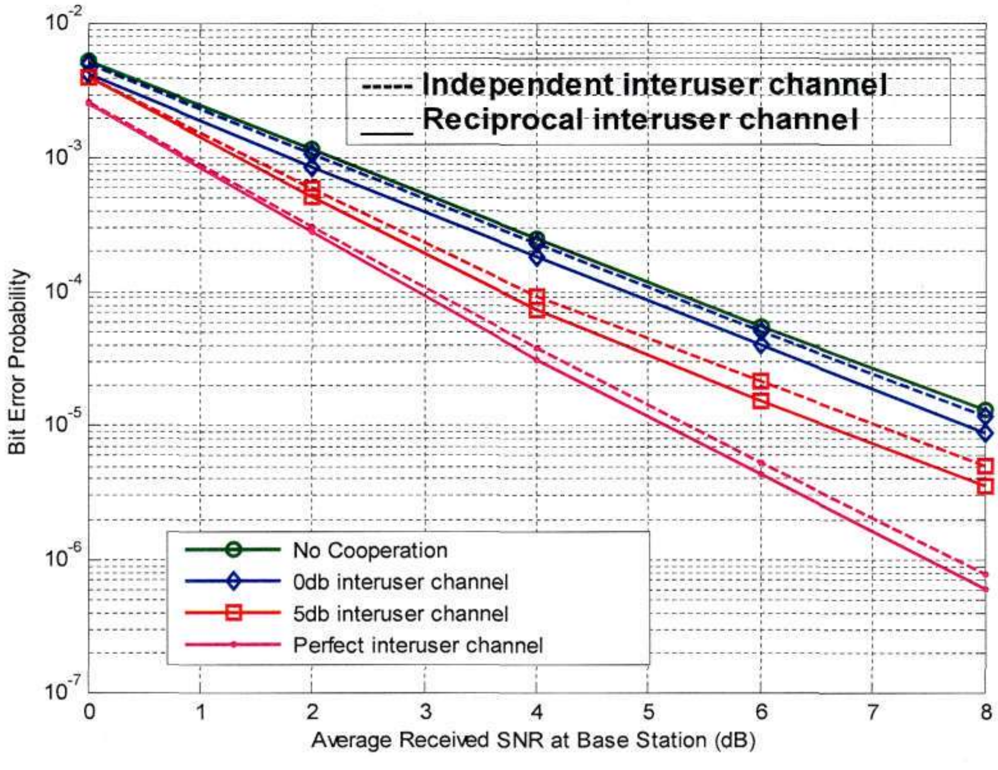

Figure 3.6: Independent vs. Reciprocal interuser channels for 50% Best Case cooperation

Figure 3.7 shows Cross Layer HARQ2 Cooperation during asymmetric uplink channels. User 1's uplink channel is fixed at 8dB and user 2's uplink varies form 0-8dB. User 2's BER improves dramatically by cooperating with user 1 even under poor interuser channel conditions. The performance of user 1 reduces slightly by cooperating under poor uplink conditions but this is negligible and improves as the uplink quality improves. Note that the interuser channel quality is set to 5dB so this also contributes to the slight degradation of BER for user 1.

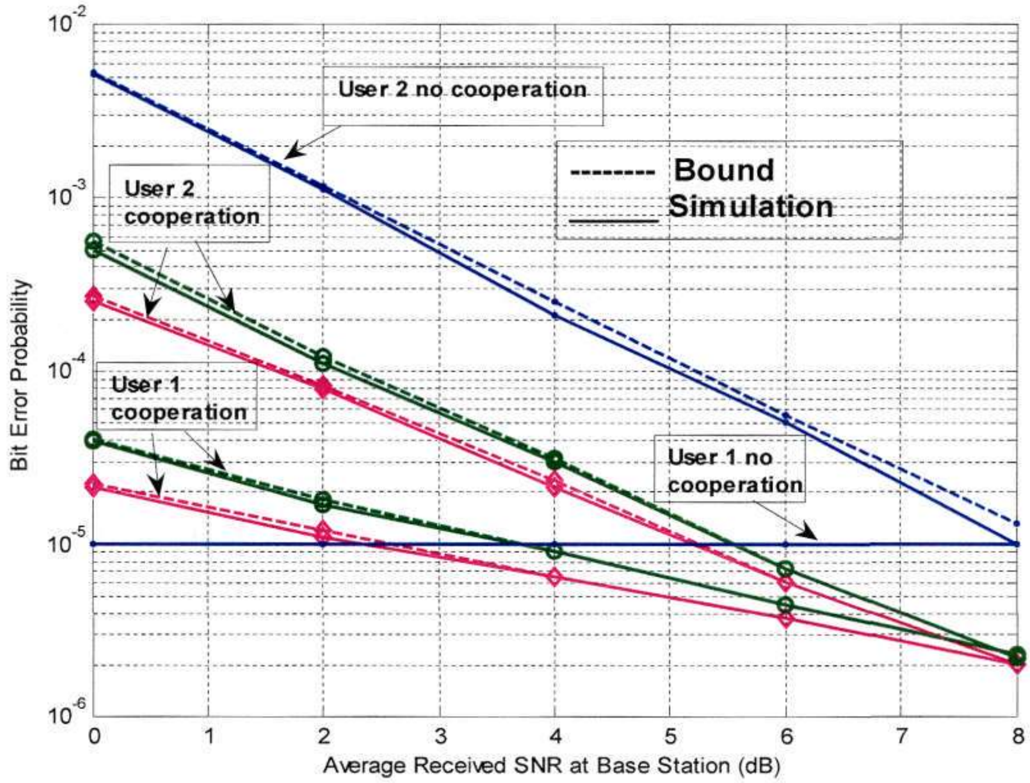

Figure 3.7: Asymmetric uplink conditions

In Figure 3.8 comparisons are made between Cross Layer HARQ 2 Cooperation and Coded Cooperation at 0dB and 5dB interuser channel conditions. Cross Layer Hybrid ARQ 2 Cooperation always performs better than Coded Cooperation even under poor interuser channel conditions with a performance gain of as high as 2.1 dB at 0dB interuser channel conditions. Note that during higher interuser channel conditions 25% cooperation outperforms 50% cooperation during high uplink conditions for both Coded Cooperation and Cross Layer HARQ 2 cooperation.

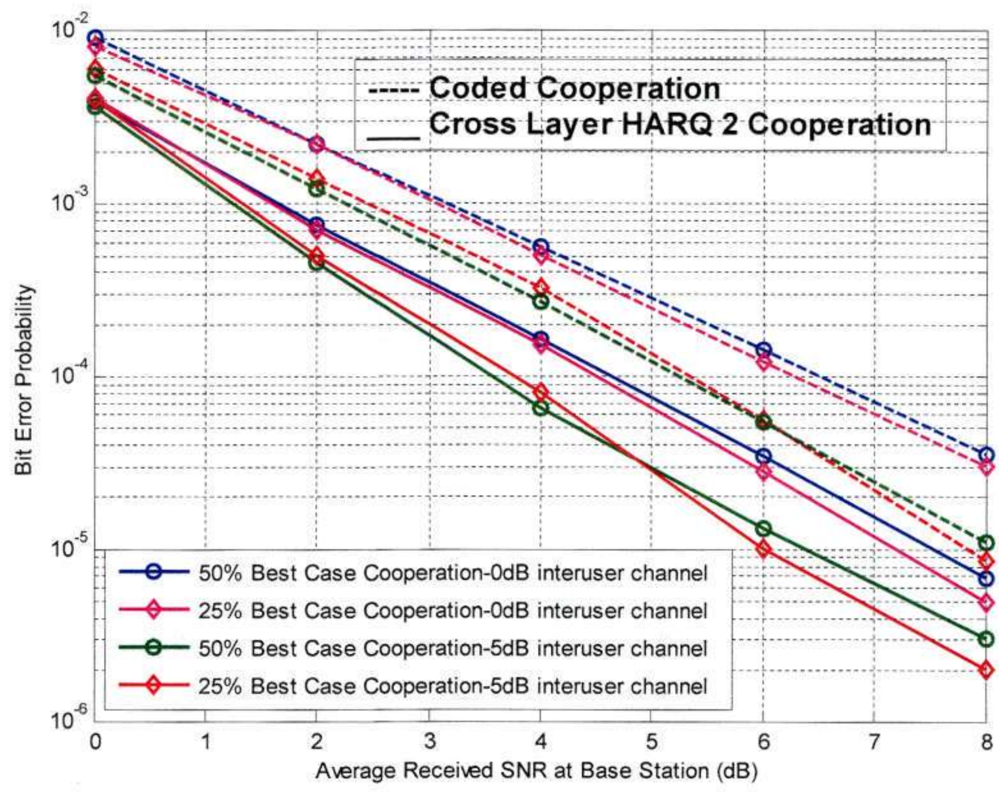

Figure 3.8: Comparison between Cross Layer HARQ 2 Cooperation and Coded Cooperation

3.8 Chapter Summary

RCPC codes were used to partition a N bit convolutional code into individual parity packets or sub code words as shown in Figure 3.1. Four incremental parity packets can be created via puncturing i.e. $B_{1,1}, B_{1,2}, B_{1,3}, B_{1,4}$ or $B_{2,1}, B_{2,2}, B_{2,3}, B_{2,4}$ for user 1 and 2 respectively. Based on the cooperation level $\lambda_{i,j}$ a partner j will receive at least $B_{i,1}, B_{i,2}$ from user i during 50% best case cooperation or $B_{i,1}, B_{i,2}, B_{i,3}$ during 25% best case cooperation.

Based on the interuser CRC states i.e. crc_1 and crc_2 and the base station CRC states i.e. $dcrc_1$ and $dcrc_2$ the partners will either cooperate, by incrementally transmitting parity, or will not cooperate - because there is no need to (i.e. base station has sent back ACK signals for both users' incremental parity transmissions). Since the base station CRC states and the interuser CRC states yield values for 4 bits, 16 possible transmission states are possible and thus 16 possible transmission scenarios occur for each user.

Analytical analysis of BER for Cross Layer Hybrid ARQ 2 Cooperation is performed using the unconditional PEP, in each transmission scenario, and summing the individual BERs for each case weighted by the case probabilities. The analytical BER bounds confirm the simulated results for Cross Layer Hybrid ARQ 2 Cooperation.

Simulations show improvements in Cross Layer Hybrid ARQ 2 Cooperation over Coded Cooperation for BER during all interuser channel conditions. The BER performance for Cross Layer Hybrid ARQ 2 Cooperation also shows gains in performance over non cooperation even at an interuser channel condition of 0 dB. As the interuser channel condition improves the performance gain of Cross Layer Hybrid ARQ 2 Cooperation improves dramatically over non cooperation.

The performance loss in BER due to the use of independent interuser channel conditions is also shown to be within 1 dB of that of reciprocal interuser channel conditions.

Chapter 4

Chapter 3 introduced Cross Layer Hybrid ARQ 2 Cooperative Diversity and presented simulation and analytical bound results with regard to BER improvements over Coded Cooperation. In this Chapter the throughput of Cross Layer HARQ 2 Cooperation is examined via simulations and by computation of theoretical throughput bounds, using principles of packet error probability and average amount of transmitted sub code words per source information packet, as mentioned in [47]. These mentioned analytical throughput principles in [47] are used as a foundation and understanding in order to be able to compute higher level throughput bounds that occurs in Cross Layer HARQ 2 Cooperation. The throughput of Coded Cooperation is also computed and compared to that of Cross Layer HARQ 2 Cooperation, showing the throughput efficiency of Cross Layer Hybrid ARQ 2 Cooperation over Coded Cooperation.

4 Throughput Evaluation of Cross Layer HARQ 2 Cooperative Diversity

4.1 Throughput Improvements in Cross Layer HARQ 2 Cooperation

Cross layer Hybrid ARQ 2 combines cooperative diversity at the physical layer and truncated ARQ at the data link layer.

The Cross Layer Hybrid ARQ 2 protocol involves a cooperating user attempting to decode its partners first two transmitted parity packets, which is evaluated based on the computation of a CRC parity check sum. By using CRC the room for introducing further performance error's into the system (via estimation of the partners transmitted signal) is minimized to zero. This is in contrast to fixed cooperation protocols such as amplify and forward and detect and forward where the cooperating partner is chosen irrespective of the interuser decoding scenario (these are termed fixed cooperation protocols). These two fixed cooperation protocols operate based on exploitation of the omni directional electromagnetic waves emitted from the source, which involves signal processing on an analog signal. Detect and forward would detect the analog transmission for the source based on thresholding of the received signal.

Choosing a suitable threshold is a difficult matter since it depends on the instances of the fading coefficients. If too high a threshold is chosen then the performance is reduced. When a low threshold is chosen then an extra amount of error is introduced into the system which would also reduce the system performance.

Coded Cooperation [17]-[19], [21]-[24], helped to solve this problem by introducing error detection codes (Cyclic Redundancy Check - CRC) into the system, so that cooperating users append CRC bits in their information packet, which is then encoded (using RCPC codes) and transmitted. The receiving partner used a Viterbi decoder to decode the RCPC code and then a CRC syndrome calculation was done to check if the source data packet is received correctly or not. In this way cooperation only takes place upon successful CRC syndrome calculations i.e. CRC parity check sum has to equal to zero.

Coded Cooperation, however, allows for continual transmission. The user's continued transmission is oblivious to the fact that the destination (base station) could have received the initial parity transmissions correctly and so continued transmissions until all parity is completed. Although Coded Cooperation results in good BER performance the protocol does not allow for efficient utilization of bandwidth i.e. in some cases there is no need to cooperate anymore.

