AVOIDANCE OF TRAPPINGS IN CONTINUAL COLLAPSE OF SPHERICAL STARS

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Avoidance of trappings in continual collapse of spherical stars

by

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As the candidate's supervisor, I have approved this dissertation for submission.

Name: Prof. R. Goswami Signed: Date:

Abstract

In this dissertation we study the physical process of a spherically symmetric perfect fluid experiencing a continuous gravitational collapse in concurrence with continuous radiating energy in an outward spacetime. Trapped surfaces are avoided and the final fate of the collapse is a flat spacetime. In addition, the collapsing matter conforms to the weak and dominant energy conditions at all epochs. Our investigation clearly unveils the purpose of the equation of state and reveals the bounds on the thermodynamic potentials the equation of state admits for such a model. We affirm that these models are generic without any of the issues and paradoxes attached to horizons and singularities, because the system of Einstein field equations accepts such a theoretical account for an open set of initial data and the equation of state function in their respective functional spaces. High resolution radio telescopes of today, should ideally detect the existence of these compact bodies in the sky.

Preface

The research presented in this dissertation was completed by the candidate while based in the Discipline of Physics, School of Chemistry and Physics in the College of Agriculture, Engineering and Science, University of KwaZulu-Natal, Westville, South Africa. The research was financially supported by the National Astrophysics and Space Science Program (NASSP).

The contents of this written report have not been submitted in any form to another university and, except where the work of others is acknowledged or referenced in the text, results composed herein is consequential of the research completed by the candidate.

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I, Terricia Govender, declare that:

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Declaration 2 - Publications

This dissertation contains components founded on the following publicised article, which has been submitted to the "General Relativity and Gravitation" scientific journal:

Publication 1

Goswami, R. and Govender, T., 2019. Role of equation of states and thermodynamic potentials in avoidance of trapped surfaces in gravitational collapse. arXiv preprint arXiv:1903.01126.

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Date: ....

Dedicated

Dedicated to my grandparents, Ma (Mummy) and Thatha (Krish)

Maduraiamma and Manival

Govender

They tended the soil, making strong the roots of which generations may blossom

Acknowledgments

I would like to thank my supervisor, Prof. R. Goswami, for his patience and guidance with this dissertation. I appreciate his support in my endeavour to find where my talents are most suited with my life aspirations.

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To my dearest Fiancé Shamik, you're the 'Agni' to my spirit. Thank you for being my pillar of strength and support. I find splendor in our intellectual conversations and spiritual aspirations. "We must be quarks because you're the charm to my strange."

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Terricia Govender

August 2019

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Chapter 1

Introduction

In this first chapter, we introduce key concepts that lay the foundations relating to the main focus of our interest. We start from the equivalence principle, extend into general relativity, gain insight into gravitational collapse, black hole physics and singularities. We explore the end result of a star's journey and take a look at an important equation deduced by Raychaudhuri.

1.1 The equivalence principle

Since 1907, Einstein released improved works on what is now known as the equivalence principle (Dieks, 2018). Einstein's equivalence principle (EEP) simply suggests that a gravitational field is equal by nature to an inertial field in an accelerated reference frame. The EEP can also be understood as an uncharged test particle moving in a vacuous space pursuing a path independent by its interior makeup and anatomy (Wu et al., 2017).

The EEP is an amalgamation of concepts. There is the weak equivalence principle

(WEP) referring to the universality of free fall. The WEP describes a given gravitational field where every form of matter accelerates at the same rate, provided that all non-gravitational laws of physics assume their special relativistic structure in such a free-fall frame of reference (Wu et al., 2017). The WEP usually refers to bodies consisting of small self-gravitational energy (Ostoma and Trushyk, 2000).

In contrast, the strong equivalence principle (SEP) is required to maintain the same acceleration as the WEP for bodies with a significant portion of their mass comprising of the compressive gravitational energy, such as neutron stars and black holes (Barausse, 2017).

The local Lorentz invariance describes non-gravitational measurements which are independent of the velocity of a freely falling frame of reference (Bize et al., 2004). Local position invariance (LPI) is similar to the local Lorentz invariance but is instead independent of the location in spacetime, where the measurements were done (Bize et al., 2004).

The theory of general relativity (GR) that Einstein developed was based on the simple and logical understanding of two main scenarios, namely the elevator and rocket thought experiments. Figure 1.1 depicts the SEP and WEP.

The rocket scenario depicts SEP, where a person inside, has no means of determining his state of motion with respect to his external surroundings. A person in a rocket, stationary on earth, would feel the same effects as a person in a rocket accelerating through space (Nobili et al., 2004). Either way, there would be no means for the person inside the rocket to deduce whether they were moving or not.

Similarly, show casing WEP, the elevator scenario depicts a person feeling the physical effects of free falling or gravitational acceleration towards Earth and a man in space

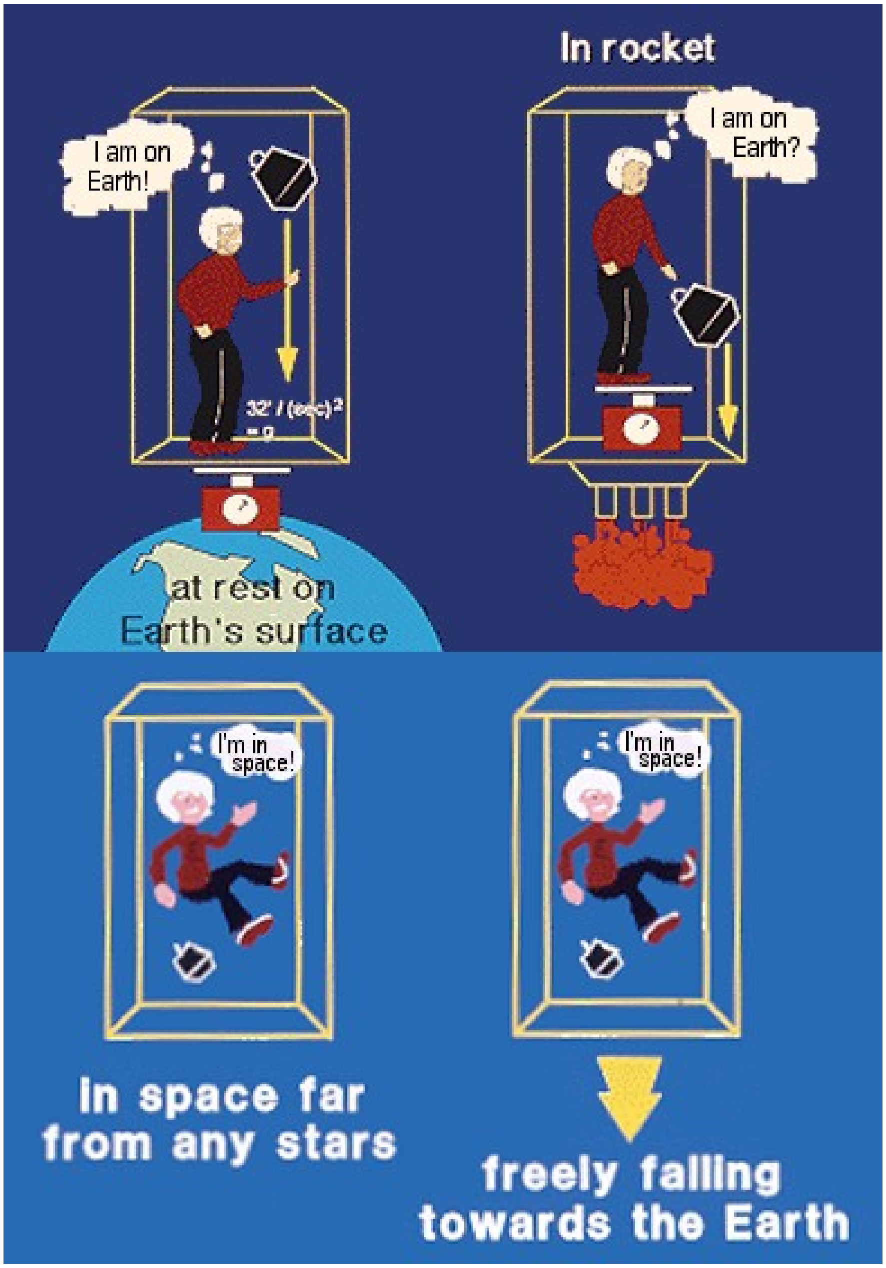

Figure 1.1: Depiction of the strong equivalence principle and the weak equivalence principle.

Time Travel Research Center:

http://www.zamandayolculuk.com/cetinbal/ HTMLdosya1/RelativityFile.htm)

experiencing weightlessness (Nobili et al., 2004). Either way, the physical effects are the same for the person in the elevator in space or on Earth.

Metric theories of gravity are originally based on EEP. In quantum physics proving the EEP has been a struggle even in recent studies. Quantum particles have been subjected to a variety of test studies holding the WEP as credible. Arguably, the SEP in classical mechanics and in non-relativistic quantum mechanics has not yet yielded a strong correlation of evidence.

Einstein's famous concept of the general theory of relativity gives birth to the popularly accepted metric theory of gravity. To understand more on Einstein's thought process regarding his GR theory, it would be of great interest to read (Dieks, 2018).

The endeavour of the unification of GR with quantum field theory has yet to achieve any progress as the EEP fails in the quantum regime (Smarandache et al., 2013). This can lead one to believe that either general relativity is geodesically incomplete or a revelation in quantum physics is required. Perhaps the macro and micro realms of physics are just incompatible to such a degree that it will forever remain a mystery.

1.2 General relativity

In 1905, Albert Einstein released a paper named, 'On the electrodynamics of moving bodies,' which implemented the foundations of his genius idea for the special theory of relativity (Einstein, 1905). He later compensated for gravity in his famous general theory of relativity (Einstein, 1915), (Ren and Schemmel, 2007).

It is highly recommended to read, 'On the Special and General Theory of Relativity (A Popular Account) (Einstein, 1917),' for a more thorough understanding of general and special relativity.

GR establishes gravity as a result of warped spacetime, where spacetime is a $4-D$

manifold comprising of three spatial dimensions and one temporal dimension (Einstein, 1916). GR unifies Newton's law of gravity and special relativity, yielding an accumulation of gravity due to mass curving spacetime, rather than a simple action of force (Einstein, 1915b). The curvature of spacetime correlates to energy and momentum of matter as well as radiation (Einstein, 1916). This relation is delineated by a system of partial differential equations best-known as the Einstein field equations (Einstein, 1916).

Spacetime is a pseudo-Riemannian manifold (Penrose, 2002). The GR description is depicted in Figure 1.2, where Earth is seen 'sitting' on the 'fabric' of spacetime, creating a 'dip' which results in the property of gravity. GR corroborates with the concepts of special relativity, linearised Newtonian physics and subsequent theories of gravity.

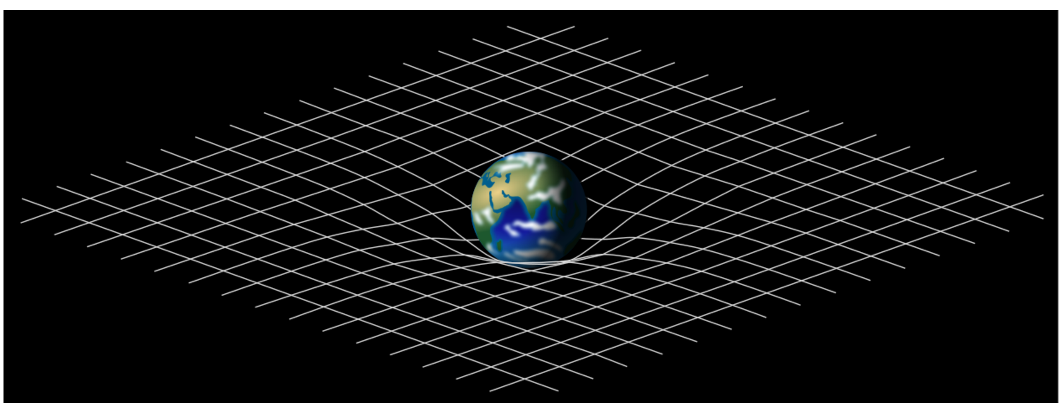

Figure 1.2: Depiction of spacetime curvature in 2D.

