The risk-return relationship and volatility feedback in South Africa: a nonparametric Bayesian approach

by

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DECLARATION

I, Nitesha Dwarika, declare that:

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ABSTRACT

The risk-return relationship is a fundamental concept in finance and economic theory and is also known as the “first fundamental law” in finance. Traditionally, the risk-return relationship is known to help assist individuals in the construction of an efficient portfolio where a desired risk and return profile is tailored to their needs. However, it is a source of much more valuable information to various market participants such as bankers, investors, policy makers and researchers alike. There are a number of investment strategies, policy frameworks, theories and asset pricing models built on the empirical result of the risk-return relationship. Hence, the topic of the risk-return relationship is of broad importance. It has been widely investigated on an international scale, especially by developed markets from as early as the 1950's, with the primary motive being to help market participants optimise their chance to earn higher returns.

According to conventional economic theory, the relationship between risk and return is a positive and linear relationship – the higher the risk, the higher the return. However, there are many studies documented in literature which show a positive or negative or no relationship at all. As a result, due to the magnitude of conflicting results over the years, this has caused an international and local debate to arise regarding the risk-return relationship. International studies have explored a number of theories and models to attempt resolving the inconclusive empirical backing of the risk-return relationship. On the other hand, the methods employed by South African studies and the volume of literature on the topic is relatively limited.

South Africa is becoming increasingly more recognised, liberalised, interactive and integrated into the international economy. Therefore, this study makes a significant contribution from a South African market perspective. This study identifies volatility feedback, a stronger measure of regular volatility, as an important source of asymmetry to take into account when investigating the risk-return relationship. Given that South Africa is an emerging market which is subject to higher levels of volatility, one would expect the presence of this mechanism to be more pronounced. Thus, this study investigates the risk-return relationship once volatility feedback is taken into account by its magnitude in the South African market.

A valuable contribution of this study is the introduction of the novel concept “asymmetric returns exposure” which refers to the risk that arises from the asymmetric

nature of returns. This measure has a certain level of uncertainty attached to it due to its latent and stochastic nature. As a result, it may be ineffectively accounted for by existing parametric methods such as regression analysis and GARCH type models which are prone to model misspecification.

The results of this study are presented according to the robustness of the approaches in the build up to the final result. First, the GARCH approach is employed and consists of a symmetric and asymmetric GARCH type models. The GARCH approach is treated as a preliminary test to investigate the presence of risk-return relationship and volatility feedback, respectively. While the GARCH type models have the ability to take into account the volatile nature of returns, asymmetries and nonlinearities remain uncaptured by the probability distributions governing the model innovations. Thus, the results of the GARCH type models are inconsistent and not statistically sound.

This motivates the use of a more robust method, namely, the Bayesian approach which consists of a parametric and nonparametric Bayesian model. The Bayesian approach has the ability to average out sources of uncertainty and measurement errors and thus effectively account for “asymmetric returns exposure”. The test results of both the parametric and nonparametric Bayesian model find that volatility feedback has an insignificant effect in the South African market. Consequently, the risk-return relationship is estimated free from empirical distortions that result from volatility feedback. The result of the parametric Bayesian model is a positive and linear relationship, in line with traditional theoretical expectations.

However, it is noteworthy that in the context of this study that the nonparametric approach is highlighted over the parametric approach. The nonparametric approach has the ability to adjust for model misspecifications and effectively account for stochastic, asymmetric and latent properties. It has the ability to take into account an infinite number of higher moment asymmetric forms of the risk-return relationship. Thus, the nonparametric Bayesian model estimates the actual fundamental nature of the data free from any predetermined assumptions or bias. According to the nonparametric Bayesian model, the final result of this study is no relationship between risk and return, in line with early South African studies.

KEYWORDS

risk-return relationship, volatility feedback, asymmetry, GARCH type models, nonparametric approach, Bayesian approach, asymmetric returns exposure, return inherent risk

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ABBREVIATIONS

ACF plot	Autocorrelation Function plot
AD test	Anderson-Darling test
ADF test	Augmented Dickey-Fuller test
AIC	Akaike Information Criterion
ALSI	All Share Price Index
APARCH	Asymmetric Power ARCH
AR model	Autoregressive model
ARCH	Autoregressive Conditional Heteroskedasticity
ARCH-LM	Autoregressive Conditional Heteroskedastic Lagrange Multiplier
BDS test	Brock, Dechert and Scheinkman test
BIC	Bayesian Information Criterion
DP	Dirichlet Process
DW test	Durbin Watson test
EGARCH	Exponential GARCH model
GARCH	Generalised Autoregressive Conditional Heteroskedasticity model
GARCH-M	GARCH-in Mean model
GED	Generalised Error Distribution
IID	Independent and Identically distribution
JB test	Jarque-Berea test
JSE	Johannesburg Stock Exchange
KPSS test	Kwiatkowski, Phillips, Schmidt and Shin test
LB test	Ljung-Box test

MA model	Moving Average model
MLE	Maximum Likelihood Estimation
NORM	Standard Normal distribution
OLS	Ordinary Least Squares
pdf	Probability density function
pp plot	Probability plot
PP test	Phillips-Perron test
Q-Q plot	Quantile-Quantile plot
Skew-t	Skew student-t distribution
Std-t	Student-t distribution
SW test	Shapiro-Wilk test
TGARCH	Threshold GARCH model
VaR	Value-at-Risk
VAR model	Vector Autoregression model

CHAPTER 1

1. INTRODUCTION

The introduction describes the rationale and motivation of this research topic. The overview captures the essence of what this research sets out to do while the background provides a more detailed discussion. The research problem narrows down the importance of this study with respect to South Africa. Thereafter, the research aims, questions and hypotheses guide the route of this research and the contribution shows the novel significance this study has to offer. Finally, the chapter concludes the structure of this study.

1.1 Overview

Although fundamental economic theory establishes a linear and positive risk-return relationship, many studies show a positive or negative, nonlinear or linear, significant or insignificant relationship (Savva and Theodossiou, 2018). Hence, there exists an inconclusive empirical backing to the theoretical risk-return relationship (Maneemaroj, Lonkani and Chingchayanurak, 2019). As a result, there is an ongoing international and local debate about the magnitude of conflicting results regarding the risk-return relationship (Savva and Theodossiou, 2018). The magnitude of the empirical risk-return relationship differs due to various factors (Maneemaroj *et al.,* 2019).

In the investigation of the risk-return relationship, it is found that there is always some dynamic component to account for variability or asymmetry that arises from price data, in order to address the omitted variable bias (Kim and Kim, 2018). That's because the assumption that price data of an entire financial system follows a normal and symmetric distribution cannot be accepted (Li, 2018). Share prices are dynamic as they constantly change over time, thus, are stochastic or random in nature which means that they can be statistically analysed but not with certain precision (Harris, 2017). Therefore, it follows that the return distribution, which is derived from price data, follows an asymmetric distribution (Gyldberg and Bark, 2019).

This research makes a significant contribution to the ongoing debate about the magnitude of the risk-return relationship by introducing the novel concept "asymmetric returns exposure" which refers to the risk that arises from the asymmetric nature of returns. The return inherent risk may be ineffectively accounted for by existing parametric methods due to its latent and stochastic nature (Jin, 2017). This is mainly due to the model's limitations and misspecifications to effectively estimate risk (Jensen and Maheu, 2018). For example, the Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model is one of the foremost methods to investigate the risk-return relationship (Madaleno and Vieira, 2018).

There has been a number of extensions of the standard GARCH model and various sources of price data variability to take into account (Cenesizoglu and Reeves, 2018). Extensions of the standard GARCH model include the EGARCH, GJR-GARCH and APARCH models among others which account for the asymmetric nature of volatility (Savva and Theodossiou, 2018). Further, sources of price data variability include volatility feedback, the leverage effect, skewness and behavioural biases (Yu, Kang and Park, 2018). However, if a model can effectively estimate risk, there is no need for such model extensions, specifications and omitted variables biases (Demirer, Gupta, Lv and Wong, 2019).

A model that satisfies these conditions is the nonparametric Bayesian approach by Jensen and Maheu (2018). The nonparametric Bayesian approach is unique to existing literature that typically uses conventional parametric methods such as the GARCH approach and regression analysis (Jensen and Maheu, 2018). The application of the Bayesian approach in practical real-life situations demonstrates its usefulness and effectiveness (Karabatsos, 2016). Given the recent pandemic of the Coronavirus disease (COVID-19), a number of studies applied the Bayesian approach and methods to contribute uncovering its properties (Linton, Kobayashi, Yang, Hayashi, Akhmetzhanov, Jung, Yuan, Kinoshita and Nishiura, 2020; Jung, Kinoshita, Thompson, Linton, Yang, Akhmetzhanov, Nishiura, 2020).

The Bayesian approach is suitable for models that understand the complexity of financial data, especially the nature of returns which is nonlinear, asymmetric, volatile, stochastic and latent (Wagenmakers, Marsman, Jamil, Ly, Verhagen, Love, Selker, Gronau, Smira, Epskamp, Matzke, Rouder and Morey, 2018). Thus, the use of a model such as the nonparametric Bayesian approach, that can fit nonlinear and asymmetric properties effectively, ensures a robust result (Demirer *et al.,* 2019). In turn, a reliable result ensures a significant contribution and progression of research, especially in a case where a problem does exist which causes an international debate to arise due to the magnitude of conflicting results (Savva and Theodossiou, 2018).

Therefore, a central theme of this study emphasises and recommends the use of nonparametric Bayesian models (Karabatsos, 2016). More so, for emerging markets which has unique market return characteristics such as heavy tails and high levels of volatility as pointed out by Herbert, Ugwuanyi and Nwaocha (2018). Volatility feedback is a measure of volatility that is persistent over time and stochastic in nature (Inkaya and Okur, 2014). This mechanism is characterised by tendencies to get stronger over time and take longer to die out, in comparison to regular volatility (Harris, Nguyen and Stoja, 2019).

In the context of the risk-return relationship, volatility feedback is driven by volatility (Chakrabarti and Kumar, 2020). Hence, it is a source of risk and poses a greater threat to an emerging market which is characterised by higher levels of volatility (Herbert *et al.,* 2018). As a result, this study specifically takes into account volatility feedback as an important source of asymmetry following the studies by Jensen and Maheu (2018), Kim and Kim (2018) and Harris *et al.,* (2019). The focus of this study is a South African market perspective since it is becoming increasingly more integrated into the international economy. This leads to the background of this study which provides a more detailed discussion.