By combining Automatic Repeat re-Quest (ARQ) at the data-link layer and cooperative diversity on the physical layer, in a cross layer approach, control can be achieved on the amount of parity to transmit by a cooperating or non cooperating user. This results in improvements in throughput.

Instead of having just the partners decoding the interuser CRC, the destination also decodes the user's parity, in groups of packets, until sufficient parity is available to successfully decode the user's information (with the help of cooperation parity packets and MRC being performed at the base station).

4.2 Throughput Analysis

The throughput can be defined to be the successful data rate, which is given by:

$$\eta = \frac{B}{B+c} \cdot \frac{p_s}{\overline{L}} \tag{4.1}$$

Where B denotes the information packet length and excludes the "c" CRC bits concatenated with the information packet. Note that p_s is defined to be the packet successful probability.

The following notation has to be taken into account during the derivation of the throughput bounds:

1. p_{pd} denotes the packet error probability for direct incremental transmission i.e. no cooperation.
2. $p_{pR}^{(1)}$ is the packet error probability for incremental retransmission for user 1.
3. $p_{pR}^{(2)}$ is the packet error probability for incremental retransmission for user 2.
4. p_{sd} denotes the symbol error probability for direct incremental transmission involving no cooperation.
5. $p_{sR}^{(1)}$ denotes the symbol error probability for incremental retransmission by user 1.
6. $p_{sR}^{(2)}$ denotes the symbol error probability for incremental retransmission by user 2.
7. $\overline{L}$ is the average amount of transmitted sub code words per source information packet.

During direct transmission i.e. no cooperation the packet successful probability p_s is given by:

$$p_s = 1 - p_{pd}{}^2 \left(p_{pR}{}^{(1)}\right)^2 \qquad \textbf{(4.2)}$$

$\overline{L}$,during direct transmission, is given by:

$$\overline{L} = p_{pd}{}^2 \left[1 + p_{pR}{}^{(1)}\right] \qquad \textbf{(4.3)}$$

Note that $p_{pd} = 1 - \left(1 - p_{sd}\right)^B$ and similarly $p_{pR}{}^{(1)} = 1 - \left(1 - p_{sR}{}^{(1)}\right)^B$ and $p_{pR}{}^{(2)} = 1 - \left(1 - p_{sR}{}^{(2)}\right)^B$ which yields the relationship between packet error probability and symbol error probability. Since BPSK is used the symbol error rate is equal to the bit error rate. During 50% best case cooperation $\overline{L}$ is given by:

$$= 1 + \left(p_{pd}\right)^2 \begin{bmatrix} p_{pR}^{(1)} + \left(p_{pR}{}^{(1)}\right)^2 + \left(1 - p_{pR}{}^{(1)}\right) + p_{pR}{}^{(1)}\left(1 - p_{pR}{}^{(1)}\right) + p_{pR}^{(1)}.p_{pR}^{(2)} + \left(p_{pR}^{(1)}\right)^2.\left(p_{pR}^{(2)}\right)^2 \\ + \left(1 - p_{pR}^{(1)}\right).\left(1 - p_{pR}^{(2)}\right) + \left(1 - p_{pR}^{(1)}\right).p_{pR}^{(2)} + p_{pR}^{(1)}.\left(1 - p_{pR}^{(2)}\right) \\ + p_{pR}^{(1)}.p_{pR}^{(2)}.\left(1 - p_{pR}^{(1)}\right).\left(1 - p_{pR}^{(2)}\right) \end{bmatrix} \qquad \textbf{(4.4)}$$

Using the approach taken in [47] the calculation of $\overline{L}$ involves computing the union of all possible transmission scenarios during Cross Layer Hybrid ARQ 2 Cooperative Diversity. This involves no cooperation, partial cooperation and full cooperation. Note that during 50% best case cooperation the source node will always first transmit two packets of parity to the base station which will be inherited or listened to by the cooperating partner node, hence the $p_{pd}{}^{2}$ term.

During 25% cooperation best case cooperation $\overline{L}$ is calculated to be:

$$\overline{L} = 1 + \left(p_{pd}\right)^{3}\left[1 + p_{pR}^{(1)} \cdot p_{pR}^{(2)} + p_{pR}^{(2)}\left(1 - p_{pR}^{(1)}\right) + p_{pR}^{(1)}\left(1 - p_{pR}^{(2)}\right) + \left(1 - p_{pR}^{(1)}\right)\left(1 - p_{pR}^{(2)}\right)\right] \quad \textbf{(4.5)}$$

During 25% best case cooperation the source node will always first transmit three packets of parity to the base station which will be inherited or listened to by the cooperating partner node, hence the $p_{pd}{}^{3}$ term.

Note that the packet error probability for retransmission by user 1 and 2 is given by $p_{pR}^{(1)} = 1 - \left(1 - p_{sR}^{(1)}\right)^{B}$ and $p_{pR}^{(2)} = 1 - \left(1 - p_{sR}^{(2)}\right)^{B}$ respectively.

The packet successful probability for 50% best case cooperation is calculated using a similar approach to [47] but now extending this to the Cross Layer HARQ 2 cooperation protocol i.e.

$$p_{s} = \left[1 - \left(p_{pd}\right)^{2}\left(p_{pR}^{(1)}\right)^{2}\right].p(\text{no cooperation}) + \left[1 - \left(p_{pd}\right)^{2}\left(p_{pR}^{(1)}\right)^{2}\left(p_{pR}^{(2)}\right)^{2}\right].p(\text{cooperation}) \quad \textbf{(4.6)}$$

The same approach can be applied for 25% best case cooperation but now taking into account that three initial parity packets are transmitted by each node to the base station before cooperation begins i.e.

$$p_{s} = \left[1 - \left(p_{pd}\right)^{3}\left(p_{pR}^{(1)}\right)\right].p(\text{no cooperation}) + \left[1 - \left(p_{pd}\right)^{3}\left(p_{pR}^{(1)}\right)\left(p_{pR}^{(2)}\right)\right].p(\text{cooperation}) \quad \textbf{(4.7)}$$

Refer to Appendix B for further insight into the derivation of p_s and $\overline{L}$. The calculation of $p(\text{cooperation})$ involves correct interuser CRC syndrome calculations. This involves evaluation of the block error probability in the interuser channel.

Assuming reciprocal interuser channels $p(\text{cooperation})$, during 50% best case cooperation, is calculated to be:

$$p(\text{cooperation}) = \left[\prod_{i=1}^{2}\left(1 - p_{block,1}\left(\gamma_{1,2,i}\right)\right)\right] \quad \textbf{(4.8)}$$

During 25% best case cooperation $p(\text{cooperation})$ is given by:

$$p(\text{cooperation}) = \left[\prod_{i=1}^{3}\left(1 - p_{block,1}\left(\gamma_{1,2,i}\right)\right)\right] \quad \textbf{(4.9)}$$

Using the block error probability bound for a convolutional code i.e. $\mathrm{p_{block}}(\gamma)$, $\mathrm{p_{block}}(\gamma) \leq 1 - \left(1 - P_E(\gamma)\right)^B$, where $P_E(\gamma)$ is the first event error probability and is bounded by:

$$P_E(\gamma) \leq \sum_{d=d_f}^{\infty} a(d) P(d \mid \gamma) \quad \textbf{(4.10)}$$

$p(\text{cooperation})$ can be calculated to be:

$$p(\text{cooperation}) = \left[\prod_{i=1}^{2}\left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d) p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^B\right] \quad \textbf{(4.11)}$$

during 50% best case cooperation using the first event error probability. Similarly $p(\text{cooperation})$ during 25% best case cooperation is given by:

$$p(\text{cooperation}) = \left[\prod_{i=1}^{3}\left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d) p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^B\right] \quad \textbf{(4.12)}$$

The inverse calculation is performed for $p(\text{no cooperation})$. Thus incorrect interuser CRC syndrome calculations are obtained by the users. This involves evaluation of the block error probability in the interuser channel. Assuming reciprocal interuser channels $p(\text{no cooperation})$, during 50% best case cooperation, is calculated to be:

$$p(\text{no cooperation}) = \left[\prod_{i=1}^{2} p_{block,1}(\gamma_{1,2,i})\right] \tag{4.13}$$

During 25% best case cooperation $p(\text{no cooperation})$ is given by:

$$p(\text{no cooperation}) = \left[\prod_{i=1}^{3} p_{block,1}(\gamma_{1,2,i})\right] \tag{4.14}$$

Following the same approach as that of the calculation of $p(\text{cooperation})$ but this time for incorrect interuser CRC results $p(\text{no cooperation})$ is calculated during 50% best case cooperation as:

$$p(no\ cooperation) = \left[\prod_{i}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)p(d \mid \gamma_{1,2,i})\right)\right)^{B}\right)\right] \tag{4.15}$$

Similarly during 25% best case cooperation $p(\text{no cooperation})$ is derived to be:

$$p(\text{no cooperation}) = \left[\prod_{i=1}^{3}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)p(d \mid \gamma_{1,2,i})\right)\right)^{B}\right)\right] \tag{4.16}$$

Using the end to end BER results derived in Chapter 3 i.e. (3.24) and (3.28), $p_{sR}^{(1)}$ and $p_{sR}^{(2)}$ - during 50% best case cooperation is given by:

$$p_{BER} = \sum_{l=1}^{2}\sum_{r=1}^{4}\sum_{j=2}^{4} p_b(\theta = l, r)_j \, p(\theta = l, r)_j \tag{4.17}$$

Since BPSK modulation is used the symbol error rate is equal to the bit error rate. Using $p_{sR}^{(1)}$ and $p_{sR}^{(2)}$ we can compute $p_{pR}^{(1)}$ and $p_{pR}^{(2)}$ as shown in (4.18) and (4.19) below:

$$p_{pR}^{(1)} = 1-\left(1-p_{sR}^{(1)}\right)^{B} \tag{4.18}$$

$$p_{pR}{}^{(2)} = 1-\left(1-p_{sR}{}^{(2)}\right)^{B} \tag{4.19}$$

The bit error rate for 25% best case cooperation is given by:

$$p_{BER} = \sum_{l=1}^{2}\sum_{r=1}^{4}\sum_{j=3}^{4} p_b\left(\theta = l,r\right)_j \, p\left(\theta = l,r\right)_j \qquad \textbf{(4.20)}$$

Using (4.20) $p_{sR}^{(1)}$ and $p_{sR}^{(2)}$ are calculated for 25% best case cooperation, since they are equal to (4.20) and thus $p_{pR}^{(1)}$ and $p_{pR}^{(2)}$ are calculated using (4.18) and (4.19) respectively.

4.3 Simulation Parameters and Discussion

Figure 4.1 shows a throughput comparison between Coded Cooperation and Cross Layer HARQ 2 Cooperation using only analytical bounds for 50% best case cooperation. The dotted lines represent Coded Cooperation and the solid lines represent Cross Layer Cooperation as shown in the legend.

At all interuser channel conditions Cross Layer HARQ 2 Cooperation always maintains a throughput improvement over Coded Cooperation. Note that at high SNR the throughput performance of Coded Cooperation degrades. As an example in Figure 4.1 the throughput of Coded Cooperation at 5dB drops below the throughput of Cross Layer Cooperation at 0db at high uplink SNRs. This is due to the fact that at high uplink SNRs the amount of parity packets transmitted during Cross Layer Cooperation reduces due to good uplink conditions whilst Coded Cooperation continues transmitting all the parity available and thus experiences a major reduction in throughput. The same throughput degradation is seen for Coded Cooperation with a perfect interuser channel i.e. the throughput of Coded Cooperation under perfect interuser channel conditions drops below the throughput of Cross Layer Cooperation at 5dB interuser channel conditions.

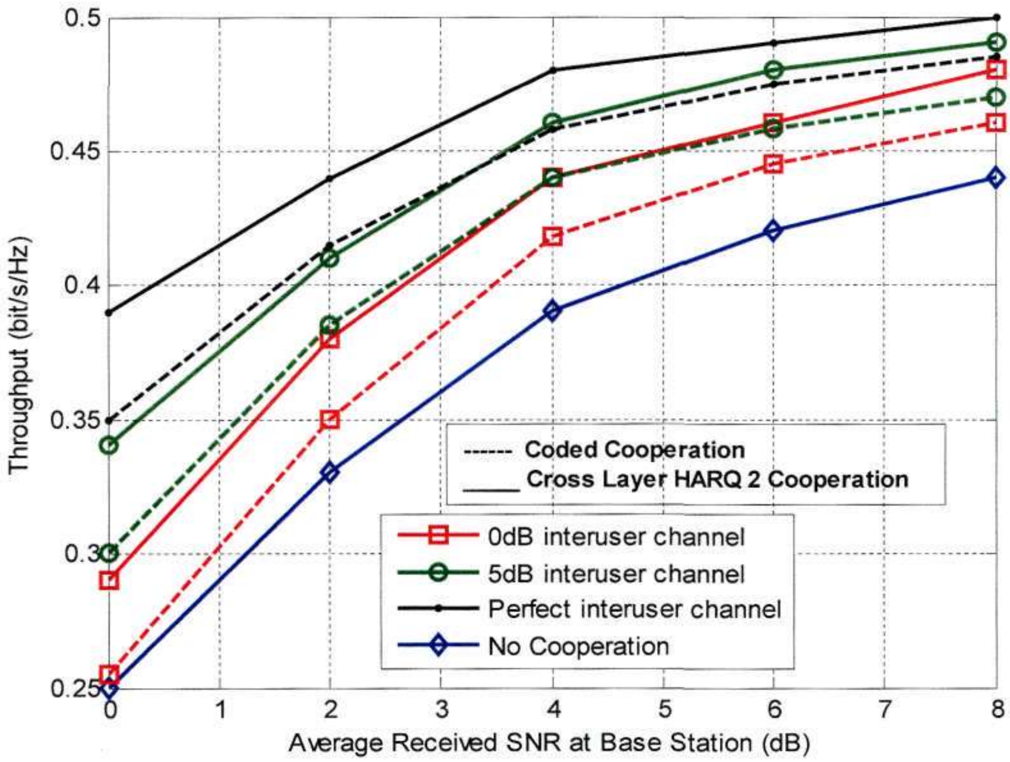

Figure 4.1: Throughput comparison between Cross Layer HARQ 2 Cooperation and Coded Cooperation for 50% best case cooperation

Similar comments can be made for Figure 4.2 from that of Figure 4.1, the only difference being that here 25% best case cooperation is being evaluated. The throughput range however reduces intuitively due to the fact that three incremental parity packets are transmitted upfront by each user as opposed to two due to the variation in cooperation level from 50% best case to 25% best case. Cross Layer HARQ 2 Cooperation again maintains a throughput performance gain over Coded Cooperation at all interuser channel conditions. Coded Cooperation again shows throughput degradation at higher interuser channel conditions due to the transmission of all available parity during good uplink conditions (this is bandwidth inefficient).