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The classical tests of GR as suggested by Einstein in 1915 (Einstein, 1915b and Einstien, 1917) are as follows:

1. The perihelion precession of the celestial orbit of the planet Mercury (Einstein 1915h, 1916 and 1917),

2. The deflection of light due to the Sun (Einstein 1914l and 1917),

3. Light having a gravitational redshift (Einstein 1917),

which have all been verified experimentally as predicted by the theory.

GR is described mathematically by a spacetime model that includes all events set out as (M, g), where M is a connected $4-$dimensional Hausdorff C^{∞} time- oriented manifold and g is a Lorentz metric on M (Hawking and Ellis, 1973). Additionally, M is paracompact on the basis that the Hausdorff condition is in conjunction with the Lorentz metric (Geroch, 1968).

We examine the pair (M, g) comprising of the whole equivalence class of spacetimes and corresponding equivalent physical attributes. We consider all diffeomorphic pairs (M', g') to (M, g) as equal. The postulate of local causality is adhered to by the spacetime. The matter field equations are valid if $U \subset M$ is a convex normal neighbourhood and p, q are points in U, hence a signal can be transmitted in U between p and q, if and only if, there exist a C^1 curve in U between p and q whose tangent is non-zero everywhere and is either null or timelike.

The aforementioned postulate distinguishes between the geometrical property of the g metric from those fields in M. The postulate of local conservation of energy and momentum must prevail for any matter field on the spacetime. The matter field equations yield an energy momentum tensor T_{ik} that is symmetric and supported by the fields, their covariant derivatives and the metric.

Characteristically, this tensor disappears in an open set $U \subset M$, if and only if, all matter fields disappear on U. The field's energy and momentum are conserved when $T^{ik}{}_{;k} = 0$, holds (Ren and Schemmel, 2007). The Ricci scalar is an appropriate scalar Lagrangian to use as inferred from the metric tensor, it's first derivatives and whereby the second derivatives are linear (Landau and Lifshitz, 1975). The principle of least action describes the connection between spacetime geometry and matter,

$$\delta(S_g + S_m) = \delta \int dV \sqrt{-g}(R + L_m) = 0, \tag{1.2.1}$$

where S_g and S_m are gravitational and matter field variables respectively, found by integrating over a four volume, R is the Ricci scalar of the spacetime and finally L_m is the matter Lagrangian. Upon simplification of (1.2.1) we obtain Einstein's field equations (where $8\pi G = c = 1$),

$$G_{ik} \equiv R_{ik} - \frac{1}{2} g_{ik} R = T_{ik}, \tag{1.2.2}$$

where R_{ik} is the Ricci tensor contracted from the four dimensional Riemann curvature tensor R^k_{ilm} of the spacetime and G_{ik} is the Einstein tensor. Einstein's equations must be met on M as this is another postulate of GR.

To summarise, we state what can arguably be the three main postulates of GR:

1. Local causality,
2. Local conservation of energy and momentum,
3. Einstein's equations holds on M.

1.3 The ultimate destiny that awaits massive stars

The inception of GR sparked scientists to then understand what becomes of a massive star which has depleted its nuclear fuel (Bonolis, 2017).

In 1931, Chandrasekhar published his findings on what is now known as the 'Chandrasekhar limit', $1.4M_{\odot}$ (Chandrasekhar, 1931). From the theory of special relativity, he deduced that there are no stable solutions for a non-rotating body of electron-degenerate matter larger than the Chandrasekhar limit (Chandrasekhar, 1935 and 1983).

However, this idea was strongly disputed mainly by the expert Arthur Eddington (Gursky et al., 2000), who influenced others against the support of Chandrasekhar's work. Eddington claimed that the collapse would not occur due to something as yet unknown. This was later confirmed for only a specific case of a white dwarf.

The Chandrasekhar limit is an upper limit to the maximum mass of an ideal white dwarf (Gursky et al., 2000). This star is stable because of the Pauli exclusion principle (Pauli, 1925). In 1932, Landau declared that for masses bigger than a critical mass and considering quantum theory, nothing stops the system from breaking down to a singular point (Landau, 1932).

On the other hand, for a star with a mass greater than this limit, it will undergo further gravitational collapse. A super red giant star will become a supernova and this will transform into either a neutron star or a black hole. In 1939, it was proposed that supernova remnants exceeding the Tolman-Oppenheimer-Volkoff limit $(1.5 - 3.0M_{\odot})$ would collapse into black holes, verifying Chandrasekhar's predictions (Heiselberg and Pandharipande, 2000).

Oppenheimer, Snyder and Datt (OSD) thoroughly investigated the collapse of a spherically symmetric, homogeneous and marginally bound dust cloud (Datt, 1938), (Oppenheimer and Snyder, 1939). The interior spacetime was regarded to be a marginally bound Friedmann-Robertson-Walker (FRW) (2.2.2), which was sliced at an appropriate boundary and paired to the static Schwarzschild exterior.

Oppenheimer and his colleagues comprehended that the singularity at the boundary of the Schwarzschild radius meant that time was standing still (Oppenheimer and Snyder, 1939). External viewers could watch the exterior of the star frozen in time, at the very moment when the collapse is reduced just enough to squeeze inside the Schwarzschild radius (Schwarszchild, 1916a), (Gursky et al., 2000).

The OSD model infers to the illustrious ideas of trapped surfaces and black holes (Joshi and Malafarina, 2011). All that remains after termination of the collapse is a vacuum Shwarzschild spacetime geometry, whereby the event horizon at the Shwarzschild radius ($r = 2m$) (which is also known as a null surface), conceals the central spacetime singularity. No null or timelike curves reach the observer at infinity, due to the boundary of the black hole at $r = 2m$ (Dowker, 2013). The trapped region of the interior collapsing spacetime emerges before the singularity, wherein outward and inward non-spacelike congruences have a negative expansion and converges in the future.

Figure 1.3 portrays a spacetime diagram for a black hole in an Oppenheimer-Snyder (OS) collapse. The apparent horizon is the boundary of the trapped surface and is concurrent with the event horizon at the boundary of the cloud. We delve deeper into the understanding of these concepts later in Chapter 3.

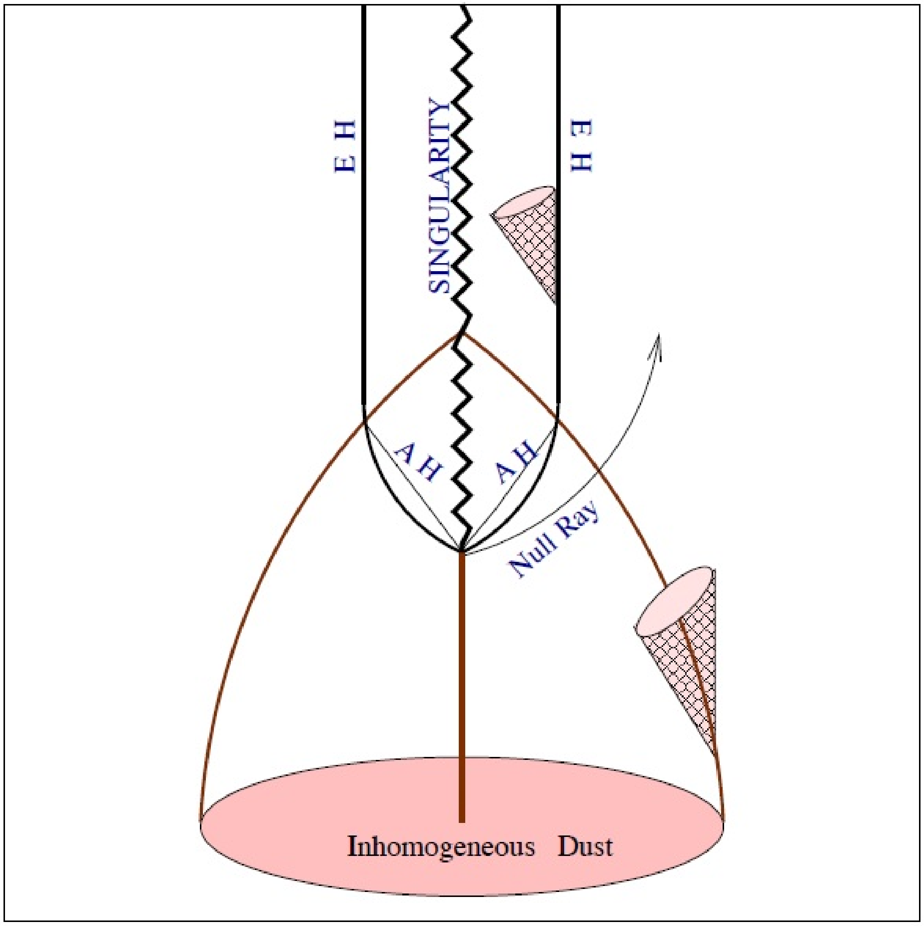

Figure 1.3: Spacetime diagram for a black hole in an OS collapse.

Ideally, the ultimate end of a gravitational collapse has to be a black hole (which hides a singularity) and not a naked singularity. Alternatively, a singularity collapsed result essentially needs to be concealed within a black hole where it is disconnected from viewers at infinity, though a more detailed mathematical proof is needed!

1.4 Gravitational collapse

When an object becomes unbalanced, due to its own gravity exceeding the internal outward pressure, it undergoes a development called gravitational collapse (Oppenheimer and Snyder 1939), (Joshi and Goswami, 2007). This occurs for a star that runs out of nuclear fuel, which sustains its temperature and pressure through the process of nucleosynthesis, or when increased matter has been obtained where its high core temperature is thus unbalanced (Joshi and Malafarina, 2011). Either way, it collapses under its own gravity. Due to degeneracy pressure, the collapse can result in a remnant such as a white dwarf.

The collapse will definitely occur if the remnant mass is greater than the Tolman-Oppenheimer-Volkoff limit ($1.5 - 3.0 M_{\odot}$) (Oppenheimer and Snyder, 1939). This retained amount of mass could be a result of an initially dense star or the remnant retained extra matter through the process of accretion. The formation of black holes are thought to be a result of the gravitational collapse of heavy stars (Oppenheimer and Snyder, 1939), (Joshi and Malafarina, 2011).

An unusual scenario of a gravitational collapse of a significantly large stellar mass was initially conceived by Steinmuiller, King and Lasota (1975): where a collapsing star is continuously radiating, thereby shedding it's mass and hence the surface of the star never breaches the Schwarzschild radius. Banerjee, Chatterjee and Dadhich (2002) also used a similar mechanism to produce a spherical collapse with heat flow and no horizons. There is no trapped region in the spacetime, and when the shells reach the central point, all the matter is radiated away leaving a flat spacetime. The exact mechanisms by which the collapsing star can transfer radiation/matter to the external spacetime was studied by numerous authors thereafter.

One can match the collapsing star with a non-comoving (evaporating) boundary to a purely radiating Vaidya exterior (De Oliveira et al., 1985), (Fayos et al., 1991), (Fayos et al., 1992), (Fayos et al., 1996), (Senovilla, 1998), (Fayos et al., 2003), (Fayos and Torres, 2004). The matching can also be done at a comoving stellar boundary to a generalised Vaidya exterior (see Goswami and Joshi 2007 and the references therein). The matter form for all the models considered in this scenario are very particular, like a special kind of self interacting scalar field (Goswami et al., 2006), or matter fields that have a specific form of negative pressure in the later stages of collapse.

Both black holes and naked singularities develop as a result of continual gravitational collapse (Joshi, 2000). There remains the issues of genericity and stability of naked singularities, though many investigations have been made concerning their astrophysical consequences.

1.5 Black hole physics

When a dense stellar body does not burn nuclear fuel and does not generate enough thermal pressure to counteract its own gravity, it undergoes a gravitational collapse, the final state of which is termed a black hole (Joshi and Malafarina, 2011). In contrast to normal stars of similar mass, compact bodies have a smaller radii owing to a stronger surface gravity.