1.2 Background

The condition of a country's financial system is a key indicator of a country's stability (Asuming, Osei-Agyei and Mohammed, 2018). This is because it forms the foundation for progression in terms of sustainable economic growth, development and job creation (SARB, 2019). While contributing to the overall stability of a country, it can also affect local and foreign investment decisions (Asuming *et al.,* 2018). In other words, the state of the financial market can either attract or repel investors, affecting the growth of the nation (Hung, 2019). Therefore, by understanding the status of a financial market, an investors decision making process and investment strategies can be assisted and molded accordingly (Nahil and Lyhyaoui, 2018). That is, in terms of what level of risk they should take with regards to an investment venture in order to optimise their chance to earn a superior return (Marozva, 2019). However, the volatile nature of a market causes forecasting key indicators such as risk and return to be difficult and more importantly, unreliable if inadequate models are used (Nahil and Lyhyaoui, 2018). Thus, forecasting risk and returns still remains a fundamental problem in any financial market (Liu, 2019). This is especially the case for an emerging market such as South Africa which is characterised by higher levels of volatility and turbulent market conditions (Herbert *et al.,* 2018).

The Johannesburg Stock Exchange (JSE) is the largest market in Africa and is the financial powerhouse of the nation (SARB, 2019). The JSE is ranked amongst the top twenty stock exchanges in the world and consists of almost 400 listed companies which are made up of a wide array of sectors (JSE, 2020). This makes South Africa highly interconnected to the global economy by various channels such as financial, trade and investment (Asuming *et al.,* 2018). In 2017, South Africa's export value amounted to \$108 billion, making the nation the 34th largest exporter in the world (OEC, 2019). The Morgan Stanley Capital International (MSCI) emerging markets index consists of twenty-four countries (MSCI, 2019). Figure 1 shows the 2019 contributing weights.

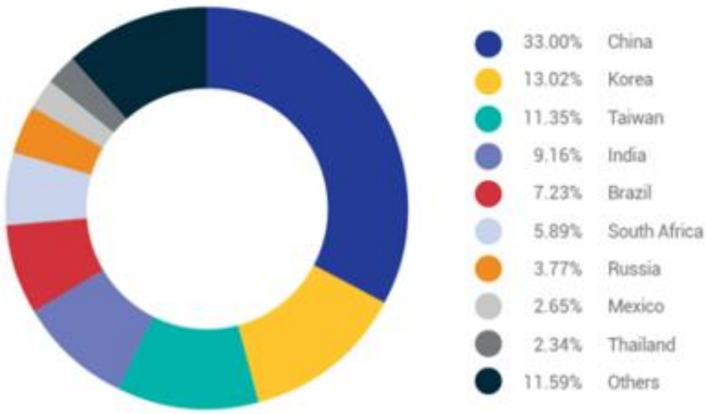

Figure 1: Weightings of MSCI Emerging Markets

Source: MSCI (2019)

From Figure 1, it can be seen that South Africa is ranked sixth out of the twenty-four countries, making it a significant contributor on an international scale. According to the Economic Development in Africa Report (2019), the nation redresses the African Continental Free Trade Area to improve the current domestic and international trading conditions. Thus, making South Africa an active participant to global and regional trade agreements with other continents and nations (Melis and Bonga-Bonga, 2019). South Africa's interactive nature is especially prevalent in its partnership with the five major emerging markets consisting of Brazil, Russia, India, China and South Africa (BRICS). According to Melis and Bonga-Bonga (2019), BRICS received the highest portion of the total $900 billion global corporate debt issued to the Emerging Market Economies in 2016. As a result, the high capital inflow led to accelerated development and growth among these nations (Melis and Bonga-Bonga, 2019). However, despite cash inflows and strengthened partnerships with respect to trade, there still exists a high degree of variation in economic activity in the South African market. The economic activity of South Africa is measured by the Gross Domestic Product (GDP).

According to the National Treasury (2020), the GDP growth rate had an estimated value of 0.3% for 2019 and an expected value of 0.9% for 2020. In contrast, the forecasted values in the previous year expected figures of 1.5% for 2019 and 1.7% for 2020 (National Treasury, 2019). The variation in GDP can be attributed to the volatile nature of the South African market which is a given characteristic of emerging markets (Herbert *et al.,* 2018). However, there are a specific number of factors that contribute to the volatile nature of the South African market which poses a risk to GDP growth. This includes consistent levels of corruption in the public and private sectors, power shortages as a result of Eskom, as well as policy and political uncertainty (National Treasury, 2019). This is an issue because South Africa relies heavily on foreign investment due to a low rate of local savings (National Treasury, 2018). Private investment contributes to approximately 60% of total investment and has been decreasing since 2015 (National Treasury, 2018). Figure 2 shows the index of private investment and business confidence in South Africa.

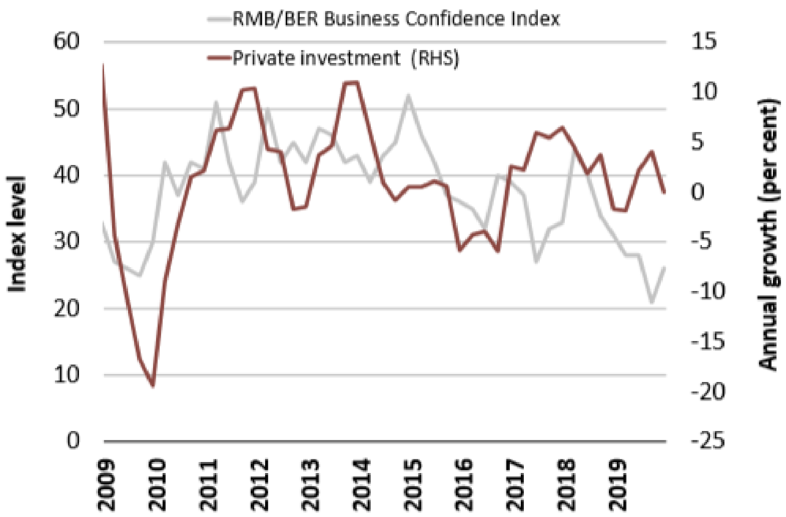

Figure 2: Index representing private investment and business confidence

Source: National Treasury (2020)

From Figure 2, over the years it can be seen that there has been a general greater decline in business and investor confidence as well as private investment. When a market is subject to volatility, which is a persistent level of risk, investors become risk averse because expected returns may mismatch with realised returns (Hussain, Murthy and Singh, 2019). Consequently, discouraging an investor's business confidence and corresponding private investment (Industrial Development Corporation, 2019). Thus, lead to low levels of economic activity and unfavourable conditions for investment ventures and exporting products as well as financial losses for the economy as a whole (Hussain *et al.,* 2019). This is because the fall in one country cannot easily be contained as a domestic event as it affects the rest of the world (Mancino and Sanfelici, 2019).

The world economy is interlinked by various markets, from emerging markets such as South Africa to advanced markets such as the United States (US). The interconnectedness of the markets allows for easy transmission of volatility known as the spillover effect (Newaz and Park, 2018). Spillover effects can lead to economic instability, major cash outflows and a potential financial crisis (Gulzar, Kayani, Xiaofeng, Ayub and Rafique, 2019). Financial crises are documented as one of the greatest sources for strong persistent levels of volatility and have a significant impact on market returns (Harris *et al.,* 2019). According to Marozva (2019), the 2007/2008 financial crisis amounted to an estimated $30 trillion worth of total losses in the equity market worldwide. The largest stock markets suffered losses around 40% to 60% in the period September 2008 to March 2009 alone (Marozva, 2019).

According to Gulzar *et al.,* (2019), the well-known Lehman Brothers, an international investment banking firm, suffered a great deal from the 2008 financial crisis. One of the accelerators for the firm's downward spiral was the impact of the transmission of bad news, a source of volatility. On 9th September 2008, there was a possibility of the Lehman Brothers being saved by the Korean Development Bank, but when talks were put on hold, the bad news spread. Consequently, there was a 45% drop in the share market, hedge fund investors pulled out and creditors ended their credit lines. As a result, the Lehman Brothers filed for bankruptcy on the 15th September 2008 and by then, the crisis had spread worldwide (Sehgal and Pandey, 2018; Gulzar *et al.,* 2019).

Emerging markets are more prone to negative effects since their markets may be underdeveloped and they do not have the sufficient funds to withstand financial losses (Gulzar *et al.,* 2019). This makes a country such as South Africa susceptible due to the low rate of local savings as well as being highly integrated with the rest of the world (National Treasury, 2018). Figure 3 shows the relative GDP growth over the years.

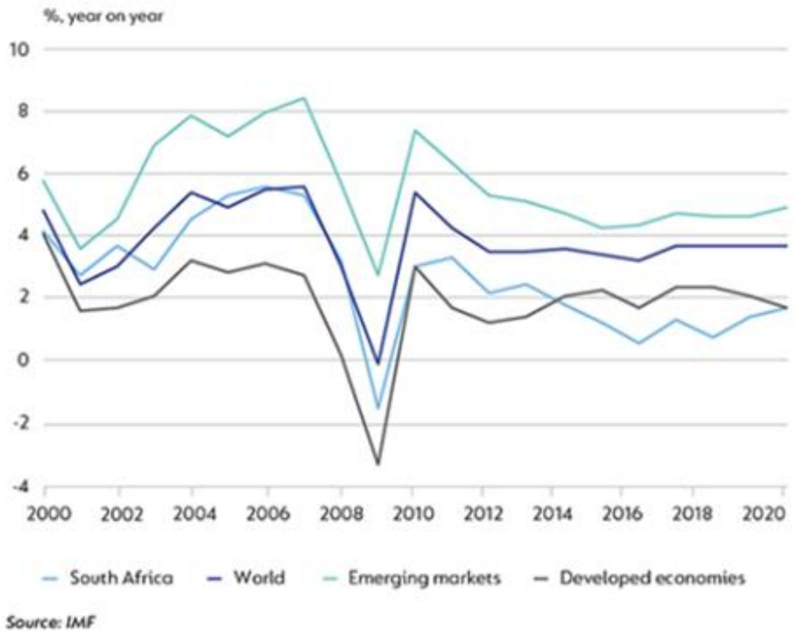

Figure 3: Relative GDP growth over the years

Source: Antelme (2018)

From Figure 3, South Africa appears to gain the least growth on a consistent basis. In contrast, the developed economies gain the most till after 2014 where they then overtake the emerging markets. The conditions of the South African market are due to its volatile nature and variation in GDP growth (National Treasury, 2019). However, it is noteworthy that an economies volatile nature can either be a source of development or stunt the growth of the economy (Hung, 2019). While persistent levels of volatility can deteriorate a market, on the other hand, it can attract investors to the possibility of earning superior returns due to fluctuations in market prices (Hussain *et al.,* 2019). In other words, mispricing's in the market are seen as an opportunity for arbitrage and to capitalise on (Gulzar *et al.,* 2019). This is because it is well known that investors often seek opportunities for arbitrage and diversification benefits based on the volatile nature of emerging markets (Hung, 2019).

