From Learner Algebraic Misconceptions

to Reflective Educator:

Three Cycles of an Action Research Project

By

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ABSTRACT

This was a qualitative study carried out with one grade 8 multicultural, multiethnic, mathematics class. This research study began with the idea of finding out whether the learners home language (especially Zulu Xhosa) could be linked to algebraic misconceptions. The 40 learners (participants) in my study had just been introduced to algebra. I chose the school and participants through "convenience sampling". This made sense since I am an educator at this particular school. I had explained the meaning of the word "variable" in depth. The concepts "like terms" and "unlike terms" had been explained. The index laws for multiplication and division of the same bases had been discussed. It was within this context that the algebra worksheet was given to the learners, in the first cycle. I examined the algebra errors made by the grade 8 learners after marking the worksheets. I linked the errors to past literature on algebraic misconceptions as well as to Bernard's (2002b) error classification list. The conclusion was that the learners were making common errors which were not affected by their home language. I spent time on reflection since the outcome was not exactly what I had anticipated (that is, I had harboured strong suspicions that English second language learners would commit more algebraic errors than the English home language learners). I then considered a possible link between culture and algebraic misconceptions. Videotaped lessons were used for this purpose. However, observations of these videotaped lessons did not produce much data. I honestly could not reach a conclusion. This formed the second cycle of my action research. Prompted by the obvious lack of interaction in the video recordings from my teaching, I changed my focus to what I, the teacher, did during the lessons, and how these actions may or may not have supported some of the algebraic misconceptions. I reflected on my teaching method and recognized the need to change to a more interactive teaching style. I needed to give the learners the space to think for themselves. I would merely facilitate where necessary. In the third cycle, I drew up a set of problems which matched the new teaching style (interactive teaching).The lessons during which the new set of problems were discussed and solved, were videotaped. These videotaped lessons were analyzed and a completely different picture emerged. The learners were absolutely responsive and showed a side of them that I had not seen before! This study came to be an action research study because I went through three cycles of reflecting, planning, acting and observing and then reflecting, re-planning, further implementation, observing and acting etc.

DECLARATION

This thesis was supervised by Professor Iben Christiansen at the University of Kwa-Zulu Natal, Pietermaritzburg Campus.

I, Rosanthia Angeline Reed, declare that:

- The research reported in this thesis, except where otherwise indicated, is my original research.
- This thesis has not been submitted in any form for any degree or diploma to any tertiary institution.
- This thesis does not contain other persons' writing, data, pictures, graphs or other information, unless specifically acknowledged as being sourced from other researchers.
- This thesis does not contain texts, graphics or tables copied and pasted from the internet, unless specifically acknowledged, and the source being detailed in the thesis and the references section.

Rosanthia Angeline Reed

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Table of Contents

Contents

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I thank my school principal, the Department of Education and the learners' parents, for granting me permission to conduct this study.

This dissertation has been completed in memory of my mother whose love continues to shine throughout my life.

Dedication

To my mother, Freda Haines

Thank you for believing in me

LIST OF TABLES

LIST OF FIGURES

ESL –English Second Language

EFL – English First Language

CHAPTER 1

INTRODUCTION

This thesis narrates my journey which started with a desire to explore the link between learners' home language and their algebra errors, but ended with my having to change my practice as an educator. It is a journey in several respects. Firstly, in the sense that it was only as the project ran into obstacles and had to change, that it became an action research project. Secondly, because I started with a strong focus on what learners do that hinders learning, the project directed me to change my implied view on what hinders and furthers learning, so that I am now more aware of my own role. Thirdly, in the sense that it moved me from a very cognitive view on learning, to a more social view.

The thesis has 5 chapters, which follow the path my journey took. In this chapter I discuss the nature of action research and the context of this study. I also summarized the study. In chapter 2, I explore the possible link between learners' home language and their errors in algebra. Why? Research has shown that grade 8 learners experience difficulties, which often result in misconceptions when moving from arithmetic to algebra (Booth, 1990 & 1995; Rosnick, 1981: Hewson, 1996; Kieran, 1992; Stacey and Mcgregor, 1997). My thoughts then began to centre on those learners who were required not only to learn algebra as such, but the language of instruction (English) as well.

In the chapters to follow, I describe the next phases. I found that all learners committed the same errors, so I went on to determine whether the learners' background culture played a role –in particular how it might influence the culture of the classroom. However, I did not obtain enough data to answer my question, which directed me to the nature of the interaction in the classroom. As a result, I went on to examine my teaching style.

Most action research, centres around one problem situation, where an intervention is planned, carried out, and evaluated – possibly leading to a new intervention. In some ways, this project could be seen to change focus, from the influence of learners' home language on their learning, to the influence of their home culture on their learning, to classroom

culture and teaching style. In other ways, however, it could be seen as centred around one concern, namely my struggles as an educator to find the various influences on learners' difficulties in algebra. I could have written it differently, but I have chosen to reflect the journey as honestly as possible, so that it can be a report for me as well as an inspiration to other educators who think that research and improving one's own teaching can or even should go hand in hand.

1.1   Nature of this study

This is an action research study embracing teaching as a praxis, since the traditional transmission approach came to be challenged over time, firstly by looking for patterns in learners' errors, then by looking for patterns of interactions, and finally adjusting the teaching to be more interactive. Action research as a method can be used in almost any educational setting where a problem involves people such as in my study where there is the educator and the learners and where undesirable outcomes require solutions (as is the case in my study). Finding the solutions becomes the task or the question to engage with.

In my case, the task varied over the three phases that the project went through, but throughout the study each task reflected an overarching interest in addressing learners' misconceptions and the errors to which these give rise.

Task 1 - Are algebraic misconceptions linked to language?

Procedure - Examine learners' errors on their worksheets and link them to existing literature and Barnard's (2002b) error classification list. Consider the impact of home language qualitatively as well as quantitatively.

Reflection – language is not a key factor.

Task 2 - Does culture play a role in creating misconceptions?

Procedure – Analysis of videotaped lessons to interrogate how social, cultural and linguistic factors played out in classroom interactions.

Reflection – classroom culture silenced learners and thus any impact of their home

culture was indiscernible. Many misconceptions could have their root in teaching rather than in what the learners bring to class.

Task 3 - What happens when I change to an interactive teaching style?

Procedure - Analysis of the second set of videotaped lessons.

Reflection – here some change of feature resulted in a more desirable outcome, for example , a change from a traditional method of teaching to an interactive teaching style had a positive effect on the learners' competencies and overall learning.

1.2 What is action research? The Nature of Action Research.

According to Cohen, Manion & Morrison (2008, p.297), "Action research is a powerful tool for change and improvement at the local level."

There are different conceptions of action research. I mention a few typical definitions from four sources in the following extracts:

Hopkins (1985, p. 32), suggests that "the combination of action and research renders that action a form of disciplined inquiry, in which a personal attempt is made to understand, improve and reform practice." Ebbutt (1985, p.156), too, regards action research as a "systematic study that combines action and reflection with the intention of improving practice." Cohen and Manion (1994, p. 186), define action research as a "small scale intervention in the functioning of the real world and a close examination of the effects of such an intervention."

Kemmis and Mc Taggart (1992, p. 10), argue that "to do action research is to plan, act, observe and reflect more carefully, more systematically, and more rigorously than one usually does in everyday life." The common features of the definitions of action research point to the fact that action research has key principles. In Kemmis and Mc Taggart (1992:22-25), they outline their ideas of the key principles of an action research study as follows in table 1.1. In this table, I have mentioned only those criteria that my research study satisfies.

According to Kemmis & McTaggart (1992, p.16) , "Action research is concerned equally with changing individuals, on the one hand, and, on the other, the culture of the groups, institutions and societies to which they belong".

Action research seriously seeks to change particular practioners' particular practices, rather than focusing on general or abstract practices. The ultimate aim of action researchers is to understand their own particular practices as they appear in their own particular circumstances, rather than reducing them to the status of the general, abstract, or the ideal. Action research is concerned with the relationships between social and educational theory and practice, but as I see it, it is fundamentally and centrally concerned with the praxis of the educator as a life - long learner.

Table 1.1 Core Principles of Action Research.

- Educational action research aims to improve education by changing it and learning from these changes.

- Action research is participatory - people work primarily towards improving their own practices (and only secondarily on other people's practices).

- Action research involves the self - reflective spiral: a spiral of cycles of planning, acting, observing, reflecting and then re-planning, further implementation, observing and reflecting....

- Action research is often collaborative: it involves those responsible for action in improving that action.

- Action research requires that people put their ideas about institutions to the test by gathering evidence which could convince them that their previous ideas were less effective.

- Action research starts small, by working through the changes which even a single person can try, and works towards bigger and broader changes.

- Action research allows us to give a reasoned justification of our educational work to others because we can show how the evidence we have gathered and the critical

> reflection we have done have helped us to create a developed, tested and critically examined rationale for what we are doing.
>
> - Action research allows the research flexibility to work with data in its various forms - it involves not only keeping records which describe what is happening as accurately as possible, but also collecting and analyzing our own judgments, reactions and impressions about what is going on.

Kemmis and McTaggart (1992:22-25).

My research study encompasses the key principles of an action research study as listed in table 1.1. The spiral of cycles (of planning, acting, observing, reflecting and then re-planning, further implementation, observing and reflecting) has been used throughout the study. My study conforms to the various definitions of action research as described in section 1.1., page 2 of this study. The common factor in these definitions is that reflection must take place and intervention must follow with the aim of improving practice. In my study, I realized that the conventional teaching style that I was using, was not working well so I changed to an interactive method, which I hope over time will result in overall learner improvement. In some ways, I had read these principles of action research but only through my own project did I come to realize the extent to which I had to challenge my own assumption and that the journey was only starting.

1.3 Context of the study

As I have already mentioned in the introduction, this research study started out investigating the possible links between the language of instruction (English) and the algebraic misconceptions in a grade 8 multicultural South African classroom.

This study was conducted at a secondary school in Kwa-Zulu Natal (previously an all Coloured[1] school). It is a co-educational school since the enrolment of 1100 learners is

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[1] Coloured is an apartheid category which refers to persons of mixed race coming from a variety of other race groups (White, Indian and African) . The San/khoi/bushmen were included in the category as were people of Chinese origin, and others. However, in this school, the learners during the apartheid years were of mixed race.

made up of both girls and boys in each classroom. Approximately 84% of the learners are African, with the remaining 16% of the learner population being made up of 2 Indian learners and 174 Coloured learners. The majority of the learners reside in Umlazi, a vast "township" in the Durban metropolis. These learners from Umlazi travel to school by train, bus or taxi. In organising the school day, timetables, activities, controlled tests and examinations, the learners from Umlazi must be considered. The remaining learners are within walking distance of the school.

There are 35 to 45 learners per class in grades 8 and 9 while in grades 10, 11 and 12 there are 30 to 35 learners per class. The school is badly in need of painting and the classrooms have broken windows. There is a shortage of furniture. The school fees are R750.00 (around US $110) per year. However, in most cases it is impossible to acquire the full school fees because the learners come either from low income families, or situations where unemployment is rife. The school works closely with a cluster of schools from the area. In this way, all control tests and examinations are standardized.

Since I am an educator at this school, I chose the school and learners in my class as participants. Sampling is not a concept generally used in Action Research, as it is assumed that the researcher-practitioner works within her/ his own practice, but this could be considered a "convenience sampling".Neuman (1977, p. 205), affirms that convenience sampling "selects anyone who is convenient." With regard to convenience sampling, Cohen et al (2000, p. 102), states that it is occasionally called ‚opportunity sampling'. Convenience sampling "involves choosing the nearest individuals to serve as respondents and continuing that process until the required sample size has been obtained."

1.4 The Action Research Cycles in this study

In this study, feedback from data was used in an ongoing cyclical process or perhaps more appropriately in a spiraling process.

In the first cycle, existing literature on algebraic misconceptions was reviewed and an error list presented the wide variety of commonly made errors that had been detected and documented in Algebra over the years. The grade 8 learners were given a few algebra problems to solve (via worksheets). The errors that the learners had made on these worksheets, were then linked to those on the error classification list (Barnard, 2002b), and

the literature readings. The purpose of this was to identify possible areas for intervention, in other words, it was part of the planning process. However, I found that all learners, regardless of their home language, committed the same errors (as per the error classification list). The English speaking learners and the Zulu[2] speaking learners made common errors – there was nothing which suggested that, the errors made were as a result of the learners' home language. Upon reflection, the home language did not create problems for this particular grade 8 research class. I therefore abandoned my original starting point, as I realized my assumptions had been incorrect. Yet it must also be recognized that this worksheet demanded fairly straightforward symbolic manipulation only, and did not allow me to interrogate language issues. I could at this stage have revisited the formulation of the worksheet. However, my readings had pointed me to how language cannot be separated from culture. I then considered whether culture played a role in creating misconceptions in algebra.

In the second cycle, my aim was to explore if learner engagement in the classroom was linked to home culture. This was inspired by reading about the ways in which home culture can lead to conflicts in the classroom culture. For this purpose, lessons were videotaped and carefully analyzed, since it would be difficult to identify such issues while also teaching. Unfortunately, very little came out of these lessons. The learners hardly spoke during the course of the lessons so the data collected were hopelessly insufficient. Upon reflection, I could not tell whether the learners' background culture played a role in creating algebraic misconceptions, but I came to realize that my teaching style was part of what had limited learner interaction. This made it clear that before I looked at what learners bring to the classroom situation, I first had to consider the classroom culture I promoted.

Upon observation of the lessons, I became aware of the fact that I did not allow the learners to think for themselves. I was too quick to take over the lessons and provide the answers to the questions. I was also not very gracious about incorrect answers, this most likely

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[2] In Zulu, the correct term for the language would be isiZulu , while amaZulu refer to Zulu people. However, I have opted for common English usage and used " Zulu" throughout, no offence or cultural insensitivity intended.

contributed to the learners' reluctance to answer questions during the lessons. Upon reflection, I recognized the real need to change the teaching style that I was using.