Black holes are characterised according to their mass M, spin parameter a, denoted such that the angular momentum of the black hole is $J = aM$ and electric charge Q (Narayan, 2005). A black hole is unlikely to have a substantial amount of electric charge as it would normally be quickly neutralised by the encompassing plasma (Narayan, 2005).

A black hole can be distinguished by M and a, where a is restricted within the range of 0 (which is non-rotating) and M (rotating at its maximum) (Narayan, 2005). The elemental natural phenomenon resulting in the formation of black holes is thought to be the gravitational collapse of dense stellar bodies like stars, but there are even stranger transformations that can evolve into black holes (Hooft, 2009).

Nothing can escape a black hole! This phenomenon is an area of spacetime that displays powerful gravitational effects. Particles and even light (being one of the consequences of electromagnetic radiation) can not escape from within the sphere of the black hole (Narayan, 2005).

GR is a theory which foresaw that with enough mass the stellar body could mold spacetime to form a black hole. Here, an event horizon is a boundary in spacetime where escape is impossible (Narayan, 2005). The only effects known are in relation to passing by objects.

1.5.1 Blackbody

A blackbody is an idealised physical body that absorbs all radiation and emits an equal amount of radiation. The most natural occurring ideal black bodies are black holes because they do not reflect any light (Narayan, 2005).

In quantum field theory, it is predicted by the curvature of spacetime that event horizons emit Hawking radiation (Hawking, 1974), (Cai et al., 2009). This radiation emission is identical to the spectrum of a blackbody which has a temperature inversely proportional to the mass of the stellar object (Cai et al., 2009). These temperatures are too high to be observed.

1.6 The Raychaudhuri equation

Initially the spacetime singularity was thought to be a simple outcome of Einstein's equations using exact symmetry conditions instead of being seen as a direct result of GR itself. It was believed that the singularity would disappear once realistic conditions were implemented. The exact symmetries of homogeneity and isotropy would not be applied.

In 1955, Raychaudhuri worked towards such a general result. The Raychaudhuri equation broadly expresses a rate of change regarding the expansion of the volume which moves congruently along the timelike geodesic curves,

$$\frac{d\theta}{d\tau} = -R_{ik}U^iU^k - \frac{1}{3}\theta^2 + 2\sigma^2 + 2\omega^2, \tag{1.6.1}$$

where θ, σ and ω are the volume expansion, shear and vorticity respectively, of the timelike congruence whose unit tangent vector is U^i and τ is the proper time along the geodesics (Raychaudhuri, 1955).

When a spacetime meets the strong energy condition $R_{ik}U^iU^k \geq 0$, the focusing effect of gravity on congruence of timelike geodesics is revealed, these of which are hypersurface orthogonal and this is where vorticity disappears. Where two neighbouring geodesics encounter each other, conjugate points along a congruence exists when $\theta \to -\infty$ at those points (Joshi, 1993). So if the strong energy condition is met, then for converging congruence of timelike geodesics, which are hypersurface orthogonal, conjugate points appear in finite proper time.

In the 1960's and 1970's, singularity theorems formulated by Hawking, Penrose and Geroch used the Raychaudhuri equation to demonstrate that the singularities produced in general relativity are not an aftermath of the exact symmetries. They presented a thorough investigation of global properties of a general spacetime. They found that under specific but common and physically reasonable conditions, such as allowing energy to be positive, the occurrence of trapped surfaces and an appropriate causality condition, influenced the outcome of the properties. The spacetime singularities will be an unavoidable characteristic in accordance to the variety of gravitation theories which describes the gravitational force.

The causal structure of spacetime along with the gravitational focusing effect on the families of timelike and null geodesic congruences are studied. Many singularity theorems substantiating the non-spacelike geodesic incompleteness in M under various sets of conditions are relevant to unique physical scenarios. The Hawking-Penrose theorem is the most generalised and relative to both a collapse model and cosmological model (Hawking and Penrose, 1970).

An effortless structure analysis demonstrates the existence of a maximal length timelike curve between particular pairs of events which occur in the spacetime (Joshi, 2007). Given any chosen geodesic, which is both past and future complete, it has to have a set of conjugate points if M meets the generic condition and an energy condition (Joshi, 2007). The manifestation of a conjugate point would refute the maximal nature of the curve, therefore obtaining a longer non-spacelike curve between the given points at the conjugate points: thus proving M to be non-spacelike and geodesically incomplete.

The exact global definition of a black hole is given with respect to a future asymptotically foreseeable spacetime M,

$$B = M - J^{-}(I^{+}). \tag{1.6.2}$$

Here B denotes the black hole region and $J^{-}(I^{+})$ is the causal past of the future null infinity (Joshi, 2007). The topological boundary of B is represented as,

$$H = \dot{J} - (I^{+}) \cap M, \tag{1.6.3}$$

where H is defined as the event horizon (Joshi, 2007). This horizon has to be an achronal surface, as indicated by $\dot{J}$, generated by null geodesics that could have past end points in M, but that have no future end points. In the Minkowski spacetime, $J - (I+) = M$ and a black hole is non-existent. Yet, in the Schwarszchild scenario, $J - (I+)$ is the region for the spacetime exterior to $r = 2m$, the event horizon is given by the null hypersurface $r = 2m$, which acts as the boundary of the black hole region $0 < r < 2m$. It can be shown that the region B is closed in M, which indicates that the event horizon is contained in B.

An event horizon in the Schwarzschild case has to be an achronal surface which has been created by null geodesics that can have past end points in M yet no occurrence of future end points (Joshi, 2007). Tried and true for a future asymptotically predictable spacetime, consisting of a black hole region, there does not exist any spacetime singularities where a trapped region is viewable from a future null infinity (Hawking and Ellis, 1973).

The OS collapse model describes an event horizon as a null hypersurface created by null geodesics which happens to arrive at the surface of the stellar body when it crosses the radius $r = 2m$, where m is the Schwarzschild mass of the stellar body (Joshi, 2007). The gradual growth of the horizon field eventually stops once it extends to the surface of the stellar body. The exterior region has an invariable area of $A = 16\pi m^2$ (Joshi, 2007)).

Strong spacetime curvature singularities possibly happening in nature, during the collapse of massive stars, are yet to be confirmed by singularity theorems; they are compelled to be hidden below the event horizon of gravity. It is also possible for these singularities to be naked and perceptible to outside observers at a far distance thus changing the mechanics of the outside universe. The changes would mean that the spacetime manifiold M would not be globally hyperbolic or future asymptotically known. The OS collapse model describes a singularity to be totally encrusted by the event horizon and can not be visible to any outside observers (Oppenheimer and Snyder, 1939).

1.7 Singularities and singularity theorems

In this chapter we take a look at the definition of a singularity and the theorems regarding it. We recognise the major problem concerning this phenomenon.

1.7.1 Singularities

In 1932, Chandrasekhar expressed that for any star undergoing collapse having exceeded his 'Chandrasekhar limit,' it will inevitably form into a singularity. According to GR, there exists a gravitational singularity situated at the center of a black hole.

This phenomenon is a region in spacetime curvature in which tidal gravitational forces become infinite (Joshi and Malafarina, 2011). This region develops into a singular point for a non-rotating black hole.

However, for a rotating black hole, it is otherwise spattered, creating a ring singularity on the rotation (Joshi and Malafarina, 2011). This singular region has zero volume for either situation, as aforementioned that all mass has been crushed by gravity. The entire mass of the black hole solution is found in this singular region. Thus, infinite density is believed to reside within this gravitational singularity (Joshi and Malafarina, 2011).

Singularities must be concealed from an observer at infinity by the event horizon of black hole. Mathematically, for a general set of initial data, the maximal Cauchy progress features a complete future null infinity (Eardley and Smarr, 1979).

For further understanding of gravitational collapse, black holes and singularities, the reader is encouraged to seek out, 'Recent Developments In Gravitational Collapse and Spacetime Singularities,' by Joshi and Malafarina (2011) and 'Spherical dust collapse in higher dimensions' by Goswami and Joshi (2004) for a general description on how such states can result in a black hole or singularities.

1.7.2 Naked singularities

It is described in GR that a naked singularity is a gravitational singularity which excludes the occurrence of an event horizon (Goswami and Joshi, 2007). In a black hole, the boundary known as the event horizon encloses the singularity entirely. Inside the event horizon light cannot escape due to the intensity of the gravitational force of the singularity. Therefore, entities inside the event horizon, including the singularity, are unobservable to outside spectators.

A naked singularity is observable to outside spectators. The naked singularity theory signifies that its existence would allow the entity, which collapses to an infinite density, to be observable. There is speculation that a naked singularity may emit light (Joshi, 2002). The fundamentals of GR would break down as the future evolution of spacetime near a singularity is unknown. Nonetheless, in black holes, an outside observer is unable to witness the spacetime near the singularity due to the event horizon boundary.

It has been demonstrated that the formation of the apparent horizon can be delayed when shear falls close enough to the singularity at a slow rate (Goswami and Joshi, 2007). This leaves the singularity to be visible by an outside observer. Conversely, when shear quickly decreases all the way to zero at the central point of the stellar body, a black hole occurs, hiding the singularity from outside observers (Goswami and Joshi, 2007).

If loop quantum gravity is accurate, research has hinted that naked singularities may naturally occur in spacetime defying the cosmic censorship conjecture. Gravitational waves at LIGO were first discovered after two black holes collided, consequently named $GW150914$ (George and Huerta, 2017). This collision however did not result in an observable naked singularity.

Yodzis, Seifert and Müler Zum Hagen showed for the first time, theoretical examples of naked singularities (Yodzi et al., 1973). They described shell crossing globally naked singularities which occur in spherical collapses of perfect fluids; where shells of matter undergo implosions. The implosions occur in a manner where fast traveling exterior shells surpass the interior shells and some curvature components expand (Joshi, 2007).

These shell crossing singularities are gravitationally weak distributional singularities and are extendable (Clarke, 1993). Shell focusing strong naked singularities were discovered at the central point of spherically symmetric collapsing structures of inhomogeneous dust, shells of radiation or perfect fluids, further validating their findings (Joshi, 2007).

Models developed by Szekeres (1960), Christodoulou (1984), Newman (1986) and numerical works by Eardley and Smarr (1979) substantiated the existence of naked singularities as the end state of a continual collapse, where the physical radius of all collapsing shells disappear. In these models, the matter form was taken to be marginally or non-marginally bound dust, assuming that the initial data functions are smooth and even profiles.

1.7.3 Singularity theorems

The singularity theorems predicting the occurrence of spacetime singularities contain three main assumptions; under which the existence of a singularity is predicted in the form of geodesic incompleteness in the spacetime. These are in the form of a typical causality condition which ensures a suitable and physically reasonable global structure of the spacetime, an energy condition which requires the positivity of energy density at the classical level as seen by a local observer and finally a condition demanding that trapped surfaces must exist in the dynamical evolution of the universe, or in the later stages of a continual gravitational collapse (Goswami and Joshi, 2007).

A trapped region in spacetime consists of trapped surfaces; whereby these 2-surfaces, both inflowing as well as outflowing wavefronts, normal to the same, have no alternative but to converge (Goswami and Joshi, 2005). These trapped surfaces result in a spacetime singularity for both a gravitational collapse or in general cosmology.

Penrose-Hawking singularity theorems deem singularities to be unavoidable in suitable physical conditions (Penrose, 1974). The strong cosmic censorship conjecture declares that GR depicts the universe as deterministic, but only without naked singularities. From the initial data the classical outcome of all observers can be predetermined.

Mathematically, the maximal Cauchy evolution of the general set or asymptotically flat initial data is locally unchanged as a typical Lorentzian manifold (Sbierski, 2014). Evolution of the universe could be predicted but infinite regions of space concealed inside event horizons of singularities are excluded. This prediction can take place by knowing its current development at a specific time, more accurately everywhere on a Cauchy surface, which is a spacelike 3-dimensional hypersurface.