Either way, the prediction of risk and return are key indicators to assist an investor's decision making process (Marozva, 2019). By forecasting these measures, investors are able to determine and establish their strategy before official announcements by rating agencies such as Moody's (Huang and Startz, 2019). Meaning, a forecast of the risk-return relationship provides an investor with a safe environment to strategise their investment ventures in order to optimise their chance to earn a superior return (Hussain *et al.,* 2019). By understanding risk, an investor's risk tolerance can be met where they can have an idea as to how they are compensated for the level of risk undertaken (Kempers, Leittersorf and Kammerlander, 2019). The actual realised returns earned then allows an investor to confirm the profitability of their investment venture (Gyldberg and Bark, 2019).

Based on fundamental economic and finance theory, one would expect the investor that takes on a higher level of risk to expect and realise a higher rate of return. According to the trade-off theory by Markowitz (1952), an investor only takes on a high level of risk if compensated by a high level of return. Further developments by Sharpe (1964), Lintner (1965) and Mossin (1966), led to the Capital Asset Pricing Model (CAPM) where the risk-return relationship originated. According to the trade-off theory and CAPM framework, the risk-return relationship is positive and linear. However, documented studies in literature show a magnitude of conflicting results regarding the risk-return relationship (Savva and Theodossiou, 2018).

The magnitude of conflicting results can be explained by the asymmetric nature of returns and volatility as follows. The assumption that price data of an entire financial system follows a symmetric bell-shaped curve cannot be accepted (Li, 2018). This is because share prices are dynamic as they constantly change over time; thus, price data are stochastic or random in nature which means that they can be statistically analysed but not with certain precision (Harris, 2017). Therefore, it follows that the return distribution, which is derived from price data, follows an asymmetric distribution (Gyldberg and Bark, 2019). An underlying cause is because the market is subject to asymmetric volatility which is an empirical regularity that has been systematically proven over time (Yu *et al.,* 2018).

Asymmetric volatility refers to the tendency where volatility increases more for negative returns than positive returns, or vice versa, for the same magnitude (Yu *et* *al.,* 2018). There are two theories that can further explain this phenomenon, namely, the volatility feedback (effect) and the leverage effect (Umutlu, 2019; Harris *et al.,* 2019, Chakrabarti and Kumar, 2020). The asymmetric volatility phenomenon is often treated synonymously with the leverage effect by the GARCH school of models (Mandimika and Chinzara, 2012). However, asymmetric volatility is strongly associated to a time varying risk premium which is linked to volatility feedback (Horpestad, Lyocsab, Molnara and Olsen, 2019). The risk premium refers to the excess or abnormal returns earned due to taking on a higher level of risk and is often used to investigate the risk-return relationship (He, He and Wen, 2018).

Despite the two theories being closely related, it is important to understand the difference between volatility feedback and the leverage effect (Mandimika and Chinzara, 2012). Both hold a relationship between price movements and volatility, where the leverage effect is negative by Black (1976) and volatility feedback is positive by Pindyck (1984). Volatility feedback is empirically favoured, in comparison to the leverage effect, mainly due to not being associated to the amount of debt a firm has (Cao, Chen and Hull 2018). Volatility feedback is a measure of volatility that is characterised by tendencies to get stronger over time and take longer to die out, in comparison to regular volatility (Harris *et al.,* 2019). According to Mancino and Sanfelici (2019), volatility feedback is a useful tool in understanding market conditions and can indicate the level of liquidity of individual securities and the market. Hence, it can be used as an indicator of market stability and assist investor decisions (Inkaya and Okur, 2014; Mancino and Sanfelici, 2019).

In the context of the risk-return relationship, the leverage effect is driven by returns, whereas volatility feedback is driven by volatility (Chakrabarti and Kumar, 2020). Thus, volatility feedback is a source of risk and poses a greater threat to an emerging market which is characterised by higher levels of volatility (Herbert *et al.,* 2018). In recent studies, it has been noted that the risk-return relationship is positive, in line with theoretical expectations when volatility feedback is taken into account. Specifically, when Kim and Kim (2018) apply a unified framework and Jensen and Maheu (2018) apply a nonparametric Bayesian approach. Therefore, this study takes into account volatility feedback as an important source of asymmetry following the studies by Jensen and Maheu (2018), Kim and Kim (2018) and Harris *et al.,* (2019).

The volatility feedback mechanism, risk and return variables are all nonlinear, inconstant and stochastic in nature (Harris, 2017). In order to observe these variables directly, previous studies impose economic restrictions or assumptions on the data (Jin, 2017). According to Karabatsos (2016), parametric methods are often based on a number of assumptions. Basic regression modelling consists of a number of different types of analyses such as prediction, causal and time series. In most research, the problems and questions are framed in terms of the dependent variable as a linear function of the independent variable/s. Additionally, the innovations are assumed to follow a normal distribution. If the data properties are in violation of such assumptions then the parameter estimates are going to be misleading. However, in the context of time series analysis, the violation of assumptions is most likely to occur because financial data has a volatile, asymmetric and nonlinear nature (Jensen and Maheu, 2018; Karabatsos, 2016).

For example, in the study by Kim and Kim (2018), returns are modelled as a linear function of volatility. However, this does not empirically hold true for both volatility and returns given the asymmetric volatility phenomenon and asymmetric returns exposure. Further, the GARCH approach has a number of nonnegativity constraints and assumptions to satisfy in order to validate the model which is tedious in nature (Jin, 2017). This school of models also limits the possibility of nonlinear and asymmetric properties due to essentially being parametric which means that it has a set number of parameters with respect to the sample size (Jin, 2017). Therefore, it has the inability to account for every possible risk-return relationship that can hold (Demirer *et al.,* 2019). As a result, this should lead researchers to use more robust methods that are available to effectively model complex data (Karabatsos, 2016).

A model that satisfies these conditions is the nonparametric Bayesian approach by Jensen and Maheu (2018). Bayesian statistics is an extensive field of study built on Bayes (1763) theorem which is defined as the probability estimation of a relationship given prior information. In this case, the relationship is between risk and return and the prior information refers to volatility feedback which is modelled from historical price data. The novelty and effectiveness of the Bayesian approach are demonstrated in its current use. Given the recent pandemic of COVID-19, a number of studies applied Bayes theorem and methods to contribute uncovering its properties (Linton *et al.,* 2020; Jung *et al.,* 2020). For example, a Bayesian regression model was used to confirm that the disease is transferrable by surfaces (van Doremalen, Bushmaker, Morris, Holbrook, Gamble, Williamson, Tamin, Harcourt, Thornburg, Gerber, Lloyd-Smith, de Wit and Munster, 2020).

The nonparametric approach is a “model free” approach where the data is estimated free from assumptions, nonnegativity or economic restraints (Jin, 2017). Studies highlight that the nonparametric approach relaxes the normality assumption, effectively accounting for asymmetric properties such as skewness, kurtosis and multiple modes (Apergis, Bonato, Gupta and Kyei, 2018). By modelling data in a nonparametric framework, this allows “the data to speak for itself” solely based on its nature and not any predetermined assumptions or bias (Jensen and Maheu, 2018). The combination of the Bayesian approach and nonparametric approach addresses the issues presented by the parametric methods effectively (Wagenmakers *et al.,* 2018). This ensures a powerful method of data estimation, a more robust result and significant contribution and progression of research, with respect to the magnitude of conflicting results regarding the risk-return relationship (Karabtsos, 2016; Jin, 2017).

There exists a rich amount of literature documented, regarding the risk-return relationship, especially in developed countries as highlighted in the study by Savva and Theodossiou (2018). However, the use of unconventional methods is still somewhat limited since the study itself employs a GARCH approach. This is mainly because the GARCH approach is one of the foremost methods used when investigating the risk-return relationship (Madaleno and Vieira, 2018; Savva and Theodossiou, 2018). While most documented studies highlight the advantages of the GARCH school of modelling based on its conventional use, they fail to investigate the model’s limitations and shortcomings. Therefore, this study highlights the superiority of the nonparametric and Bayesian approach which is effective in modelling real world data (Karabatsos, 2016; Jensen and Maheu, 2018; Wagenmakers *et al.,* 2018).

To conclude the background, Griffin, Kalli and Steel (2018), note that nonparametric Bayesian modelling is a relatively unconventional approach in the fields of economics and finance. In the Bayesian approach, the theory and empirical model are closely related by means of prior information (Herath, 2019). This is the main drawback of the Bayesian approach because prior information can be modelled from a source of subjectivity such as prior beliefs or experience (Bartlett and Keogh, 2016). However, this limitation can easily be overcome by taking into account relatively mathematically convenient mechanisms such as volatility feedback as in this study. Additionally, by using objective prior model specifications in the model implementation stage (Karabatsos, 2016; Waldmann, 2018).

Essentially, one drawback should not be the reason for the lack of use of such a robust approach that is most likely to provide more accurate results. This is especially so, in comparison to the conventional quantitative finance methods, such as regression analysis and the GARCH approach, which has a number of limitations that the Bayesian and nonparametric approach easily overcomes (Karabatsos, 2016; Griffin *et al.,* 2018; Wagenmakers *et al.,* 2018). Thus, this study aims to resolve the ongoing risk-return relationship debate by highlighting and applying the novel nonparametric Bayesian approach. This leads to the research problem of this study which narrows its focus on a South African market perspective.

1.3 Research Problem

In a financial market, volatility arises from changes in price data as a result of the reaction and response of investors to information in the form of news (Hussain *et al.,* 2019). Strong levels of volatility over a long period of time are considered undesirable as it can result in severe market instability and major cash outflows (Gulzar *et al.,* 2019). However, a fair amount can attract both local and foreign investors, increase overall investment activity and stimulate economic growth (Mancino and Sanfelici, 2019). In other words, volatility is a source of mispricing's from which investors can identify and capitalise on in order to make a profit (Hussain *et al.,* 2019). Volatility feedback is a measure of volatility and is of key interest because it takes longer to die out, in comparison to regular volatility (Mancino and Sanfelici, 2019).