In the third cycle, I critically examined the teaching style that I used. I decided I needed to change from a teaching style that relied exclusively on transmission of knowledge from the educator to the learners, to an interactive, more learner centred method.

An interactive teaching approach required learners to think and increasingly take control of their learning. I had to learn to play the role of facilitator and not take over the classroom situation completely all the time. However, this facilitation had to go hand in hand with some teaching or intervention. Allowing the learners space to think, discuss and come up with the solutions themselves, would contribute to a greater feeling of self-worth.

To this end, a second set of lessons were videotaped and observed. As time was running out, I could only start this cycle and thus recorded fewer lessons and collected less evidence than what would have been required to give a substantial evaluation of the change in teaching. The questions that were asked during these videotaped lessons, were constructed in such a way that the learners could relate comfortably to the language used – these questions were within the learners' background experiences, thus thinking and discussion should go more smoothly. This was done in order to give the learners the opportunity to build up their self-confidence so that they would be more willing to speak out and answer, and mirrored my learning process. Ideally, I should have continued into algebra in the same way, but this had to fall outside of the project owing to time constraints.

In the chapter that follows, I will engage with the first cycle, which mostly considers preparing for an intervention. This cycle allows for language issues in the teaching and learning of Mathematics. I took each of the errors that the learners made on a worksheet given to them in the first action research cycle and classified them according to the error classification list developed from the literature, found in table 2.1.

Chapter 2:   First Cycle: Learners' errors in algebra- a language issue?

In the first cycle, my starting point was, as described in chapter 1, to explore the possible link between learners' home language and their errors in algebra. I gave each of the learners in my grade eight research class, a worksheet on which a few algebra problems appeared. The learners were required to solve these problems. I collected these worksheets and marked them. The errors made by these learners were linked to the error classification list (Barnard, 2002b), as well as to the existing literature on algebraic misconceptions. The error classification list appears under the literature review for this cycle.

2.1. Problem Identification

Language plays a major role in how learners interpret and make sense. Vygotsky (1934, p.213), stated, "It is not merely the context of a word that changes, but the way in which reality is generalized and reflected in a word." Vygotsky (1934, p 218), goes on to say that "thought is not merely expressed in words; it comes into existence through them. Every thought tends to connect something with something else, to establish a relation between things." This is relevant to this study since the lessons involve a large number of words in specific contexts. Learners whose home language is not English, are forced to connect the words to something they already know so as to "make sense" of the words. Before applying the mathematics in a particular context, the learners have to understand the meaning of each word in English.

Whereas it would seem a reasonable assumption that learners attending an English medium school would be sufficiently fluent in English for this not to be a factor, research shows that this assumption is problematic. I will return to that point below.

2.1.1 Background / early reflection: observing learners' difficulties.

Many learners find algebra confusing, pointless and difficult to understand. They know that algebra has to do with letters, however, they do not understand what the letters refer to, or why they are used. The existence of letter symbols is frequently taken for granted, (and in algebra teaching untouched by the later reforms) not much is said about the origin of the idea of using letters to stand for variables in mathematics, nor why using variables might be useful (Booth, 1995).However, at the time I started this project, I was unaware of other ways of teaching algebra. I had also not realized that a teaching method could be a

source of misconceptions. I had taught in a very procedural way and I observed how learners (in answering the algebra problems on their worksheets) viewed letters as objects or things which could simply be gathered up together. For example, expressions such as 3x + 2y were incorrectly simplified as 5xy.

These learners felt that an answer should be a single term. The learners looked to arithmetic for justification where an answer would not be left as 3+2 but would be written as 5 (Booth, 1995). The old finding that learners " do not understand the use of letters in equations,(they) tend to view the use of letters in equations as labels that refer to concrete entities" (Rosnick, 1981,p.418), was confirmed in my class. Many of the learners see the equal sign as a symbol separating the expressions on either side, they do not see the sign as a symbol showing the equivalence between the right and left hand sides of the equation. Consequently, they are reluctant to accept statements like 2a+5a=3a + 4a (Bansilal, 2006). The learners also experienced difficulties with cancelling, for example, in $\frac{2x+4y}{2}$ the learners cancelled the scalar in the first term of the numerator with the denominator and gave x +4y as the answer. I observed the difficulties mentioned above and began to wonder whether the source of these difficulties was a language issue. I even considered a few Zulu terms to see whether they somehow linked up with the errors made by the learners in my grade eight research class.

Taking the word *even* in mathematics (even numbers for instance) – in Zulu the term for "even", in the mathematical context is "lingana." However, from the home background language, "lingana" would mean "equal in size" in the context of, perhaps, trying on a pair of shoes or fitting. Thus, sometimes when code switching is used in the classroom, the Zulu version of a particular English word, may conjure up a slightly different meaning. This could lead to a conceptual conflict.

When having difficulties in structuring new meanings in the new context, the learners may experience a gap in the coherence and the continuity of their living experiences, appearing as cultural discontinuities (Nieto, 1999) and hence as a cultural conflict and, as mentioned by Gorgoriò and Planas,(2000, p.53) the social, cultural and linguistic aspects of mathematics cannot be separated.

Let us take for example, the word "hlanganisa". In Zulu it means "to calculate the total of" or "include" in a mathematical context. However, in a Home Economics context, where we are concerned with "putting together with something else" or "mixing together", the Zulu word is "Engeza", according to the multilingual mathematics dictionary (2003). From the background home language the learners come with the knowledge that "hlanganisa" means "to join".

Thus, in keeping with the idea of joining perhaps, on many occasions, when a question such as add: 3x+5y is given, and the equivalent Zulu translation of the instruction: "hlanganisa" is mentioned, learners write 8xy as the answer. This is confusing – a word or instruction in one language could mean something totally different in another language. For example " hlanganisa", an isiZulu word, means "join" in English. However, 3x + 5y should not be joined to give 8xy as an answer. However, this type of answer is given by all the learners, irrespective of their home language. The existing schemas of grade eight learners mean they are familiar with the process of obtaining a single number as an answer from addition in arithmetic, for example, 11+20=31, so an explanation which is independent of the broader context of the learners is that they transfer this knowledge to an algebraic problem such as 3x+5y, and come up with 8xy (Booth, 1995 and Kieran, 1992). So the question remained, to what extent does the home language influence the kind of errors learners make?

According to Gorgoriò and Planas (2000, p.52), a valid starting point is "to consider the cultural contribution of ethnic minorities and different social groups as a source of richness to be maintained and shared". More knowledge about how language impacted on learner conceptions would allow me to approach the teaching differently. In keeping with this statement, the Zulu word for "divide" (Hlukanisa) seems to be more apt in describing the operation, as compared with the English version.

Many learners, when confronted with the following fraction:

$\frac{x+4y}{2}$ say $\frac{x+4y}{2} = x + 2y$.

In the learners' existing schemas, division in arithmetic involves "cancelling" e.g. $\frac{20}{4}=5$

The learners thus transfer this knowledge to 2 and 4 and ignore x + y in the fraction $\frac{x+4y}{2}$

However, the isiZulu word "Hlukanisa" from the learners' home background

knowledge means "separate", thus, the learners would be better equipped in

answering $\frac{x+4y}{2}$ in that they would first separate it into $\frac{x}{2} + \frac{4y}{2}$

Learners may, for example, be given three temperature graphs each drawn in different colours (red, blue and green). The learners may be asked to find the difference between Monday's temperature of the blue graph and Tuesday's temperature of the red graph or the sum of Wednesday's temperature of the green graph and Thursday's temperature of the red graph. Within a social learning theory, this would influence the learning since the same word is used for the colours blue and green in Zulu, namely "eluhlaza". Anecdotal evidence on this particular issue confirms that language informs thinking : even after years of writing in English, some people with Zulu or any other related language as their mother tongue indicate that they have to stop and think to tell the difference between "green" and "blue".

Thus isiZulu speaking learners might confuse the two words. We find thus that one language can be richer in some ways (as in the division example) but, poorer in other ways (as in this example of colours), so the desired learning may be furthered or hindered.

2.1.2 Research Question

Does language impact on algebraic misconceptions? What interventions could assist me in using the learners' home languages as resources rather than see them as hindrances?

2.2. Planning

2.2.1 What is in the literature?

Research on learning algebra has moved through different stages, as I will discuss below, reflecting the dominant view on what algebra is, but there is also a change in focus within education from learner to educator to teaching. Researchers have described algebra in a number of different ways. I will mention some of them: Wagner and Kieran (1989, p. 221) see algebra as a) generalized arithmetic, b) a set of rules and c) a representation system.

Barbara von Ameron (2003) and Stacey, Chick and Kendal (2004) broadened this view to four perspectives: a) generalized arithmetic, b) a tool to solve problems, c) the study of relationships, and d) the study of structures. However, for school algebra, the study of structures is often not seen as relevant (though this view could be challenged), generalized arithmetic and the use of algebra in problem solving being fore-grounded. Tall and Razali (1993), hold the view that many learners consider algebra to be a process of memorizing rules and procedures, and indeed it has often been taught in that way. The learners in the grade 8 research class that I worked with, also viewed algebra as a process of memorizing rules and procedures, and as I later realized this was not disconnected from my way of teaching. Much research on early algebra learning focused on the description that algebra was generalized arithmetic. The focus was on arithmetic and the link between algebraic symbols or letters (Wagner and Kieran, 1989).

Up to the 1950s, algebra was viewed as a tool for manipulating symbols and solving problems (Kieran, 2007). Thus the research centred on the difficulties that learners had with the procedures and notation. The field of learning algebra was used to study memory and skills development by researchers in the 1950s and the 1960s. In the late 1970s, the focus shifted towards making algebra meaningful to learners and investigating the meaning that learners made of algebra.

According to Thom (1973), the construction of meaning rather than the question of rigour is the central problem facing mathematics education. How is such meaning to be constructed? Algebraic reasoning is enhanced when there is a strong foundation in problem solving contexts (Kieran, 2007). Bell (1996) is of the view that problem solving does not only consist in forming and solving equations (as in many words problems) but can be seen from the perspective of explorations leading to more general results which is the core business of all mathematics. Focusing on general results fosters a more comprehensive understanding of the use of variables and mistakes such as substituting a=1 into an expression, because ‚a' is the first letter of the alphabet, can be avoided. On the other hand, as Bell (1996) claims, avoiding the use of problem solving contexts in teaching, robs the learners of a complete understanding of algebra. These statements by Kieran and Bell make me realize that I need to devote more classroom time to problem solving in order to pave the way for an indepth understanding of algebra among my learners.

Gorgoriò and Planas (2000) are also of the view that it is difficult to disentangle the social and cultural conflicts of a multi-ethnic mathematics classroom from the language issue. The social, cultural and linguistic aspects of mathematics teaching and learning are integrated. It is for this reason that I tried to choose problems that were within the background experiences of all the learners in the grade 8 research class that I worked with.

Language has to do with communication at different levels. The construct of transparency, is central in the literature concerning the different learning strategies used to face the discontinuities experienced by English second language students (Gorgoriò & Planas, 2000, p. 54). Educators need to make explicit the norms that regulate the dynamics of the mathematics classroom and of mathematical practices. However, too much focusing on language in order to minimize cultural conflicts may produce a loss in the mathematical problem under consideration (Adler, 1999). According to Wong (1982), educators in multilingual settings usually address their students using a simplified language register. This, however, does not guarantee that learners have better access to the mathematical content, but may interfere with the acquisition of rich mathematical concepts by obscuring them. On the other hand, Adler (1999) expresses a view that difficulty in verbalizing mathematics does not necessarily mean that no mathematical thinking is taking place. I agree with Adler's statement. I found that in the grade 8 research class that I worked with, some learners could not verbalise their responses to particular questions yet, when given the opportunity, they were capable of writing down their responses correctly.

According to Pirie and Kieran (1992, p.8), "language is the mechanism by which educators and pupils alike, attempt to express their mathematical understandings to each other. It is well accepted that individuals construct understandings that differ not only from one another but that are likely to differ also from the meaning intended by the originator of a particular communication."

Van Hiele (1986. p.86), expressed his opinion regarding language in his statement, "Language is important because by mention of a word, parts of a structure can be called up." By asking students to explain their solutions, the educator gets the opportunity to distinguish among conceptual conflicts, differences in social and cultural experiences and

communication problems. I agree with this statement. I used the method of getting the learners to explain their solutions in the grade 8 class that I researched and from this, I obtained some of the data that I required.   To communicate mathematically, English second language students have to work with abstractions and symbols (which occur throughout algebra).

Pimm (1987,) explores some connections between language and mathematics, between everyday and specialist use of language, and between terminology and compensation. "We should not think of a mathematical register as consisting solely of terminology or of the development of a register as simply a process of adding new words". (Halliday, 1975 a, p.65. in Pimm, 1987, p76).

Within the South African context, the majority of learners are taught mathematics through the medium of English, even though they are not yet fully fluent in this language of learning and teaching (LoLT). Educators and learners prefer English (Setati in press), since they feel that this is an international language which contributes to a good way of life in all respects. However, the South African language in education policy (LiEP) promotes multilingualism (Department of Education, 1997) and surveys show that the use of learners' home languages is encouraged (Moschkovich,2002).

The argument that the learners' home languages are a resource for Mathematics learning (Adler, 2001; Moschkovich, 1999; Setati, 2005;Setati & Adler, 2000), is perceived to be an obstacle preventing multilingual learners from becoming competent in English. As Sachs (1994, p.1) pointed out, in South Africa, " all language rights are rights against English."

Recent research on the use of code switching (for example , Adendorff , 1993; Adler, 1998, 2001; Moschkovich,1996, 1999; Setati, 1998; Setati, 2005; Setati & Adler, 2000) , encourages the recognition of home languages in oral communication as this leads to success in Mathematics. South African Mathematics education needs to appreciate the multilingual learners' ability across languages (Barker, 1993).