1.8 Purpose and intention

In this dissertation, we examine the role of an equation of state for an isentropic perfect fluid stellar body, which must be conducive for a collapse formation delaying or preferably excluding any trapping of matter but in collaboration with the fulfillment of the weak and dominant energy conditions (Joshi, 2007). The perfect fluid form of matter is used because it acknowledges various physically realistic equations of state and has been studied thoroughly within astrophysical environments.

Our investigation clearly shows the classes of equation of state functions and the bounds on the thermodynamic potentials which creates such a phenomenon. We explicitly tie in our findings to the enthalpy and acceleration potentials of the collapsing matter.

We demonstrate that these classes have non-zero measure in the function space. Thus, these models are generic in nature and detached of the issues or paradoxes affiliated with horizons (Black Hole Paradox) and the infinite density spacetime singularities.

In these models, no trapped surfaces are created as the collapse evolves in time. Therefore, the matter of the stellar body simply escapes or radiates away during the last stages of gravitational collapse (Joshi, 2007). We impose physically reasonable conditions for such a case to occur. Regularity of the initial data and weak energy condition is required. We can permit pressure to become negative.

The latest high resolution radio telescopes (like the Event Horizon Telescope) should, in principle, detect the presence of these dense bodies in the sky as the observational signatures of these continually collapsing but non-trapped compact bodies will vary from those of a black hole.

The dissertation has been organised as follows: In the following chapter, we define some basic concepts, have a look at a few historically important spherically symmetric solutions and unravel the general solution for the collapsing perfect fluid in terms of the thermodynamic quantities. We set out field equations along with the regularity and energy conditions for the collapsing matter. Chapter 3 comprises of the conditions for no-trapping; we delve into the different terms regarding trapped regions and we well-define our results in association with the thermodynamic potentials of the collapsing matter. Lastly, in the final chapter we discuss two main problems, the singularity problem and the black hole paradox, where we explain how we have resolved them. We then make some concluding points concerning the ultimate end state of the models we have regarded here.

Chapter 2

Spherically symmetric solutions of the field equations

We start this chapter with some noteworthy basic concepts and dive into some well established spherically symmetric solutions such as the Schwarzschild solution. This allows us to then build on our implementation of finding new solutions to spherically symmetric perfect fluids and a metric in terms of a function of thermodynamic quantities.

2.1 Basic concepts

The geometry of spacetime is formulated by the metric,

$$ds^2 = g_{ab}dx^a dx^b, \tag{2.1.1}$$

where g_{ab} is the metric tensor. The curvature of spacetime is described by the Riemann curvature tensor,

$$R_{efbc} = g_{ea}R^a{}_{fbc}, \tag{2.1.2}$$

determined by,

$$R^d{}_{abc} = \partial_b \Gamma^d{}_{ac} - \partial_c \Gamma^d{}_{ab} + \Gamma^e{}_{ac}\Gamma^d{}_{eb} - \Gamma^e{}_{ab}\Gamma^d{}_{ec}. \tag{2.1.3}$$

An array of numbers which formulate a metric connection are called Christoffel symbols. This supports the study of the geometry of the metric. The Christoffel symbols are determined by,

$$\Gamma^a{}_{bc} = \frac{1}{2} g^{am} \left(\frac{\partial g_{cm}}{\partial x^b} + \frac{\partial g_{mb}}{\partial x^c} - \frac{\partial g_{bc}}{\partial x^m} \right). \tag{2.1.4}$$

The Ricci tensor or trace component of the Riemann tensor is affined to the changing volume of a region of space. The Ricci tensor depicts the volume magnification effects. By further contracting two indices from (2.1.4), we obtain the Ricci tensor,

$$R_{ab} = g^{cd} R_{cadb}. \tag{2.1.5}$$

Contracting the Ricci tensor (2.1.5) provides us with the Ricci scalar curvature,

$$R = g^{ab} R_{ab}. \tag{2.1.6}$$

The Einstein tensor is determined from the Ricci tensor (2.1.5) and Ricci scalar (2.1.6) as,

$$G_{ab} = R_{ab} - \frac{1}{2} R g_{ab}. \tag{2.1.7}$$

The law of the gravitational field is,

$$R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4} T_{ab}, \tag{2.1.8}$$

which is the equivalent of the Newtonian theory of the gravitational field (34) being,

$$\nabla^2 \phi = 4\pi G \rho. \tag{2.1.9}$$

The equation (2.1.9) is known as Poîsson's equation for gravity.

The gravitational field transfers energy and momentum to the matter. GR relates the geometry of spacetime to matter through Einstein's field equations where we take $c = 1$ resulting in,

$$R_{ab} - \frac{1}{2} R g_{ab} = -8\pi T_{ab}, \tag{2.1.10}$$

where T_{ab} is the energy-momentum tensor.

For further understanding, it is suggested that the reader have a look at the original works (Einstein and Grossman, 1913), (Einstein and Fokker, 1914), (Einstein, 1914i) and especially ('The Foundation of the General Theory of General Relativity,' Einstein, 1916e).

2.2 Some well-known spherically symmetric solutions

Conclusions for the various dynamical collapse models can be made. The collapse always creates curvature generated spheres of heat featured by diverging curvatures and densities. Trapped surfaces may not develop early enough to always hide this process from an external observer.

2.2.1 Schwarzschild solution

In 1916, Karl Schwarzschild calculated the exact solution to the Einstein field equations (Schwarszchild, 1916a and 1916b). It defined the gravitational field of a point mass and a spherical mass. The disturbing outcome of this solution is a singularity, which is a direct result of the Schwarzschild radius, where terms of the Einstein equations turned out to be infinite (Schwarszchild, 1916a and 1916b)!

Later, it was revealed that by using different coordinates, the singularity vanished. Eddington claimed it was possible that a star with a mass, whose density is not great, could be compressed into the Schwarzschild radius (Gursky, Ruffini and Stella, 2000). Only in 1933, was it understood, by Lemaître, that the singularity of this Schwarzschild radius was not a physical coordinate singularity.

This solution has been popularly used in experimentations to prove the predictions of GR. This is a hollow outer result where the Ricci tensor disappears and is compatible at the boundary to the inner result residing within the structure. The metric coordinates is written in the coordinates (t, r, θ, ϕ) as,

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{2.2.1}$$

where $r > 2m$.

This solution (2.2.1) is asymptotically flat. The spacetime is spherically symmetric, therefore it is static and the gradient is a timelike killing vector. When likened to Newtonian theory, m is taken as the gravitational mass, as calculated from infinity, of the object making the field. Spherically symmetric solutions of the vacuum field equations are locally isometric to the Schwarzschild solution (as this is a unique solution) (Birkhoff, 1923).

2.2.2 Friedmann Robertson-Walker

In cosmology, the FRW solutions inside the model of an isotropic and homogeneous universe can be dealt with mathematically (Friedmann, 1922), (Robertson, 1933), (Walker, 1944). The Robertson-Walker geometry adapted significant scale homogeneity and isotropy of the universe. On the premise of symmetry, the spacetime metric in co-moving coordinates (t, r, θ, ϕ) is set out as,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]. \tag{2.2.2}$$

This line element contains a constant k describing the characteristic curvature of the 4−space. The values of $k \in \{-1, 0, 1\}$ represent negative, zero and positive curvature respectively.

2.2.3 Lemaître, Tolman and Bondi

Lemaitre, Tolman and Bondi introduced inhomogeneity into the equation (2.2.2) and found an exact cosmological solution to the Einstein's equations for spherically symmetric dust-like matter. This is known as the LTB metric which is written in co-moving coordinates (t, r, θ, ϕ) as,

$$ds^2 = -dt^2 + \frac{R'^2}{1 + r^2 b_0(r)} dr^2 + R^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{2.2.3}$$

where $R(t, r)$ is the given radius of the area of the collapsing shells of dust and $b_0(r)$ is the energy distribution function for the shells.

There are various exact solutions to the Einstein equations which are found using different symmetrical treatments. The system of differential equations are extremely non-linear and because of their intricacy, a fully realised general solution is yet to be discovered.

The spacetime diagram for a naked singularity in LTB collapse is depicted in Fig 2.1. In the design the inhomogeneous dust, the null ray, the singularity, the event horizon and the apparent horizon can be seen.

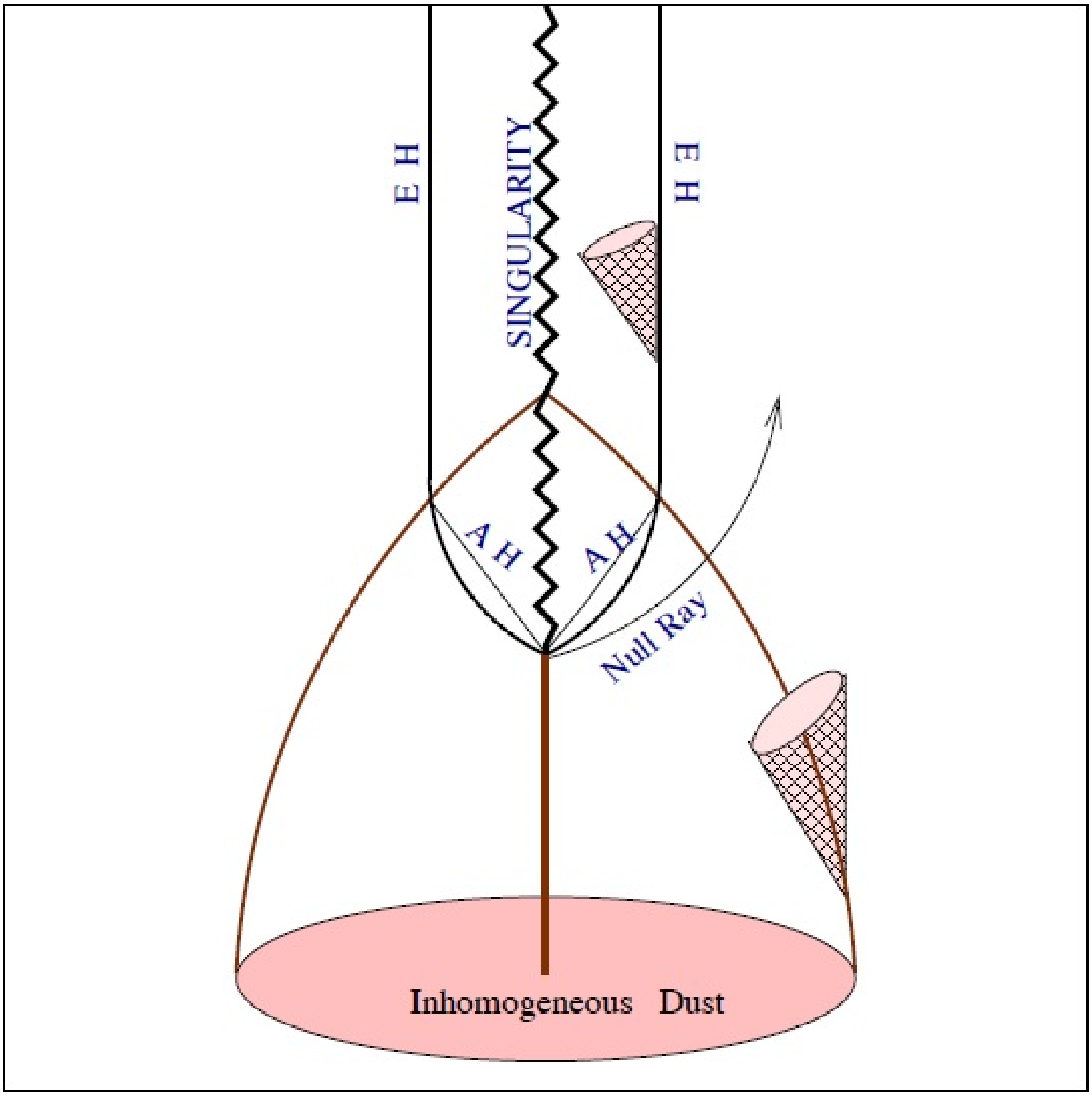

Figure 2.1: Spacetime diagram for a naked singularity in a LTB collapse.