Typically, higher levels of volatility and turbulent conditions are found in an emerging market (Herbert *et al.,* 2018). The returns of an emerging market are characterised by higher levels of volatility, heavy tails and better forecast ability (Herbert *et al.,* 2018). Hence, the presence of volatility feedback in an emerging market such as South Africa can be expected to be more pronounced. The presence of the volatility feedback mechanism can pose a serious threat to the South African market and have negative financial implications (Hussain *et al.,* 2019). This is because private investment contributes to around 60% of total investment and South Africa relies heavily on foreign investment because of a low rate of local savings (National Treasury, 2018).

Thus, volatility feedback is not only a useful market characteristic but also an important source of asymmetry to take into account when investigating the risk-return relationship (Jensen and Maheu, 2018). By taking volatility feedback into account, this assists the estimation of the risk-return relationship which is of paramount importance as it forms the basis of a number of strategies (Vo, Pham, Pham, Truong and Nguyen, 2019). There are a number of theories and asset pricing models built upon the empirical result of the risk-return relationship (Liu, 2019). Therefore, a more robust result can contribute to model developments in stochastic volatility modelling and efficient risk estimation in the fields of portfolio and risk management (Jin, 2017).

A holistic view of the risk-return relationship and volatility feedback provides useful information to various market participants. Specifically, this includes investors, financial analysts, portfolio managers and arbitrageurs when setting up an investment venture or strategy. It can assist their decision making with respect to the following: When to go long (buy) or short (sell), whether to invest, trade or hedge, decide the optimal time of entry or exit of the market, the level of risk to undertake and the amount to invest in. Therefore, the research problem of this study can be summarised in a statement as follows: Volatility feedback is an important source of asymmetry that should be taken into account when investigating the risk-return relationship. This leads to the following research aims and questions of this study.

1.4 Research Aims

- To investigate the magnitude of volatility feedback in the market.

The presence of volatility feedback is determined by its magnitude where it is characterised by tendencies to get stronger over time and take longer to die out (Harris *et al.,* 2019). On the other hand, the absence of volatility feedback can be shown to get weaker and die out over time (Jensen and Maheu, 2018).

- To investigate the risk-return relationship provided volatility feedback is taken into account by its magnitude.

The risk-return relationship is investigated once volatility feedback is taken into account by either its presence or absence based on its magnitude.

1.5 Research Questions

The research aims lead to the following research questions to arise which is addressed by the methodology:

- What is the magnitude of volatility feedback?
- What is the risk-return relationship provided volatility feedback is taken into account by its magnitude?

1.6 Research Hypotheses

From the research aims and questions, the research hypotheses are set up:

The null hypothesis that volatility feedback is evident in the market is tested against an alternative hypothesis of no evidence of volatility feedback in the market.

Should evidence of volatility feedback exist, the secondary hypothesis that volatility feedback has no effect on the risk-return relationship is tested against an alternative hypothesis that volatility feedback affects the relationship.

In the case of the alternative hypothesis, the magnitude of this effect is assessed. In a case where there is no significant evidence of volatility feedback in the market, the risk-return relationship is analysed free from empirical distortions that result from volatility feedback.

1.7 Contribution

This study is of importance as it has a valuable contribution in a number of ways:

It provides guidance to investors in their decision making which leads to more efficient and effective strategies (Nahil and Lyhyaoui, 2018). Investors can have a more accurate idea of the type of returns they can gain for their given level of risk undertaken (Gyldberg and Bark, 2019). This can be in terms of investing in the JSE or in a positive net present value project (Hussain *et al.,* 2019). Essentially, they can forecast their compensation for risk with greater accuracy and mould their investment strategy accordingly (Nahil and Lyhyaoui, 2018). The improved forecasting efficiency can lead to improved economic growth on a macroeconomic level (Liu, 2019).

The nonparametric Bayesian approach by Jensen and Maheu (2018), is the first study applied to the US market with respect to the risk-return relationship, to the best of the authors knowledge. Applying the proposed method to the South African market can allow for a comparative analysis of results. This is due to the unique market characteristics of an emerging market relative to a developed market as pointed out by Herbert *et al.,* (2018).

This study highlights the limitations and shortcomings of the GARCH family (Jin, 2017). Thus, stating that it can be considered as irrelevant and obsolete along with other conventional methods with regards to estimating the risk-return relationship (Jensen and Maheu, 2018). In turn, this study highlights the improvements of the Bayesian and nonparametric approach over the simple GARCH models in order to prove its superiority (Jensen and Maheu, 2018). Thus, encouraging the use of unconventional nonparametric Bayesian models (Karabatsos, 2016).

This study makes a significant and novel contribution to the local and international ongoing debate about the magnitude of the risk-return relationship. That is, by the introduction of the novel concept of "asymmetric returns exposure" which refers to the risk that arises from the asymmetric nature of returns.

Finally, this study addresses a gap in existing literature with respect to the risk-return relationship and the effects of volatility feedback. The lack of existing literature in South Africa over the years, with respect to the risk-return relationship topic, is highlighted in the studies by Mandimika and Chinzara (2012) and Savva and Theodossiou (2018).

1.8 Structure

The structure of this research is organised into five chapters:

Chapter 1 introduces the background and motivation for this research study.

Chapter 2 highlights the disparity in existing literature and points out the existing gaps which are then addressed in the form of a critical analysis.

Chapter 3 refers to the methodology employed.

Chapter 4 presents the empirical results and related discussion from the model output.

Chapter 5 concludes the study with a discussion of the results in the context of the research objectives. Further, the limitations of this study are noted and recommendations for future research purposes are proposed.

CHAPTER 2

2. LITERATURE REVIEW

The literature review consists of a theoretical framework, an empirical review and a summary of critical analysis. The overview captures the essence of what this literature sets out to achieve. The theoretical framework provides an overall understanding, by the review of various strands of literature and theories, to convey the importance of the risk-return relationship and volatility feedback. The empirical review shows previous empirical evidence, that has made use of alternative approaches from basic regression analysis to the GARCH approach, in the investigation of the risk-return relationship. Finally, the chapter concludes with a critical analysis to consolidate the literature review.

2.1 Overview

Both the theoretical framework and empirical evidence of the literature review discusses the risk-return relationship from the onset before introducing the volatility feedback mechanism. The different risk-return relationships are backed up by various theories (Chari, David, Duru and Zhao, 2018). However, this study focuses on the reason for the magnitude of varying results. From the empirical evidence, the results of the risk-return relationship are reviewed, listing the conventional methods typically used such as regression analysis, the VAR model, causality tests and the GARCH approach (Madaleno and Vieira, 2018). The gap in literature is highlighted which is the inconclusive empirical backing of the theoretical risk-return relationship as well as the model's limitations, shortcomings and misspecifications. However, this review also highlights recent unconventional methods which are nonlinear and nonparametric, particularly the nonparametric Bayesian approach by Jensen and Maheu (2018).

The theories from the theoretical framework by Maneemaroj *et al.,* (2019) and Jensen and Maheu (2018), are noted and critiqued from the results and the limitations of the models. Thus, the limitations of the models, in conjunction with the response to the above theories, allow for a critical analysis and a significant contribution to the ongoing debate about the magnitude of the risk-return relationship. It does this by narrowing down the causal factors contributing the inconclusive results to a single theoretically focused research question and a single empirical research problem. The theoretical research question refers to why existing studies are misestimating the risk-return relationship, contributing to the inconclusive results. The empirical research problem focuses on the cause and how it can be solved practically. This is done by a critical analysis which consolidates the literature and explains the novel concept “asymmetric returns exposure” to address the empirical research problem.

2.2 Theoretical Framework

The theoretical framework defines, links and discusses the main concepts and theories such as risk, return, their relationship, trade-off theory and volatility feedback.

2.2.1 Risk and Return

Risk is defined as the possibility of a future event deviating from an expected outcome where the greater the possibility of deviation implies a greater level of risk (Kempers *et al.,* 2019). For an investor, this is the possibility of failing to realise an expected rate of return for an investment venture (Hussain *et al.,* 2019). Further, the probabilities of possible future outcomes can be estimated given prior information (Aliu, Pavelkova and Dehning, 2017). This means that risk allows an individual to have some probability of knowledge, whereas in contrast, uncertainty does not (Aliu *et al.,* 2017). An understanding of risk is vital to all market participants in a financial system, especially in the decision making process (Kempers *et al.,* 2019).

According to the CAPM framework by Sharpe (1964), there are two types of risk, namely, systematic and unsystematic risk, which makes up total risk. Systematic risk is also known as undiversifiable risk, market risk or volatility and is a market inherent risk (Charles and Okoro, 2019). This means that the entire market and all the securities within it are exposed to risk factors that arise from the market such as the interest rate, currency rate and monetary policy (Gyldberg and Bark, 2019). Although an investor may not completely keep clear of systematic risk by means of diversification, it can be managed by hedging or by a proper security allocation strategy (Aliu *et al.,* 2017). On the other hand, unsystematic risk is also known as diversifiable risk, specific risk or residual risk and is a company or industry inherent risk (Charles and Okoro, 2019). This means that the securities that an individual invests in are exposed to risk factors associated with the firm or industry such as a change in management or regulation, respectively (Gyldberg and Bark, 2019). However, unsystematic risk can be reduced by means of diversification (Aliu *et al.,* 2017).

The standard measures used to quantify risk are often captured by beta for total systematic risk, which is specifically characterised by the CAPM, and standard deviation or variance for total risk (Charles and Okoro, 2019). Theoretically, variance is an appropriate risk estimator only when the return distribution is normal; however, empirically this is not always the case (Sehgal and Pandey, 2018). Thus, the quantification of risk can pose a challenge to researchers (Chiang and Zhang, 2018). As a result, a certain criterion is often set to support their approach or why studies tailor a risk estimator to cater for such statistical conditions which may be stochastic in nature (Vo *et al.,* 2019). For example, in the context of the risk-return relationship, the four standard types of risk measures that are typically used are historical, implied, conditional and realised variance (Jin, 2017). Historical and realised variance which are computed from historical data are considered to be inflexible, have limited forecast ability and low explanatory power (Park, Ryu and Song, 2017). Implied variance is proposed as a better risk measure due to its ability to capture investor behaviour and future firm prospects (Bekiros, Jlassi, Naoui and Uddin, 2017).

However, from a financial perspective, implied variance is limited in that it does not account for the risk that arises from macroeconomic fundamentals (Khan, Rehman, Khan and Xu, 2016). On the other hand, many studies document conditional variance as their risk measure as characterised by the GARCH approach (Madaleno and Vieira, 2018). However, the use of conditional variance may require certain assumptions and constraints to be imposed to the data (Kim and Kim, 2018). In contrast, realised variance is a data driven measure due to its random, stochastic nature and better forecast ability (Maneemaroj *et al.,* 2019; Noguchi, Aue and Burman, 2016; Zhang and Lan, 2014). Hence, the realised variance risk measure is used in models that are able to incorporate such properties, unlike a normal-type GARCH model (Chiang and Zhang, 2018). Realised variance is also a popular choice in nonparametric Bayesian modelling due to being in line with a model free approach (Jensen and Maheu, 2018).