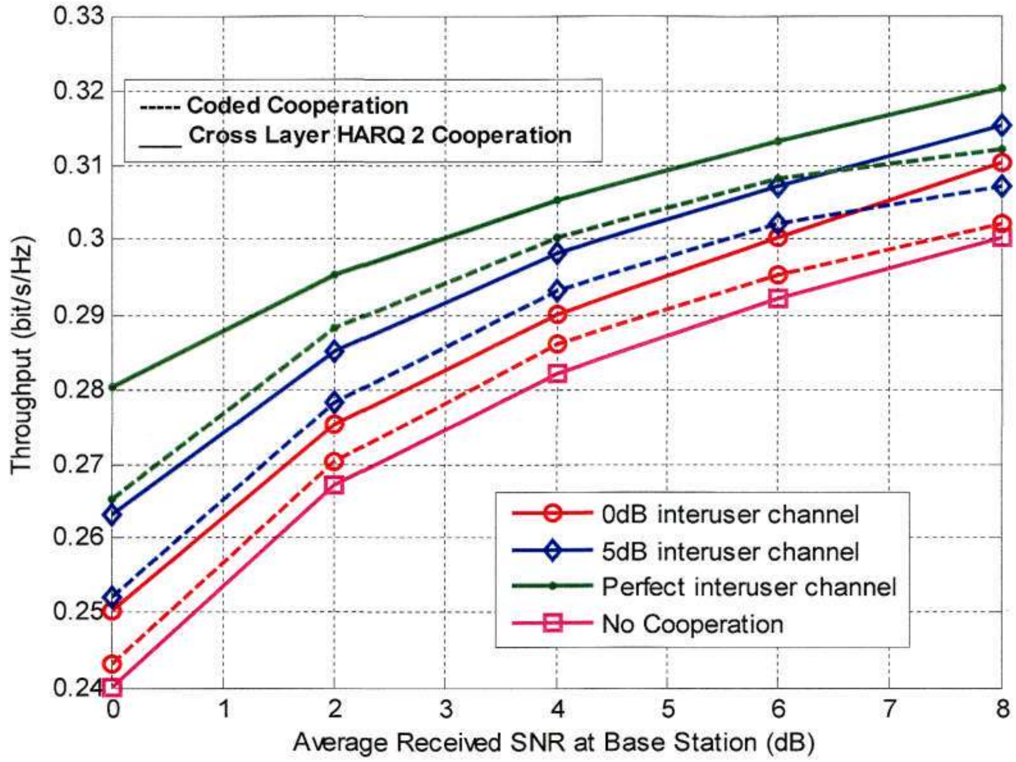

Figure 4.2: Throughput comparison between Cross Layer HARQ 2 Cooperation and Coded Cooperation for 25% best case cooperation

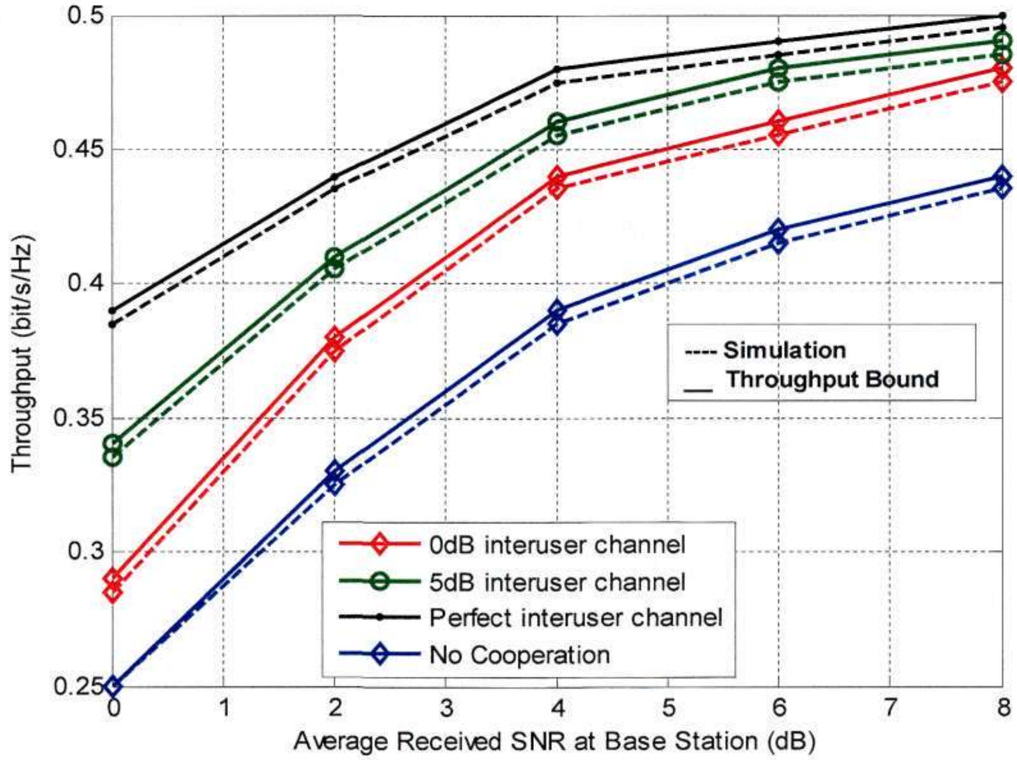

Figure 4.3: Theoretical throughput bound vs. throughput simulation for 50% best case cooperation

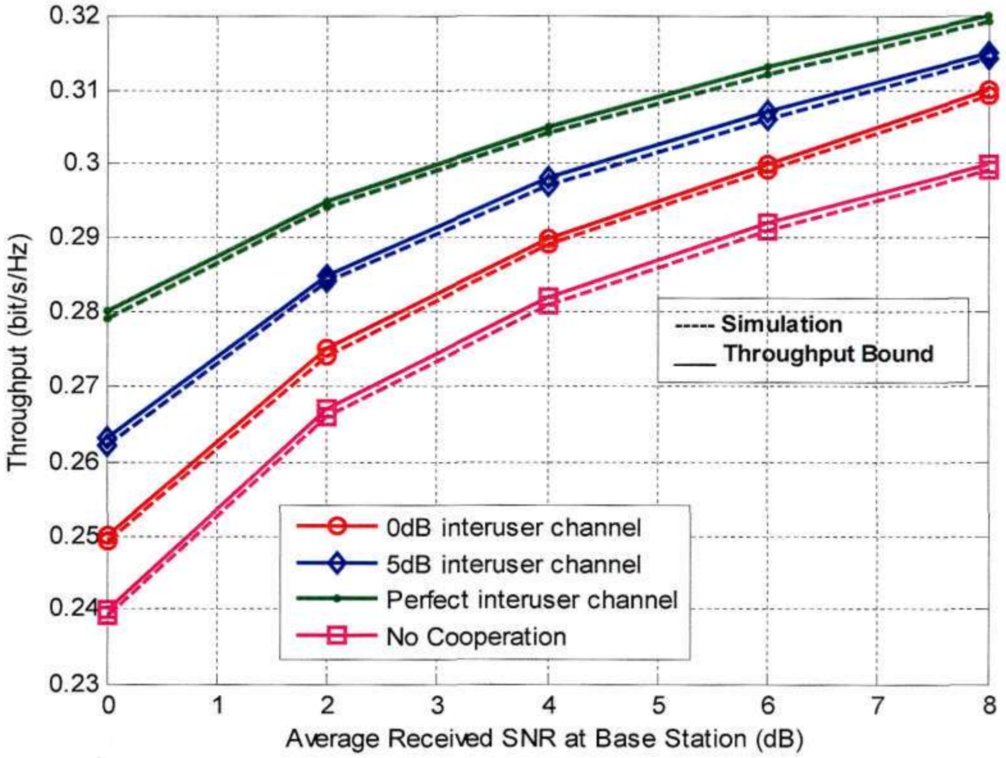

Figure 4.4: Theoretical throughput bound vs. throughput simulation for 25% best case cooperation

Figures 4.3 and 4.4 show the performance comparison between the simulated and analytical bounds Cross Layer HARQ 2 Cooperation for both 50% and 25% best case cooperation. The analytical results confirm the simulations at all interuser channel conditions to within 0.5dB's of performance variation. The dotted lines show the simulations of the throughput and the solid lines represent the analytical bounds - as shown in the legends.

4.4 Chapter Summary

Due to the bandwidth saving nature of Cross Layer HARQ 2 Cooperation, throughput bounds are developed in order to analyse the performance of the protocol and to compare it to Coded Cooperation. Higher throughput performance is observed for Cross Layer HARQ 2 Cooperation over Coded Cooperation under both 50% and 25% best case cooperation levels.

The packet successful probability and the average amount of retransmitted sub code words are used to compute the throughput bound. Cross Layer HARQ 2 Cooperation always maintains a competitive throughput performance gain over Coded Cooperation due to the incremental nature of the protocol over Coded Cooperation, which in some cases transmits parity when it is not needed.

Cross Layer HARQ 2 Cooperation is thus bandwidth efficient whilst also using CRC to adapt to channel conditions. In this way BER and throughput are improved during Cross Layer HARQ 2 Cooperation over Coded Cooperation since Coded Cooperation will always result in a user transmitting all its available parity without checking or realizing that sufficient parity has been transmitted to the base station - for the decoding of a particular information bit block. This results in the throughput of Coded Cooperation at higher interuser SNR channel conditions to drop below the throughput of Cross Layer Hybrid ARQ 2 Cooperation with lower interuser SNR channel conditions during higher uplink SNR conditions.

The simulations of Cross Layer Hybrid ARQ 2 cooperation match the analytical bounds closely for both 25% and 50% best case cooperation.

Chapter 5

5 Conclusions and Future Work

5.1 Dissertation Achievements

Pioneer work in the area of cooperative diversity as explained in Chapter 1 involved users repeating the transmitted parity of their partners to the base station in an effort to create spatial diversity via repetition. There were two signalling methods of repetition, which were Detect and forward and Amplify and forward. Amplify and forward allowed for a user to first amplify the partner's transmitted parity and then forward it to the base station. Detect and forward on the other hand involved a user first estimating every transmitted parity symbol of its partner and then transmitting the estimated signals to the base station.

Amplify and forward however assumed that interuser fading coefficient knowledge is available at the base station. Obtaining a method of passing or estimating this information to the base station is not a trivial exercise. Note also that at poor interuser channel conditions the user amplifies the partner's noise as well which will degrade the BER performance at the receiver.

The introduction of error into the system is also provided for by the detect and forward signalling method. Again at low interuser channel conditions the probability of the user detecting the partner's transmitted parity (via threshold estimation) correctly is low and thus the user transmits an incorrect estimate of the partner's signal to the base station.

Coded Cooperation took care of the error introduction into the system, by the cooperating users, even before decoding is performed at the base station. This was catered for by using channel coding and error detection codes such as CRC. The user only cooperated with a partner if the CRC syndrome calculation of the partner's initial parity transmission (to the base station which is inherited by the user) was successful. This allowed for Coded Cooperation to have a competitive edge over Detect and forward, Amplify and forward and also non cooperation transmission. Even under poor interuser channel conditions Coded Cooperation performed at least as well as non cooperation. Coded Cooperation showed massive improvements in BER, as the interuser channel quality improved, over non cooperation.

In Chapter 2 the Coded Cooperation scheme was examined in a slow fading channel. The protocol was simulated based on the work done in [24], which involved an RCPC implementation of Coded Cooperation. The Coded Cooperation system was then used as a baseline system with the introduction of a new protocol to follow. The theoretical analysis done on Coded Cooperation was split into various parts in order to obtain an end to end BER bound. The bit and block error rates had to be determined first, which was used to derive the interuser case transmission probability. The conditional PEP had to be determined for the uplink channels based on the cooperative case transmission probability. The end to end BER was then calculated using a weighting between the case probabilities and the BER for a specific case probability. The simulations and BER bounds confirmed each other. The theoretical analysis done for the BER in Coded Cooperation helped outline and lay the foundation for much more complex Cross Layer BER theoretical analysis proposed in Chapter 3.