2.3 Finding new spherical symmetric solutions for a perfect fluid

Now we give a general framework to find new spherically symmetric solutions for a perfect fluid. We consider here the spacetime geometry, the Einstein equations and the regularity conditions.

2.3.1 Spacetime geometry

Let's begin with the spacetime metric. This metric describes the shape and structure of all spacetime. It consists of the time, distance, volume, curvature and the angle. The past and future can be distinguished. The spacetime is described by a four-dimensional pseudo-Riemannian manifold M.

The metric tensor is presented as a covariant, second-degree, symmetric tensor on M, usually represented by g. The metric has to be non-degenerate having a signature of $(-, +, +, +)$. The manifold M that provides this specific metric is a form of the Lorentzian manifold.

2.3.2 The Einstein equations

We construct a general class of solutions to the Einstein field equations. We begin at the initial epoch where matter is considered as a regular isentropic perfect fluid, having a linear equation of state, $p = k\rho$. At the initial surface we consider a positive pressure for the fluid because of the energy conditions. As the collapse progresses, the perfect fluid is gradually allowed to diverge from being completely isentropic. The collapse and Einstein equations influence a change in the equation of state.

In this class of solutions, as the collapse proceeds, the pressure decreases monotonically and eventually becomes negative as it borders on the singularity. The physical process of trapped surfaces is thus avoided in the spacetime. The weak energy condition is conserved during the entire collapsing period. A generalised Vaidya exterior spacetime is made comparable to the spherical collapse of the interior. The energy of the collapsing structure dissipates through radiation until it is comparable to a Minkowski spacetime.

We introduce a general line element for a spherically symmetric matter distribution of a collapsing perfect fluid in co-moving spherical coordinates ($x^i = t, r, \theta, \phi$) as,

$$ds^2 = -e^{2\nu(t,r)}dt^2 + e^{2\psi(t,r)}dr^2 + R^2(t,r)d\Omega^2, \tag{2.3.1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi$. We note that $\nu(t,r)$ and $\psi(t,r)$ represent arbitrary functions of the co-moving shell radius r at initial epoch t.

For a perfect fluid, the energy-momentum tensor is represented as,

$$T^r_r = T^\theta_\theta = T^\phi_\phi = p(t,r), \tag{2.3.2}$$

$$T^t_t = -\rho(t,r). \tag{2.3.3}$$

The symbols ρ and p correspond to the energy density and pressure respectively for the matter cloud. The weak energy conditions are thereafter fulfilled by the matter fields. In affect, the energy density (as measured by any local spectator) is non-negative and hence for any time-like vector V^i we must hold,

$$T_{ik}V^iV^k \geq 0, \tag{2.3.4}$$

which simplifies to $\rho \geq 0$ and $\rho + p \geq 0$.

We define the Misner-Sharp mass for the collapsing star (which is the mass inside a given co-moving radius r, at a given time t) as,

$$F = R(1 - R^{,a}R_{,a}) = R(1 - G + H), \tag{2.3.5}$$

where,

$$G(t,r) = e^{-2\psi}\left(R'\right)^2, \tag{2.3.6}$$

$$H(t,r) = e^{-2\nu}(\dot{R})^2. \tag{2.3.7}$$

In terms of the mass function, the Einstein field equations are given as,

$$F' = \rho R^2 R', \tag{2.3.8}$$

$$\dot{F} = -p R^2 \dot{R}, \tag{2.3.9}$$

$$p' = -\nu' \left(\rho + p\right), \tag{2.3.10}$$

$$R' \dot{G} = 2 \dot{R} \nu' G. \tag{2.3.11}$$

We note that $(\dot{})$ and $(')$ distinguishes between the partial derivatives with respect to t and r respectively.

The Einstein field equations, with regards to Einstein's GR theory, accounts for the fundamental interaction of gravitation due to mass and energy warping spacetime. For our given scenario, we have obtained these equations to further our specific investigation of these gravitational collapse models.

2.3.3 The regularity conditions

We impose the condition $F(t_i, 0) = 0$ to preserve regularity at the initial epoch. Since we want to examine the collapsing class of solutions to the Einstein equations, we must enforce the condition $\dot{R} < 0$ on the area radius R. This enforcement results in the area radius of all shells of the continual collapsing cloud to monotonically diminish to zero (forming the spacetime singularity), $R(t_s(r), r) = 0$ and where time $t = t_s(r)$ is the time taken for a shell labelled r to reach the singularity. At the time $t = t_i$, the radius $R = r$, due to the use of the scaling freedom for the radial coordinate r.

We proceed with an arbitrary scaling function,

$$a(t, r) \equiv \frac{R}{r}, \tag{2.3.12}$$

resulting in,

$$
\begin{aligned}
R(t,r) &= ra(t,r), & (2.3.13)\\
a(t_i,r) &= 1, & (2.3.14)\\
a(t_s(r),r) &= 0, & (2.3.15)
\end{aligned}
$$

where $\dot{a} < 0$. The regularity conditions suggests that $F \approx r^3$ close to the center. The normal structure of F is thus,

$$F(t,r) = r^3 M(r,a), \tag{2.3.16}$$

where we have $M(r,a)$ to be some general function as restricted by the regularity conditions and energy conditions.

The regularity condition is a concept where stochastic occurrences corroborate symmetry when sufficiently recurrent or where adequately similar stochastic occurrences demonstrate geometrical regularity. We now have set conditions regarding the structure of our gravitational collapse model.

2.4 The metric as a function of thermodynamic quantities

It is important to find the metric as a function of thermodynamic quantities because it is a more physically accurate description of the variables, such as density and pressure, involved in the gravitational collapse model.

To obtain the general solution of the metric functions in terms of the thermodynamic quantities at any epoch, we perform a change of variables from (t,r) to (a,r). In that case for any function $\Phi(t,r)$ we must have,

$$\dot{\Phi} = \Phi_{,a}\dot{a}, \tag{2.4.1}$$

$$\Phi' = \Phi_{,r} + \Phi_{,a}a_{,r}. \tag{2.4.2}$$

Also for integrating $G^0_1 = T^0_1$, for equation (2.3.11) we define,

$$\frac{\nu'}{R'} \equiv A(r,a)_{,a}, \tag{2.4.3}$$

where $A(r,a)$ is an arbitrary function of the coordinates r and a. Now we can directly integrate equation (2.3.11) to obtain,

$$G(r,a) = [1 + r^2 b_0(r)]e^{2rA}, \tag{2.4.4}$$

where $b_0(r)$ is a free function of integration. Therefore from (2.3.7) we have,

$$H(t,r) = e^{-2\nu}\left(\dot{R}\right)^2, \tag{2.4.5}$$

$$e^{2\psi} = \frac{(R')^2}{[1 + r^2 b_0(r)]e^{2rA}}. \tag{2.4.6}$$

Now we can rewrite the definition of the Misner-Sharp mass in terms of these new variables in the following way:

$$G - H = 1 - \frac{F}{R}, \tag{2.4.7}$$

$$[1 + r^2 b_0(r)]e^{2rA} - e^{-2\nu}(\dot{R})^2 = 1 - \frac{r^3 M}{ra}, \tag{2.4.8}$$

$$a(\dot{a})^2 e^{-2\nu} = M + a\left(\frac{1 - e^{2rA}}{r^2}\right) - e^{2rA}ab_0(r), \tag{2.4.9}$$

$$\sqrt{a}(\dot{a}) = -e^{\nu}\sqrt{e^{2rA}ab_0(r) + ah - M}. \tag{2.4.10}$$

Note that the negative sign in (2.4.10) is distinctive of a collapse model. We also define the function h as,

$$h(r,a) = \frac{e^{2rA} - 1}{r^2}. \tag{2.4.11}$$

Integrating (2.4.10) we obtain the equation for the time taken for a shell labelled 'r', to reach the epoch 'a' as,

$$t(a,r) = \int_a^1 \frac{\sqrt{a}da}{e^{\nu}\sqrt{e^{2rA}ab_0 + ah + M}}. \tag{2.4.12}$$

The above equation (2.4.12) is the solution for the scaling function $a(t,r)$ in the integral form. This immediately gives the singularity curve $t_s(r)$, which is the collapse end time where the shell labelled 'r' diminishes to a zero area radius ($R = 0$),

$$t_s(r) = \int_0^1 \frac{\sqrt{a}da}{e^{\nu}\sqrt{e^{2rA}ab_0 + ah + M}}. \tag{2.4.13}$$

We now have to relate the function $A(r,a)$ to the thermodynamic variables of the collapsing matter. To do so, we first write $G^0_0 = T^0_0$ and $G^1_1 = T^1_1$ as equations in terms of the new variables. In order to do this, we start with (2.3.10) and (2.3.9),

$$\rho = \frac{F'}{R^2R'}, \tag{2.4.14}$$

$$p = -\frac{\dot{F}}{R^2\dot{R}}. \tag{2.4.15}$$

Using (2.3.13) we find its variations,

$$R^2 = r^2a^2, \tag{2.4.16}$$

$$R' = a + ra', \tag{2.4.17}$$

$$\dot{R} = \dot{a}r. \tag{2.4.18}$$

Using (2.3.16) we find its variations,

$$\dot{F} = r^3M_{,a}\dot{a}, \tag{2.4.19}$$

$$F' = 3r^2M + r^3[M_{,r} + M_{,a}a']. \tag{2.4.20}$$

For density we then find,

$$\rho = \frac{F'}{R^2 R'}, \tag{2.4.21}$$

$$= \frac{3M + r[M_{,r} + M_{,a}a']}{a^2[a + ra']}. \tag{2.4.22}$$

For pressure we find,

$$p = -\frac{\dot{F}}{R^2 \dot{R}}, \tag{2.4.23}$$

$$= -\frac{M_{,a}}{a^2}. \tag{2.4.24}$$

Substituting (2.4.24) into the density equation (2.4.22) and rearranging, we solve for a',

$$\rho = \frac{3M + rM_{,r} + rM_{,a}a'}{a^3 + ra^2a'}, \tag{2.4.25}$$

$$a'[ra^2\rho - rM_{,a}] = 3M + rM_{,r} - \rho a^3, \tag{2.4.26}$$

$$a' = \frac{3M + rM_{,r} - \rho a^3}{ra^2(\rho + p)}. \tag{2.4.27}$$

To solve A we first find a' from the definition (2.4.3) and (2.3.10) to obtain,

$$A_{,a} = \frac{\nu'}{R'}, \tag{2.4.28}$$

$$A_{,a}a + A_{,a}ra' = -\frac{(p_{,r} + p_{,a}a')}{p + \rho}, \tag{2.4.29}$$

$$a' = \frac{-p_{,r} - (p + \rho)A_{,a}a}{r(p + \rho)A_{,a} + p_{,a}}. \tag{2.4.30}$$

We equate (2.4.27) and (2.4.30),

$$\frac{3M + rM_{,r} - \rho a^3}{ra^2(\rho + p)} = \frac{-p_{,r} - (p + \rho)A_{,a}a}{r(p + \rho)A_{,a} + p_{,a}}. \tag{2.4.31}$$

We then use (2.4.31)to solve for $A_{,a}$,

$$[3M + rM_{,r} - \rho a^3][r(\rho+p)A_{,a} + p_{,a}] = [-p_{,r} - (\rho+p)A_{,a}a][ra^2(\rho+p)], \tag{2.4.32}$$

$$A_{,a} = \frac{[3M + rM_{,r} - \rho a^3]r(\rho+p) + (\rho+p)a[ra^2(\rho+p)]}{-p_{,r}[ra^2(\rho+p)] - p_{,a}[3M + rM_{,r} - \rho a^3]}, \tag{2.4.33}$$

$$A_{,a} = \frac{-p_{,r}[ra^2(\rho+p)] - [3M + rM_{,r} - \rho a^3]p_{,a}}{[3M + rM_{,r} - \rho a^3][r(\rho+p)] + [ra^3(\rho+p)^2]}, \tag{2.4.34}$$

where $p_{,r} = c_s^2\rho_{,r}$, $p_{,a} = c_s^2\rho_{,a}$ and c_s is the speed of sound.