Durkin (1991, p.3) states that "Mathematics education begins in language, it advances and stumbles because of language, and its outcomes are often assessed in language." This quotation emphasizes the importance of language as a resource in the teaching and learning of mathematics. The major problem in South African multilingual classrooms is the fact that many learners are not sufficiently fluent in English to enable them to tackle mathematical tasks set in English, which is the LoLT in their classrooms.

The effect of language on the learning of mathematics is a widely researched and debated topic not only internationally, but also in South Africa (Howie, 2002; Setati, 2002). Poor language skills are often associated with low attainment in mathematics and, in addition to that, mathematics has its own set of language patterns, symbols and vocabulary. A major part of developing an understanding of mathematics involves learning to handle these and making connections between symbols and their corresponding terminology and meaning (Haylock, 1991). Daniels and Anghileri (1995) stress that, speech and written language are the tools of mathematical dialogue. The development of some mathematical thought may be hampered through a lack of access to these tools. As Dockrell and McShane (1992) point out, when solving a problem it is crucial that the learner first understands the problem before planning and executing a method for solving it.

In research carried out by Baxter, Woodward and Olsen (2001), it was indicated that whole class discussions are often dominated by verbal, capable learners, while the low attainers tend to remain passive. When they do in fact respond, their answers are typically simple and at times incomprehensible (Rall, 1993; Chard, 1999, as cited in Baxter et al., 2002). I agree with this statement since, in the grade 8 class that I researched, I also found that only a few learners who were capable in mathematics, dominated the group discussions.

Particular errors in Algebra

Firstly, I distinguish between "errors" and "misconceptions". Many learner mistakes are the symptoms of misconceptions. Learners may misunderstand a concept and this leads to such mistakes. For example, from arithmetic, learners understand that addition means "putting together", thus 5+2=7. When they first come across an algebraic expression such as 5x + 2y, they apply the same reasoning and say, the answer is 7xy. The difference is that errors that do not have their basis in misconceptions are less systematic – they may result

from misreading a question, from a short lapse in attention span, from missing a symbol in writing down the answer, etc.

This section discusses algebraic errors identified in the literature, where both misconceptions and other types of errors are included though the focus is mostly on misconceptions.

Categories of algebra errors were generated from mathematics educators in Great Britain by Tony Barnard (2002 a; 2002 b; 2002 c). Barnard's classification lists were compiled from experienced educators' observations. Although these observations were based on experience rather than research, it is a list commonly referred to in literature on algebra errors. Its shortcoming is that it does not make a distinction between the natures of the conception leading to the error. This classification is useful for my study as it contains errors that I encountered in my grade 8 research class. The errors are summarized in table 2.1 (the numbering is mine).

Table 2.1: Errors in Algebra

MISUNDERSTANDING THE MEANING OF VARIABLES
V1. A letter stands for a specific value
V2. A letter stands for the name of an object
V3. Letters are objects that can be "gathered" e.g. $3n + 4 = 7n$
V4. Expressions with letters should be a single term
V5. Letters have number values according to their place in the alphabet, e.g. $a = 1$
V6. Not knowing how to operate on „unknowns" e.g. adding $3x + 4x$ to get $7x$
ERRORS REGARDING EQUIVALENT FORMS AND THE MEANING OF THE EQUAL SIGN
E1. Not knowing when to change form e.g. $(a + b)^2$ or $a^2 + 2ab + b^2$
E2. Using equals signs to link unequal steps e.g. $x^2 = 2x$
E3. Seeing the equals sign as a „find the answer' command rather than a sign of equivalence
E4. Being unaware of equivalence e.g. between $2a + 5a$ and $7a$
MISINTERPRETATION OF   QUESTIONS
Q1. Creating and solving an equation when asked to simplify an expression
Q2. Not being aware of mathematics communication skills other than learning vocabulary, such as

using mathematical reasoning, discussing mathematics

Q3. Problems with not understanding questions given in words

OVER-GENERALISING

G1. Over-generalizing distributive laws e.g. $\sin (a+b) = \sin a + \sin b$

G2. False linearity e.g. $(a +b)^2 = a^2 + b^2$

CONFUSION BETWEEN OPERATIONS

O1. e.g. $2+ a^2 = 2a^2$

ERRORS WITH MINUS SIGNS

N1. e.g. $(-x)^2 = -x^2$

MISAPPLIED RULES

R1. e.g. $2^2 a = (2a)^2$

ERRORS WITH SIMPLIFYING FRACTIONS

F1. e.g. $\frac{2h^2}{2} = 2^2$

ERRORS WITH THE WRONG APPLICATION OF ORDER

B1. e.g $2xy + 3xy \times \underline{z} = 5xyz$
xy

All of these errors appear to reflect misconceptions where the symbols are being operated on without understanding what they refer to.

Misunderstanding the meaning of variable.

  Learners make errors with simplifications because they misunderstand the different meanings of algebraic variables (Booth, 1988; Linchevski & Herscovics, 1996; Stacey McGregor, 1997). Appreciating the fact that a single variable can represent many quantities simultaneously ("any quantity") is difficult (Wagner & Kieran, 1989). Lesley Booth (2001) explains that errors such as $2x + 5y = 7xy$ and $3n + 4 = 7n$ or $3n4$ (made by 45% of the 13- year-olds in her study) is a direct result of viewing variables as objects that can be gathered together (p.109).

An interpretation of $3 + a = 3a$ is the result of putting 3 and ‚a' together. The belief crops up that an answer could not be left as $3 + a$, it should be a single term, hence $3a$ (Booth 1990). A student must appreciate that $3n + 4$ is a numerical object, i.e., the number that is 4

greater than 3 times the number n. It is also a process that can be performed for a given numerical value of n (thus creating a functional relationship: n yields 3n + 4). This is a common problem when learners are introduced to algebra for the first time, resulting in the process-object duality situation which will be discussed in the theoretical framework. Booth (1995) makes the statement that letters in algebra are viewed by many learners as shorthand notations for names of objects. For example, apple + apple +apple = a+a+a which is then 3 apples, thus 3a.

Errors regarding Equivalent Forms and the meaning of the Equal sign

Booth (1988; 2001) reported that focusing on a numerical answer in arithmetic encourages the learners to view the equals sign as an instruction to perform an operation rather than showing algebraic relation. The learners thus combine any available numbers, for example $11x + 5 = 18x$ (Booth, 2001). This error appeared regularly in my grade 8 research class. Booth (2001) found that learners who are doing algebra for the first time, are oblivious to the fact that the same quantity can be expressed in different ways, for example $5a + 2a = 7a$. This was also true in my grade 8 research class.

Misinterpretation of Questions

Mestre and Gerace (1986) maintained that creating an equation and solving instead of merely simplifying an expression when asked to do so, was a misinterpretation that existed mostly among second language students. I found this error to be prevalent in my teaching across all grades. I also found that this error occurs in the case of all learners regardless of their home language. Perkins and Simmons (1988) referred to four main causes of algebraic misunderstandings: a) content, b) problem solving, c) epistemic, and d) inquiry. Many of the misconceptions occur at content level, which is the preferred level at which most educators teach.

Misapplied Rules

Tony Barnard (2002 b), identified errors such as "when multiplying, add the indices, as in 2^2 x a = $(2a)^3$, ignoring that the bases are different. This error was common in my grade 8 research class.

Errors with Simplifying Fractions

Tony Barnard (2002 b) specified errors from cancelling incorrectly, for example,
$\frac{x+y}{y} = x + 1$ or $\frac{x+y}{y} = x$ (the latter also treating y as an entity different from number with the implicit scalar 1).

In the analysis, the errors made by my grade 8 research class, are illustrative of this list of errors and error classification referred to above in table 2.1.

2.2.2 Theoretical Framework

Before discussing any learning theories, I will describe how my evolving theoretical standpoint started. My perspective at the start of the project was that learners from home language backgrounds other than English would be more prone to committing the algebraic errors, as are listed in table 2.1. I was also of the viewpoint that culture played a role in creating algebraic misconceptions. I have gradually changed my views as I will discuss later on in this thesis.

Process – Object View of Algebra

Carolyn Kieran (1992) and Anna Sfard (1995) both concluded that the transition from arithmetic to algebra was primarily a shift from procedural operations and numbers, e.g.13-7 = 6 where the answer is viewed as an object, to process – oriented operations on algebraic expressions that yield other algebraic expressions rather than numbers, as answers e.g.

2x + 5y +x =3x+5y; 5a + 6b is both an object (it stands for the number 5a + 6b), and a process that can be performed for given values of a and b.

The + sign complicates the appreciation of the object nature of 5a + 6b, a two termed expression, compared to a single termed expression. Anna Sfard (1992) describes operational as seeing e.g. 3x+2 as an action which could be performed for a given x value and ‚structural' as seeing 3x + 2 as an object that could itself be operated on by say, multiplying it by 3.

As with many other mathematical constructs, algebraic expressions have a dual nature as processes or objects. This gives a framework for analyzing learners' errors in a more systematic way than the empirically based list in table 2.1. This theory (Process-Object duality) helped in the design of questions and incorrect answers that could be reached from

a process – only view, e.g. the error of adding 2x + 5y to give 7xy - this is a process view that prescribes to the idea that all operations should be carried out in order to yield a 'final answer'. Such a single term answer is the only form of object accepted within a process view.

The Pseudo - Structural Approach

One perspective, associated with constructivism, is that a pseudo – structural approach (where algebra becomes a collection of meaningless symbol manipulation) to learning algebra, occurs when skills and concepts are placed in compartments rather than being linked (Sfard & Linchevski, 1994, p.223). For example, 3x + 2 and 3p + 2 are seen as different functions even if they differ only in the variable used, that is in their symbolic representation. This is reinforced by Tall and Thomas (1991) who state that learners camouflage their difficulties in routine activities that give correct answers when they are unable to understand concepts. This is in line with the constructivist notion that learners create meaning from their experiences, but not always the meaning that the educator intended. This is confirmed by Epstein (2002, p.3), when he identified nine general principles of constructivism, one of which is, "learning is contextual: we learn in relationship to what else we know, what we believe, our prejudices and our fears."

My study embraces the basic view of socio- constructivism. This implies the staunch conviction that language is important in the process of knowledge construction and acquisition, which happens in interaction with both the social and the physical environment. The process/object view and the pseudo/structural approaches to algebra learning are important to this study since they provide us with a framework for analyzing learners' errors.

2.2.3. Constructing the worksheet

To explore learners' misconceptions more systematically, I gave them a worksheet which contained tasks that could give rise to many common misconceptions. Retrospectively, I can of course see that some of the formulations of the tasks are problematic, but I am being true to the process I went through.

In constructing the worksheet (see Appendix A), I made sure that the questions included could be matched with the algebra errors listed in table 2.1. Questions 1 and 2 on the

worksheet could be matched with V3 according to the error classification list in table 2.1. Questions 4 and 5 on the worksheet could be matched to V1 according to the error list in table 2.1. Question 6 could be matched to V5 while questions 7 and 8 could be matched to F1 in the error list of table 2.1. However, there were questions on the worksheet that could not be matched to the remaining errors in the list in table 2.1. The reason was that the grade 8 learners had just been introduced to algebra when I began with this research project, thus there was a limited quantity of algebra that had been completed.

2.3 Results: Comparing learner worksheet answers

The 40 grade 8 learners participating in this research project have attended English Medium Schools from grade 1. There were 25 Zulu mother tongue speakers, 10 Xhosa mother tongue speakers and 5 were English first language speakers.   Perhaps, in future, a similar research project could be conducted with grade 8 learners who have not attended English Medium Schools from grade 1, but who have been taught with a more conceptual focus. This is one of the shortcomings in the first cycle. I will return to this at the end of this chapter.

In the first question on the worksheet the learners were required to add $2x+5y$[3] There were 5 English second language (ESL) learners and 2 English first language (EFL) learners who gave the answer as being 7xy.The idea of letters standing for objects or words resurfaces (e.g. a-apple) and when combined with a number, 3a would then be 3 apples. Another interpretation of 3a is the result of putting together 3 and ‚a'. Once again, the belief crops up that an answer could not be left as 3+ a, it should be a single term, hence 3a (Booth, 1990). Thus it is clear that the error made by these learners confirm what theory and past observations indicate. What stands out here is that learners appear to be operating on the symbols rather than considering the content/meaning to which the symbols refer.

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[3] I realize now the problem with this question which appears to imply that it is possible to perform such an addition. Not only would ‚simplify' have been more appropriate than ‚add', perhaps a question where it would be possible to simplify at least some terms would have been more appropriate etc. eg; 2x + 5y+ 4x.

This is in keeping with the errors that learners make in other countries.

In the second question on the worksheet, the learners were asked to determine

xy-10 if x =4 and y=5. Table 2.2 shows different responses and their frequency. In the third

column, I have indicated the type of error according to the classification in table 2.1. All of

the errors represent a process or pseudo-object conception of algebraic expressions

Table 2.2. Learners' responses to questions on worksheet.

Simplify	RESPONSES	CLASSIFICATION OF ERROR	FREQUENCY
1) 2x + 5y	7xy	Letters are objects that can be "gathered"	5 ESL and 2EFL 18% of the learners gave this answer
2) Determine xy-10 if x = 4 and y=5	xy-10 = 45-10 = 35	Confusion between operations	4 ESL and 3 EFL 18% of the learners gave this answer
3a) Evaluate 5a + 3b – 2c if a = 1, b=2, c= 3 b) 2a-3b +c if a =2, b=3 & c=1	a) 5a + 3b - 2c = 51 + 32 – 23 = 60 15 learners did not answer this question because they claimed that a=1, b=2 and c=3 is already in ‚a' above so how can their values now change.	Confusion between operations A letter stands for a specific value.	3 EFL and 4ESL 18% of the learners gave this answer
4) Under what condition is a + b +d = a +c +d?	The learners said that this was impossible since b is not equal to c because they are different letters.	A letter stands for a specific value.	7 learners 5 ESL and 2EFL 18% of the learners gave this answer
5) My book has p	p-q= -1	Letters have number	4 ESL and 5 EFL

pages. I have read q pages. How many pages must I still read?	p is the 16th letter and q is the 17th letter of the alphabet so 16-17=-1	values according to their place in the alphabet.	23% of the learners gave this answer
6) Simplify: $\dfrac{8a^3b^2}{4a^2b}$	$2ab^3$	Errors with simplifying fractions	3 ESL and 4 EFL 18% of the learners gave this answer
7) Simplify: $\dfrac{4a^3\ b^2}{8a^2b}$	$2ab$	Errors with simplifying fractions	4 ESL and 3 EFL 18% of the learners gave this answer

The above errors made by the grade 8 learners in their algebra worksheets match those mentioned in table 2.1, in particular, error types V1 and V3.