The BER performance curves under asymmetric uplink conditions show that even the user with the good uplink condition (i.e. user 1 uplink SNR=20dB) exhibits BER performance gains by cooperating with a user with a poor uplink SNR for both 50% and 25% cooperation. This is not a very intuitive result.

The interuser assumption of reciprocal channels, in Coded Cooperation, was qualified by comparisons between reciprocal and independent interuser channel conditions. At worst case a 2.5 dB loss in performance is experienced at 50% cooperation by the use of an independent as opposed to a reciprocal interuser channel.

In Chapter 3 a new protocol is developed, called Cross Layer Hybrid ARQ 2 Cooperative Diversity, in an effort to improve the BER as well as the throughput of the system jointly. Coded Cooperation, although it exhibits impressive BER performance over non cooperation and pioneer cooperative diversity methods, does not posses the most efficient use of bandwidth during cooperation. During Coded Cooperation users continue cooperating and transmitting parity for their partners when it might not be required, because the base station could have already decoded the user's parity correctly.

By introducing HARQ 2 into the system and combining this with Cooperative diversity the users only have to transmit parity for their partners when it is required. The base station provides feedback on whether a specific user's information has been decoded correctly or not through the use of ACK or NACK signals. In this way a user does not have to transmit a full parity frame of N bits if not required for a set of K information bits. This results in improvements in throughput as well as BER over Coded Cooperation.

Simulations are performed for the BER of Cross Layer Hybrid ARQ 2 Cooperation and these are confirmed via computation of analytical bounds. These analytical bounds are a bit more complicated since the cooperative case probabilities are dependant not only on the interuser CRC states but also the base station CRC states. Hence the BLER has to be computed for the interuser channel as well as the uplink channels. The BER has to then be determined using the PEP for each transmission case for the uplink channel and weighted based on each transmission case probability and the amount of parity packets transmitted.

Both analytical and simulated results show that Cross Layer Cooperation always maintains a BER performance advantage over Coded Cooperation even under the worst interuser channel conditions. Cross Layer Cooperation also shows impressive gains over non cooperation even under poor interuser channel conditions. During asymmetric uplink conditions the BER performance of user 1 reduces slightly by cooperating with user 2 (user 1 uplink SNR = 8dB) however this is negligible and the performance increases again as the uplink channel quality improves. Note that the interuser channel quality of 5dB also contributes to the BER reduction of user 1 at low uplink SNRs.

The assumption of using reciprocal instead of independent interuser channels is qualified by comparing the BER performance loss between reciprocal and independent interuser channel conditions, for the end to end BER during Cross Layer Cooperation. The BER performance loss is less than 1 dB for both 50% and 25% best case cooperation.

Chapter 4 looks at throughput analysis of Cross Layer Hybrid ARQ 2 Cooperation under various interuser channel conditions. Analytical throughput bounds are derived based on the packet successful probability and the average amount of transmitted sub code words per source information packet. Since Cross Layer Cooperation is incremental in nature the throughput is improved dramatically over Coded Cooperation since efficient use of bandwidth is performed. This is done by users selectively only transmitting incremental parity for their partners if it is required. This is not the case with Coded Cooperation and results in the throughput of Coded Cooperation, at higher interuser SNR channel conditions, dropping below the throughput of Cross Layer Hybrid ARQ 2 Cooperation, with lower interuser SNR channel conditions, during higher uplink SNR conditions. During the throughput analysis presented in Chapter 4 the throughput of Coded Cooperation, at 5 dB and perfect interuser channels, drops below the throughput of Cross Layer Hybrid ARQ 2 Cooperation, at 0 dB and 5 dB interuser channel conditions, for uplink SNRs starting from as low as 4 dB's.

The analytical throughput bounds are confirmed via simulations for Cross Layer HARQ 2 Cooperation.

Chapters 3 and 4 demonstrate the performance of a new Cross Layer Cooperation protocol that improves BER and throughput, two important QoS metrics that will be extensively demanded with the ever so dynamic Next Generation Broadband Wireless Network requirements. Cross Layer Cooperation will allow for the support of real-time applications that have very stringent QoS demands with regard to throughput and BER. Cross Layer Cooperation allows for improved signal quality of a user at the base station with a throughput improvement and a BER improvement all at the same transmit power, rate and bandwidth as that of non cooperation but at the same time also maintaining a performance (BER and throughput) edge over Coded Cooperation.

5.2 Future Work

In [39] the concept of multiple source cooperation is born. Extending the amount of users, using Cross Layer HARQ 2 Cooperation, would be an interesting future study. This can be extended in a multiple source context with the use of Low Density Parity Check (LDPC) codes. In the multi user regime cluster size, inter and intra cluster routing become very important and also very complicated. To manage the amount of users in a cluster and to create a cluster boundary is difficult because when the amount of users increase, the throughput reduces. Also inter cluster routing introduces delays in transmission. These issues define research areas individually in their own right.

Another rich area currently in the cooperative diversity area is partner choice during multiple source cooperation. Choosing the correct partner can be done using channel estimation techniques as well as using other metrics. Extending Cross Layer Hybrid ARQ 2 Cooperation to a multiple source cooperation setup, with well defined node clusters and proper intra and inter cluster routing - catering for high code rates and high diversity orders will result in outstanding performance results.

5.3 Current Literature Contributions to the Research Area

Conference Papers:

1. S.R. Beharie, H. Xu and F. Takawira, "Cross Layer Hybrid ARQ 2 Cooperative Diversity in Next Generation Wireless Networks," SATNAC 2008, South Africa.
2. S.R. Beharie, H. Xu and F. Takawira, "Cross Layer HARQ 2 Cooperation with Throughput Improvement" Submitted to *IEEE Wireless Communications and Networking Conference (WCNC)*, Hungry, April 2009.

References

1. Sendonaris, E. Erkip, and B. Aazhang, "Increasing uplink capacity via user cooperation diversity," in *Proc. of IEEE Int. Symp. on Info. Theory*, Cambirdge, MA, p.156, Aug. 1998

2. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity–Part I: System description," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.

3. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity–Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, pp.1939–1948,Nov.2003.

4. N. Ahmed and B. Aazhang, "Cooperative Communications in the Fading Channel," *SPIE Conference on Noise in Communication Systems*, USA, May 2005.

5. Sendonaris, Advanced Techniques for Next-Generation Wireless Systems, Ph.D. thesis, Rice University, May 1999.

6. J. N. Laneman and G. W. Wornell, "Energy-Efficient Antenna Sharing and Relaying for Wireless Networks," in *Proc. IEEE Wireless Comm. and Networking Conf.* (WCNC),Chicago,IL,Sept.2000.

7. J. N. Laneman, G. W. Wornell, and D. N. C. Tse, "An Efficient Protocol for Realizing Cooperative Diversity in Wireless Networks," in *Proc. IEEE Int. Symp. Information Theory (ISIT)*, Washington, DC, June 2001.

8. J. N. Laneman and G. W. Wornell, "Distributed Space-Time Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks," *IEEE Trans. Inform. Theory*,vol.49,no.10,pp.2415-2525,Oct.2003.

9. J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behaviour," *IEEE Trans. Inform. Theory*, vol.50,no.12,pp.3062-3080,Dec.2004.

10. J. N. Laneman, "Network Coding Gain of Cooperative Diversity," in Proc. *IEEE Military Comm. Conf. (MILCOM)*, Monterey, CA, Nov. 2004, invited paper.

11. D. Chen and J. N. Laneman, "Noncoherent Demodulation for Cooperative Wireless Systems," in *Proc. IEEE Global Comm. Conf. (GLOBECOM)*, Dallas, TX, Nov. 2004.

12. D. Chen and J. N. Laneman, "Modulation and Demodulation for Cooperative Diversity in Wireless Systems,"*IEEE Trans. Wireless Commun.*, vol. 5, no. 7, pp. 1785 – 1794, July 2006.

13. J. N. Laneman and G. W. Wornell, "Distributed Spatial Diversity Techniques for Improving Mobile Ad-Hoc Network Performance," in *Proc. ARL FedLab Symp. on Adv. Telecomm. & Inform. Distrib. Prog. (ATIRP)*, College Park, MD, Mar. 2000.

14. J. N. Laneman and G. W. Wornell, "An Efficient Protocol for Realizing Distributed Spatial Diversity in Wireless Ad-Hoc Networks," in *Proc. ARL FedLab Symp. on Adv. Telecomm. & Inform. Distrib. Prog. (ATIRP)*, College Park, MD, Mar. 2001.

15. J. N. Laneman, "Cooperative Diversity in Wireless Networks: Algorithms and Architectures," Ph.D. dissertation, Massachusetts Institute of Technology, Cambridge, MA, Aug. 2002.

16. Nosratinia, T. E. Hunter, A. Hedayat, "Cooperative communication in wireless networks," *IEEE Communications Magazine*, Volume 42, Issue 10, pp. 74 - 80, Oct. 2004.

17. T. E. Hunter and A. Nosratinia, "Cooperative Diversity through Coding," *Proc. IEEE ISIT*,Laussane,Switzerland,Jul. 2002.

18. T. E. Hunter and A. Nosratinia, "Coded Cooperation under slow fading, fast fading, and power control," in *Proc. Asilomar Conference on Signals, Systems and Computers*,Nov.2002.

19. T. E. Hunter and A. Nosratinia, "Performance analysis of Coded Cooperation diversity," in *Proc. IEEE ICC*, vol.4, pp. 2688 – 2692, May 2003.

20. M. Janani, A. Hedayat, T. E. Hunter, and A. Nosratinia, "Coded Cooperation in wireless communications: Space-time transmission and iterative decoding," *IEEE Trans. Signal Processing*, vol. 52, pp. 362-371, Feb. 2006.

21. T. Hunter and A. Nosratinia, "Diversity through Coded Cooperation," *IEEE Transactions on Wireless Communications*, vol. 5, pp. 283-289, Feb 2006.

22. T. Hunter, S. Sanayei, and A. Nosratinia, "Outage analysis of Coded Cooperation," *IEEE Transactions on Information Theory*, vol. 52, pp. 375-391, 2006.

23. T. Hunter and A. Nosratinia, "Distributed Protocols for User Cooperation in multi-user wireless networks,"*In Proceedings of 2004 GLOBECOM Communication Theory Symposium*, vol. 6, pp. 3788-3792, December 2004.

24. T. Hunter, " Coded Cooperation: A new framework for user cooperation in wireless networks," Ph.D. dissertation, University of Texas, Dallas, May 2004.

25. Stefanov and E. Erkip, "Cooperative information transmission in wireless networks," In *Proceedings of 2002 Asian-European Information Theory Workshop*, Breisach, Germany, Jun. 2002.

26. A.Stefanov and E. Erkip, "On the performance analysis of cooperative space-time coded systems," In *Proceedings of IEEE Wireless Communications and Networking Conference (WCNC)*, New Orleans, Lousiana, March 2003.

27. Stefanov and E. Erkip, "Cooperative Space-Time Coding for Wireless Networks," In *Proceedings of IEEE Information Theory Workshop*, La Sorbonne, Paris, France, April 2003.

28. E. Erkip, "Cooperative Communications in Wireless Systems," Presented at *DIMACS Workshop on Network Information Theory* (invited talk), Rutgers, NJ, March 2003.

29. M. Yuksel and E. Erkip, "Diversity in relaying protocols with amplify and forward," In *Proceedings of GLOBECOM Communication Theory Symposium*, San Francisco, Dec.2003.

30. M. Yuksel and E. Erkip, "Diversity gains and clustering in wireless relaying," In *Proceedings of 2004 International Symposium on Information Theory*, Chicago, Jun. 2004.

31. D. Gunduz and E. Erkip, "Joint source-channel cooperation: Diversity versus spectral efficiency," In *Proceedings of 2004 International Symposium on Information Theory*, Chicago,Jun.2004.

32. A. Stefanov, E. Erkip, "Cooperative coding for wireless networks," *IEEE Trans. on Communications*, vol. 52, pp. 1470 - 1476, Sept. 2004.

33. Z. Lin, E. Erkip and A. Stefanov, "An asymptotic analysis on the performance of Coded Cooperation systems," In *Proceedings of 2004 Fall Vehicular Technology Conference*,LosAngeles,Sept.2004.

34. M. Yuksel and E. Erkip, "Broadcast strategies for the fading relay channel," In *Proceedings of 2004 Military Communications Conference*, Monterey, November 2004.

35. Z. Lin, E. Erkip and A. Stefanov, "Cooperative regions for coded cooperative systems," In *Proceedings of 2004 GLOBECOM Communication Theory Symposium*, Dallas,Dec.2004.

36. X. Xu, Y. Wang and E. Erkip, "Layered cooperation for wireless multimedia communications," In *Proceedings of Picture Coding Symposium*, San Francisco, Dec. 2004.

37. D. Gunduz and E. Erkip, "Opportunistic cooperation and power control strategies for delay-limited capacity," To appear, *Proceedings of the 2005 Conference on Information Sciences and Systems*, Baltimore, Mar. 2005.

38. X. Xu, D. Gunduz, E. Erkip and Y. Wang, "Layered cooperative source and channel coding," To appear, *Proceedings of 2005 ICC Multimedia Communication and Home Networking Symposium*, Seoul, Korea, May 2005 (invited paper).

39. O. Shalvi, "Multiple source Cooperation diversity," *IEEE Communications Letters*, vol. 8, Issue 12, pp. 712 - 714, Dec. 2004.