Therefore, we can write the function $A(r,a)$ as,

$$A = \int_1^a \frac{-p_{,r}[ra^2(\rho+p)] - [3M + rM_{,r} - \rho a^3]p_{,a}}{[3M + rM_{,r} - \rho a^3][r(\rho+p)] + [ra^3(\rho+p)^2]}da. \tag{2.4.35}$$

Now we can find the metric function e^ν from (2.3.10),

$$A_{,a} \equiv \frac{\nu'}{R'}. \tag{2.4.36}$$

. We find ν first,

$$\nu = \int A_{,a}R'dr, \tag{2.4.37}$$

$$= \int \left(\frac{-p_{,r}[ra^2(\rho+p)] - [3M + rM_{,r} - \rho a^3]p_{,a}}{[3M + rM_{,r} - \rho a^3][r(\rho+p)] + [ra^3(\rho+p)^2]}\right)(a + ra')dr, \tag{2.4.38}$$

$$= \int \left(\frac{-p_{,r}[ra^2(\rho+p)] - [3M + rM_{,r} - \rho a^3]p_{,a}}{[3M + rM_{,r} - \rho a^3][r(\rho+p)] + [ra^3(\rho+p)^2]}\right)\left(\frac{3M + rM_{,r} + pa^3}{a^2(\rho+p)}\right)dr. \tag{2.4.39}$$

Therefore the function e^ν is represented as,

$$e^\nu = \exp\left\{\int_0^r \frac{XY}{Z}\right\}dr, \tag{2.4.40}$$

where firstly $X = -p_{,r}[ra^2(\rho+p)] - [3M + rM_{,r} - \rho a^3]p_{,a}$, secondly $Y = 3M + rM_{,r} + a^3p$ and thirdly $Z = [ra^2(\rho+p)^2][3M + rM_{,r} - \rho a^3 + a^3(\rho+p)]$.

We note that the solution of M depends on the equation of state $p = p(\rho)$. Once the equation of state is supplied, then all the metric functions can be written explicitly in terms of the thermodynamic quantities of the collapsing matter field.

By knowing the evolution of thermodynamic quantities we can immediately obtain the metric quantities as we have demonstrated in this section. We can calculate the outcome of density, pressure etc. of the collapse model to better understand the dynamics of the structure as well as the expected physical outcomes. This can help us understand the natural characteristics of such a particular scenario.

Chapter 3

Conditions for no-trapping

In this chapter, we seek to understand what a trapped surface is and enlighten ourselves on the concepts of an apparent horizon versus an event horizon. We delve into spherically symmetric spacetimes, understand how trapped surfaces can be avoided and summarise our findings with regards to the condition where no-trapping occurs.

3.1 Trapped surfaces, apparent horizons and event horizons

When a continually collapsing star crosses it's own Schwarzschild radius, it gets trapped (Schwarzschild, 1916b). For all the collapsing shells, ingoing as well as outgoing null wavefronts, normal to these shells, converge; hence the matter must collapse to a central singularity. Existence of these closed trapped 2-surfaces (the collapsing shells after crossing the Schwarzschild radius) is key to all the singularity theorems developed by Hawking, Penrose and Geroch (Hawking and Ellis, 1973).

The process of formation of trapped surfaces, trapped regions and the boundary of these trapped regions, is also central to black hole physics. In the context of GR, these were first highlighted by (Datt, 1938), (Oppenheimer and Snyder, 1939), for collapse of pressureless homogeneous dust. It was shown that the entire star gets trapped some time before the formation of the central singularity, hence the central singularity can never be seen by distant observers.

Although the Oppenheimer, Snyder and Datt (OSD) model is extremely idealised, Penrose argued that any realistic gravitational collapse should be qualitatively similar to this model which became his famous Cosmic Censorship Conjecture (CCC) (Penrose, 1969).

3.1.1 Definition of a trapped surface

The idea of closed trapped surfaces is used in black hole solutions, depicting the inner region of an event horizon. Closed trapped surfaces was a concept described by Roger Penrose in 1965 (Penrose, 1965).

A trapped surface is a 2-dimensional embedded spatial surface, whereby the product of the traces of their two future-directed null, second fundamental forms, is positive at every point. In the physical sense it describes two families of future-directed null geodesics originally orthogonal to the surface area and at the same time diverging or converging. This idea is interpreted mathematically as sub-manifolds of co-dimension 2 in any Lorentzian manifold (V, g) of arbitrary dimension D (Mars and Senovilla, 2003).

Regarding a black hole, a trapped surface describes a region where light can only converge towards and not diverge away from the surface. The boundary consisting of all the trapped surfaces around a black hole is termed as an apparent horizon. Trapped surfaces can be spacelike, timelike or null (Nielsen, 2014). A trapped region in the spacetime is formed of trapped surfaces.

Trapped null surfaces are a set of points described as a closed surface on which outward pointing light rays converge. The apparent horizon is a trapped null surface, encompassing the black hole. It is a compact, orientable, spacelike surface where its outward pointing normal vectors are found. A surface is trapped when every null congruence orthogonal to a spacelike 2-surface has negative expansion (Booth, 2005).

3.1.2 Apparent horizon

From the concept of a trapped null surface emerges the concept of an 'apparent horizon'. An orientable, spacelike and compact surface has two independent lightlike, normal, future headed paths (Booth, 2005). The case of a spacelike sphere in Minkowski space has light-like vectors directed inward and outward on the radial path. The inward directed, lightlike, normal vectors converge. Whereas, the outward directed lightlike normal vectors diverge. If it so happens that the inward directed and outward directed, lightlike, normal vectors converge, then the surface is known as a trapped surface (Booth, 2005).

A surface acting as the boundary between outward directed light rays moving outwards and outward directed light rays moving inward, is called an apparent horizon.

Apparent horizons are not constant characteristics of a spacetime and are different from event horizons. Light is trapped in a black hole only when there exists an event horizon. Light does not travel away from a black hole within an apparent horizon. It is presently conceivable for light to travel away from a black hole such that it is outside the apparent horizon.

Nonetheless, due to the expanding mass of the black hole, this light is trapped inside the event horizon. Hence, light rays at the present moment in time are bounded by the apparent horizon of a black hole. Light rays in the future are restricted by the event horizon of the black hole (Booth, 2005). The event horizon is the absolute outermost boundary of a black hole.

For a stellar collapse developing into a black hole, an event horizon forms before an apparent horizon. As the black hole subdues, both horizons draw closer and asymptotically transform into one surface.

The existence of an apparent horizon may only reside within an event horizon. Apparent horizons rely on the location and separation of spacetime into space and time. The Schwarzschild geometry can be 'sliced' so that there is not, and will never be, an apparent horizon, whereas an event horizon typically exists.

3.1.3 Event horizon

An event horizon, (typically related to black holes), is the boundary at which nothing can leave due to the strong gravitational pull of a massive entity. The emission of light within the event horizon can never be observed from outside this boundary.

An entity moving towards the exterior of the event horizon seems to decelerate and appears to be incapable of infiltrating the horizon, appearing more redshifted over time. Therefore, it appears that the wavelength of the entity's light emission increases as it draws nearer to the event horizon. On the contrary, the moving entity does not endure any unusual effects and actually infiltrates the event horizon in a fixed period of real time.

Inside the event horizon, the speed of light is exceeded by the black hole's escape velocity. All lightlike paths and all paths in the forward light cones of particles within the horizon, are distorted. Therefore, these particles move deeper into the hole. A particle within the event horizon is destined to move into the hole and travel ahead in time on certain spacetime coordinate systems.

Event horizons interpreted by GR are considered geometrically incomplete. Many scientists suspect that there must be a link between GR and quantum mechanics. Today, the main influence of quantum effects is considered to be that event horizons have a temperature, thus emitting radiation.

Black holes emit what is called, Hawking radiation. Such phenomena constitutes to the need of understanding the subject of black hole thermodynamics. For rapidly moving particles, this occurs as the Unruh effect, where the surrounding space of particles are seemingly full of matter and radiation.

Thus, for a better comprehension of event horizons, a theory of quantum gravity is needed. The $M-$theory and loop quantum gravity theory are possibilities to complete the theory of GR to solve the singularity and/or black hole paradoxes.

3.2 Avoiding trapped surfaces

3.2.1 Spherically symmetric spacteimes

It is well known that for any spherically symmetric spacetimes, a shell labelled 'r' is trapped, if the Misner-Sharp mass enclosed by the shell is greater than the area radius of the shell. Therefore the spherical 2-surface labelled by the co-ordinate 'r' is trapped if $F > R$; whereas when $F < R$ the surface is not trapped. It is obvious then, that the boundary of the trapped region or the *apparent horizon* is described by the equation,

$$F = R. \tag{3.2.1}$$

3.2.2 The avoidance of trapped surfaces

For a continual collapsing matter cloud, regularity conditions imply the avoidance of trapped surfaces at the initial epoch. When the boundary of this cloud is $r = r_b$ and we enforce the condition $M_0(r_b)^2 < 1$, then the avoidance of trapped surfaces for any shell $r \leq r_b$ occurs at $t = t_i$. If we want to avoid trapping in the complete spacetime we must ensure that throughout the spacetime,

$$F < R. \tag{3.2.2}$$

This obviously implies that

$$G - H > 0. \tag{3.2.3}$$

Using the definition for the function H, we then obtain,

$$G - e^{-2\nu}(\dot{R})^2 \quad > \quad 0, \tag{3.2.4}$$

$$\left(\sqrt{G} - e^{-\nu}\dot{R}\right)\left(\sqrt{G} + e^{-\nu}\dot{R}\right) \quad > \quad 0. \tag{3.2.5}$$

Since $\dot{R} < 0$ during collapse, we then find (3.2.5) to be,

$$-e^{-\nu}\dot{R} \quad < \quad \sqrt{G}. \tag{3.2.6}$$

3.2.3 Summary of the conditions for no-trapping

We can now summarise the conditions for no-trapping in the following way:

Proposition 1. *Consider a continually collapsing spherically symmetric perfect fluid* *with an energy density $\rho(r,a)$ and pressure $p(r,a)$ where we have $r \in [0, r_b]$ and $a \in [0,1]$. If the following conditions are satisfied:*

1. *$\rho > 0$ and $\rho + p \geq 0$, $\forall\ r \in [0, r_b]$ and $\forall\ a \in [0,1]$,*

2. *$p(r,1) > 0$ for all $r \in [0, r_b]$,*

3. *$r\sqrt{e^{2rA}b_0 + ah + M} < \sqrt{a(1+r^2b_0)}e^{rA}$, $\forall\ r \in [0, r_b]$ and $\forall\ a \in [0,1]$, where the function $A(r,a)$ is given by equation, (2.4.35),*

then the collapsing spacetime will be devoid of any trappings irrespective of the weak energy condition being satisfied by the collapsing matter.