Financial market returns are used to determine whether or not a trading strategy is profitable (Gyldberg and Bark, 2019). Investors often use CAPM to determine a rate of return to compensate for a level of risk taken which originates from the trade-off theory, the idea of higher the risk the higher the return (He *et al.,* 2018). Under the framework of the CAPM, there exists a direct relationship between expected returns and systematic risk (Sharpe, 1964). Financial securities that do not correspond to this relationship act as a source of price data variability (Gyldberg and Bark, 2019). Further, returns are subject to risk often as a result of uncertainty in the market (Apergis *et al.,* 2018). This calls the validity of the Efficient Market Hypothesis (EMH) by Fama (1970) into question. The EMH states that in an efficient market, prices contain all available information (Fama, 1970). Consequently, no securities are mispriced under the EMH, making excess returns impossible to realise consistently (Lehoczky and Schervish, 2018). As a result, this causes a more realistic approach to strategies and models in the estimation of risk and return (Apergis *et al.,* 2018).

According to Li (2018), returns of the entire financial market follow a normal distribution for two reasons. First, the Central Limit Theorem by de Moivre (1733), states that for a sample drawn from a distribution with a finite mean and variance, for a sufficiently large sample, tends to a normal distribution. Second, market stability arises from investor sentiment and individual risk preferences, which follow a positively skewed distribution (Li, 2018). This area of the distribution is favoured by investors due to being able to achieve higher payoffs (Yao, Wang, Cui and Fang, 2019). According to Casella and Gulen (2018), there exists a substantial amount of evidence in literature that financial market returns can be forecast. However, forecasting returns by time series and behavioural models are subject to a number of factors that cause returns to deviate from a normal distribution (Cenesizoglu and Reeves, 2018; Casella and Gulen, 2018). Contributing factors that affect returns include volatility feedback, the leverage effect, inefficient information, behavioural biases and different investor sentiment (Yu *et al.,* 2018). All of which, in turn, affect the risk-return relationship.

2.2.2 The Risk-Return Relationship

The risk-return relationship is a fundamental concept in finance and economic theory (Vo *et al.,* 2019). It is also known as the "first fundamental law of finance" (Liu, 2019). According to Modern Portfolio Theory (MPT) by Markowitz (1952), the variables of the risk-return relationship explain the construction of an efficient portfolio. Steyn and Theart (2019) emphasises the importance of MPT in portfolio and risk management where it provides a framework to quantify and understand the variables, risk and return as well as their relationship. Specifically, that risk can be reduced by means of diversification and higher returns can only be attained by higher risk (Steyn and Theart, 2019). Further developments of MPT by Sharpe (1964), Lintner (1965) and Mossin (1966), led to CAPM which provides a simplified explanation of MPT. First, CAPM introduced the two types of risk, systematic and unsystematic risk, as discussed, to provide a practical understanding of risk. Second, following MPT, unsystematic risk should be diversified away, leaving an investor with an opportunity to a higher return from systematic risk (Rutterford and Sotiropoulos, 2016).

According to Steyn and Theart (2019), CAPM explains the risk-return relationship by an equilibrium in which there is a linear relationship. According to the CAPM, the expectation of excess returns in a portfolio is a function of systematic risk and market excess returns. That is, by understanding the risk-return relationship in a market, an investor has insight as to whether they have an opportunity to optimise their chance of earning a superior rate of return. Essentially, from both MPT and CAPM, the risk-return relationship demonstrates the traditional risk-return trade-off in which an investor can only earn a superior return if they undertake a higher level of risk. By following this theory and understanding the empirical risk-return relationship in the market, an investor can construct an efficient portfolio in order to meet their desired risk profile and expected rate of return (Rutterford and Sotiropoulos, 2016; Steyn and Theart, 2019).

Forecasting the risk-return relationship is of paramount importance as it forms the basis of a number of strategies by investors, financial institutions, asset pricing models and policy frameworks (Vo *et al.,* 2019). There are a number of theories and models built upon it (Liu, 2019). Such theories include the underlying idea that the risk-return relationship is a requirement in the modelling of valuation techniques such as the Discounted Cashflow Model and the Contingency Claims Approach to name a few (Sehgal and Pandey, 2018). Financial institutions are able to determine and implement proper cash flow strategies in terms of borrowing and lending (Liu, 2019). When predicted for a specific market, it can help determine profitable strategies and curb market risk (Vo *et al.,* 2019). It assists policy makers in the construction of regulatory and policy frameworks (Mandimika and Chinzara, 2012). Further, it can also be used as an indicator of investor behaviour in terms of risk profiling (Dicle, 2018). Finally, the estimation of the risk-return relationship can assist in the prediction of a potential financial crisis according to Sehgal and Pandey (2018).

Based on literature, there are four existing types of the empirical risk-return relationship which are positive, nonlinear or curvilinear, negative and none (Savva and

Theodossiou, 2018). The underlying theories and graphs are briefly explained for each type of relationship:

For a positive risk-return relationship, a rational investor has the ability to choose their risk-return preference based on a wide array of investment choices (Dicle, 2018). The trade-off theory suggests that an investor is risk averse whereby a low level of risk undertaken results in a corresponding level of return and vice versa for a risk taking investor (Chari *et al.,* 2018). This is supported by the expected utility theory which states that an investor makes a choice that maximises utility, which is similar to a measure of satisfaction, and minimises loss (Rutterford and Sotiropoulos, 2016). Figure 4 shows the expected utility function.

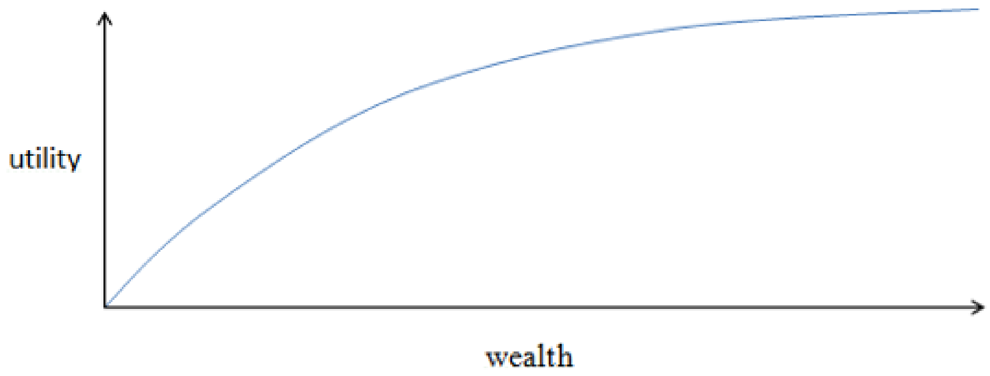

Figure 4: Expected utility function

Source: du Preez (2011)

From Figure 4, the conventional utility function has a concave shape which has a positive slope. The shape of the function demonstrates an investor's preference to a higher rate of returns, in comparison to a lower rate (Chari *et al.,* 2018). However, there is also a diminishing marginal utility which means that an investor's preference for higher returns increases but at a decreasing rate (Rutterford and Sotiropoulos, 2016). Since the graph is measured over total wealth, this suggests an investor's risk averse behaviour is symmetric for both gains and losses (Dicle, 2018).

On the other hand, a nonlinear risk-return relationship is explained by the prospect theory. The prospect theory by Kahneman and Tversky (1979), states that investors are risk seeking in unstable market conditions but risk averse in stable conditions. This is because the prospect of gain outweighs the prospect of loss and an investor makes a decision to ensure maximum gain and minimum loss. The prospect theory is essentially where an investor is more likely to take on a higher level of risk to avoid losses and ensure gains (He *et al.,* 2018; Kahneman and Tversky, 1979). Figure 5 shows a utility function.

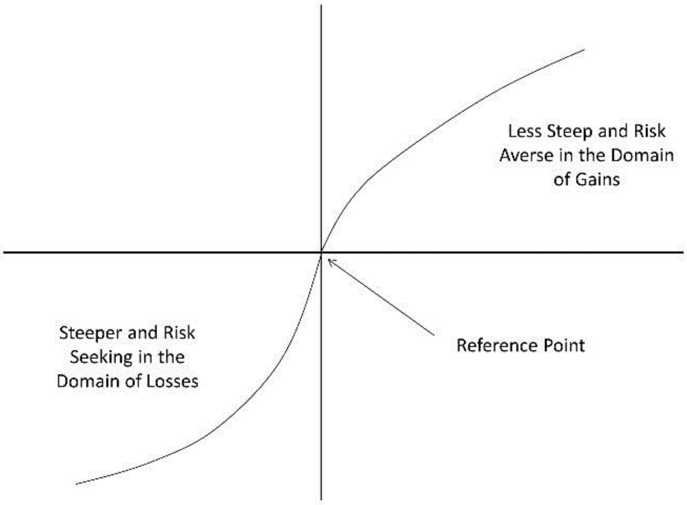

Figure 5: Utility function

Source: Phillips and Pohl (2017)

From Figure 5, the utility function is s-shaped and the positive part of the function is concave in the region of gains. This is similar to the expected utility function in Figure 4 above. From Figure 5, the negative part of the function is convex in the region of losses reflecting risk averse behaviour of an investor. Since the graph is measured over both losses and gains, this suggests an investor's risk averse behaviour is asymmetric where gains are favoured over losses (Dicle, 2018).

Chari *et al.,* (2018) highlights that a negative risk-return relationship is considered as paradoxical based on traditional theoretical literature. This is because a negative risk-return relationship is contrary to expectations founded on conventional economic theory and traditional literature (Chari *et al.,* 2018). Thus, it is also known as Bowman's (1980) Paradox which is explained by the prospect, behavioural and agency theory.

According to Patel, Shamsi and Asim (2018), in a firm setting, managers take on projects that do not match their risk profiles. Specifically, they take on risky projects when the firm is performing negatively, to advance their careers by improving their reputation in the labour market. Their actions are not necessarily in the interest of improving the value of the firm and increasing the wealth of the shareholders. But rather in line with their own self-interest because their actions suggest the ability to take on risks which can lead to a positive effect on their careers. Managers could simply pay out dividends if they cannot find profitable projects or investments. However, if they take on risky projects and if such risky projects consistently fail to meet expected returns, the value of the firm is negatively affected. Thus, problems arise between managers and shareholders if actual returns fail to meet the expected returns of shareholders (Chari *et al.,* 2018; Patel *et al.,* 2018). It consequently leads to a conflict of interest known as the agency theory developed by Mitnick (1973) and Ross (1973). Figure 6 explains Bowman's paradox by testing hypotheses that consolidate the theory of the negative risk-return relationship.