40. T. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. IT-25, pp. 572–584, Sept. 1979.

41. P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inf. Theory*, vol. 46, pp. 388–404, Mar. 2000.

42. M. Grossglauser and D. N. C. Tse, "Mobility increases the capacity of ad-hoc wireless networks," *IEEE/ACM Trans. Netw.*, vol. 10, pp.477–486, Aug. 2002.

43. P. Gupta and P. R. Kumar, "Toward an information theory of large networks: An achievable rate region," *IEEE Trans. Inf. Theory*, vol. 49, pp. 1877–1894, Aug. 2003.

44. T. Hunter and A. Nosratinia, "Diversity through Coded Cooperation," *IEEE trans. On Wireless Commun.*, vol. 5, pp. 283-289, Feb. 2006.

45. N. Agarwal and Dr. J.A. Cobb, "Coded Cooperative ARQ Protocol in Wireless Networks," *Term Paper*, www.utdallas.edu/~niraj/Courses/ACN/Term_Paper.doc.

46. E. Malkamaki and H. Leib, "Evaluating the Peformance of Convolutional Codes over Block Fading Channels," IEEE Trans. On Information Theory, vol. 45, no. 5,pp. 1643-1646, July 1995

47. L. Dai and K.B. Letaief, "Throughput Maximization of Ad-hoc Wireless Networks Using Adaptive Cooperative Diversity and Truncated ARQ," *Manuscript submitted to IEEE Trans on Commun.*, 2005, last accessed 29/05/2008, http://repository.ust.hk/dspace/bitstream/1783.1/2252/1/0_throughputmaximization_cooperativediversity_IEEE_Trans_Commun.pdf.

48. S. A Zummo, "Peformance Analysis of Coded Cooperation Diversity in Wireless Networks," *Jounal of Wireless Communications and Mobile Computing*, 2005.

49. Z. Wang and G. Giannakis, "A Simple and General Parameterization Quantifying Peformance in Fading Channels," IEEE Trans on Commun., vol. 51, Issue 8, pp. 1389-1398, Aug. 2003.

50. V. Tarokh, H. Jafarkhani and A.R. Calderbank, "Space-Time Block Coding for Wireless Communiactions: Peformance Results," IEEE Journal on Selected Areas in Commun.,vol 17,no.3, pp. 451-460, Mar. 1999.

51. G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Technical Journal*, 1(2): pp. 41-59, 1996.

52. G.J. Foschini and M.J. Gans, "On limits of wireless communication in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 311-335, 1998.

53. S. M. Alamouti, "A simple transmit diversity technique for wireless communications," IEEE *Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

54. C. E. Shannon, "A mathematical theory of communications," *Bell System Technical Journal*, vol. 27, pp. 379-423 and 623-656, 1948.

55. Wittneben, "Base station modulation diversity for digital SIMULCAST," *in Proc. IEEE VTC*, pp. 505-511 May 1993.

56. V. Tarokh, N. Seshadri and A. R. Calderbank, "Space-time block codes for high data rate wireless communication performance analysis and code construction," *IEEE Trans. on Inform. Theory*, vol. 44, pp. 744-765, Mar. 1998.

57. J. Hagenauer, "Rate-compatible punctured convolutional codes (RCPC codes) and their applications," *IEEE Transactions on Communications*, vol. 36, no. 4, pp. 389-400, Apr. 1988.

58. Lifang Li and Andrea J. Goldsmith, "Capacity and optimal resource allocation for fading broadcast channels - Part II: Outage Capacity," *IEEE Trans. Inform. Theory*, 47(3):1103-1127, Mar. 2001.

59. J. W. Craig, A new, simple, and exact result for calculating the probability of error for two-dimensional signal constellations," in Proc. IEEE MILCOM, McLean, VA, pp. 571-575, Oct. 1991.

60. M. K. Simon and M.-S. Alouini, Digital Communication over Fading Channels:A Uni_ed Approach to Performance Analysis. New York: John Wiley and Sons, 2000.

61. Viterbi and J. K. Omura, Principles of Digital Communication and Coding. New York: McGraw-Hill, 1979.

Appendix A

Derivation of the End to End BER Bounds for Cross Layer Hybrid ARQ 2 Cooperative Diversity

Derivation of the 50% Best Case Cooperation End to End BER Bound

During 50% best case cross layer cooperation the end to end BER is given by (3.24):

$$p_{BER} = \sum_{l=1}^{2}\sum_{r=1}^{4}\sum_{j=2}^{4} p_b\left(\theta = l, r\right)_j \, p\left(\theta = l, r\right)_j .$$

Note that to derive the end to end BER bound for user 1 one needs to calculate the case probability ($p\left(\theta = l, r\right)_j$, $l \in \{1,2\}, r \in \{1,4\}, j \in \{2,4\}$) for each case where $dcrc_1$ is equal to 0 i.e. a NACK is received for parity transmitted for user 1 (This would be the same as calculating the BER of user 2). This includes cases 1.1 to 2.4, which is highlighted in blue in Table 3.1, drawn below again for convenience.

Case	$dcrc_1$	$dcrc_2$	crc_1	crc_2
1.1	0	0	0	0
1.2	0	0	0	1
1.3	0	0	1	0
1.4	0	0	1	1
2.1	0	1	0	0
2.2	0	1	0	1
2.3	0	1	1	0
2.4	0	1	1	1
3.1	1	0	0	0
3.2	1	0	0	1
3.3	1	0	1	0
3.4	1	0	1	1
4.1	1	1	0	0
4.2	1	1	0	1
4.3	1	1	1	0
4.4	1	1	1	1

One also needs to calculate the BER during the individual case probability i.e. $p_b\left(\theta = l, r\right)_j$ - $l \in \{1,2\}, r \in \{1,4\}, j \in \{2,4\}$ and the PEP for each instance of $p\left(\theta = l, r\right)_j$. Based on (3.24), the individual case probabilities ($p_b\left(\theta = l, r\right)_j$) are computed and the individual BER i.e. $p_b\left(\theta = l, r\right)_j$ for each case, for a specific set of incremental packets is computed. The

PEP for the individual BER - based on j is also computed. These equations are listed below and are substituted into (3.24) to obtain the end to end BER for 50% best case cooperation.

Case 1.1

$$p\left(\theta=1.1 \mid \gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,2}\right)_{2Trans}=$$
$$\left[\prod_{i=1}^{2} p_{block,1}\left(\gamma_{1,2,i}\right)\right]^{2}\cdot\prod_{j=1}^{2} p_{block,1}\left(\gamma_{1,j}\right)\cdot\prod_{k=1}^{2} p_{block,2}\left(\gamma_{2,k}\right)$$

(A. 1)

using $\mathrm{p_{block}}(\gamma)\leq 1-\left(1-P_E(\gamma)\right)^B$

(A. 2)

$$p\left(\theta=1.1 \mid \gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,2}\right)_{2Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{1,2,i}\right)\right)\right)^{B}\right)\right]^{2}\cdot\prod_{j=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{1,j}\right)\right)\right)^{B}\right)$$
$$\prod_{k=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{2,k}\right)\right)\right)^{B}\right)$$

(A. 3)

$$P_b(\theta=1.1)_{2trans}\leq\int_0^{\infty}\int_0^{\infty}\min\left[\frac{1}{2},\frac{1}{k_c}\sum_{d=d_{free}}^{\infty}c(d)\,p\left(d\mid\gamma_{1,1},\gamma_{1,2},\theta\right)\right]\cdot\prod_{i=1}^{2}p\left(\gamma_{1,i}\right)d\gamma_{1,i}$$

(A. 4)

$$p\left(d\mid\gamma_{1,1},\gamma_{1,2}\right)=Q\left(\sqrt{2\sum_{i=1}^{2}d_{1,i}\gamma_{1,i}}\right)$$

(A. 5)

$$p\left(\theta=1.1 \mid \gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3}\right)_{3Trans}=$$
$$\left[\prod_{i=1}^{2} p_{block,1}\left(\gamma_{1,2,i}\right)\right]^{2}\cdot\prod_{j=1}^{3} p_{block,1}\left(\gamma_{1,j}\right)\cdot\prod_{k=1}^{3} p_{block,2}\left(\gamma_{2,k}\right)$$

(A. 6)

$$p\left(\theta = 1.1 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}\right)_{3Trans} =$$

$$\left[\prod_{i=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B}\right)\right]^{2} \cdot \prod_{j=1}^{3}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,j}\right)\right)\right)^{B}\right)$$

$$\prod_{k=1}^{3}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,k}\right)\right)\right)^{B}\right)$$

(A. 7)

$$P_b(\theta = 1.1)_{3trans} \le \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \min\left[\frac{1}{2}, \frac{1}{k_c} \sum_{d=d_{free}}^{\infty} c(d)\, p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \theta\right)\right] \cdot \prod_{i=1}^{3} p\left(\gamma_{1,i}\right) d\gamma_{1,i}$$

(A. 8)

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}\right) = Q\left(\sqrt{2\sum_{i=1}^{3} d_{1,i}\gamma_{1,i}}\right)$$

(A. 9)

$$p\left(\theta = 1.1 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}, \gamma_{2,4}\right)_{4Trans} =$$

$$\left[\prod_{i=1}^{2} p_{block,1}\left(\gamma_{1,2,i}\right)\right]^{2} \cdot \prod_{j=1}^{4} p_{block,1}\left(\gamma_{1,j}\right) \cdot \prod_{k=1}^{4} p_{block,2}\left(\gamma_{2,k}\right)$$

(A. 10)

$$p\left(\theta = 1.1 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}, \gamma_{2,4}\right)_{4Trans} =$$

$$\left[\prod_{i=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B}\right)\right]^{2} \cdot \prod_{j=1}^{4}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,j}\right)\right)\right)^{B}\right)$$

$$\prod_{k=1}^{4}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,k}\right)\right)\right)^{B}\right)$$

(A. 11)

$$P_b(\theta = 1.1)_{4trans} \le \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \min\left[\frac{1}{2}, \frac{1}{k_c} \sum_{d=d_{free}}^{\infty} c(d)\, p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}, \theta\right)\right] \cdot \prod_{i=1}^{4} p\left(\gamma_{1,i}\right) d\gamma_{1,i}$$

(A. 12)

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}\right) = Q\left(\sqrt{2\sum_{i=1}^{4} d_{1,i}\gamma_{1,i}}\right)$$

(A. 13)

Case 1.2

$$p\left(\theta=1.2\,|\,\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,1}\right)_{2Trans}=$$
$$\left[\prod_{i=1}^{2}p_{block,1}\left(\gamma_{1,2,i}\right)\cdot\prod_{j=1}^{2}\left(1-p_{block,1}\left(\gamma_{2,1,j}\right)\right)\right]\cdot\prod_{k=1}^{2}p_{block,1}\left(\gamma_{1,k}\right)\prod_{l=1}^{2}p_{block,2}\left(\gamma_{2,l}\right)$$

(A. 14)

$$p\left(\theta=1.2\,|\,\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,1}\right)_{2Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{1,2,i}\right)\right)\right)^{B}\right)\cdot\prod_{j=1}^{2}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{2,1,j}\right)\right)\right)^{B}\right]$$
$$\prod_{k=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{1,k}\right)\right)\right)^{B}\right)\cdot\prod_{l=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{2,l}\right)\right)\right)^{B}\right)$$

(A. 15)

$$P_{b}\left(\theta=1.2\right)_{2trans}\leq\int_{0}^{\infty}\!\!\int_{0}^{\infty}\min\left[\frac{1}{2},\frac{1}{k_{c}}\sum_{d=d_{free}}^{\infty}c(d)\,p\left(d\,|\,\gamma_{1,1},\gamma_{1,2},\theta\right)\right]\cdot\prod_{i=1}^{2}p\left(\gamma_{1,i}\right)d\gamma_{1,i}$$

(A. 16)

$$p\left(d\,|\,\gamma_{1,1},\gamma_{1,2}\right)=Q\left(\sqrt{2\sum_{i=1}^{2}d_{1,i}\gamma_{1,i}}\right)$$

(A. 17)

$$p\left(\theta=1.2\,|\,\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3}\right)_{3Trans}=$$
$$\left[\prod_{i=1}^{2}p_{block,1}\left(\gamma_{1,2,i}\right)\cdot\prod_{j=1}^{2}\left(1-p_{block,1}\left(\gamma_{2,1,j}\right)\right)\right]\cdot\prod_{k=1}^{3}p_{block,1}\left(\gamma_{1,k}\right)\prod_{j=1}^{3}p_{block,2}\left(\gamma_{2,j}\right)$$

(A. 18)

$$p\left(\theta=1.2\,|\,\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3}\right)_{3Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{1,2,i}\right)\right)\right)^{B}\right)\cdot\prod_{j=1}^{2}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{2,1,j}\right)\right)\right)^{B}\right]$$
$$\prod_{k=1}^{3}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{1,k}\right)\right)\right)^{B}\right)\cdot\prod_{l=1}^{3}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{2,l}\right)\right)\right)^{B}\right)$$

(A. 19)

$$P_b(\theta=1.2)_{3trans} \le \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty \min\left[\frac{1}{2},\frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)\,p\left(d\mid\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{2,3},\theta\right)\right]\cdot\prod_{i=1}^{3} p(\gamma_{1,i})p(\gamma_{2,3})\,d\gamma_{1,i}d\gamma_{2,3}$$

(A. 20)

$$p\left(d\mid\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{2,3}\right)=Q\left(\sqrt{\left(2\sum_{i=1}^{3} d_{1,i}\gamma_{1,i}+2d_{2,3}\gamma_{2,3}\right)}\right)$$