The second condition above, is to ensure that the collapse commences with positive pressure. However, in the process of collapse, the pressure can be negative without violating the energy conditions. The above conditions become remarkably simple in the case of a homogeneous collapse, given by the FRW metric,

$$ds^2 = -dt^2 + a^2(t)\left(dr^2 + r^2 d\Omega^2\right). \tag{3.2.7}$$

In this case, the equation $G^0_0 = T^0_0$ can be directly integrated to give,

$$M = \frac{1}{3}\rho a^3. \tag{3.2.8}$$

We note that ρ is increasing as t is increasing and a is decreasing as t is increasing. Therefore,

$$\frac{d\rho}{da} < 0. \tag{3.2.9}$$

Integrating the energy conservation equation,

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + p(\rho)\right), \tag{3.2.10}$$

gives,

$$-3\ln a = \int \frac{d\rho}{\rho p(\rho)} + c, \tag{3.2.11}$$

where c is some arbitrary constant. Rearranging equation (3.2.11) we have,

$$a^{-3} = c_1 \exp\left[\int \frac{d\rho}{\rho + p(\rho)}\right]. \tag{3.2.12}$$

Consequently,

$$a = c_1 \exp\left[-\frac{1}{3}\int \frac{d\rho}{\rho + p(\rho)}\right]. \tag{3.2.13}$$

Substituting (3.2.13) into (3.2.8) we have,

$$M = \frac{1}{3}\rho a^3 = \frac{c_1}{3}\rho \exp\left[-\int \frac{d\rho}{\rho + p(\rho)}\right]. \tag{3.2.14}$$

The condition for no trapping is now given by (as F is a monotonic function of r at any given epoch),

$$\frac{{r_b}^3 M}{r_b a} < 1. \tag{3.2.15}$$

Thus, the mass is represented as,

$$M < \frac{1}{{r_b}^2 a}. \tag{3.2.16}$$

Using equations (3.2.13), (3.2.14) and (3.2.16) the condition becomes,

$$\frac{c_1}{3}\rho \exp\left[-\int \frac{d\rho}{\rho + p(\rho)}\right] < \frac{c_1}{{r_b}^2} \exp\left[-\frac{1}{3}\int \frac{d\rho}{\rho + p(\rho)}\right]. \tag{3.2.17}$$

Simplification of equation (3.2.17) gives us,

$$\rho < \frac{3}{{r_b}^2} \exp\left[\frac{2}{3}\int_{\rho_0}^{\rho} \frac{d\rho}{\rho + p}\right], \tag{3.2.18}$$

where ρ_0 is the initial density from which the collapse starts and ρ is the final density of the collapse end state. To show that the set of equations of state which ensures such behaviour is indeed non-empty and an open set, we explore the equation of state,

$$p(\rho) = p_0 + p_1\rho, \tag{3.2.19}$$

where p_0 and p_1 are constants. Then we can directly integrate equation (3.2.18) to get the condition,

$$\rho < \frac{3}{{r_b}^2} \exp\left[\frac{2}{3}\int \frac{d\rho}{(\rho + p(\rho))}\right], \tag{3.2.20}$$

$$\rho < \frac{3}{{r_b}^2} \exp\left[\frac{2}{3}\left(\frac{1}{1+p_1}\right)\ln(p_0 + \rho + p_1\rho)\right], \tag{3.2.21}$$

$$1 < \frac{3}{{r_b}^2}\frac{(p_0 + \rho + p_1\rho)^{\left(\frac{2}{3(1+p_1)}\right)}}{\rho}. \tag{3.2.22}$$

We immediately see that for the above condition to be satisfied we must have $p_1 < 0$. We can always choose p_0 and p_1 in a certain manner such that at the initial epoch, pressures remain positive but at later stages become negative (although the energy conditions are still satisfied).

A summary of the conditions for no-trapping models is represented in Table 3.1. We have set out each equation of state (EOS) with their respective possible outcomes in accordance to their initial conditions. We note that ρ_0 is the initial density from which the collapse starts and ρ is the final density of the collapse end state.

The first two cases can not have a non-trapped solution without violating the strong energy condition at the initial epoch; therefore they are not viable models that work in accordance to the desired outcome. In the third case, the EOS has a possibility of not being trapped if and only if the given conditions in the table are met. The fourth EOS is not trapped only when $p_2 < 0$ and both initial conditions are absolutely satisfied. Our ideal models for which no-trappings occur are thus the third and fourth cases; but only when those necessary conditions are satisfied.

Equation of state	Possibility	Initial Condition
$p = p_0$	Not Possible	N/A
$p = p_1\rho$	Not Possible	N/A
$p = p_0 + p_1\rho$	Possible when $p_0 > 0$, $p_1 < 0$	$p_0 + p_1\rho_0 > 0$,
$p = p_0 + p_1\rho + p_2\rho^2$	Trapped when $p_2 > 0$ and Not trapped when $p_2 < 0$	$p_0 + p_1\rho_0 + p_2\rho_0^2 > 0$ and $4p_0p_2 > p_1^2 + 2p_1 + 1$

Table 3.1: Different equations of states

For each EOS their outcomes are different. The possibility of trapping is related to the co-dependency of the EOS and the initial condition. We can create a number of EOS with various variables; but the more variables added for a better physically realistic EOS, the more difficult it can be to find any possible physical solutions, if any.

3.3 Relating to the thermodynamic potentials

3.3.1 Enthalpy and acceleration potential

Reversible flows of isentropic perfect fluids, as outlined in (Ellis et al., 2012), have to involve the barotropic equation of state $p = p(\rho)$, with enthalpy W and acceleration potential $\mathcal{A}$, all of which are characterised herewith:

$$W = \exp\left\{\int_{\rho_0}^{\rho} \frac{d\rho}{3(\rho+p)}\right\}, \tag{3.3.1}$$

$$\mathcal{A} = \exp\left\{\int_{p_0}^{p} \frac{dp}{(\rho+p)}\right\}. \tag{3.3.2}$$

3.3.2 Energy and momentum conservation

The potentials (3.3.1) and (3.3.2) correlate to the energy and momentum conservation equations respectively. As a matter of fact, from equation (2.3.10) we can instantly notice this at a given epoch $t = t_0$, which we write as,

$$\mathcal{A} = e^{-\nu}. \tag{3.3.3}$$

3.3.3 Homogeneous versus inhomgeneous spacetimes

For homogeneous spacetimes it is forthwith verified that $\mathcal{A} = 1$ and the kinematics of spacetime is controlled specifically by the matter enthalpy. Definitive of the matter enthalpy, the condition for no-trapping for homogeneous matter can be declared accurately in the following way;

Proposition 2. *For a collapsing spherically symmetric homogeneous perfect fluid cloud, if the matter energy density is stringently confined by the matter enthalpy by the succeeding relation,*

$$\rho < \frac{3}{{r_b}^2} W^2\,, \tag{3.3.4}$$

then the formation of trapped surfaces would strictly not be allowed to form in the spacetime.

The relation is far more intricately vexing for inhomogeneous spacetimes. Even so, at any given epoch, we can associate the function $\mathcal{A}$ to the acceleration potential as,

$$[\ln \mathcal{A}]' = -A_a \left(a + \frac{3M + rM_{,r} - \rho a^3}{a^2(\rho + p)} \right). \tag{3.3.5}$$

In addition with regards to inhomogeneous spacetimes, the matter enthalpy is delineated in terms of the metric functions as,

$$W = \exp\left\{-\frac{1}{3}\int_{t_0}^{t} e^{-\nu}\left(\dot{\psi} + 2\frac{\dot{a}}{a}\right) dt\right\}. \tag{3.3.6}$$

Using the above two equations (3.3.5) and (3.3.6), we can implicitly relate these potentials to Proposition 1. This helps us to understand the involvement of these potentials in the avoidance of trapped surface formation.

It can be understood that enthalpy and acceleration potentials correlate to the energy and momentum conservation equations respectively. For homogeneous spacetimes, the condition for no-trapping is explicitly related to enthalpy. For inhomogeneous spacetimes, we obtain an acceleration potential and enthalpy relation; which we can be implicitly related to the potentials of Proposition 1. So, for both the homogeneous and inhomogeneous cases, it is possible to avoid any trappings as long as their respective propositional conditions are satisfied.

Chapter 4

Discussions and conclusions: the final fate of the collapse

In this dissertation, we studied the physical process of a spherically symmetric perfect stellar fluid undergoing a continuous gravitational collapse together with a continuously radiating energy in an outward spacetime.

4.1 The two main problems

The major problems which we have worked to overcome are the naked singularity problem and the black hole paradox. The existence of naked singularities is tackled if we assume the cosmic censorship conjecture to be true. The black hole paradox is a problem due to information loss.

4.1.1 Cosmic censorship conjecture

In 1969, Penrose idealised a theorem now known as the CCC (Penrose, 1969). In GR, the form of singularities are described by two mathematical conjectures, namely the weak and strong cosmic censorship hypotheses. It suggests that singularities that form in a general gravitational collapse should necessarily be enveloped by event horizons and stay unnoticeable by outside observers.

The conjecture is yet to be proved, though it is a very popular open problem in gravitational physics and general relativity. The applied theory of black hole dynamics has numerous astrophysical applications and the proof of the CCC would validate this.

In general conditions, the collapse or asymptotic predictability is inaccurate. We decide on a spacetime manifold inclusive of a naked singularity which consequently is a solution to Einstain's equations, as long as we define $T_{ik} = G_{ik}$. This means that at the lowest limit, particular conditions, like the energy condition on the energy momentum tensor regarding the matter is definitely essential (Joshi, 2007).

Penrose had his own ideas about the structure of naked singularities versus black holes. Interpretations of his ideas are depicted for the specific case of the LTB model in Figure 4.1 and Figure 4.2. The main difference that can be noted is that the event horizon (EH) takes longer to form in Figure 4.1, leaving the singularity exposed but eventually covered by the horizon as seen by the covered horizon (CH). The event horizon in Figure 4.2 is quickly formed and the collapsing cloud is never visible at any point in time.

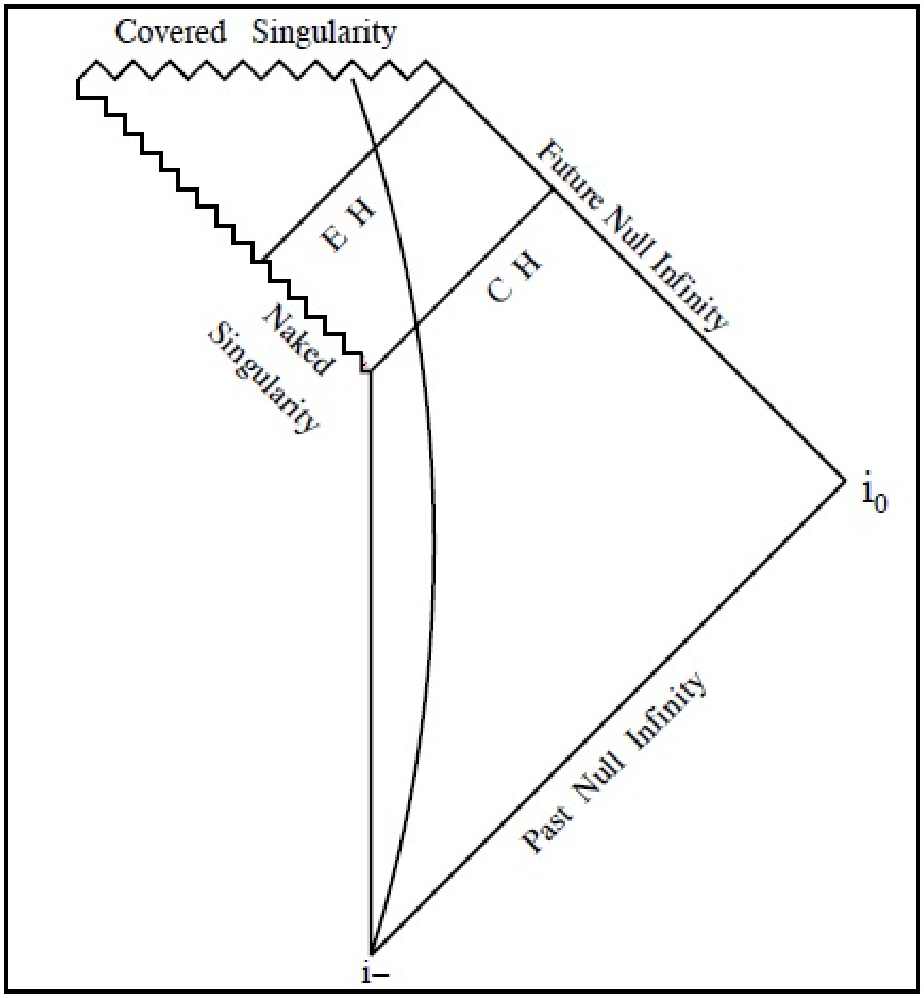

Figure 4.1: Penrose diagram of a naked singularity in a LTB collapse.

A reasonable cosmic censorship could be reached if we find an exact characterization of limitations on the matter field but such particular conditions are currently not known.