In question 2 of the worksheets, the learners had to determine the value of xy-10, if x = 4 and y = 5. Six of the learners, replaced x with 4, y with 5 and then said 45 – 10 = 35. Thus, they fail to see the implied multiplication sign in xy. When I have taught this in the past, I have emphasized that if there is no sign to indicate the operation being performed, we assume that it is multiplication, so that x x y would simply be written as xy. These learners appear not to be aware of the implicit multiplication, thus seeing xy as 45. I would consider this an O1 type error. Despite going back to the timetables, these learners were finding difficulty with the concepts and procedures in algebra, but not with the language of instruction itself. This also pointed me to the role of the educator in promoting errors. Perhaps it is wise to write 3 x a for some time before introducing the abbreviated form 3a, until learners stop viewing conjoining as indicating addition or simply joining of symbols (cf. Booth, 1995).

In questions 3(a) & 3 (b) of the worksheets, the learners were asked to evaluate firstly 5a + 3b – 2c if a = 1,b =2,c=3 and, secondly, 2a-3c+c if a=2, b=3 and c=1. Ten learners wanted to know how can a = 1 and then a = 2 or c = 3 and then c = 1. Learners erroneously became convinced that a particular letter must be replaced by one specific value

(a type V 1 error) - I realized then that, as educators, we are not explaining clearly enough what the variables mean, nor why they are used. Booth (1995) believed that the origin and usefulness of using letters should be emphasized.

A number of learners seemed content to interpret variables as specific unknowns rather than generalized numbers or open variables, for example, question 4 on the worksheets given to my grade 8 research class was: under what condition is $a+b+d = a+c+d$?

Instead of the answer $b = c$, the learners said it was impossible since b and c represent different numerical values (numbers). This ties in with the statement by Booth (1984) that learners are not appreciative of the fact that the same value can be represented in several ways. This is a pseudo- object understanding of variables and a type V1 error.

Upon re-examining the questions posed on the worksheets, I began to focus on the problem: under what condition is a $a+b+d = a+c+d$? I realized then that this question was too generalized for these grade 8 learners though it did allow for some of their existing errors to be addressed. By asking grade 8 learners questions such as this, I am creating problems in the algebra learning process.

Question 5 on the worksheet was:

My book has p pages, I have read q pages. How many pages must I still read? When I questioned the learners, 9 of them answered that p is the sixteenth letter of the alphabet, and q is the seventeenth letter of the alphabet, thus p-q = 16-17.   This as well is in keeping with past research studies on algebraic misconceptions. (Rosnick, 1981; Booth, 1990 and 1995; Kieran, 1992; Hewson, 1996; Stacey and McGregor, 1997) and is a type V5 error according to table 2.1.Basically, this boils down to the fact that the learners do not fully comprehend what a variable is.

Question 6 on the worksheet was:

Simplify: $\dfrac{8a^3b^2}{4a^2b}$ [7 learners answered incorrectly: $2ab^3$]

Twenty eight learners answered correctly: 2ab. However, the very next question (Q7) was:
Simplify: $\dfrac{4a^3\ b^2}{8a^2b}$

This question was incorrectly answered as 2ab by seven learners. There is an assumption here that the smaller number must always and only be divided into the larger number, an assumption, which could be traced back to arithmetic. All these errors were common

amongst all learners, irrespective of their home language. This caused me to reflect on the investigation as well as my own assumptions.

2.4 Reflection on first action research cycle

2.4.1 Reflections on the worksheet

When considering the worksheet again, I realized that despite my efforts in designing the worksheet, the learners were asked questions that required them merely to carry out procedures and the questions did not allow the learners to think conceptually or to explain. From the errors that I saw, I realized that the worksheet was not inclusive enough. There were no questions on the worksheet which matched the errors E1; E2;E3;E4;B1;N1; R1;Q1;Q2;G1; and G2.

2.4.2 Reflections on the findings

Returning to question one in which 2x+5y must be added, seven learners answered 7xy. Looking under the frequency column in table 2.2, the English first language speakers, the Zulu speakers as well as the Xhosa speaking learners gave 7xy as the answer.

With regard to questions 2, 3,4,6and 7:

The above mentioned algebra errors were common to 7 of the grade 8 learners participating in the research regardless of their home language, be it English, Zulu, Xhosa etc. Home language did not seem to matter. Why was this the case? Perhaps because the algebra is already somewhat removed from the everyday language (algebra is a language on its own), or perhaps the learners in this class are all fairly competent in English at this stage. All learners struggle with the technical language of algebra. However, this brought to the fore another potential source of the errors, namely the teaching. As I read and reflected on the types of errors I saw in the class, I became aware that I had been focusing on the learners as the source of the errors. Perhaps the answer was not that simple? I was still convinced that context played a role in the learning of algebra. Question 5 was incorrectly answered by 9 of the grade 8 learners.

I then considered whether these algebraic errors were a result of language in a bigger context. Was it an issue of culture? This became the focus of the second cycle. I will discuss that in chapter 3.

Chapter 3: Second Cycle: Learners' errors in algebra and their classroom engagement-a culture issue?

In the first action research cycle, by examining the learners' algebra errors that they made on the worksheets that I had given to them, I found that the learners had made common algebra errors, regardless of their home language. In this chapter (the focus of the second action research cycle), I considered whether the learners' algebraic misconceptions were an issue of culture, as well as issues of cultural conflict in the classroom situation.

Considering this now, it would have been more in line with my original concern to explore the roots of learners' misconceptions instead, which also would have brought me to confront my own teaching. However, my supervisor had engaged me on seeing language as integrated with cultural issues, so I continued (using culture) to explore whether my assumptions that the learners' background influences their participation and learning, were justified.

3.1 Planning: What is in the literature?

There is a lot to say about the role of culture. Firstly, a distinction between home culture and school culture is in place. However, at the onset of this cycle in the study, I was interested in exploring the learners' home culture and how this impacted on their engagement with content as well as their interaction in the classroom.

How did I set out to look at the role of culture? This was done by the video recording of lessons and careful observation of these lessons. At the very beginning, the learners were aware of the video camera and focused on it. It appeared that they soon got used to it, yet I recognize that the presence of the camera could mean that both the learners and the educator act differently- an inevitable validity issue. One camera was used by a gentleman that I had hired. He moved around the room and photographed each group as the learners discussed. Video recording of the lessons would be useful in that I could view these lessons repeatedly at any given time in order to check whether I could pick up on data I might have overlooked at a previous viewing. The data would be, to name a few, issues of disruption of the learning process, the atmosphere created in the classroom (conducive to

learning or not), and the type of proficiency created. Furthermore, it would enable me to see the ways in which the learners engaged with each other, me and the materials used, in line with the study of Gorgoriò and Planas (2000).

Research points to the fact that mathematical ideas in their broader sense exist in all cultures, but which ideas are emphasized, how they are expressed and their content, varies from culture to culture (Bishop, 1988). The common concern for mathematics educators in these varied contexts is to give meaning to the mathematical knowledge and practices which learners bring with them to the classroom by virtue of having lived in a particular community and culture (Allen, and Johnston-Wilder, 2004).

Anne Watson (1996) concerned herself with a particular aspect of classroom culture, that of educators' informal assessment of students' mathematics. She believed that educators' assessments reflect their values and, like Ernest (1998), believes that this impacts on the classroom culture. These different forms of assessment could result in a social inequity and contribute to a discriminatory curriculum. I agree with this statement. As mathematics educators, many of us are inclined to cling to questions from textbooks, which tend to disregard the fact that learners come from different backgrounds in terms of economics, language and culture. We assume that, all learners have the same background experiences.

Cooper & Dunne (2000) were particularly interested in the effects of social class on pupils' learning. In their 2000 article they were concerned with the 'realistic' tasks in the National Curriculum tasks in England. They found that social class and gender differences were pronounced when 'realistic' tasks were used. They felt that learners from lower social classes performed better in a task that is decontextualised. This was because the learners did not have the cultural experience or the "linguistic habitus" (Zevenbergen, 2001), to understand the game of answering realistic questions, which were not part of their home experience and did not exist in the discourse of the lower social class of learners. The middle class learners were advantaged. This is disturbing at a time when some colleagues are arguing for more 'realistic' tasks to be included in the National Curriculum tests.

The socio- cultural perspective is an example of a contemporary model of learning that advocates changes in classroom practices, allowing all participants to communicate more interactively, while giving due consideration to the link between classrooms and the

cultural practices of the communities in which these schools are situated. The centrality of community in socio-cultural theory reflects the view that knowledge acquisition is a stepping stone to more complete participation in the practices, beliefs, conventions and values of a community of practitioners and not primarily as the acquisition of mental structures (Goos, Galbraith & Renshaw, 1996).

However, Gorgoriò and Planas (2000, p.53), mention that "*When minority language students join a mathematics class, they often find different norms, regulating both the social dynamics of the mathematics classroom and the mathematical practices. Discontinuities in understanding new words and new meanings can turn into a wide variety of cultural conflicts and disruptions of the learning process*".
Since the minority language students are at a point where they are at a loss with regard to comprehension, they often display behaviour associated with failure (Gorgoriò & Planas, 2000) .

Gorgoriò and Planas (2000) are of the view that it is difficult to disentangle the social and cultural conflicts of a multi-ethnic mathematics classroom from the language issue. The social, cultural and linguistic aspects of mathematics teaching and learning are integrated. Language has to do with communication at different levels. Language is a communication tool that goes beyond the translation of words. Potential mismatches may exist between the cultural practices that learners bring to the learning situation, and the classroom culture that the educator promotes.

 Goos, Galbraith and Renshaw (1994; 1996) regarded the idea of the classroom community as central, where gaining knowledge is seen as a way of getting to know mathematics. This community sees everyone as having a voice and learners are authors of their own mathematics. This is an important aspect of the cultural practices that the learners can bring to the classroom. In direct contrast to this, the culture that I had promoted in the grade 8 class that I taught was such that the learners were not afforded autonomy (my teaching was like filling empty vessels). Thus learners had no chance of being authors of their own mathematics. Just as Buxton (1981) reported, I expected the learners to develop techniques and skills with single correct answers to questions,

therefore, the majority of learners most likely saw themselves as unsuccessful learners of mathematics. The learners had been silenced.

Returning to Goos, Galbraith and Renshaw (1994; 1996), the notion of a ‚community of practice' would take place effectively if the roles of both educator and learners changed. Goos and colleagues believed that a combination of mathematics and pedagogic knowledge was needed by educators in the form of long term continuing professional development so that mathematics classrooms could become communities of learners.

Returning to the notion that learners bring various cultural practices to the learning situation, Gorgoriò and Planas (2000) illustrated an example of the influence of cultural values. Raima (a 15-year-old girl) seemed to understand mathematics only when working in small groups. She could not understand the language of her mathematics educator when the whole class was addressed. However, according to her values, it was not acceptable to tell an adult (her educator) that she could not understand her. Raima could not be convinced that asking for clarification was essential for her learning. This can be classified as a cultural conflict since Raima's cultural values (not questioning an adult were in direct contrast to those of the classroom). Here we find that cultural values interfere with communication by preventing students from asking for help. A similar situation existed in the grade 8 research class that I worked with. Learners would not ask for clarification if they did not understand something. Many of the learners (like Raima) came from a culture where adults were not to be questioned.

On the other hand, in the grade 8 research class that I worked with, instead of encouraging the learners to talk and seek clarification, they were subjected to a classroom culture of educator transmission where they did not get the opportunity to be actively involved in their own learning. This resulted in the learners not seeking clarification.

Cultural values may cause learners to refuse help, or even to help others. This is illustrated in Gorgoriò and Planas (2000). Sheraz (a 15-year-old Pakistani boy) came up with a solution which was different to those produced by the other students. Sheraz refused the opportunity to present his solution. His family had left a position of high social class in Pakistan – he resented this. He expected to return to Pakistan so his efforts at school were minimal. He wanted nothing to do with his classmates, even with the boy Sahid who was

also from Pakistan. Sheraz stated "I am here because of war, he is here because he is poor." Once again, we find that cultural values play an important role in different situations such as the classroom.

In any social situation people tend to behave in ways that are relevant to that culture, but with possible transfer from their home culture due to its primacy. What comes to mind when recalling my own high school experiences in a convent school for girls is the fact that our various cultures that we brought from our different backgrounds, often clashed with those of the convent school classrooms. For example, from our background cultures, we were given the opportunity to explain or state our cases if we were falsely accused of being responsible for having done something that we knew nothing about. However, it was another matter altogether in the school classroom. We were deemed to be rude if we attempted any explanations. This is one of the situations which captures the essence of what a cultural conflict is (conflict between the learners' background cultures and those of the school classrooms). In the culture of the classroom the type of behaviour that is encouraged is that which conforms to learning expectations and discourages disruption. The way in which learners' behaviour and achievement are perceived by their educators and peers in the classroom has an impact on the way they view themselves as learners of mathematics (Allen and Johnston - Wilder, 2004). This resulted in learners viewing themselves as unsuccessful learners of mathematics. I now realize that I created a similar situation in the grade 8 class that I worked with, in that I had tried to enforce particular norms and standards of behavior to the extent that the learners were not afforded the space and autonomy to develop their conceptual skills. I did not use the cultural aspects that the different learners brought to the classroom to enrich the learning experiences of the whole class.