Figure 6: Theoretical model

Source: Chari *et al.,* (2018)

From Figure 6, Hypothesis 1 (H1-) states that the negative risk-return relationship is intensified by a manager's risk-taking behaviour. This is in order to improve his professional reputation and not add value to the firm or enhance the wealth of shareholders (Patel *et al.,* 2018). Hypothesis 2 (H2+) encapsulates a number of solutions to counteract the negative risk-return relationship and align the interests of managers and shareholders by means of governance. Hence, the risk-return paradox can be resolved by the establishment of governance mechanisms such as the market monitoring corporate control, establishing incentives and having organisational owners (Chari *et al.,* 2018).

When there is no risk-return relationship, this indicates that the risk-return relationship is insignificant (Apergis *et al.,* 2018). This often occurs in international studies that incorporate a number of countries which have different market return characteristics that can affect the final result of the risk-return relationship (Savva and Theodossiou, 2018). The returns of emerging markets are characterised as having higher levels of volatility, heavy tails and better forecast ability, in comparison to developed markets (Herbert *et al.,* 2018). Therefore, it is more useful in the context of this study to investigate countries with similar market return characteristics such as BRICS which consists of emerging markets only (Adu, Alagidede and Karimu, 2015). The analysis of similar markets allows for better statistical inference and comparative analysis between them (Sultan, 2018). Similarly, when investigating the risk-return relationship on an aggregate market level, it is important to consider the sectors within the market (Khan *et al.,* 2016). This is because the heterogeneity among firms, in terms of firm characteristics such as leverage and capital structure, can affect risk estimation (Horpestad *et al.,* 2019).

According to existing documented literature, studies show a magnitude of conflicting results with respect to the risk-return relationship (Chari *et al.,* 2018). From a broad perspective, results vary from study to study as a result of different choices such as data frequency, sample period and model specification (Savva and Theodossiou, 2018). To narrow it down, the magnitude of the empirical risk-relationship can be explained by two respective theories by Maneemaroj *et al.,* (2019) and Jensen and Maheu (2018).

Maneemaroj *et al.,* (2019) notes that there are five areas of concern that give rise to the different empirical outcomes regarding the risk-return relationship. The first is the type of frequency of return data, particularly high frequency data which is a source of unaccountable noise (Khan *et al.,* 2016). Second, the proxy for expected returns cannot be equivalent to historical returns (Koutmos, 2012). Third, the use of historical returns to represent expected returns should contain a sample period that is at least

200 years (Maneemaroj *et al.,* 2019). Fourth, the risk associated with return is due to information, and the reaction of investors in response to good and bad news are not the same (Yu *et al.,* 2018). Finally, the return distribution is asymmetric and heavy tailed (Herbert *et al.,* 2018). The first three factors surround the returns variable and the latter two are with respect to model specifications in capturing risk (Maneemaroj *et al.,* 2019).

According to Jensen and Maheu (2018), conventional methods found in existing literature that typically use the GARCH approach and regression analysis may be misestimating risk, contributing to the problem of inconclusive results. The novel nonparametric Bayesian approach overcomes the problems presented by the parametric methods. Further, there are a number of factors that contribute to price data variability such as volatility feedback, the leverage effect, inefficient information, behavioural biases and different investor sentiment (Yu *et al.,* 2018). However, this study accounts specifically for volatility feedback, which has been identified as an important source of asymmetry to take into account, in determining a positive risk-return relationship. That is, in line with the theoretical expectations, following the studies by Jensen and Maheu (2018), Kim and Kim (2018) and Harris *et al.* (2019).

2.2.3 Volatility Feedback and the Risk-Return Relationship

The assumption that price data of an entire financial system follows a normal and symmetric bell-shaped curve by Li (2018) cannot be accepted. This is because the market is subject to volatility which is a strong persistence of risk and has an asymmetric nature (Yu *et al.,* 2018). The phenomenon of asymmetric volatility is an empirical regularity that has been systematically proven over time (Yu *et al.,* 2018). Two traditional theories that aim to explain this phenomenon are the leverage effect and volatility feedback (Jin, 2017; Umutlu, 2019; Harris *et al.,* 2019). The GARCH school of models treat the leverage effect synonymously with asymmetric volatility (Mandimika and Chinzara, 2012). However, asymmetric volatility is strongly associated to the time varying nature of the risk premium which is strongly associated with volatility feedback (Horpestad *et al.,* 2019). It is important to understand the difference between the two theories since they are closely related (Mandimika and Chinzara, 2012). Both describe a relationship between volatility and price movements, but the leverage effect describes a nonlinear relationship, whereas volatility feedback describes a positive and linear relationship (Jensen and Maheu, 2018). There is also a difference in terms of the direction of causality for both effects. For the leverage effect, price movements lead to changes in volatility, whereas for volatility feedback, volatility leads to changes in price movements (Chakrabarti and Kumar, 2020).

Both theories are further discussed before volatility feedback is linked to the risk-return relationship. For the leverage effect in a firm, an increase in risk (debt to equity ratio) results in a decrease in return (equity value) and a drop in share price (Cao *et al.,* 2018). According to Herbert *et al.,* (2018), there exists a substantial amount of literature documented on the leverage effect in various types of markets and countries. However, there is still no agreement as to what are the driving forces that affect the magnitude of the leverage effect. Research suggests the triggers of the leverage effect are trading volumes and behavioural biases, herding behaviour in particular, but this is still questionable (Ong, 2015; Newaz and Park, 2018; Herbert *et al.,* 2018).

According to Chakrabarti and Kumar (2020), the leverage effect and volatility feedback are based on the fundamental factors of the firm. Although the underlying theory caters for leverage, there is no indication as to whether the leverage effect is really associated to the amount of debt a firm has (Carr and Wu, 2017). The leverage effect may also contribute more strongly to the magnitude of the risk-return relationship due to the heterogeneity among firms found on a sectoral level, affecting the outcome on a market level (Khan *et al.,* 2016). However, capital structure and expected returns may have a negligible effect on volatility as opposed to a negative effect (Horpestad *et al.,* 2019; Aboura and Chevallier, 2018). One of the main reason's volatility feedback is empirically favoured, in comparison to the leverage effect, is that it is not associated to the amount of debt a firm has (Cao *et al.,* 2018). Moreover, this study specifically treats volatility feedback as a measure of volatility and asymmetry, in line with Jensen and Maheu (2018), Kim and Kim (2018) and Harris *et al.* (2019).

For volatility feedback, a volatility shock in the market causes an increase in expected risk, a rise in return and a drop in share price (Pindyck, 1984). Volatility feedback is defined as a measure of volatility that is persistent over time and stochastic in nature (Inkaya and Okur, 2014). It is characterised by tendencies to get stronger over time and take longer to die out relative to regular volatility (Harris *et al.,* 2019). Volatility feedback is also described as a predictor of unanticipated errors as a result of macroeconomic fundamentals which affects returns (Kim and Kim, 2018). The volatility feedback mechanism is a positive relationship between price movements and volatility, in line with the theoretical risk-return relationship (Jensen and Maheu, 2018).

According to Umutlu (2019), volatility feedback is based on two assumptions, asymmetric volatility and the trade-off theory. The first assumption is asymmetric volatility which refers to the tendency of volatility to increase more for negative returns than positive returns, or vice versa, for the same magnitude (Yu *et al.,* 2018). Assumption two refers to the trade-off theory where an investor only takes on a high level of risk if compensated by a high level of return; hence, a positive risk-return relationship (Markowitz, 1952). Now, given that volatility is a priced factor in the market, meaning, for a price movement in shares there is a relation to risk (Khan *et al.,* 2016).

For a volatility shock in the market, that is, for a strong persistence of risk, actual volatility persists and signals future volatility by asymmetric volatility (Umutlu, 2019). Asymmetric volatility refers to the tendency of volatility to increase more for negative returns than positive returns, or vice versa, for the same magnitude (Yu *et al.,* 2018). From that increase in volatility, results an increase in return by the trade-off theory which refers to the idea – the higher the risk, the higher the return (Jensen and Maheu, 2018). Hence, volatility feedback closely reflects the risk-return relationship by definition (Horpestad *et al.,* 2019). This is why the volatility feedback mechanism can also be described as a risk premium that is an increasing function of volatility in the market (Apergis, Barunik and Lau, 2017).

Triggers of volatility feedback include good and bad news which leads to the persistence of actual volatility (Ong, 2015). It is said that the more liberalised a nation becomes the greater the ease of information and volatility transmission (Toraman, iGde, Bugan and Kilic, 2016). This contributes to the volatility spillover effect which refers to the transmission of volatility shocks (Newaz and Park, 2018). The volatility spillover phenomenon plays a role across markets on an international scale and can provide useful information since volatility is associated with portfolio and diversification risk (Toraman *et al.,* 2016). Therefore, volatility feedback plays an important role in the field of risk management as well as the allocation, pricing and diversification of asset securities in portfolio management (Herbert *et al.,* 2018).

Mancino and Sanfelici (2019) note that volatility feedback is a useful tool in understanding market conditions and can indicate the level of liquidity of individual securities and the market. Hence, it can be used as an indicator of market stability and assist investor decisions. The presence of strong volatility feedback suggests recessionary market conditions, whereas a weak presence indicates expansionary market conditions (Inkaya and Okur, 2014; Mancino and Sanfelici, 2019). This affects price movements and suggests the rising or falling of market prices, respectively (Aboura and Wagner, 2016). Forecasting this measure allows an investor to strategise their investment to improve the probability of realising a profitable return (Huang and Startz, 2019). For example, in the presence of strong volatility feedback which indicates recessionary market conditions, prices are expected to fall (Aboura and Wagner, 2016). Therefore, an investor may immediately short sell their shares to minimise their future potential loss (Khan *et al.,* 2016).

In the context of the risk-return relationship, the leverage effect is driven by returns (Chakrabarti and Kumar, 2020). On the other hand, volatility feedback is driven by volatility and is, therefore, a greater source of risk (Mancino and Sanfelici, 2019). Volatility feedback is a positive and linear relationship between price movements and volatility by definition and the assumptions by Umutlu (2019). This is in line with the theoretical risk-return relationship by Markowitz (1952), Sharpe (1964), Lintner (1965) and Mossin (1966). The volatility feedback mechanism has the ability to counteract cumulative price movements, such as an increase or decrease in price, and thus accounts for asymmetry (Yu *et al.,* 2018). Hence, volatility feedback has been identified as an important source of asymmetry to take into account in determining a positive risk-return relationship, in line with the theoretical expectations (Jensen and Maheu, 2018; Kim and Kim, 2018 and Harris *et al.,* 2019).