(A. 21)

$$p\left(\theta=1.2\mid\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3},\gamma_{2,4}\right)_{4Trans}=$$
$$\left[\prod_{i=1}^{2} p_{block,1}\left(\gamma_{1,2,i}\right)\cdot\prod_{j=1}^{2}\left(1-p_{block,1}\left(\gamma_{2,1,j}\right)\right)\right]\cdot\prod_{k=1}^{4} p_{block,1}\left(\gamma_{1,k}\right)\prod_{l=1}^{4} p_{block,2}\left(\gamma_{2,l}\right)$$

(A. 22)

$$p\left(\theta=1.2\mid\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3},\gamma_{2,4}\right)_{4Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty} a(d)\,p\left(d\mid\gamma_{1,2,i}\right)\right)\right)^{B}\right)\cdot\prod_{j=1}^{2}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty} a(d)\,p\left(d\mid\gamma_{2,1,j}\right)\right)\right)^{B}\cdot\right]$$
$$\prod_{k=1}^{4}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty} a(d)\,p\left(d\mid\gamma_{1,k}\right)\right)\right)^{B}\right)\cdot\prod_{l=1}^{4}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty} a(d)\,p\left(d\mid\gamma_{2,l}\right)\right)\right)^{B}\right)$$

(A. 23)

$$P_b(\theta=1.2)_{4trans} \le \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty \min\left[\frac{1}{2},\frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)\,p\left(d\mid\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,3},\gamma_{2,4},\theta\right)\right]\cdot\prod_{i=1}^{4}\prod_{j=3}^{4} p(\gamma_{1,i})p(\gamma_{2,j})\,d\gamma_{1,i}d\gamma$$

(A. 24)

$$p\left(d\mid\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,3},\gamma_{2,4}\right)=Q\left(\sqrt{\left(2\sum_{i=1}^{4} d_{1,i}\gamma_{1,i}+2\sum_{j=3}^{4} d_{2,j}\gamma_{2,j}\right)}\right)$$

(A. 25)

Case 1.3

$$p\left(\theta=1.3\mid\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,1}\right)_{2Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right)\cdot\prod_{j=1}^{2} p_{block,1}\left(\gamma_{2,1,j}\right)\right]\cdot\prod_{k=1}^{2} p_{block,1}\left(\gamma_{1,k}\right)\prod_{l=1}^{2} p_{block,2}\left(\gamma_{2,l}\right)$$

(A. 26)

$$p\left(\theta=1.3 \mid \gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,1}\right)_{2Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{1,2,i}\right)\right)\right)^{B}\prod_{j=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{2,1,j}\right)\right)\right)^{B}\right)\right]$$
$$\prod_{k=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{1,k}\right)\right)\right)^{B}\right)\cdot\prod_{l=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{2,l}\right)\right)\right)^{B}\right)$$

(A. 27)

$$P_b\left(\theta=1.3\right)_{2trans}\le\int_0^{\infty}\int_0^{\infty}\min\left[\frac{1}{2},\frac{1}{k_c}\sum_{d=d_{free}}^{\infty}c(d)\,p\left(d\mid\gamma_{1,1},\gamma_{1,2},\theta\right)\right]\cdot\prod_{i=1}^{2}p\left(\gamma_{1,i}\right)d\gamma_{1,i}$$

(A. 28)

$$p\left(d\mid\gamma_{1,1},\gamma_{1,2}\right)=Q\left(\sqrt{2\sum_{i=1}^{2}d_{1,i}\gamma_{1,i}}\right)$$

(A. 29)

$$p\left(\theta=1.3\mid\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3}\right)_{3Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right)\cdot\prod_{j=1}^{2}p_{block,1}\left(\gamma_{2,1,j}\right)\right]\cdot\prod_{k=1}^{3}p_{block,1}\left(\gamma_{1,k}\right)\prod_{l=1}^{3}p_{block,2}\left(\gamma_{2,l}\right)$$

(A. 30)

$$p\left(\theta=1.3\mid\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3}\right)_{3Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{1,2,i}\right)\right)\right)^{B}\prod_{j=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{2,1,j}\right)\right)\right)^{B}\right)\right]$$
$$\prod_{k=1}^{3}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{1,k}\right)\right)\right)^{B}\right)\cdot\prod_{l=1}^{3}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{2,l}\right)\right)\right)^{B}\right)$$

(A. 31)

Note that no BER is observed here, for user 1, since user 1 transmits parity for user 2 in this incremental parity slot i.e. $p_b\left(\theta=1,3\right)_{3Trans}=0$.

$$p\left(\theta=1.3\mid\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3},\gamma_{2,4}\right)_{4Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right)\cdot\prod_{j=1}^{2}p_{block,1}\left(\gamma_{2,1,j}\right)\right]\cdot\prod_{k=1}^{4}p_{block,1}\left(\gamma_{1,k}\right)\prod_{l=1}^{4}p_{block,2}\left(\gamma_{2,l}\right)$$

(A. 32)

$$p\left(\theta = 1.3 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{2,1,1}, \gamma_{2,1,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}, \gamma_{2,4}\right)_{4Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B} \prod_{j=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,1,j}\right)\right)\right)^{B}\right)\right]$$
$$\prod_{k=1}^{4}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,k}\right)\right)\right)^{B}\right) \cdot \prod_{l=1}^{4}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,l}\right)\right)\right)^{B}\right)$$

(A. 33)

Note that no BER is observed here, for user 1, since user 1 transmits parity for user 2 in this incremental parity slot i.e. $p_b\left(\theta = 1,3\right)_{4Trans} = 0$.

Case 1.4

$$p\left(\theta = 1.4 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{2,1}, \gamma_{2,1}\right)_{2Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right)\right]^{2} \cdot \prod_{j=1}^{2} p_{block,1}\left(\gamma_{1,j}\right) \prod_{k=1}^{2} p_{block,2}\left(\gamma_{2,k}\right)$$

(A. 34)

$$p\left(\theta = 1.4 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{2,1}, \gamma_{2,1}\right)_{2Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B}\right]^{2}$$
$$\prod_{j=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,j}\right)\right)\right)^{B}\right) \cdot \prod_{k=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,k}\right)\right)\right)^{B}\right)$$

(A. 35)

$$P_b\left(\theta = 1.4\right)_{2trans} \le \int_0^{\infty}\int_0^{\infty} \min\left[\frac{1}{2}, \frac{1}{k_c} \sum_{d=d_{free}}^{\infty} c(d)\, p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \theta\right)\right] \cdot \prod_{i=1}^{2} p\left(\gamma_{1,i}\right) d\gamma_{1,i}$$

(A. 36)

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}\right) = Q\left(\sqrt{\left(2\sum_{i=1}^{2} d_{1,i}\gamma_{1,i}\right)}\right)$$

(A. 37)

$$p\left(\theta=1.4 \mid \gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3}\right)_{3Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right)\right]^{2}\cdot\prod_{j=1}^{3}p_{block,1}\left(\gamma_{1,j}\right)\prod_{k=1}^{3}p_{block,2}\left(\gamma_{2,k}\right)$$

(A. 38)

$$p\left(\theta=1.4 \mid \gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3}\right)_{3Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B}\right]^{2}$$
$$\prod_{j=1}^{3}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)p\left(d \mid \gamma_{1,j}\right)\right)\right)^{B}\right)\cdot\prod_{k=1}^{3}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)p\left(d \mid \gamma_{2,k}\right)\right)\right)^{B}\right)$$

(A. 39)

$$P_{b}\left(\theta=1.4\right)_{3trans}\leq\int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\min\left[\frac{1}{2},\frac{1}{k_{c}}\sum_{d=d_{free}}^{\infty}c(d)p\left(d \mid \gamma_{1,1},\gamma_{1,2},\gamma_{2,3},\theta\right)\right]\cdot\prod_{i=1}^{2}p\left(\gamma_{1,i}\right)p\left(\gamma_{2,3}\right)d\gamma_{1,i}d\gamma_{2,3}$$

(A. 40)

$$p\left(d \mid \gamma_{1,1},\gamma_{1,2},\gamma_{2,3}\right)=Q\left(\sqrt{\left(2\sum_{i=1}^{2}d_{1,i}\gamma_{1,i}+2d_{2,3}\gamma_{2,3}\right)}\right)$$

(A. 41)

$$p\left(\theta=1.4 \mid \gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3},\gamma_{2,4}\right)_{4Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right)\right]^{2}\cdot\prod_{j=1}^{4}p_{block,1}\left(\gamma_{1,j}\right)\prod_{k=1}^{4}p_{block,2}\left(\gamma_{2,k}\right)$$

(A. 42)

$$p\left(\theta=1.4 \mid \gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3},\gamma_{2,4}\right)_{4Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B}\right]^{2}$$
$$\prod_{j=1}^{4}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)p\left(d \mid \gamma_{1,j}\right)\right)\right)^{B}\right)\cdot\prod_{k=1}^{4}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)p\left(d \mid \gamma_{2,k}\right)\right)\right)^{B}\right)$$

(A. 43)

$$P_b\left(\theta=1.4\right)_{4trans} \le \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty \min\left[\frac{1}{2},\frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)\,p\left(d\mid\gamma_{1,1},\gamma_{1,2},\gamma_{2,3},\gamma_{2,4},\theta\right)\right]\cdot\prod_{i=1}^{2}\prod_{j=3}^{4} p\left(\gamma_{1,i}\right)p\left(\gamma_{2,j}\right)d\gamma_{1,i}d\gamma_{2,j}$$

(A. 44)

$$p\left(d\mid\gamma_{1,1},\gamma_{1,2},\gamma_{2,3},\gamma_{2,4}\right)=Q\left(\sqrt{\left(2\sum_{i=1}^{2}d_{1,i}\gamma_{1,i}+2\sum_{j=3}^{4}d_{2,j}\gamma_{2,j}\right)}\right)$$

(A. 45)

Case2.1

$$p\left(\theta=2.1\mid\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,1}\right)_{2Trans}=$$
$$\left[\prod_{i=1}^{2}p_{block,1}\left(\gamma_{1,2,i}\right)\right]^2\cdot\prod_{j=1}^{2}p_{block,1}\left(\gamma_{1,j}\right)\prod_{k=1}^{2}\left[1-p_{block,2}\left(\gamma_{2,k}\right)\right]$$

(A. 46)

$$p\left(\theta=2.1\mid\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,1}\right)_{2Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{1,2,i}\right)\right)\right)^B\right)\right]^2\cdot$$
$$\prod_{j=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{1,j}\right)\right)\right)^B\right)\cdot\prod_{k=1}^{2}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\mid\gamma_{2,k}\right)\right)\right)^B$$

(A. 47)

$$P_b\left(\theta=2.1\right)_{2trans} \le \int_0^\infty\int_0^\infty \min\left[\frac{1}{2},\frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)\,p\left(d\mid\gamma_{1,1},\gamma_{1,2},\theta\right)\right]\cdot\prod_{i=1}^{2}p\left(\gamma_{1,i}\right)d\gamma_{1,i}$$

(A. 48)

$$p\left(d\mid\gamma_{1,1},\gamma_{1,2}\right)=Q\left(\sqrt{2\sum_{i=1}^{2}d_{1,i}\gamma_{1,i}}\right)$$

(A. 49)

$$p\left(\theta=2.1\mid\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3}\right)_{3Trans}=$$
$$\left[\prod_{i=1}^{2}p_{block,1}\left(\gamma_{1,2,i}\right)\right]^2\cdot\prod_{j=1}^{3}p_{block,1}\left(\gamma_{1,j}\right)\prod_{k=1}^{3}\left[1-p_{block,2}\left(\gamma_{2,k}\right)\right]$$

(A. 50)

$$p\left(\theta = 2.1 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}\right)_{3Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B}\right)\right]^{2} \cdot$$
$$\prod_{j=1}^{3}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,j}\right)\right)\right)^{B}\right) \cdot \prod_{k=1}^{3}\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,k}\right)\right)\right)^{B}$$

(A. 51)

$$P_b\left(\theta = 2.1\right)_{3trans} \leq \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \min\left[\frac{1}{2}, \frac{1}{k_c} \sum_{d=d_{free}}^{\infty} c(d)\, p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \theta\right)\right] \cdot \prod_{i=1}^{3} p\left(\gamma_{1,i}\right) d\gamma_{1,i}$$

(A. 52)

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}\right) = Q\left(\sqrt{2\sum_{i=1}^{3} d_{1,i}\gamma_{1,i}}\right)$$

(A. 53)

$$p\left(\theta = 2.1 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}, \gamma_{2,4}\right)_{4Trans} =$$
$$\left[\prod_{i=1}^{2} p_{block,1}\left(\gamma_{1,2,i}\right)\right]^{2} \cdot \prod_{j=1}^{4} p_{block,1}\left(\gamma_{1,j}\right) \prod_{k=1}^{4}\left[1 - p_{block,2}\left(\gamma_{2,k}\right)\right]$$

(A. 54)

$$p\left(\theta = 2.1 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}, \gamma_{2,4}\right)_{4Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B}\right)\right]^{2} \cdot$$
$$\prod_{j=1}^{4}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,j}\right)\right)\right)^{B}\right) \cdot \prod_{k=1}^{4}\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,k}\right)\right)\right)^{B}$$

(A. 55)

$$P_b\left(\theta = 2.1\right)_{4trans} \leq \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \min\left[\frac{1}{2}, \frac{1}{k_c} \sum_{d=d_{free}}^{\infty} c(d)\, p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}, \theta\right)\right] \cdot \prod_{i=1}^{4} p\left(\gamma_{1,i}\right) d\gamma_{1,i}$$