Gender, ethnic background, class and language are but some of the dimensions of diversity found in mathematics classrooms. An increasing number of mathematics educators are exploring the significance of these diverse dimensions, particularly with respect to issues of equity and social justice in different spaces of the mathematics education system. Not all dimensions of diversity have received the same attention in the mathematics education literature, and these dimensions have been investigated from many different perspectives, not all of which integrate a critical perspective. Debates in gender, class, race, culture and

language often coincide.

Michael Clyne (1998), in a paper entitled "Managing Language Diversity and Second Language Programmes in Australia", made some suggestions concerning diversity in LOTE (language other than English) programmes. Clyne concerned himself with the pressure on educators, who have to cope with diversity in the LOTE classroom. This surely represented an added burden. He suggested the encouragement of collaboration between ethnic communities and education systems. The use of peer group support, according to him, was invaluable because many young people from the various language groups could be motivated to become educators. Information exchange between parents, educators, caregivers, researchers and others, and the many who assumed several of these roles in the transmission and management of language diversity, was needed. The multi cultural and ethnic media had an important role to play in providing suitable and appropriate input for language acquisition, maintenance and development for the young in programmes that were of interest to them.

In the paper, Michael reported on two studies recently completed by his institute and ended with a heartfelt plea for diversity, tolerance of diversity and a recognition that the state ought to help bear the costs of what may constitute the common good.

Many complex debates and reactions to Michael Clyne indicated that inclusion (although worthy) was not easily managed alongside policies to safeguard cultural diversity and plurilingualism. Economic imperatives worked against diversity, both at the level of individual choices and government policy.

What would I look for in the observations?

In my observations of the second set of videotaped lessons, I would look for the "cultural conflicts, disruptions of the learning process and failure manifestations" mentioned by Gorgoriò and Planas (2000, p. 53), and how these related to the learners' home culture as well as the classroom culture. I would check the atmosphere that had been created in the classroom – was it conducive to learning? Were the learners willing to respond and answer questions? What type of proficiency did I promote? These were the questions that I would ask myself as I observed the videos.

3.2 Acting and observing the interactions of the classroom.

Before commencing with the 5 lessons that were to be videotaped, I organized my 40 (multicultural & multilingual) Grade 8 learners into 8 groups of 5.The learners were grouped in such a way that each group had at least 2 ‚good' learners (in Mathematics), 2 ‚average' learners and one ‚weak' learner[4] in as far as this pattern could possibly be achieved. The groupings were made for the purpose of teaching, as well as the purpose of research : the learners who were good in mathematics would help those who were weaker, both in understanding the mathematics as well as the language used in the questions that were asked. I expected that this would result in interactions which I could then analyse for " cultural conflicts" and " disruptions". While at the time, I thought this would help make the learners feel comfortable, relaxed and not threatened during the research process, I recognize now that it could have caused a disruption in itself. I did not want to group the learners according to home language even if this made more sense in terms of this research, as I felt that this would have created division. These learners were very sensitive to "racial discrimination." I was subtly reminded of this every now and again.

Research by McCullum, Hargreaves and Gipps (2000) into pupils' view of learning found that learners wanted a classroom that had a relaxed and happy atmosphere where they could ask the educator for help without ridicule. I had to acknowledge that the atmosphere in my Grade 8 research classroom was the exact opposite – the learners were tense and appeared to avoid attention, perhaps hoping that I would not direct questions at them. The learners preferred mixed ability grouping because this gave them a range of people with whom they could discuss their work. It appeared that these learners were suggesting that they would like to have worked in a collaborative community, a community of practice. (Allen, and Johnston – Wilder, 2004).

A group leader was appointed in each group. It seemed that the term "leader" as in group leader conjured up different connotations, one of which is "chief", a term synonymous with "power", "absolute authority" and "wisdom" in some cultures. There were looks of envy in five of the groups when I announced the names of the group leaders. Furthermore,

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[4] Based on their past performance in Mathematics.

as the lessons progressed the learners in each group, followed the instructions of the group leaders. During the observations of the videotaped lessons some learners looked to the group-leaders to come up with all the acceptable ideas for a successful discussion, while other groups appeared to want the leader to sit back and let the "subjects" do all the work. Nieto (1999) states that difficulty in structuring new meanings in a new context causes learners to experience a gap in the coherence and continuity of their living experiences, leading to cultural discontinuities, then cultural conflicts which, in turn, cannot be divorced from the language issue. When I first noticed 8 learners sitting isolated, not participating in the group discussions, I thought this might be for this reason. Two of these learners looked out of the window, three of them waved at the video camera while the remaining three concentrated on chewing bubblegum. They showed no interest whatsoever in what was happening in the lesson and, in particular, the group discussions. I was unsure as to whether this was an example of a cultural conflict, whether they were not motivated, or whether I had not managed to engage or interest them.

However, the observation did not tell me to what extent they felt unable to adapt to the social norms of a classroom in a school several of them had spent years in. Again, I had expected that the theoretical lenses provided by the literature would have helped me unpack and understand my learners' behaviour and learning better, but I found it impossible to do so with the data I had. The videotaped lessons also revealed that I was not gracious about incorrect answers. The learners certainly seemed to have picked up on this. They became withdrawn and reluctant to answer. I praised correct answers only, so there was very little conversation during the first lot of videotaped lessons. My attitude towards the learners' answers may have led to the learners believing that mathematics is only about correct answers. It also set a norm in the classroom of focusing on correct answers, not on process or meaning-making, and of valuing the swift production of answers.

I soon realized that it was not a language problem since these same learners understood exactly what was said when they were asked to pay attention and participate. However, the technical language of algebra is another matter altogether. Yet when I look more closely at my teaching of algebra, I came to new realizations; not about learners' difficulties with the discourses of algebra or the disruptions when confronted with an authoritarian social norm. My realizations were about the type of proficiency I promoted, as well as the cultural norm that I encouraged for the classroom. It was when looking at these videos that

I for the first time started to become aware of the possibility that I may actually be encouraging misconceptions. I will engage this further in the next section.

3.3 Procedural focus in the teaching – encouraging misconceptions?

Two of the problems that needed to be solved during one of the video-taped lessons were:

Simplify: 1 $\frac{24x^3y^2}{3x^2y}$ and   2 $\frac{3x^2y}{24x^3y^2}$

I emphasized that the variable is placed where the index is greater. For example, the index of x in the numerator of $\frac{24x^3y^2}{3x^2y}$ is 3, so the answer,

x will go in the numerator. Likewise, y will also go in the numerator since the index of y is also greater in the numerator of $\frac{24x^3y^2}{3x^2y}$ so the answer to

$\frac{24x^3y^2}{3x^2y}$ is $\frac{8yx}{1}$ or 8xy.

Conceptually, the reason for this is that the greater index signifies a greater number of multiplicands, only some of which are cancelled out. While I had explained that previously, in this lesson I only directed the learners to follow a procedure, without reference to the underlying concept of exponents signifying repeated multiplication:

The learners then responded that the answer to $\frac{3x^2y}{24x^3y^2}$ is 8xy

I focused entirely on the variables and, in doing so, took for granted that the learners knew what to do with the numerical coefficients. However, fractions were still a problem for the grade 8 learners.

They did not realize that $\frac{3}{24} = \frac{3\div3}{24\div3} = \frac{1}{8}$

so the answer to $\frac{3x^2y}{24x^3y^2}$ would be $\frac{1}{8xy}$

According to Kilpatrick et al (2001,p.116), "no existing terminology can completely capture all aspects of expertise, competency, knowledge and facility in mathematics." However, the term mathematical proficiency was settled upon to encapsulate what was necessary for the successful learning of mathematics. Mathematical proficiency consists of 5 components or strands as described by Kilpatrick et al (2001). I have included these

strands of mathematical proficiency in table 3.1, where I have used my own numbering system.

Table 3.1: The 5 strands of mathematical proficiency.

A1. Conceptual Understanding	Comprehension of mathematical concepts, operations and relations.
A2. Procedural Fluency	Skill in carrying out procedures flexibility, accurately, efficiently and appropriately.
A3. Strategic Competence	Ability to formulate, represent, and solve mathematical problems.
A4. Adaptive Reasoning	Capacity for logical thought, reflection, explanation and justification.
A5. Productive Disposition	Habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy.

These 5 strands are interwoven and occur simultaneously in the development of proficiency in mathematics. In order to help children acquire mathematical proficiency, mathematics instruction should address all 5 strands mentioned in table 3.1.

Research and theory in cognitive science corresponds to the ideas represented by these five strands. Of extreme importance is the manner in which learners link pieces of knowledge as this influences their understanding and their ability to solve problems, and this again is connected to the emotions or productive disposition of the learners. Good understanding is the result of knowledge being stored and arranged in such a way that it can be easily recalled and applied when needed. Comparisons cannot always be drawn between the strands of Mathematical proficiency and the kinds of knowledge investigated by those concerned with learning. However, the five strands are important inside and outside mathematics education (Kilpatrick et al, 2001).

According to Kilpatrick et al (2001), a certain amount of skill is crucial to understanding many mathematical concepts. This understanding can be deepened and strengthened using procedures thus strands A1 and A2 of table 3.1 are interwoven.

Returning to the questions posed during the video-taped lessons in the second action research cycle, chapter 3, particularly $\frac{24x^3y^2}{3x^2y}$ (to which the learners answered 8xy) and $\frac{3x^2\,y}{24x^3y^2}$ (to which the learners answered 8xy as well), I realize now that I should have applied a stronger conceptual focus, where the learners would not have had to remember cumbersome rules:

$$\frac{24x^3y^2}{3x^2y} = \frac{24x.x.x.y.y}{3.x.x.y} = \frac{8xy}{1}$$

$$\frac{3x^2\,y}{24x^3y^2} = \frac{3.x.x.y}{24.x.x.x.y.y} = \frac{1}{8xy}$$

Strand A1 (in table 3.1 of the 5 strands of mathematical proficiency) was not applied in the questions $\frac{24x^3y^2}{3x^2y}$ and $\frac{3x^2\,y}{24x^3y^2}$

According to Kilpatrick et al (2001, p.118), conceptual understanding refers to "an integrated and functional grasp of mathematical ideas", while procedural fluency refers to "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately and efficiently" (Kilpatrick et al, 2001, p.121).

In the task, procedural fluency (strand A2 in table 3.1) was emphasized. With regard to the remaining strands (A3, A4 and A5) as described in table 3.1, the learners were not encouraged to represent the problems their own way (A3), provide explanations (A4) or to see mathematics as meaningful (A5). Not surprisingly, the learners did not display any self-confidence or ability to solve mathematical problems. The learners also did not offer any justification or explanations on the rare occasions that they did answer. However, it was also clear that I did not encourage such activity. This caused me to examine my teaching style. In chapter 4, I will return to the 5 strands of mathematical proficiency in order to determine whether they had been addressed in my new teaching style. I will also investigate the effect of the application of these 5 strands on the mathematical learning of

my grade 8 research class.

During the course of the second videotaped lesson, I revised addition of integers with the learners (in preparation for the lesson on addition of algebraic expressions). Some of the statements I made misled the learners, for example, " a negative and a negative gives a positive". Besides treating the rule as an isolated procedure, this caused the learners to say -3+-2 = +5 because, from their background experiences, "and" implies "add". The focus on rules may thus actively have directed the learners away from making sense of the new concepts and objects (such as negative numbers). I was suddenly becoming more aware of my own role in creating misconceptions amongst the learners!

 In the third videotaped lesson, I again lost my focus and forgot that I was dealing with grade 8's who were learning algebra for the first time. I took things for granted and confused the learners. I said, "When multiplying, add the indices", without stressing that the bases must be the same. Again, a procedural focus and misleading (a type R1 error as found in table 2.1).

Perhaps that was part of the reason for the response to the problem $3x \times 4y^2$ being $12x^3$ in some cases, $12y^3$ in other cases and $12xy^3$ in a few instances. If so, it is an indication not only of my teaching but of the meta-learning that the learners have accumulated over the years: they seem to act within a classroom mathematics norm where mathematics is about following rules.

3.4 Reflection

Returning once more to the previous 5 videotaped lessons, I realized that the data I obtained was too limited to work with, in terms of telling me more about the possible link between learners' home language, culture and their algebraic errors. What was I going to do now? The supervisor of my thesis had also viewed the videotaped lessons and agreed that the data was insufficient – I was then advised to have a few more lessons video-taped, with more interaction so as to allow for in-depth analysis of any disruptions. There were a number of unfortunate incidents that occurred during the first set of videotaped lessons as I described above, and I was determined to change my practice, not just for the sake of the research but to improve my facilitation of learning as well . I felt that I, as the educator, could perhaps be more flexible. I needed to practice more patience with the learners, allowing them time to think instead of hastily taking over the classroom situation

completely. The learners were not given a chance to think for themselves. I honestly believed that no learning would take place if I did not take charge. In turn, the learners were quite prepared to leave everything to me, and not take responsibility for their learning. This was the culture of the classroom. Perhaps it had not caused any obvious disruptions because it was the culture of the school as well?

My belief had been "if I, the educator, don't take over, nothing will happen, learners don't need to think for themselves." This fits in with the culture that some of the Grade 8 learners of the research class grew up in where the norm is that children are expected to keep quiet and listen to adults. According to 2 of the Grade 8 learners in the research class, in their cultures children are not even allowed to defend themselves. If a child tries to clarify his/her point he/she is deemed to be disrespectful. Children are also not allowed to question adults. As a result of these cultural beliefs the learners are taught from very young that the adult (educator) should do all the talking while the learners listen attentively.