2.2.4 Summary of Theoretical Framework

The asymmetric nature of volatility is an underlying cause for the inconclusive risk-return relationship results (Maneemaroj *et al.,* 2019). The two foremost theories in explaining the asymmetric volatility phenomenon are the leverage effect and volatility feedback (Yu *et al.,* 2018). However, from the above theory, volatility feedback is more closely linked to the risk-return relationship by definition (Umutlu, 2019). It is a greater source of risk and is empirically favoured, in comparison to the leverage effect (Mancino and Sanfelici, 2019). This is because it is in line with the theoretical risk-return relationship and it is not associated to the amount of debt a firm has (Cao *et al.,* 2018). Volatility feedback is not just a useful market characteristic but also an important source of asymmetry when investigating the risk-return relationship (Jensen and Maheu, 2018). Although the documented theoretical framework of volatility feedback is limited, the empirical section provides more insight to the importance of accounting for this measure when investigating the risk-return relationship.

2.3 Empirical Review

The empirical review demonstrates the gap in existing literature, motivation for this study and describes the rationale of the proposed data and methodology. International literature is reviewed then local South African literature followed by a critical analysis to consolidate the literature review.

2.3.1 International Evidence

According to Chou (1988), the US market is the largest market in the world from which market participants have been seeking various ways to earn superior returns from as early as the 1950's. From 1958 onwards, the nature of the share market became a central focus due to the negative impact on returns earned by investors (Liu, 2019). To investigate volatility, the procedure used by previous studies at the time was a two-stage Ordinary Least Squares (OLS) method as documented by French, Schwert and Stambaugh (1987), Pindyck (1984) and Pagan and Ullah (1988). However, this is essentially a linear parametric model which does not adequately account for higher moment asymmetric properties (Madaleno and Vieira, 2018). Thus, the GARCH-M model in conjunction with maximum likelihood estimation (MLE) instead proves to be a more robust method as it addresses the drawback of OLS (Chou, 1988). The MLE method estimates parameters from the actual data allowing for nonlinear parameters (Madaleno and Vieira, 2018).

In an early US study conducted by Chou (1988), the returns from the New York Stock Exchange (NYSE) index for the sample period July 1962 to December 1985 is analysed at a weekly frequency. A GARCH (1, 1) MLE and linear AR (1) method is used, where the correlation coefficient in AR (1) acts as the parameter $(\alpha + \beta)$ found in the standard GARCH model, to capture the persistence of volatility. A plot of the NYSE returns shows the clustering nature of volatility where high returns follow high returns and low return follow low returns. The AR (1) method explicitly captures volatility clustering; however, the GARCH method provides more robust parameter estimates. The AR (1) method gives less robust parameter estimates due to its inability to capture a high level of volatility persistence over time. Chou (1988) concludes that the relationship between risk and return is time varying whereby the relationship changes over time.

More importantly, the study by Chou (1988), highlights the importance of using nonlinear models to capture market return characteristics since variance does have implications on returns earned. Specifically, the parameters reflect a strong impact of variance on the market resulting in negative returns earned (Chou, 1988). This finding is in contrast to Poterba and Summers (1986), who states that volatility is temporary and has a negligible effect. Chou (1988) further states the parameter estimates are found to be sensitive to data frequency. That is, the reason for the finding by Poterba and Summers (1986), is because of the use of monthly data instead of weekly which holds stronger for persistent levels of volatility. This is essentially due to the MLE method which has an improved ability to capture volatile properties, in comparison to the previous documented studies by French *et al.,* (1987) and Pindyck (1984) and Pagan and Ullah (1988).

The OLS method has been shown to be inadequate in capturing nonlinear properties by the model parameters, in contrast to MLE which is a nonlinear method by Chou (1988). Likewise, the AR (1) and VAR model which are parametric models are not designed to fit higher moment properties effectively (Demirer *et al.,* 2019). A parametric model has a set number of parameters with respect to the sample size (Jin, 2017). Consequently, it is limited in its ability to account for every possible risk-return relationship that can hold (Demirer *et al.,* 2019). This includes asymmetric forms with properties such as skewness, kurtosis and multiple modes (Apergis *et al.,* 2018). A model cannot be effective in modelling data with properties it is not designed or specified to take into account (Jensen and Maheu, 2018). This is especially relevant to the OLS since higher moment properties lie outside of its design parameters which can lead to biased parameter estimates (William and Ligori, 2016).

However, the drawbacks of the OLS method can mainly be attributed to the assumptions that they are based on. According to Conradt, Finger and Bokusheva (2015), the dependent variables are assumed to be constant and normally distributed.

Additionally, the innovations are assumed to be normally distributed as well as have a constant variance (Conradt *et al.,* 2015). According to William and Ligori (2016), while improvements have been made, such improvements still involve imposing constraints to the parameters or omitting some of the parameters. It is further noted that a more favourable approach would be to retain all the parameters without omission but such an approach has not been explored as yet (William and Ligori, 2016). While William and Ligori (2016) notes such an unexplored approach in the context of OLS, Conradt *et al.,* (2015) highlights that the nonparametric approach overcomes these issues. The nonparametric approach is a model free approach and models the data free from assumptions and restrictions (Jin, 2017; Conradt *et al.,* 2015).

Thus, Umutlu (2019) employs a parametric and nonparametric VAR model to investigate the relationship between market returns and idiosyncratic volatility. Idiosyncratic volatility is similar to unsystematic risk where risk exposure arises from the firm or industry such as a change in management or regulation, respectively (Gyldberg and Bark, 2019). Umutlu (2019) analyses monthly data consisting of nineteen local indexes of thirty-seven nations on an international aggregate level for the period 1973 to 2015, a sample of 42 years. Measures of volatility are model dependent and independent as well as a further four sub samples are analysed to investigate the possibility of a nonlinear risk-return relationship. Despite the use of subsamples, a parametric and nonparametric approach, results still reveal no risk-return relationship with the VAR model. However, the study did conclude strong support for the presence of volatility feedback during recessionary and high volatility periods (Umutlu, 2019).

Accordingly, one would expect a strong risk-return relationship during periods of extreme volatility (Umutlu, 2019). However, in contrast, Harris *et al.,* (2019), shows that the risk-return relationship dissipates due to the persistent effects of volatility feedback and the leverage effect. UK monthly data for the sample of July 1962 - December 2005 are analysed using a Value at Risk (VaR) model. A VaR model focuses on the tails of a distribution where extreme events such as the 2008 financial crises are likely to occur. A Markov switching model is further applied to account for the changing nature of volatility over time, for an extended period of July 1962 to December 2016. This is to account for the original period, the 2008 financial crisis and the remaining subsequent years. The addition of the Markov switching model is often used to account for the different regimes of volatility (Harris *et al.,* 2019).

However, according to Chang, Choi and Park (2017), the Markov switching model is considered impractical and unrealistic for two reasons. One, because the forecast of the model is based on the current state and not the underlying time series. Two, the Markov chain estimates the regimes in isolation of other parts of the model (Chang *et al.,* 2017). This implies a frequentist approach by Herath (2019), where theory is either accepted or rejected based on empirical results. In contrast, to the Bayesian approach which has the ability to account for prior information before making an estimation. In particular, where the theory and empirical model are closely related by means of prior information (Karabatsos, 2016; Herath, 2019). Hence, the Markov switching model is considered more effective when it is used in conjunction with the Bayesian approach according to Chang *et al.,* (2017) and Kang (2014).

Harris *et al.,* (2019) finds that there is a strong presence of volatility feedback and the leverage effect due to the persistent volatile periods. After accounting for both effects, a positive risk-return relationship is found at all states of volatility (Harris *et al.,* 2019). This finding supports the theory by Jensen and Maheu (2018) and Kim and Kim (2018), that when accounting for a source of asymmetry such as volatility feedback, this can result in a positive risk-return relationship. Chakrabarti and Kumar (2017) specifically investigates which theory is foremost in explaining the risk-return relationship of the Indian share market. The theories include behavioural theory, volatility feedback and the leverage effect (Chakrabarti and Kumar, 2017).

Chakrabarti and Kumar (2017) analyse daily data sets for the period 3 March 2008 - 31 August 2015, an approximate sample of 7 and a half years. The risk and return variables are obtained from the National Stock Exchange (NSE). Returns are obtained from Nifty which is the Indian share market index and consists of over twenty-three sectors. Implied volatility is the risk measure used due to its popularity in recent studies based on developed markets according to Chakrabarti and Kumar (2017). This risk measure is a forward-looking value that captures investor behaviour and future prospects of a firm (Bekiros *et al.,* 2017). It is in contrast to realised variance which is a recent preferred risk measure due to its ability to capture the stochastic nature of risk (Maneemaroj *et al.,* 2019).

Chakrabarti and Kumar (2017) employ a VAR model and Granger causality tests where volatility feedback is found to have the strongest explanatory power, in comparison to the other two theories. However, the VAR result and model is concluded to be ineffective because of its inability to account for extreme values and asymmetric properties. The Granger causality test is supported by a nonparametric framework where results reveal weak evidence for volatility feedback explaining the risk-return relationship (Chakrabarti and Kumar, 2017). Thus, a quantile regression analysis is applied which is effective in accounting for extreme values, in comparison to the VAR model and OLS method, according to Chakrabarti and Kumar (2017). The quantile regression finds the behavioural theory to be the dominant factor in explaining the risk-return relationship.

However, according to Waldmann (2018), the determination of the parameters for quantile regression is more difficult, in comparison to other regression types such as normal or generalised. In the context of model implementation, specific to the R software, the number of iterations has to be chosen in order to have optimal parameter estimates. This refers to the process of repetitive resampling by trial and error which can be tedious in nature (Karabatsos, 2016; Waldmann, 2018). Waldmann (2018) further recommends a Bayesian approach to enhance the method of interest; however, the priors should be as noninformative as possible. In other words, the prior should be objective and not guided by a source of subjectivity which overcomes the main limitation of the Bayesian approach (Bartlett and Keogh, 2016). The Bayesian method is often recommended to aid other methods because it has the ability to average out uncertainty affecting parameters (Kang 2014; Chang *et al.,* 2017; Waldmann, 2018).