(A. 56)

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}\right) = Q\left(\sqrt{2\sum_{i=1}^{4} d_{1,i}\gamma_{1,i}}\right)$$

(A. 57)

Case 2.2

$$p\left(\theta = 2.2 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{2,1,1}, \gamma_{2,1,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{2,1}, \gamma_{2,1}\right)_{2Trans} =$$
$$\left[\prod_{i=1}^{2} p_{block,1}\left(\gamma_{1,2,i}\right) \cdot \prod_{j=1}^{2}\left(1 - p_{block,1}\left(\gamma_{2,1,j}\right)\right)\right] \cdot \prod_{k=1}^{2} p_{block,1}\left(\gamma_{1,k}\right) \prod_{l=1}^{2}\left[1 - p_{block,2}\left(\gamma_{2,l}\right)\right]$$

(A. 58)

$$p\left(\theta = 2.2 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{2,1,1}, \gamma_{2,1,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{2,1}, \gamma_{2,1}\right)_{2Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1 - \left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B}\right) \cdot \prod_{j=1}^{2}\left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,1,j}\right)\right)\right)^{B}\right]$$
$$\prod_{k=1}^{2}\left(1 - \left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,k}\right)\right)\right)^{B}\right) \cdot \prod_{l=1}^{2}\left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,l}\right)\right)\right)^{B}$$

(A. 59)

$$P_b\left(\theta = 2.2\right)_{2trans} \leq \int_0^\infty \int_0^\infty \min\left[\frac{1}{2}, \frac{1}{k_c} \sum_{d=d_{free}}^{\infty} c(d)\, p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \theta\right)\right] \cdot \prod_{i=1}^{2} p\left(\gamma_{1,i}\right) d\gamma_{1,i}$$

(A. 60)

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}\right) = Q\left(\sqrt{2 \sum_{i=1}^{2} d_{1,i} \gamma_{1,i}}\right)$$

(A. 61)

$$p\left(\theta = 2.2 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{2,1,1}, \gamma_{2,1,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}\right)_{3Trans} =$$
$$\left[\prod_{i=1}^{2} p_{block,1}\left(\gamma_{1,2,i}\right) \cdot \prod_{j=1}^{2}\left(1 - p_{block,1}\left(\gamma_{2,1,j}\right)\right)\right] \cdot \prod_{k=1}^{3} p_{block,1}\left(\gamma_{1,k}\right) \prod_{l=1}^{3}\left[1 - p_{block,2}\left(\gamma_{2,l}\right)\right]$$

(A. 62)

$$p\left(\theta = 2.2 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{2,1,1}, \gamma_{2,1,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}\right)_{3Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1 - \left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B}\right) \cdot \prod_{j=1}^{2}\left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,1,j}\right)\right)\right)^{B}\right]$$
$$\prod_{k=1}^{3}\left(1 - \left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,k}\right)\right)\right)^{B}\right) \cdot \prod_{l=1}^{3}\left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,l}\right)\right)\right)^{B}$$

(A. 63)

$$P_b(\theta=2.2)_{3trans} \le \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty \min\left[\frac{1}{2},\frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)\,p(d\mid\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{2,3},\theta)\right]\cdot\prod_{i=1}^{3} p(\gamma_{1,i})p(\gamma_{2,3})\,d\gamma_{1,i}d\gamma_{2,3}$$

(A. 64)

$$p(d\mid\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{2,3}) = Q\left(\sqrt{\left(2\sum_{i=1}^{4} d_{1,i}\gamma_{1,i} + 2d_{2,3}\gamma_{2,3}\right)}\right)$$

(A. 65)

$$p(\theta=2.2\mid\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3},\gamma_{2,4})_{4Trans} =$$
$$\left[\prod_{i=1}^{2} p_{block,1}(\gamma_{1,2,i})\cdot\prod_{j=1}^{2}\left(1-p_{block,1}(\gamma_{2,1,j})\right)\right]\cdot\prod_{k=1}^{4} p_{block,1}(\gamma_{1,k})\prod_{l=1}^{4}\left[1-p_{block,2}(\gamma_{2,l})\right]$$

(A. 66)

$$p(\theta=2.2\mid\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3},\gamma_{2,4})_{4Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty} a(d)\,p(d\mid\gamma_{1,2,i})\right)\right)^{B}\right)\cdot\prod_{j=1}^{2}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty} a(d)\,p(d\mid\gamma_{2,1,j})\right)\right)^{B}\right]$$
$$\prod_{k=1}^{4}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty} a(d)\,p(d\mid\gamma_{1,k})\right)\right)^{B}\right)\cdot\prod_{l=1}^{4}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty} a(d)\,p(d\mid\gamma_{2,l})\right)\right)^{B}$$

(A. 67)

$$P_b(\theta=2.2)_{4trans} \le \int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty \min\left[\frac{1}{2},\frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)\,p(d\mid\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,3},\gamma_{2,4},\theta)\right]\cdot\prod_{i=1}^{4}\prod_{j=3}^{4} p(\gamma_{1,i})p(\gamma_{2,j})\,d\gamma_{1,i}d\gamma_{2,j}$$

(A. 68)

$$p(d\mid\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,3},\gamma_{2,4}) = Q\left(\sqrt{\left(2\sum_{i=1}^{4} d_{1,i}\gamma_{1,i} + 2\sum_{j=3}^{4} d_{2,j}\gamma_{2,j}\right)}\right)$$

(A. 69)

$$p(\theta=2.3\mid\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,1})_{2Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1-p_{block,1}(\gamma_{1,2,i})\right)\cdot\prod_{j=1}^{2} p_{block,1}(\gamma_{2,1,j})\right]\cdot\prod_{k=1}^{2} p_{block,1}(\gamma_{1,k})\prod_{l=1}^{2}\left[1-p_{block,2}(\gamma_{2,l})\right]$$

(A. 70)

$$p\left(\theta = 2.3 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{2,1,1}, \gamma_{2,1,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{2,1}, \gamma_{2,1}\right)_{2Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B} \prod_{j=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,1,j}\right)\right)\right)^{B}\right)\right]$$
$$\prod_{k=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,k}\right)\right)\right)^{B}\right)\cdot \prod_{l=1}^{2}\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,l}\right)\right)\right)^{B}$$

(A. 71)

$$P_b\left(\theta = 2.3\right)_{2trans} \le \int_0^{\infty}\int_0^{\infty} \min\left[\frac{1}{2}, \frac{1}{k_c} \sum_{d=d_{free}}^{\infty} c(d)\, p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \theta\right)\right] \cdot \prod_{i=1}^{2} p\left(\gamma_{1,i}\right) d\gamma_{1,i}$$

(A. 72)

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}\right) = Q\left(\sqrt{2\sum_{i=1}^{2} d_{1,i}\gamma_{1,i}}\right)$$

(A. 73)

$$p\left(\theta = 2.3 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{2,1,1}, \gamma_{2,1,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}\right)_{3Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right) \cdot \prod_{j=1}^{2} p_{block,1}\left(\gamma_{2,1,j}\right)\right] \cdot \prod_{k=1}^{3} p_{block,1}\left(\gamma_{1,k}\right) \prod_{l=1}^{3}\left[1-p_{block,2}\left(\gamma_{2,l}\right)\right]$$

(A. 74)

$$p\left(\theta = 2.3 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{2,1,1}, \gamma_{2,1,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}\right)_{3Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B} \prod_{j=1}^{2}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,1,j}\right)\right)\right)^{B}\right)\right]$$
$$\prod_{k=1}^{3}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,k}\right)\right)\right)^{B}\right)\cdot \prod_{l=1}^{3}\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,l}\right)\right)\right)^{B}$$

(A. 75)

$$P_b\left(\theta = 2.3\right)_{3trans} \le \int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \min\left[\frac{1}{2}, \frac{1}{k_c} \sum_{d=d_{free}}^{\infty} c(d)\, p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \theta\right)\right] \cdot \prod_{i=1}^{3} p\left(\gamma_{1,i}\right) d\gamma_{1,i}$$

(A. 76)

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}\right) = Q\left(\sqrt{2\sum_{i=1}^{3} d_{1,i}\gamma_{1,i}}\right)$$

(A. 77)

$$p\left(\theta=2.3\,|\,\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3},\gamma_{2,4}\right)_{4Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right)\cdot\prod_{j=1}^{2}p_{block,1}\left(\gamma_{2,1,j}\right)\right]^{2}\cdot\prod_{k=1}^{4}p_{block,1}\left(\gamma_{1,k}\right)\prod_{l=1}^{4}\left[1-p_{block,2}\left(\gamma_{2,l}\right)\right]$$

(A. 78)

$$p\left(\theta=2.3\,|\,\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{2,1,1},\gamma_{2,1,2},\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\gamma_{2,1},\gamma_{2,2},\gamma_{2,3},\gamma_{2,4}\right)_{4Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{1,2,i}\right)\right)\right)^{B}\prod_{j=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{2,1,j}\right)\right)\right)^{B}\right)\right]$$
$$\prod_{k=1}^{4}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{1,k}\right)\right)\right)^{B}\right)\cdot\prod_{l=1}^{4}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{2,l}\right)\right)\right)^{B}$$

(A. 79)

$$P_b\left(\theta=2.3\right)_{4trans}\le\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\min\left[\frac{1}{2},\frac{1}{k_c}\sum_{d=d_{free}}^{\infty}c(d)\,p\left(d\,|\,\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4},\theta\right)\right]\cdot\prod_{i=1}^{4}p\left(\gamma_{1,i}\right)d\gamma_{1,i}$$

(A. 80)

$$p\left(d\,|\,\gamma_{1,1},\gamma_{1,2},\gamma_{1,3},\gamma_{1,4}\right)=Q\left(\sqrt{2\sum_{i=1}^{4}d_{1,i}\gamma_{1,i}}\right)$$

(A. 81)

Case 2.4

$$p\left(\theta=1.4\,|\,\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,1}\right)_{2Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-p_{block,1}\left(\gamma_{1,2,i}\right)\right)\right]^{2}\cdot\prod_{j=1}^{2}p_{block,1}\left(\gamma_{1,j}\right)\prod_{k=1}^{2}\left(1-p_{block,2}\left(\gamma_{2,k}\right)\right)$$

(A. 82)

$$p\left(\theta=2.4\,|\,\gamma_{1,2,1},\gamma_{1,2,2},\gamma_{1,1},\gamma_{1,2},\gamma_{2,1},\gamma_{2,1}\right)_{2Trans}=$$
$$\left[\prod_{i=1}^{2}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{1,2,i}\right)\right)\right)^{B}\right]^{2}$$
$$\prod_{j=1}^{2}\left(1-\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{1,j}\right)\right)\right)^{B}\right)\cdot\prod_{k=1}^{2}\left(1-\min\left(1,\sum_{d=d_{free}}^{\infty}a(d)\,p\left(d\,|\,\gamma_{2,k}\right)\right)\right)^{B}$$

(A. 83)

$$P_b(\theta = 2.4)_{2trans} \le \int_0^{\infty}\int_0^{\infty} \min\left[\frac{1}{2}, \frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)\, p(d \mid \gamma_{1,1}, \gamma_{1,2}, \theta)\right] \cdot \prod_{i=1}^{2} p(\gamma_{1,i})\, d\gamma_{1,i}$$

(A. 84)

$$p(d \mid \gamma_{1,1}, \gamma_{1,2}) = Q\left(\sqrt{2\sum_{i=1}^{2} d_{1,i}\gamma_{1,i}}\right)$$

(A. 85)

$$p(\theta = 2.4 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3})_{3Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1 - p_{block,1}(\gamma_{1,2,i})\right)\right]^2 \cdot \prod_{j=1}^{3} p_{block,1}(\gamma_{1,j}) \prod_{k=1}^{3}\left(1 - p_{block,2}(\gamma_{2,k})\right)$$

(A. 86)

$$p(\theta = 2.4 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3})_{3Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p(d \mid \gamma_{1,2,i})\right)\right)^B\right]^2$$
$$\prod_{j=1}^{3}\left(1 - \left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p(d \mid \gamma_{1,j})\right)\right)^B\right) \cdot \prod_{k=1}^{3}\left(1 - \min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p(d \mid \gamma_{2,k})\right)\right)^B$$

(A. 87)

$$P_b(\theta = 2.4)_{3trans} \le \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} \min\left[\frac{1}{2}, \frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)\, p(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{2,3}, \theta)\right] \cdot \prod_{i=1}^{3} p(\gamma_{1,i}) p(\gamma_{2,3})\, d\gamma_{1,i}\, d\gamma_{2,3}$$

(A. 88)

$$p(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{2,3}) = Q\left(\sqrt{\left(2\sum_{i=1}^{3} d_{1,i}\gamma_{1,i} + 2d_{2,3}\gamma_{2,3}\right)}\right)$$

(A. 89)

$$p(\theta = 2.4 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}, \gamma_{2,4})_{4Trans} =$$
$$\left[\prod_{i=1}^{2}\left(1 - p_{block,1}(\gamma_{1,2,i})\right)\right]^2 \cdot \prod_{j=1}^{4} p_{block,1}(\gamma_{1,j}) \prod_{k=1}^{4}\left(1 - p_{block,2}(\gamma_{2,k})\right)$$