However, these cultural beliefs are now changing due to the new democracy in our country, as well as the Outcomes Based Education System and the learner centered pedagogy encouraged after 1994, where learners are encouraged to talk and think for themselves. The result may be that these learners find themselves in a culturally conflicting situation where the norms and values they grew up with are different from those encouraged in the educational system. However, we (educators and learners alike) have to change with the times (in particular since the introduction of the National Curriculum Statement). In the next few videotaped lessons, I would try to teach (as per advice) in such a way that I would not take over completely when the learners struggled. I would play a far less dominant role, taking extra care to give praise wherever it is due and be extra cautious about criticism. I needed to ensure that the learners became emancipated to the extent of being critical of their learning. The time for being mere recipients of the values of a higher authority was over. It would also give me a chance to see if there would be cultural conflicts and/or learning disruptions, when the norms of the classroom changed.

In the new set of videotaped lessons, I would change my classroom management style. I would leave almost all the talking and discussion to the learners. I would work towards creating a classroom situation and atmosphere where the myth that "learners do not need to

think for themselves" was challenged. My belief that "if I do not take over, nothing will happen" needed some serious rethinking. The learners' attitude that "it is the educator's job to explain" needed to undergo a serious change as well.

The videos did not help me answer my question because I did most of the talking! I should perhaps have pursued my question on the role of cultural interactions, but I became engrossed in whether I could change my teaching to engage the learners more, only then would I be able to eventually see the extent to which culture would play out in my classroom. This led me to a new question: Could I make my teaching more learner centred, allowing for more interaction? This question became the focus of the third action research cycle.

Chapter 4 Third action research cycle

4.1 Planning

As has been mentioned in the reflection part of chapter 3, the videos did not help me answer my question from the second cycle, that is, I could not tell that culture contributed to the creation of algebraic misconceptions or to disruptions in the classroom culture. While observing the videos, however, it struck me that the teaching style that I adopted could definitely do with a change. Could I make my teaching more learner centred, allowing for more interaction and appealing to a broader spectrum of the strands of mathematical proficiency? This question was the focus of my third action research cycle.

I used videotaped lessons once again but this time around, the questions that were asked during the course of the lessons, were quite different from those that appeared in the algebra worksheets of action research cycle one. Firstly, I realized that I needed to ease into the algebraic work and the use of variables. Secondly, I wanted tasks that provided some entry points for students and gave them more possibilities for choosing approaches and representations. I did try to make them more relevant to the learners' context, but I recognized that I needed to work more on this aspect. Furthermore, I attempted to change the style of interaction with the learners, as described below.

In drawing up the problems that would be discussed in the second set of videos, I took into account the strands of mathematical proficiency (as per table 3.1 in chapter 3), I considered how the tasks (problems) would address the strands of mathematical proficiency. In answering the first part of question 1, for instance, "conceptual understanding" (strand A1 of table 3.1) of percentages was necessary to obtain the correct answer here. In skillfully and accurately multiplying decimals to obtain the correct answer to the first part of question 1, the learners would demonstrate "procedural fluency" (strand A2 of table 3.1). Should the learners formulate and solve the problem in the first part of question 1, and also be able to apply this procedure to a similar problem such as in the second part of question 1, both of which required learners to translate between representations, the learners would be demonstrating "strategic competence" (strand A3 of table 3.1). In order to answer question 1 or any of the other questions, the learners need to be able to think logically, reflect, explain and justify, thus demonstrating "adaptive reasoning" (strand A4 of table 3.1). If I could encourage learners to work out most of the

solutions on their own, "productive disposition" would be addressed.

If a classroom has a culture that values learners creating their own mathematics and becoming authors of mathematics, then the learners are more likely to become positioned as successful learners of mathematics. For this to happen you need a community of learners working together collaboratively and creatively. There needs to be a shift in the way some educators view the nature of mathematics and an examination of the value they place on assessment and target setting. For a community of practice to flourish, learners need to develop autonomy and be able to recognize for themselves that they are creating and understanding mathematics. (Allen and Johnston – Wilder 2004, p.7).

In this context, autonomy would refer to learners working freely on their own, for example, exploring and developing various methods of solving problems. From these solutions they could be guided to deduce rules which, in turn, could assist them in solving additional problems. In changing my teaching style, I subscribed to the view that learners needed to work together collaboratively and creatively as well as to develop personal autonomy.

A large body of literature has been amassed over the years highlighting the fact that learning is facilitated when learners are encouraged to link new information to their prior knowledge and thereby generate new understandings (Fennema, Carpenter and Peterson, 1989; Greeno, 1989; Lampert, 1986; Noddings, 1990; von Glasersfeld, 1987). It is for this reason that I was determined to take full advantage of the knowledge that learners bring to the learning situation from their various backgrounds.

Learners come to the classroom with a range of background experiences and cultures. I intended to encourage the learners to share these experiences with their peers. For instance, learners whose parents have businesses may know more about calculating VAT, a percentage calculation.

4.2 Acting/Observing

I drew up a set of tasks which would give the learners opportunities to discuss and finally come up with their own results and findings. The tasks (questions 1 to 5, see Appendix B) were referring to situations that the learners may encounter in their lives. I constructed the tasks in this manner so that the learners would be able to see mathematics as useful and

worthwhile, thus addressing the mathematical proficiency strand of ‚productive disposition' as mentioned in table 3.1 of chapter 3. Although these tasks were different in nature from the algebra tasks that were given in the previous set of worksheets (in cycle one), the expectation was that lots of conversation and discussions would take place in the solution of these tasks. The whole idea was to get the learners talking and change the classroom culture to encourage more meaningful engagement with mathematics and the learning of mathematics. The language used in the tasks was in keeping with the normal language used in any algebra task. There was no unnecessary simplification. At the beginning of the new set of videotaped lessons, I pointed out to the learners that I would say as little as possible. I wanted the learners to discuss, lead, advise and correct one another. The whole idea was that the learners take responsibility for their own learning. I kept repeating that I did not want to talk much, the learners needed to do all the talking. Even though their answers may not all be quite correct, they should not hold back, but put their ideas and thoughts forward.

A camera was used by a gentleman I had hired. Five videos were recorded. Originally, when I planned time for these videotaped lessons, I expected the length of the periods to be 60 minutes as per the normal timetable. However, owing to an unexpected development (a week of educator team building workshops) the length of the periods was reduced to 30 minutes. As a result, I had to cut down on the number of questions and resolved to discuss whatever problems were covered during these shortened periods.

Noluthando[5] read out the first part of problem 1 in the new set of videotaped lessons. She read: "if a bag costs R68.40, excluding VAT, determine x and y in the equation: R68.40 + x = y. (let x represent the VAT and y represent the cost of the bag inclusive of VAT). Noluthando said "I know that VAT is 14%. I also know that 14% means 14/100. I then take 14, divide it by 100, and then multiply the answer by R68.40. My answer works out to R9.58 and then I round it off to R10.00.

Noluthando's working:

$$\frac{14}{100} \times \frac{R68.40}{1} = R9.58$$

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[5] The names in the transcripts have been changed in order to protect the learners' identities.

Thus x = R10.00, if I round it off to the nearest rand." Noluthando then said, "to find y which is the cost of the bag including VAT, I added R68.40 to R10.00 and this gave me R78.40."

I then said: "Do not round off, since in the shop situation, they work with numbers like R19.99, they do not round off." I asked one of the learners:"Lindsey, do you have a different approach to that of Noluthando?" None of the learners responded in the affirmative which probably meant that all the learners had worked out this problem just as Noluthando had done, or that they did not want to share answers that they were not sure of, or that they had not done the task. There were a number of possible reasons as to why the learners did not respond.

I said. "I do not want to talk too much. You must take responsibility. Do not worry about whether your answer is correct or not. The important thing is that you talk so that I can pick up whether you understand the language used in these problems." As we moved on to the second part of question 1, Vuyisa took the lead in the discussion. The second part of question 1 was: *the shop owner wants to know how she can calculate the VAT of anything she sells*. Vuyisa said: "Basically, you take 14, divide by 100 and then multiply by whatever the given price of the article is." I asked Vuyisa to give an example. He said: "Suppose a calculator is R80.00 (without VAT) the VAT will be 14% of R80.00, that is,

$$\frac{14}{100} \times \frac{80}{1} = R11.20$$

Another learner, Charles, then said "The price of the calculator with VAT will then be 80 + R11.20 = R91.20." Nomfundo said that, to calculate VAT, one needs to say 14% over 100 x the given price. I said to the learners: "Can you tell me how you can improve on what Nomfundo said?" A few learners replied "% means over 100, so we do not say 14% over 100, but simply, 14 over 100 i.e. not 14% / 100 but 14/100. Here the learners showed their ability to spot errors and improve on what their peers have said. Reflecting on the second part of question 1, the learners displayed their ability to think logically, explain and justify. This addressed the strand „adaptive reasoning' of table 3.1 in chapter 3 (the strands of mathematical proficiency). The learners were able to skillfully solve the second part of problem 1, showing understanding of the mathematical concepts and operations involved.

Thus the strands „conceptual understanding', 'procedural fluency' and „strategic competence' of table 3.1 in chapter 3 (the strands of mathematical proficiency) were covered. However, I do note that the learners all fell back on previously learned methods for calculating percentages, but that none of them made the connection to multiplication by 1.14 or 0.14.

From my observation of the discussion of question 1 above, I can safely say that the language presented no problem at all for the learners who spoke. This question was not an algebraic question, it was arithmetic. However, this question demonstrated that the learners were able to discuss in the language of instruction and were able to express themselves adequately (whatever their mother tongue was).

The learners did all the discussion, the working out of the problems and came up with the desired results and findings. I had encouraged them to think for themselves and had created a classroom atmosphere that was relaxed, yet conducive to learning. The learners did not show signs of experiencing any language problems which impeded their learning experiences and the solution of the problem in question 1. The learners showed interest in their work. There was no chewing of bubblegum and no staring out the window during these videotaped lessons. I was amazed that the classroom culture could be on the path to changing so quickly.

Levani read out question 2: "Edgars has a 33⅓ % price reduction on a pair of jeans that originally costs R540.00. Consider. 540 – x = y.   What is the new price (sale price) of the jeans? (Hint: first calculate x which is the reduction of the jeans in rands and cents. Thereafter, calculate y which is the actual sale price of the jeans). Explain in words, exactly what must be done to answer this question." Levani explained: "You take 33 ⅓, divide it by 100 and then multiply by R540.00.

$$\frac{33\frac{1}{3}}{100} \times \frac{540}{1} = \text{R}179.82$$

R179.82 is the reduction on the pair of jeans. In order to get the sale price of the jeans, you subtract R179.82 from R540.00 and you get R360.18 which is the sale price of the jeans or Silvana said, "Basically, 33⅓ can be written as $\frac{100}{3}$. I then took $\frac{100}{3}$, multiplied it by R540.00 and then divided everything by 100,

$$\left(\frac{100}{3} \times \frac{540}{1}\right) \div \frac{100}{1} = R180.00$$

Thereafter, I subtracted R180.00 from R540.00 which gives me the sale price of the jeans, R360.00.

Prosperous said "I took 33.3, divided it by 100, and then multiplied by R540.00, i.e.

$$\frac{33.3}{100} \times \frac{R540}{1} = R179.82$$

which is the reduction price of the jeans, then I subtracted R179.82 from R540.00 and I got the new sale price of the jeans, which is R360.18. Once more, there is nothing to suggest that the language used in this problem, is above the understanding of the learners, be they English speaking, Zulu or Xhosa speaking. The learners have done all the discussion, and the problem solving on their own and they feel good about themselves, so the myth that nothing will happen if the educator does not take over, has been challenged and proved incorrect. Yet I, as the educator, still have an important role to play in deepening the learners' understanding. In this case, I could have asked the learners to explain the discrepancy between the answers.

Londiwe read question 3: "A man has R1640.00 available to spend on tools. He buys an electric saw, which costs 45% of the spending money and a hammer which costs 20% of the price of the electric saw. Calculate x and y in the following equation:

R1640.00 – x = y (let x be 45% of the total spending money)

Calculate a and b in the following equation:

R738.00 – a = b (let a be 20% of R738).

After purchasing these items, how much money remains? Explain, in words what needs to be done to answer this question." Londiwe answers: "So you take R1640.00 (spending money), multiply it by 45 and divide by 100, i.e. $\frac{R1640}{1} \times \frac{45}{100} = R738.00.$

Then, you take R738.00, multiply it by 20 and divide by 100 to get the price of the hammer – Londiwe pauses to use her calculator. The answer is R147.60. To get the amount of money left over, you add R738.00 to R147.60 this gives R885.60 (pauses again to use the calculator). Then take R885.60 and subtract it from R1640.00 you get (pauses again to use the calculator) R754.40.

Andile said "you cannot add the electric saw and hammer together because they are not the same." I asked the rest of the class if they agreed with what Andile had said. The learners did not respond for quite some time – they were obviously unsure or unwilling to comment. I pointed out that "we are adding the *price* of the electric saw to the *price* of the hammer, this we can do because they are both costs involved." In retrospect, I now realize I could have been more patient and creative in my effort to draw a response from the learners instead of providing them with an answer. I could have asked them what happens when they go shopping – the total cost of whatever items they buy is obtained by adding the prices of the items together. Just as in the case of algebra where "like terms" can be added, the price of the bread can be added to the price of the milk, if these are the items being bought. Andile continued "I followed Londiwe's procedure, i.e., I found 45% of R1640.00 and got R738.00. Thereafter, I found 20% of R738.00 and got R147.60. I then added R738.00 to R147.60 and got R885.60. I then subtracted R885.60 from R1640.00 and got R754.40."

I asked the learners whether they agreed with Londiwe and Andile and there was a unanimous "Yes, Miss." It is difficult to determine whether this was indeed the case or they were simply agreeing habitually, as a result of the classroom culture that had prevailed for many years.

Nicholas read question 4: "If you invest R5000.00 for 3 years at 10% Simple Interest p.a., how much money will you have at the end of the investment period?" Nicholas said, "We use the simple interest formula $A = P(1+i.n)$". He explained that P represents the money that is being invested, in that particular case, the R5000.00. Furthermore, the i represents

the rate of interest while n represents the number of years that the R5000.00 is invested for. Nicholas then explained

A= 5000 (1+0,10x3)

   = 5000 (1+0,30)

   = 5000 (1,3)

   = R1500. R1500.00 is the Simple Interest amount. We then add R5000.00 to R1500.0 and find that R6500.00 is the amount after the investment period.