The endeavour for the proof or disproof of the CCC increased the studies of collapse scenarios apart from the exact homogeneous dust collapse case. Such studies are useful to investigate the possibilities coming up, so as to gain further understanding involving the problems of the final destiny of a continual gravitational collapse model (Joshi, 2007).

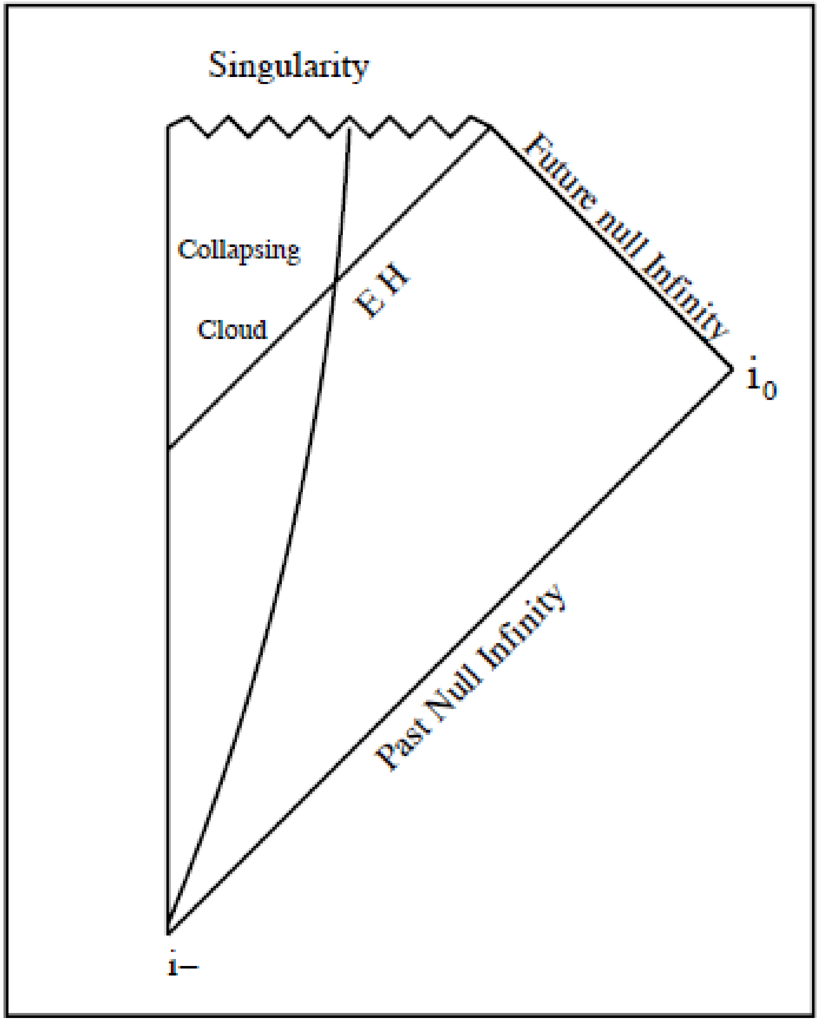

Figure 4.2: Penrose diagram for a black hole.

This censorship conjecture has no formal mathematical proof/disproof as it stands. However, there are numerous counterexamples for which the idealised picture of the OSD collapse does not hold. There are possibilities of naked singularities that can be visible to far away observers, before being trapped (see for example (Joshi, 2007) and the references therein).

In all these examples the formation of trapped surfaces are delayed by the presence of matter shear or Weyl curvature (Goswami et al., 2014), so that part of the singularity becomes naked. In all these counterexamples, however, the trapped surfaces are present in the spacetime.

The weak hypothesis suggests that a singularity would not be visible from a future null infinity and so no naked singularities exist in the universe, except for the Big Bang singularity.

The nature of physics seems governed in a foreseeable manner. Yet, the physical nature of singularities is unfamiliar and causality would cease for observable naked singularities.

The CCC is often assumed when dealing with black hole event horizons. These two conjecture variations are mathematically independent, since there exists spacetimes for which the weak cosmic censorship conjecture is true but for which the strong cosmic censorship conjecture fails and this applies conversely (Wald, 1999).

Examples that disregarded the censorship were theoretically deduced in the collapse of scalar fields, self-similar or non-similar perfect fluids and null radiation (Choptuik, 1993), (Brady, 1995), (Villas da Rocha and Wang, 2000).These investigations demonstrated that naked singularities do form from regular initial data and physically suitable equations of state. These cases, and not just isolated trajectories but families of non-spacelike geodesics, form out of the naked singularity, providing a non-zero measured set of trajectories coming out (Joshi, 2007).

If just one null geodesic manages to escape, it transforms to an individual wavefront being discharged from the singularity; therefore the singularity would seem to be naked exclusively at an instant to a far away percipient. (Joshi and Dwivedi, 1992). Emission of a non-zero measure of non-spacelike curves made it a significant occurrence. Such reviews can be further read by referring to (Israel, 1984), (Penrose, 1998), (Harada et al., 2002).

Joshi and Dwivedi conducted a general analysis of dust collapses demonstrating the breach of the CCC (Joshi and Dwivedi, 1992). Naked singularities can be disregarded on the basis that perfect fluids or dust are not primal states of matter (Joshi, 2007). The energy momentum tensor of the matter field satisfies physically reasonable conditions like the energy conditions and a well stated initial value expression for the paired Einstein field equations, which are vastly used in exploring astrophysical processes. Studies on spherical or quasi-spherical collapse of dustlike matter by various authors were done over the years (Waugh and Lake, 1988), (Singh and Joshi, 1996), (Joshi and Krolak, 1996).

Generally, for such a collapse scenario, the end state is either a naked singularity or a black hole (a covered singularity). Our resolution to these two major problems (where the outcome is not a singularity nor a black hole) is where no trapping occurs and matter dissipates. Thus, trapped surfaces are avoided and the final fate of the collapse is a flat spacetime.

In addition, the collapsing matter conforms to the weak and dominant energy conditions at all epochs. Our investigation clearly unveils the purpose of the equation of state and reveals the bounds on the thermodynamic potentials that it admits for such a model.

We affirm that these models are generic without any of the issues and paradoxes attached to horizons and singularities. This is true, because the system of Einstein field equations accept such a theoretical account for an open set of initial data and the equation of state function in their respective functional spaces.

4.1.2 Black hole information paradox

GR together with quantum mechanics calculations reveal that black holes suffer from information loss. Physical information can suddenly cease to exist in a black hole, causing a chain reaction of various physical attributes to follow suit. This breaks the unitarity principle which demands that information on the quantum level be conserved (Okon and Sudarsky, 2014).

The Copenhagen explanation is a widely accepted interpretation of quantum mechanics. It suggests that information on a whole regarding a system is inscribed within its wave function; it is only when a measurement is made that the wave-function collapses to a single eigenstate of the relevant measurement operator (Okon and Sudarsky, 2014). The unitary operator dictates the evolution of the wave function.

4.2 Conditions on EOS, thermodynamic quantities and potentials

The fundamental characteristic of these models are: the collapsing dust cloud continuously expels radiation and matter in an external radiating spacetime, such that the cloud never extends over its own Schwarzschild radius. Therefore, to find the concluding outcome we must take into consideration the external radiating spacetime. One of the most common spacetimes that can be matched with the interior across a co-moving boundary $r = r_b$, is the generalised Vaidya spacetime,

$$ds_+^2 = -\left(1 - \frac{2\mathcal{M}(v, r_v)}{r_v}\right) dv^2 + 2dvdr_v + r_v^2 d\Omega^2 \,. \qquad (4.2.1)$$

This spacetime accounts for a combination of *Type I* and *Type II* matter fields and thus is perfectly appropriate for our model. Matching the first and second rudimentary expressions across a co-moving matching surface gives,

$$\left[\frac{F}{R}\right]_{int} = \left[\frac{2\mathcal{M}(v, r_v)}{r_v}\right]_{ext} . \tag{4.2.2}$$

By intentional design, the LHS of the equation (4.2.2) is confined to be less than unity and similarly the same applies for the RHS. Hence, for any observer located at the exterior of the spacetime, the boundary of the generalised Vaidya mass $\mathcal{M}(v, r_v)$, to the generalised Vaidya radius r_v (at the central singularity $r_v = 0$), must be a non-negative value which is less than unity. Hereby invoking two conceivable end states:

1. If the limit is non-zero, a naked conical singularity will originate at the center. These are found to be weak curvature singularities which can be resolved by the extension of the spacetime through them.

2. If the limit tends to zero, then a singularity does not form and the collapse terminates with a flat spacetime.

In both of the cases above, the collapse concludes into a flat spacetime.

Figure 4.3 depicts a schematic diagram of the complete spacetime we have perused here. A crucial factor to mention here is that there exists open sets of equation of state functions in the functional space and furthermore open sets of initial data, for which these models are possible.

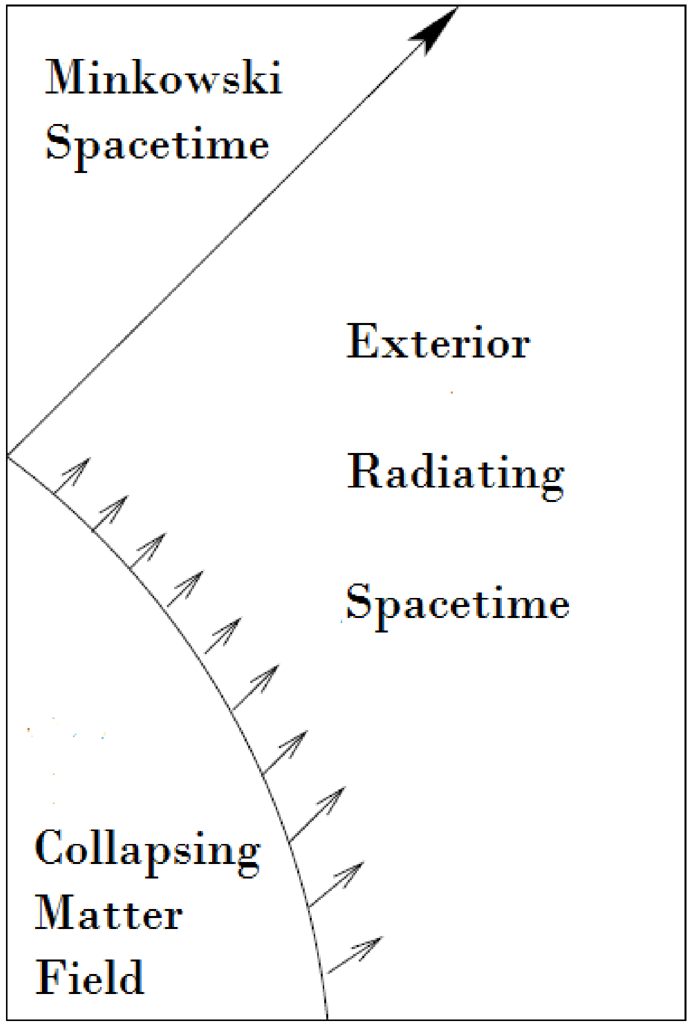

Figure 4.3: A schematic diagram of the complete spacetime.

The classes of these models have non-zero measure in the respective function spaces. This establishes that these models are generic in nature and are removed of the issues and paradoxes concerning horizons and singularities. Thus, in principle, these can describe specific forms of collapsing astrophysical bodies. Detection of these majestic objects are realised today, by the advancement of magnificent high resolution telescopes around the world.

4.3 Future work and questions

In the previous two chapters, we introduced the conditions on the equations of state, thermodynamic quantities and potentials that ensures no occurrence of trapped surfaces in the collapsing perfect fluid spacetime. The next obvious question is, "What will be the ultimate result of such a collapse?" An important question to address, because if the final fate is a strong curvature naked singularity (the singularity has to be naked in the absence of trapping), then these models would emphatically be of non-physical consequence.

We can only continue to tweak our models and compare them to observational data in order to discover the answers that await us.

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