(A. 90)

$$p\left(\theta = 2.4 \mid \gamma_{1,2,1}, \gamma_{1,2,2}, \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}, \gamma_{2,1}, \gamma_{2,2}, \gamma_{2,3}, \gamma_{2,4}\right)_{4Trans} =$$

$$\left[\prod_{i=1}^{2}\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,2,i}\right)\right)\right)^{B}\right]^{2}$$

$$\prod_{j=1}^{4}\left(1-\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{1,j}\right)\right)\right)^{B}\right) \cdot \prod_{k=1}^{4}\left(1-\min\left(1, \sum_{d=d_{free}}^{\infty} a(d)\, p\left(d \mid \gamma_{2,k}\right)\right)\right)^{B}$$

(A. 91)

$$P_b(\theta = 2.4)_{4trans} \le \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\min\left[\frac{1}{2}, \frac{1}{k_c}\sum_{d=d_{free}}^{\infty} c(d)\, p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}, \gamma_{2,3}, \gamma_{2,4}, \theta\right)\right] \cdot \prod_{i=1}^{4} p\left(\gamma_{1,i}\right) d\gamma_{1,i} \cdots$$
$$\prod_{j=1}^{3} p\left(\gamma_{2,j}\right) d\gamma_{2,j}$$

(A. 92)

$$p\left(d \mid \gamma_{1,1}, \gamma_{1,2}, \gamma_{1,3}, \gamma_{1,4}, \gamma_{2,3}, \gamma_{2,4}\right) = Q\left(\sqrt{\left(2\sum_{i=1}^{4} d_{1,i}\gamma_{1,i} + 2\sum_{j=3}^{4} d_{2,j}\gamma_{2,j}\right)}\right) \quad \textbf{(A. 93)}$$

The 25% Best Case Cooperation End to End BER Bound

In order to sustain brevity but not lose generality this BER bound will not be derived but will be explained rather. Equation (3.28) given below is the end to end BER bound for 25% best case cross layer cooperation.

$$p_{BER} = \sum_{l=1}^{2}\sum_{r=1}^{4}\sum_{j=3}^{4} p_b\left(\theta = l, r\right)_j p\left(\theta = l, r\right)_j$$

Note that the only difference between (3.28) above and (3.24) is the change in limits (due to 25% best case cooperation) of j i.e. $j \in \{3,4\}$ and hence when computing $p_b\left(\theta = l, r\right)_j$ ($l \in \{1,2\}, r \in \{1,4\}, j \in \{3,4\}$) the PEP for each $p_b\left(\theta = l, r\right)_j$ and $p\left(\theta = l, r\right)_j$ is taken into account. This occurs due to the user initially transmitting 3 incremental packets during 25% best case cooperation. After having computed $p_b\left(\theta = l, r\right)_j$, $p\left(\theta = l, r\right)_j$ and the PEP for each $p_b\left(\theta = l, r\right)_j$, they need to be substituted into (3.28) above to obtain the end to end BER for 25% best case cooperation.

Appendix B

Derivation of throughput bounds for Cross Layer Hybrid ARQ 2 Cooperative Diversity

Derivation of Non Cooperative Direct Transmission Throughput Bound

Figure B.1 shows the possible transmission scenarios during direct transmission without cooperation.

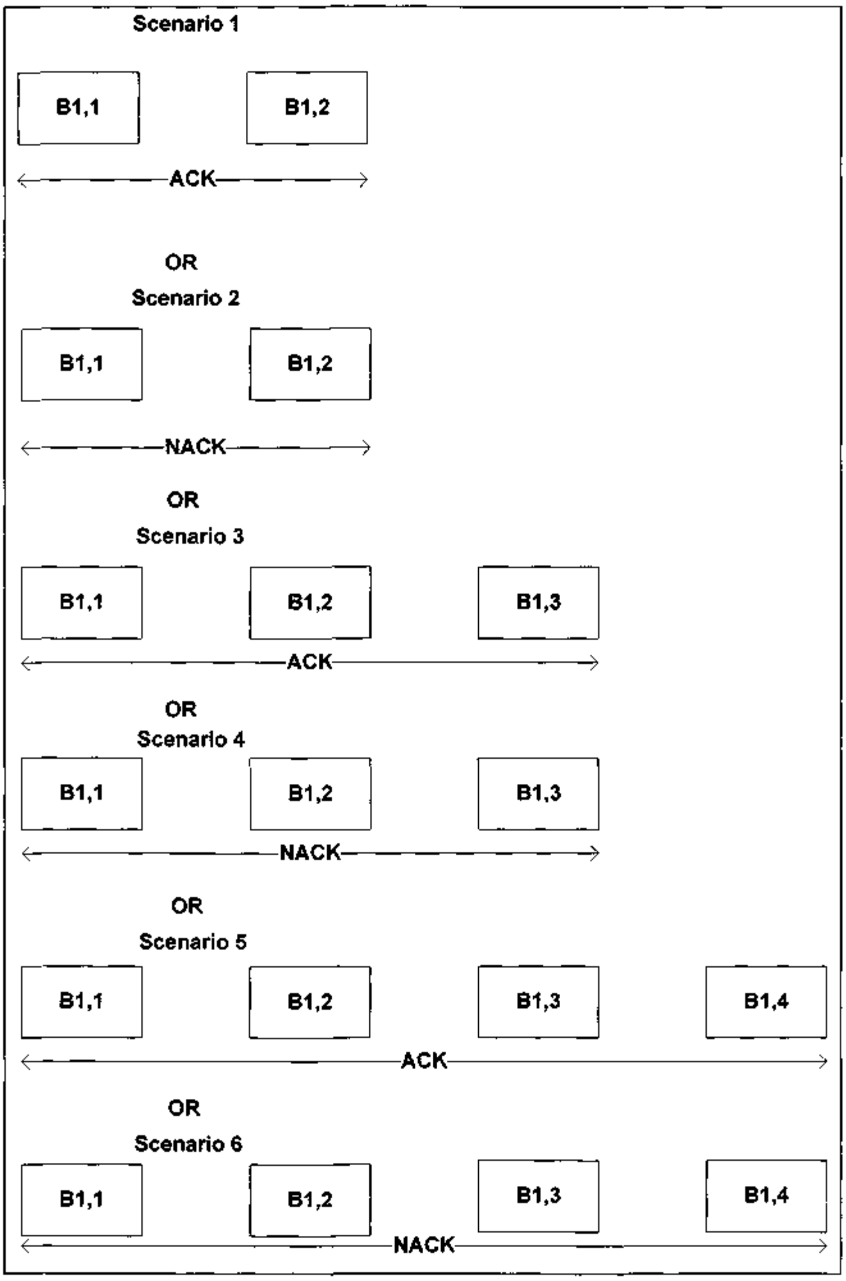

Figure B.1: Transmission Scenarios during Non Cooperative Direct Transmission

The transmission scenarios are vital for the computation of $\overline{L}$. The definition of throughput is given by (4.1) i.e.

$$\eta = \frac{B}{B+c} \cdot \frac{p_s}{\overline{L}}$$

The computation of $\overline{L}$ involves the computation of all transmission scenarios whilst the computation of p_s, the packet successful probability, is trivial. During non cooperation $\overline{L}$ is computed to be the union of all the possible transmission scenarios shown in Figure B.1.
The scenario probabilities are listed below:

Scenario 1: $\left(1-p_{pd}{}^{2}\right)$

Scenario 2: $p_{pd}{}^{2}$

Scenario 3: $p_{pd}{}^{2}\left(1-p_{pR}{}^{(1)}\right)$

Scenario 4: $p_{pd}{}^{2} p_{pR}{}^{(1)}$

Scenario 5: $p_{pd}{}^{2} p_{pR}{}^{(1)}\left(1-p_{pR}{}^{(1)}\right)$

Scenario 6: $p_{pd}{}^{2}\left(p_{pR}{}^{(1)}\right)^{2}$

$\overline{L}$ is calculated to be the union of the above probabilities i.e.

$$\begin{aligned}\overline{L} &= \left(1-p_{pd}{}^{2}\right)+p_{pd}{}^{2} p_{pR}{}^{(1)}+p_{pd}{}^{2}\left(p_{pR}{}^{(1)}\right)^{2}+p_{pd}{}^{2}+p_{pd}{}^{2}\left(1-p_{pR}{}^{(1)}\right)+p_{pd}{}^{2} p_{pR}{}^{(1)}\left(1-p_{pR}{}^{(1)}\right) \\ &= p_{pd}{}^{2}\left[1+p_{pR}{}^{(1)}\right]\end{aligned} \tag{B.1}$$

Using the packet successful probability of Equation (4.2) for direct transmission i.e.

$$p_s = 1-p_{pd}{}^{2}\left(p_{pR}{}^{(1)}\right)^{2}$$

Equation (4.2) and (B.1) can be substituted into (4.1), listed below for convenience, in order to calculate the throughput of direct transmission.

$$\eta = \frac{B}{B+c} \cdot \frac{p_s}{\overline{L}}$$

Note that B is equal to 128 and c is equal to 16. Using a similar approach and drawing the possible transmission scenarios during 50% and 25% best case cooperation $\overline{L}$ can be calculated during cooperation also. After $\overline{L}$ and p_s are determined the throughput can be calculated by substituting $\overline{L}$ and p_s into (4.1).

$\overline{L}$, during 50% cooperation, is derived to be:

$$
\begin{aligned}
\overline{L} &= \left(1-p_{pd}^{\ 2}\right)+p_{pd}^{\ 2}p_{pR}^{\ (1)}+p_{pd}^{\ 2}\left(p_{pR}^{\ (1)}\right)^2+p_{pd}^{\ 2}+p_{pd}^{\ 2}\left(1-p_{pR}^{\ (1)}\right)+p_{pd}^{\ 2}p_{pR}^{\ (1)}\left(1-p_{pR}^{\ (1)}\right) \\
&\quad +\left(p_{pd}\right)^2 p_{pR}^{(1)}.p_{pR}^{(2)}+\left(p_{pd}\right)^2\left(p_{pR}^{(1)}\right)^2.\left(p_{pR}^{(2)}\right)^2+\left(p_{pd}\right)^2\left(1-p_{pR}^{(1)}\right).\left(1-p_{pR}^{(2)}\right)+\left(p_{pd}\right)^2\left(1-p_{pR}^{(1)}\right).p_{pR}^{(2)} \\
&\quad +\left(p_{pd}\right)^2 p_{pR}^{(1)}.\left(1-p_{pR}^{(2)}\right)+\left(p_{pd}\right)^2 p_{pR}^{(1)}.p_{pR}^{(2)}.\left(1-p_{pR}^{(1)}\right).\left(1-p_{pR}^{(2)}\right) \\
&= 1+\left(p_{pd}\right)^2\left[\begin{array}{l}
p_{pR}^{(1)}+\left(p_{pR}^{\ (1)}\right)^2+\left(1-p_{pR}^{\ (1)}\right)+p_{pR}^{\ (1)}\left(1-p_{pR}^{\ (1)}\right)+p_{pR}^{(1)}.p_{pR}^{(2)}+\left(p_{pR}^{(1)}\right)^2.\left(p_{pR}^{(2)}\right)^2 \\
+\left(1-p_{pR}^{(1)}\right).\left(1-p_{pR}^{(2)}\right)+\left(1-p_{pR}^{(1)}\right).p_{pR}^{(2)}\ +p_{pR}^{(1)}.\left(1-p_{pR}^{(2)}\right) \\
+p_{pR}^{(1)}.p_{pR}^{(2)}.\left(1-p_{pR}^{(1)}\right).\left(1-p_{pR}^{(2)}\right)
\end{array}\right]
\end{aligned}
$$

(B.2)

During 50% best case cooperation p_s is calculated to be:

$$p_s=\left[1-\left(p_{pd}\right)^2\left(p_{pR}^{(1)}\right)^2\right].p\left(\text{no cooperation}\right)+\left[1-\left(p_{pd}\right)^2\left(p_{pR}^{(1)}\right)^2\left(p_{pR}^{(2)}\right)^2\right].p\left(\text{cooperation}\right)$$

as in (4.6).

$\overline{L}$, during 25% cooperation, is derived to be:

$$
\begin{aligned}
\overline{L} &= \left(1-\left(p_{pd}\right)^3\right)+\left(p_{pd}\right)^3 p_{pR}^{(1)}+\left(p_{pd}\right)^3+\left(p_{pd}\right)^3\left(1-p_{pR}^{(1)}\right)+\left(p_{pd}\right)^3 p_{pR}^{(1)}.p_{pR}^{(2)}+\left(p_{pd}\right)^3\left(1-p_{pR}^{(1)}\right).p_{pR}^{(2)} \\
&\quad +\left(p_{pd}\right)^3.p_{pR}^{(1)}.\left(1-p_{pR}^{(2)}\right)+\left(p_{pd}\right)^3\left(1-p_{pR}^{(1)}\right).\left(1-p_{pR}^{(2)}\right) \\
&= 1+\left(p_{pd}\right)^3\left[1+p_{pR}^{(1)}\cdot p_{pR}^{(2)}+p_{pR}^{(2)}\left(1-p_{pR}^{(1)}\right)+p_{pR}^{(1)}\left(1-p_{pR}^{(2)}\right)+\left(1-p_{pR}^{(1)}\right)\left(1-p_{pR}^{(2)}\right)\right]
\end{aligned}
$$

(B.3)

During 25% best case cooperation p_s is calculated to be:

$$p_s=\left[1-\left(p_{pd}\right)^3\left(p_{pR}^{(1)}\right)\right].p\left(\text{no cooperation}\right)+\left[1-\left(p_{pd}\right)^3\left(p_{pR}^{(1)}\right)\left(p_{pR}^{(2)}\right)\right].p\left(\text{cooperation}\right)$$

as in Equation (4.7).