I asked the question "Suppose you do not know the Simple Interest formula, how else could you work this problem out?" Levani answered! "Take the R5000.00 multiply it by 10 and divide the answer by 100, that is, $\frac{10}{100} \times \frac{R5000}{1} = R500.00$

This R500.00 is the interest for one year. Since there are 3 years, I now multiply R500.00 by 3 to give me an answer of R1500.00. In order to find the amount of money in the account at the end of the investment period, I then add R5000.00 and R1500.00 to give R6500.00". A comment from Pinky, one of the learners, "Miss it is easier to understand Levani's method."   The learners' preference for multi-step solutions using simpler calculations was clear from their approaches to all the tasks, but no more so than in question 5. Learners discussed question 5 for quite some time (± 10 minutes) before there was some sort of remark made. This is an important point: many times, as an educator, I feel the pressure to move on quickly and thus do not allow learners much time to think and discuss a single question, with a possible detriment to their learning.   Mthokozisi read out question 5: "5 men take 8 days to complete a task. How long will two men take to complete the same task if they work at the same pace?"

I kept asking for someone to lead the discussion until Mthokozisi said, "5-3 = 2 and 8+3=11.

To get to 2 men, you must say 5-3,   this implies that you must add 3 to 8 days so then 2 men complete the task in 11 days. Eight other learners agreed with Mthokozisi. I then changed the question slightly – in actual fact, I changed 8 days to 10 days. In retrospect, I realized that this was totally unnecessary since there are no fractions in the answer whether we use 8 days or 10 days. Vuyisa said, "In reality, 5 men should complete the task in a shorter time that 2 men would, but, mathematically, 5 men take 10 days but 2 men complete in 4 days.

This was Vuyisa's calculation:

$$5 men \quad 10\ days$$

$$2\ men \quad x\ days$$

$$5x \;=\; 2\text{x}10$$

$$5x \;=\; 20$$

$$x \;=\; 4$$

I then said to Vuyisa , with a smile, "So you are saying mathematics is removed from reality. You are saying that in reality 2 men take a longer time to complete the task but, mathematically, 2 men take only 4 days. So what are you saying about mathematics, that it is not real?"

The learners laughed and took this comment in good spirit. Nobody was malicious or nasty towards Vuyisa. The atmosphere was relaxed. This was unlike the first set of videotaped lessons where the learners became withdrawn after giving incorrect answers. My change of attitude rubbed off onto the learners, there was no mocking of incorrect answers.

I said, "Ok let's start from this point, 2 men must take a longer time to complete the task. Look at the numbers $\begin{smallmatrix}5 & 10\\ 2 & x\end{smallmatrix}$ in that pattern. How can we multiply so that x represents a number bigger than 10? Remember to create an equation so that the product of 2 numbers is equal to the product of the other two numbers." Of course, looking back, I can see that this was slipping into giving procedural directives again, since I did not engage why this approach would work. I could instead have asked the learners how many working days the job required, or something of that nature.

Prosperous said, "I say 5x x = 2 x10, 5x = 20 so x=4. Now add these 4 days to 10 so 2 men take 14 days." I then said, "O.K. people, think of the following statements:

"An increase in the number of loaves of bread bought is accompanied by an increase in cost.

A car that travels faster than another car will complete a particular journey quicker than another car that is travelling slower. If I just give you the numbers

$$\begin{matrix}5 & 10\\ 2 & x\end{matrix}$$

create an equation and see how you can multiply 2 numbers so that the answer is equal to the product of the other 2 numbers, and x ends up being a value greater than 10."

The learners discussed quietly among themselves. I then asked, "has anyone got something for me yet – trial and error. Just now Vuyisa said

$$\begin{array}{cc} 5 & 10 \\ 2 & x \end{array}$$

$$5x \, x = 2x10$$

$$5x = 20$$

$$x = 4$$

"You said that we cannot accept this because x should be bigger than 10".

Patuxolo then said, "Keep the arrangement,

$$\begin{array}{cc} 5 & 10 \\ 2 & x \end{array}$$

Then divide 5 by 5 to get 1, multiply 10 by 5 to get 50.

So, 1 man takes 50 days. Then take 1 and multiply it by 2 to get 2, and take 50 and divide by 2 to get 25. So then 2 men take 25 days." I said, "Your answer is quite correct. How can we solve this problem in a shorter way?" Prosperous said, "swop x and 10, i.e., rearrange

$$\begin{array}{cc} 5 & x \\ 2 & 10 \end{array}$$

then, say, 2x x = 5 x 10, 2x = 50, thus x = 25"

However, the learners in the class said that the arrangement did not accurately capture what was said in the question. The question, in other words, had been altered to: 5 men take x days and 2 men take 10 days. Vuyisa, basically repeated what Patuxolo said, "all men work at the same pace. 1 man takes (10x5) which is 50 days, i.e. divide 5 by 5 to get 1 and multiply 10 by 5 to get 50 days. The number of men is inversely proportional to the number of days."   I said "I like the words inversely proportional. We can also say indirectly proportional."

5 men   10 days

2 men   x days

We tried cross multiplying and we did not get x greater than 10."

Emelinah then said,   "5---------10

                    2--------- x days

Say 2x x = 5 x 10

$$2x = 50$$

$x = 25,$ so then 2 men take 25 days"

Emelinah displayed strategic competence here. She was able to formulate a problem as well as solve it.

I said, "Very good. This is an example of inverse or indirect proportion, as Vuyisa mentioned earlier on. A decrease in the number of men is accompanied by an increase in the number of days taken to complete the task."

Question 4 and question 5 were arithmetic problems. These problems (question 5) could be solved arithmetically, however, during the lesson we concentrated on creating equations and solving for the variable. In questions 4 and 5, all 5 strands of mathematical proficiency („conceptual understanding', 'procedural fluency', „strategic competence', „adaptive reasoning' and „productive disposition') of table 3.1 in chapter 3, were addressed.

Why do I say that „conceptual understanding' was covered in question 4? Unlike Nicholas had done, Levani was able to represent and solve the simple interest problem without using a formula. Levani displayed the ability to represent a mathematical situation in a different way. The learners skillfully solved problem 4 accurately, thus demonstrating „procedural fluency.'

The learners were able to formulate mathematical problems, represent them and solve them. For example, Emelinah and Patuxolo were able to solve problem 5 in different ways, yet obtaining the same answer. As I will mention later on in the reflections, I did not address the mathematical proficiency strand of „conceptual understanding (in question 2) which emphasizes the ability to represent mathematical situations in different ways.

Once again, upon observation of the lessons there was sufficient evidence that the learners understood the language during lessons and there were no problems whatever the learners' home language was. The learners had shown their willingness to take responsibility for their learning and there was nothing which suggested that they expected me to do all the explaining and talking. In fact, they enjoyed themselves immensely and were grateful for my faith in them, as mentioned by one of the learners, Siyanda, who said "Miss, it feels good when I can understand a question and I am given an opportunity to explain how to get to the answer."

4.3. Reflection

In keeping with the beliefs of Artzt & Armour- Thomas (2002), I realized that in my teaching, I needed to use reflection and self assessment procedures. I began to view myself as an agent in my own learning and development. I was willing and able to take responsibility for my actions in the classroom. I carefully planned what I intended to do before the lessons, during the lessons as well as after the lessons. I thought deeply about various aspects of my teaching, this was the first step towards becoming learner centred in my teaching!

Upon observing the videotaped lessons in which I had applied a new teaching strategy, I was struck by the vast improvement in the way in which the learners interacted with one another. In this way learners' ideas were reinforced and supported, yet challenged. The learners listened to one another, questioned one another, responded to one another, debated and gave explanations and justifications for their ideas. There were also no language problems nor cultural issues that cropped up during these videotaped lessons.

In all questions (question 1 to 5), the learners showed comprehension of mathematical concepts and skill in carrying out procedures. They displayed the ability to formulate, represent and solve mathematical problems, while reflection, explanation and justification were essential components of the videotaped lessons.

In question 5, for instance, "conceptual understanding" (strand A1 of table 3.1) was demonstrated in the sense that the learners showed comprehension of mathematical concepts, operations and relations. The learners displayed the ability to formulate, represent and solve problem 5 skillfully (actually better than what I had directed them to) and accurately, thus demonstrating "procedural fluency" and "strategic competence" (strands A2 and A3 respectively of table 3.1). The learners were able to demonstrate different ways of arriving at the answer to question 5 (for example, Patuxolo and Emelinah). In solving problem 5, the learners showed capacity for logical thought, reflection, explanation and justification, and in so doing, "adaptive reasoning" (strand A4 of table 3.1) was demonstrated. Furthermore, in question 5, explorations took place to some extent. This was in line with the advanced stage of teaching (see A3 of table 5.1), one

of the stages of teaching indentified by Goldsmith and Schifter (1997) which I will discuss later.

Upon reflection, I realize now that in the case of question 2 (the question about the sale price of the jeans), I put constraints on the method of solution. The idea was not to confuse the learners with a blizzard of solution methods. I did not focus on exploring different methods for solving the problem. I encouraged the learners to use multistep approaches. In retrospect, I accept that I could have done things differently. I could have guided the learners so that they could have picked up on other ways of solving the problem. For example, $33\frac{1}{3}$ % is $\frac{1}{3}$ which meant that the new (sale) price of the jeans would be $\frac{2}{3}$ of the old price. Thus, in question 2, the 5 strands of mathematical proficiency (as per table 3.1 in chapter 3) were not completely addressed. I thus faced the risk that learners would develop competence with the procedure but would not understand what they were doing or why. Also, linking many different approaches develops links to other aspects of mathematics (in this case fractions, but it could also have been to decimals) and this is core to conceptual understanding.

In the second action research cycle, I was not able to conclude that culture plays a role in creating algebraic misconceptions, but became aware of the prevailing classroom culture. I then began to focus on the teaching style that I adopted. I am determined to become a reflective, competent mathematics educator. To reach this status, the strands of mathematical proficiency have to be taken into account whenever lessons and assessments are planned, and learners have to be encouraged to develop meaningful mathematics. From this lesson, it appears to me that taking into account the strands of mathematical proficiency when preparing a mathematics lesson and drawing up questions, makes a world of difference to the learners' success in mathematics.

In an endeavour to become mathematically proficient, I am still working towards understanding how learners' mathematical comprehension develops and how students learn mathematics. Thereafter, I must forge connections between the two.

Chapter 5: Conclusion/Final Comments

What does it mean to be a competent mathematics educator? Let us picture two lessons (probability) as illustrated in Arzt and Armour-Thomas (2002). In the first lesson, the educator supplies the rules and definitions, the learners take notes, the educator does some examples while the students watch. Thereafter, the learners are given problems to work on. The educator then does corrections with the class and answers any queries the learners might have. (This was the procedure that I followed in the first action research cycle, and the way I had been teaching for years). In the second lesson, the educator engages the learners in a game of dice. The learners came to some conclusions regarding the fairness of the game. The learners work in pairs and then have a class discussion until they figure out the concepts of probability, underpinning the game. The educator helps the student organise and formalize their ideas. The educational reform movement recommends the type of teaching exemplified in the second lesson – which does not mean that the learners are left to their own devices: it clearly requires skilful guidance from the educator. Instead of dispensing information to passive learners, the educator of the second lesson provides an interesting problem to learners who are actively engaged in the problem solving process with one another. The educator oversees guides and facilitates the learners' construction of new knowledge.

Goldsmith and Schifter (1997) identified three stages of teaching, which have been summarized in table 5.1. (my own numbering system has been used).

Table 5.1 The three stages of teaching as identified by Goldsmith and Schifter (1997).

| A1. The initial stage | This stage is characterized by traditional instruction where the educator strongly believes that students learn best by receiving clear information transmitted by a knowledgeable educator. |

A2. Subsequent stages	The educator is more focused on helping students build on what they understand rather than helping them in the sole acquisition of facts. The instruction is founded on the educator's belief that students should take greater responsibility in their own learning.
A.3 The advanced stage.	Instruction is in line with the reform movement recommendations. The educator arranges experiences for students in which they actively explore mathematical topics, learning both the hows and whys of mathematical concepts and processes. The educator is motivated by the belief that, given appropriate settings and with careful guidance, students are capable of constructing deep and connected mathematical understanding.

My teaching in the first and second action research cycles, closely resembled the initial stage of teaching as described in A1 of table 5.1. In these action research cycles I subscribed to the popular belief that the educator's role is to transmit mathematical content, demonstrate procedures and solve sample problems, while the students learn mathematical content by paying attention to the educator and remembering what they were told.

The most important characteristic of the subsequent stages of teaching (A2 of table 5.1) is the fact that instruction is based on the educator's belief that students should take greater responsibility for their own learning.

In the second action research cycle, I considered whether culture was a source of these algebra errors. I observed a set of videotaped lessons very carefully in search of answers. However, I did most of the talking, stifling any form of responses that the learners might have offered. My actions were not in keeping with the recommendations of the Professional Standards for Teaching Mathematics (National Council of Teachers, 1991) and the most recent Principles and Standards for School Mathematics(National Council of Teachers, 2000), which emphasized that educators should create opportunities that stimulate, guide and encourage learners to make connections among mathematical concepts, construct mathematical ideas, solve problems through reasoning, and take responsibility for their own learning. Rather than seeing this as a personal issue of not changing my practice in accordance with official guidelines, it indicates to me the inertia of practice and school cultures.

Dewey (1933, p.9) defined reflective thinking as "The active, persistent and careful consideration of any belief or supposed form of knowledge in light of the grounds that support it." For transformative teaching to take place, reflection is necessary and educators should acknowledge the problems that are exposed as a result of the reflective process. The reasons for the acknowledged problem must be laid bare, plausible alternatives need to be explored and eventually a change in thinking and subsequent classroom action should take place. This is exactly what happened in my third action research cycle. I recognized that there was a problem. The teaching style that I adopted impacted negatively on the learners' understanding, response and attitude to algebra. I felt compelled to do something about this situation urgently. I turned to an extensive use of reflective and self assessment processes. This resulted in a transformation in my teaching. I was determined to become a competent mathematics educator. Research suggests that learning is an active problem-solving process in which social interaction plays a critical role. (Cobb, 1986; Vygotsky, 1978). It is for this reason that I encouraged the learners to discuss the problems that had been constructed during the set of videotaped lessons that formed the core of the third action research cycle.

My findings (from my research study of a grade 8 multicultural, multi-ethnic algebra class) were that all the learners made common algebraic errors regardless of their home language. The learners in my grade 8 research class did not experience problems with the language of instruction (English), used in the algebra class. Was this because these learners in this class had attended English Medium Schools from grade 1? Perhaps further research could focus on those learners who had not attended English Medium School from Grade 1? Would they experience problems with the language (English) in the algebra class? Or is the language issue of a more general nature, not specific to particular topics in mathematics?

With regard to whether the learners' background culture played a role in causing disruptions in the learning situation, I could not conclude much, since the data I obtained were hopelessly insufficient.

Most importantly, in the third action research cycle, I learnt what it means to be a reflective, competent mathematics educator. I am working steadily towards reaching this status. Perhaps many other colleagues (mathematics educators) could also work towards becoming competent, reflective mathematics educators in order to improve the teaching and learning of mathematics.

In terms of educators going in the direction of such a cycle, the starting point would be to become learner focused since this helps to advance quickly from the initial stage of teaching (where content is the main focus), to the advanced stages of teaching (where learner understanding is the main focus). It would be beneficial to educators if they engaged in thinking processes indicative of understanding and planning before teaching a lesson. For example, it would make sense for educators to think about their goals for the learners and thereby become conscious of their knowledge and beliefs about the learners, pedagogy and the content. The educators should also think about the difficulties that the learners are likely to encounter in their efforts to attain the goals. Educators, on completing a lesson, could rethink lesson goals and reconsider what the educators and learners said, did and felt during a lesson. This reflective phase is likely to uncover difficulties or problems which, if the educator does not address, may impede progress toward self improvement in teaching. Self assessment plays a pivotal role in enabling educators to become learner centred. Self assessment describes the kind of evaluative questions

educators ask themselves as they reflect on their teaching after completing lessons.

In the third action research cycle, I began to change my teaching approach in an effort to procure higher achievement among the learners.I began to see learning as something that happens in the interplay between the learners and myself. My own learning was cyclical in nature, beginning with determining whether language is linked to algebra errors, then investigating whether culture plays a role in creating algebraic misconceptions and, finally, ending up with something entirely different, namely, changing my teaching to an interactive style! The next stage that I want to reach in my teaching is to be able to address all mathematical strands of proficiency when preparing my lessons, tasks and assignments. I want to move towards the third kind of educator (the advanced stage educator of table 5.1), who provides learners with exploratory opportunities in mathematics and believes that learners are capable of constructing deep and connected mathematical understanding, given suitable circumstances. I am presently a mathematics educator (initial stage educator of table 5.1) who strongly believes that students learn best by receiving clear information that is transmitted by me as can be seen in the videotaped lessons of the third action research cycle. However, I plan to continue my reflections and action research on my journey towards becoming a mathematics educator (advanced stage educator of table 5.1), where I hope to guide my learners through investigations.

In an attempt to find out the possible causes of learners' misconceptions and resulting errors in algebra, I learned invaluable lessons regarding proper teaching methods. I view this new awareness of appropriate teaching styles as a starting point for further research towards understanding conceptual development of learners' algebraic thinking. Research and literature on algebraic learning and teaching currently focuses on how to prevent these errors. I definitely want to be part of this movement. A careful study of the literature on how learners' algebraic thinking should be developed could go a long way towards reducing misconceptions and resulting errors.

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APPENDIX A

Name And Surname   **Grade : 8:   .**

Algebra Worksheet

1. Simplify

2x +5y

2. Determine

xy-10 if x =4 and y=5

3. Evaluate:

a) 5a+3b-2c if a=1, b=2, c=3

b) 2a-3b+c if a= 2, b=3 and c=1

4. Under what condition is a + b +d = a+ c+ d?

5. My book has p pages. I have read q pages. How many pages must I still read?

6. Simplify:

$$\frac{8a^3b^2}{4a^2b}$$

7. Simplify:

$$\frac{4a^3\ b^2}{8a^2b}$$

APPENDIX B

Problems

1. If a bag costs R68.40, excluding vat, determine x and y in the following equation:

    R68.40 +x = y

    [Let x represent the VAT and y represent the cost of the bag inclusive of VAT.]

2. Edgards has a $33\frac{1}{3}$ % price reduction on a pair of jeans that originally

    costs R540, 00

    What is the new price (sale price) of the jeans?

    Consider : 540 – x = y.

3. A man has R1640.00 to spend on tools, he buys an electric saw, which costs 45% of the spending money and a hammer which costs 20% of the price of the electric saw. **Calculate x and y in the following equation:**

    R1640,00 –x =y

    Let x be 45% of the total spending money.

    Calculate a and b in the following equation:

    Let a be 20% of R738,00

4. If you invest R5000.00 for 3 years at 10% simple interest p.a., how much money will you have at the end of the investment period?

5. 5 men take 8 days to complete a task. How long will 2 men take to complete the same task if they work at the same pace?

Figure 6.1

<u>Algebra Worksheet</u>   <u>Grade: 8:</u> _____

ESL

1. <u>Simplify</u>
   $2x + 5y$

   $= 7xy$ ⟩

ESL

1. <u>Simplify</u>
   $2x + 5y$
   $= 7xy$ ✗

ESL

1. <u>Simplify</u>
   $2x + 5y$
   $= 7xy$ ✗

ESL

1. <u>Simplify</u>
   $2x + 5y = 7xy$ ✗

ESL

1. <u>Simplify</u>
   $2x + 5y$
   $= 7xy$ ✗

EFL

1. <u>Simplify</u>
   $2x + 5y$
   $7xy$ ✗

EFL

1. <u>Simplify</u>
   $2x + 5y$
   $= 7xy$ ✗

ESL

2. <u>Determine:</u>
   $xy - 10$ if $x = 4$ and $y = 5$
   $= 45 - 10 = 35$ ✗

ESL

2. <u>Determine:</u>
   $xy - 10$ if $x = 4$ and $y = 5$
   $= 45 - 10 = 35$ ✗

ESL

2. <u>Determine:</u>
   $xy - 10$ if $x = 4$ and $y = 5$
   $45 - 10 = 35$ ✗

ESL

2. <u>Determine:</u>
   $xy - 10$ if $x = 4$ and $y = 5$
   $45 - 10$
   $= 35$ ✗

EFL

2. <u>Determine:</u>
   $xy - 10$ if $x = 4$ and $y = 5$
   $= xy - 10$
   $= 45 - 10$
   $= 35$

EFL

2. <u>Determine:</u>
   $xy - 10$ if $x = 4$ and $y = 5$
   $= 45 - 10 = 35$

EFL

2. <u>Determine:</u>
   $xy - 10$ if $x = 4$ and $y = 5$
   $= 45 - 10 = 35$ ✗

Figure 6.2

Algebra Worksheet

EFL

3. Evaluate:
   a) $5a + 3b - 2c$ if $a = 1$, $b = 2$, $c = 3$

   $5 \cdot 1 + 3 \cdot 2 - 2 \cdot 3 = 60$ ✗

   b) $2a - 3b + c$ if $a = 2$, $b = 3$ and $c = 1$

   impossible, because a already equal 1, b equal 2 and c equal 3 in 3(a)

EFL

3. Evaluate:
   a) $5a + 3b - 2c$ if $a = 1$, $b = 2$, $c = 3$
   $= 51 + 32 - 23$
   $= 66$ ✗

   b) $2a - 3b + c$ if $a = 2$, $b = 3$ and $c = 1$
   impossible, because a already equals 1, b equals 2 and c equals 3 in 3(a)

EFL

3. Evaluate:
   a) $5a + 3b - 2c$ if $a = 1$, $b = 2$, $c = 3$
   $= 51 + 32 - 23 = 66$ ✗

   b) $2a - 3b + c$ if $a = 2$, $b = 3$ and $c = 1$
   impossible, because a already equal 1, b equals 2 and c equal 3 in 3(a)

ESL

3. Evaluate:
   a) $5a + 3b - 2c$ if $a = 1$, $b = 2$, $c = 3$
   $= 51 + 32 - 23$
   $= 60$ ✗
   b) $2a - 3b + c$ if $a = 2$, $b = 3$ and $c = 1$
   $=$ impossible, because a already equals 1, b equals 2 and c equals 3 in 3(a)

ESL

3. Evaluate:
   a) $5a + 3b - 2c$ if $a = 1$, $b = 2$, $c = 3$
   $51 + 32 - 23 = 60$ ✗

   b) $2a - 3b + c$ if $a = 2$, $b = 3$ and $c = 1$
   impossible, because a already equals 1, b equals 2 and c equals 3 in 3 (a)

ESL

3. Evaluate:
   a) $5a + 3b - 2c$ if $a = 1$, $b = 2$, $c = 3$

   $5 \cdot 1 + 3 \cdot 2 - 23 = 60$ ✗

   b) $2a - 3b + c$ if $a = 2$, $b = 3$ and $c = 1$

   impossible, because a already equals 1, b equals 2 and c equals 3 in 3(a)

ESL

3. Evaluate:
   a) $5a + 3b - 2c$ if $a = 1$, $b = 2$, $c = 3$
   $= 51 + 32 - 23 = 60$ ✗

   b) $2a - 3b + c$ if $a = 2$, $b = 3$ and $c = 1$
   Impossible, because a already equals 1, be equals 2 and $c = 3$ in 3(a)

Figure 8.3

Algebra Worksheet

ESL

4. Under what condition is a + b + d = a+c+d?
Impossible, b and c are different letters so they can't be equal.

ESL

4. Under what condition is a + b + d = a+c+d?
Impossible, b and c are different letters so they cannot be equal

ESL

4. Under what condition is a + b + d = a+c+d?
Impossible, b and c are different letters so they cannot be equal

ESL

4. Under what condition is a + b + d = a+c+d?
Impossible, b an c are different letters so they cannot be equil

ESL

4. Under what condition is a + b + d = a+c+d?
= Impossible, b and c are different letters so they cannot be equal

EFL

4. Under what condition is a + b + d = a+c+d?
Impossible, b and c are different letters so they cannot be equal

EFL

4. Under what condition is a + b + d = a+c+d?
Impossible, because b and c are different letters so they cannot be equal

Figure 6.4

Figure 6.4   Algebra Worksheet   Grade: 8: _____   81

ESL

5. My book has p pages. I have read q pages. How many pages must I still read?

$p - q = -1$ X

P 16th letters q 17th letter   so 16 -17 = -1

ESL

5. My book has p pages. I have read q pages. How many pages must I still read? $p - q = -1$

p 16th letter, I 17th letter

so 16-17 = -1

ESL

5. My book has p pages. I have read q pages. How many pages must I still read?

$p - q = -1$ X

P 16th letter   q 17th letter   so 16 -17 = -1

ESL

5. My book has p pages. I have read q pages. How many pages must I still read? $p - q = -1$ X

P 16th letter   q 17th letter so 16-17 = -1

EFL

5. My book has p pages. I have read q pages. How many pages must I still read? $p - q = -1$ X

P 16th letter q 17th letter so 16-17 = -1

EFL

5. My book has p pages. I have read q pages. How many pages must I still read? $p - q = -1$

p 16th letter q 17th letter

so 16 - 17 = -1

EFL

5. My book has p pages. I have read q pages. How many pages must I still read? $p - q = -1$

p 16th letter q 17th letter so 16-17=-1

EFL

5. My book has p pages. I have read q pages. How many pages must I still read?

$p - q = -1$ X

p 16th letter q 17th letter

so 16-17 = -1

EFL

5. My book has p pages. I have read q pages. How many pages must I still read?

$p - q = -1$ X

p 16th letter q 17th letter

so 16 - 17 = -1

Figure 6.5

ESL

6. Simplify:
$$\frac{8a^3 b^2}{4a^2 b}$$
$= 2ab$ ✗

ESL

6. Simplify:
$$\frac{8a^3 b^2}{4a^1 b}$$
$= 2ab$ ✗

ESL

6. Simplify:
$$\frac{8a^3 b^2}{4a^2 b}$$
$2ab^2$ ✗

EFL

6. Simplify:
$$\frac{8a^3 b^2}{4a^2 b}$$
$2ab^3$ ✓

EFL

6. Simplify:
$$\frac{8a^3 b^2}{4a^2 b}$$
$2ab^2$ ✗

EFL

6. Simplify:
$$\frac{8a^3 b^2}{4a^2 b}$$
$2ab^3$ ✓

EFL

6. Simplify:
$$\frac{8a^3 b^2}{4a^2 b}$$
$= 2ab^3$ ✓

ESL

7. Simplify:
$$\frac{4a^3 b^2}{8a^2 b}$$
$= 2ab$ ✗

ESL

7. Simplify:
$$\frac{4a^3 b^2}{8a^2 b}$$
$2ab$ ✗

ESL

7. Simplify:
$$\frac{4a^3 b^2}{8a^2 b}$$
$= 2ab$ ✗

ESL

7. Simplify:
$$\frac{4a^3 b^2}{8a^2 b}$$
$= 2ab$ ✗

EFL

7. Simplify:
$$\frac{4a^3 b^2}{8a^2 b}$$
$2ab$ ✗

EFL

7. Simplify:
$$\frac{4a^3 b^2}{8a^2 b}$$
$2ab$ ✗

EFL

7. Simplify:
$$\frac{4a^3 b^2}{8a^2 b}$$
$2ab$ ✗