The Granger causality test is another parametric model that is often employed to investigate the risk-return relationship (Apergis *et al.,* 2018). Similar to the Bayesian approach, the nonparametric framework is often used in conjunction with other methods such as the VAR model in order to aid capturing nonlinear properties (Umutlu, 2018; Demirer *et al.,* 2019). This is shown in the previous study by Chakrabarti and Kumar (2017), who applies the nonparametric approach in the context of Granger causality tests. Similarly, Apergis *et al.,* (2018), employs the same method of nonparametric Granger causality tests to the monthly data sets of twenty-four international defense firms. The variables analysed are the geopolitical risk index, realised variance and returns (Apergis *et al.,* 2018). Realised variance is used since it is line with the nonparametric approach in being model free, thus, aiding capturing asymmetric properties (Noguchi *et al.,* 2016). According to Apergis *et al.,* (2018), the nonparametric approach is used to account for nonlinearity in price data before applying the causality test. However, despite the application of the nonparametric approach, the results indicate no risk-return relationship. The study concludes the causality approach is unreliable and highlights the importance of accounting for nonlinearity before establishing the risk-return relationship to avoid model misspecification (Apergis *et al.,* 2018).

In contrast, to the result found by Apergis *et al.,* (2018), Demirer *et al.,* (2019) finds a significant relationship between risk and return in the US market. Demirer *et al.,* (2019) applies a number of models in order to investigate the relationship between equity return dispersion and share market volatility. Linear and nonlinear as well as bivariate and multivariate causality tests are employed to the sample July 1963 - February 2017. The share and market returns are obtained from the Center for Research in Security Prices (CRSP) value-weighted index return. The one-month Treasury bill (T-bill) rate is used to proxy the risk-free rate for calculating excess returns. The nonlinear bivariate and multivariate tests are found to be more robust as these models were able to account for the causal impact of return variance on returns and volatility. The study concludes that by accounting for the variance in returns, this improves risk estimation and contributes to the improvement of volatility models in predicting the risk-return relationship. However, this can only be performed by nonlinear models with the necessary model specifications to account for asymmetry (Demirer *et al.,* 2019).

This finding is supported by Madaleno and Vieira (2018), where the results are model-sensitive of which the volatility model, namely, the GARCH approach is the most robust. Madaleno and Vieira (2018) employ a number of models when investigating the risk-return relationship of Portugal. The daily price data of thirty-three enterprises listed on the Portuguese stock exchange for the sample period 31 December 2007 to 28 April 2017 are analysed. The methodology consists of regression analysis by OLS, the GARCH approach, VAR analysis and Granger causality tests. With respect to regression analysis by OLS, most of the results revealed no relationship between risk and return (Madaleno and Vieira, 2018). This result supports the theory of the inadequacy of regression analysis by Jensen and Maheu (2018) and the inadequacy of the linear OLS method by Chou (1988). Regression analysis is a basic parametric method which is inadequate in fitting financial data which is volatile in nature (Jin, 2017). Overall, the nonlinear GARCH-M model finds a positive risk-return relationship and is supported by the VAR model and Granger causality test.

Findings such as those by Madaleno and Vieira (2018), which are found to be limited advocate the use of nonlinear models such as the GARCH approach. According to Khan *et al.,* (2016) and Savva and Theodossiou (2018), the nonlinear GARCH models are regarded as a more suitable approach when investigating the risk-return relationship. The GARCH type models have the ability to account for a number of market characteristics which can explain the asymmetry shown in price data (Savva and Theodossiou, 2018). Such market characteristics include heavy tails, the clustering nature of volatility and asymmetric effects (Khan *et al.,* 2016). As a result, the GARCH approach is one of the foremost methods documented in previous existing literature when investigating the risk-return relationship (Madaleno and Vieira, 2018).

Khan *et al.,* (2016) employs the three most common GARCH type models – GARCH (1, 1), EGARCH and GARCH-M to the period June 1998 to June 2012, a sample of 14 years. Monthly returns are obtained from the Karachi Stock Exchange (KSE) 100 Index and twenty-three sectors of the Pakistani equity market. The choice of using monthly data is mainly to prevent the effects of time lags in price movements which usually has a greater impact for high frequency data. The study aims to analyse volatility and the risk-return relationship, on an aggregate market level and disaggregate sectorial level where the sectors are selected based on accessibility (Khan *et al.,* 2016).

The GARCH (1, 1) model has the ability to capture the clustering nature of volatility found in price data by capturing its persistence (Savva and Theodossiou, 2018). Khan *et al.,* (2016) finds that the GARCH (1, 1) model reveals persistent volatility but at different levels across the sectors. In order to capture further market characteristics, there have been a number of extensions of the standard GARCH model (Feng and Shi, 2017). The first of these is the EGARCH model, modified to capture the asymmetric nature of volatility and the leverage effect (Adu *et al.,* 2015). According to Khan *et al.,* (2016), the presence of asymmetry and the leverage effect is found to be weak by the EGARCH model, on both an aggregate and disaggregate level.

Specifically, only six sectors out of the total twenty-three reveal a presence of the leverage effect (Khan *et al.,* 2016).

The GARCH-M model is used for the pricing of risk which reveals whether return has a relation to risk (Savva and Theodossiou, 2018). Khan *et al.,* (2016) finds that the GARCH-M model reveals that risk is priced and that there exists a positive risk-return relationship, particularly at the sectoral level. Ten sectors are found to be positive and three are negative, with the remaining ten having no relationship. However, the GARCH-M model is limited because it does not account for asymmetric volatility (Maneemaroj *et al.,* 2019). Therefore, the risk premium remains constant for a specified period of time according to He *et al.,* (2018). Khan *et al.,* (2016) recommends the analysis of economic fundamentals, when investigating the risk-return relationship, since these are driving forces of asymmetric volatility and may affect returns differently.

Park *et al.,* (2017) applies a DCC-MGARCH model to the daily data sets of the Korean market for the period 2004 to 2013. The study analyses the variables KOSPI200 returns, VKOSPI implied volatility measure and four macroeconomic variables - risk-free rates, term spreads, credit spreads and exchange rates. The use of the asymmetric GARCH type model is confirmed by a sign and size bias which shows that the standard GARCH model has not adequately captured risk. The final findings of the study by Park *et al.,* (2017), reveal mixed results where return in relation to the macroeconomic factors vary based on the type of regression analysis and specifications. Further, the high correlation between the macroeconomic variables can also result in the problem of multicollinearity (Park *et al.,* 2017). Multicollinearity refers to a state in which several independent variables exhibit a high level of linear correlation, which can affect model fit and results (Khan *et al.,* 2016). Additionally, the choice of macroeconomic variables analysed are often guided by an underlying subjective approach (Messis, Alexandridis and Zapranis, 2019). This suggests a bias in the chosen macroeconomic fundamentals in explaining the risk-return relationship (Park *et al.,* 2017).

With respect to the DCC-MGARCH model, it is a complex model which forms part of the multivariate GARCH family to detect transmissions of volatility from one market or sector to another (Savva and Theodossiou, 2018). However, it is still a parametric model which is subject to the limitations of the univariate GARCH approach (Jin, 2017). This includes the nonnegativity constraints and the inability to effectively account for asymmetric properties (Demirer *et al.,* 2019). Another extension of the standard GARCH model is the GJR-GARCH model which has an additional term to capture possible asymmetries (Maneemaroj *et al.,* 2019). Specifically, in response to news which is a source of volatility and where the type of news has an asymmetric effect on volatility (Hussain *et al.,* 2019). However, this asymmetric effect is a given empirical regularity that has been systematically proven over time (Yu *et al.,* 2018). Maneemaroj *et al.,* (2019) applies the GJR-GARCH model to a sample of ten years for twenty-four stocks of the Thailand market. The study highlights the importance of variable choice in leading to the final result of the risk-return relationship (Maneemaroj *et al.,* 2019). The risk measure is realised variance because it is found to increase the predictive power of the test according to Zhang and Lan (2014).

Following the theory of Koutmos (2012), Maneemaroj *et al.,* (2019), argues the proxy for the return variable, where expected returns cannot equal historical returns. Therefore, a CAPM model is used to generate the expected return values. However, like capital structure, expected returns might have a negligible effect on volatility as opposed to a negative effect (Horpestad *et al.,* 2019; Aboura and Chevallier, 2018). Nonetheless, Maneemaroj *et al.,* (2019) finds a negative risk-return relationship when historical returns are used and a positive relationship when expected returns are used. However, this study does not account for a source of price data variability, creating an omitted variable bias as pointed out by Kim and Kim (2018). On the other hand, Savva and Theodossiou (2018) accounts for the omitted variable bias by taking into account skewness in their study, a measure of asymmetry found in price data. Skewness is found to be the main reason for the varying results regarding the risk-return relationship found in the US market (Savva and Theodossiou, 2018).

Due to the magnitude of risk-return relationship results, Savva and Theodossiou (2018) documents an international review of existing literature in an attempt to explain the ongoing debate. The Q-GARCH, GJR-GARCH and EGARCH type models are applied to the data sets of forty-eight global share markets at varying frequencies - daily, weekly and monthly. Standard returns are analysed against time varying volatility, instead of excess returns due to the inaccessibility and unavailability of a high frequency risk-free rate. Since the results of QGARCH and EGARCH are similar, the results focus on the GJR-GARCH model. The total risk-return relationship is estimated by the combined effect of a pure and skewed risk premium. For the total forty-eight markets included in their sample, the following majority of markets show no risk-return relationship at their corresponding data frequency, respectively: Forty-three markets at daily frequency, forty-two at weekly and thirty-seven at monthly. Overall results indicate no relationship between risk and return; however, the risk-return relationship is shown to be stronger for monthly data (Savva and Theodossiou, 2018).

In contrast, the study by Liu (2019) finds daily data to be the most effective in capturing the risk-return relationship. Liu (2019) uses a GARCH-M model along with in-sampling and out-sampling to investigate the risk-return relationship of the Chinese market. The in-sample refers to a forecast made based on the same set of data from which the parameters are estimated. Whereas, an out-sample refers to using a smaller dataset by excluding some of the observations. Liu (2019) took into account lagged returns since returns are subject to delays in response to new information. Both the Shanghai and Shenzen Stock Exchange indices of the aggregate Chinese stock market are analysed for the sample period 4 January 2000 to 21 May 2018. Varying data set frequencies are taken into account, namely, intraday, 2-day, 3-day, 5-day,10-day,15-day and 20-days. Results reveal a risk-return relationship that changes over time and concludes intraday data as the most robust, in comparison to the other frequencies.

A comparative analysis reveals the model taking into account the lagged returns is more robust than the one without. However, this not the only means to account for the lagged nature of returns since Khan *et al.,* (2016) uses monthly data to overcome this problem. Liu (2019) further concludes that although out-sampling improves prediction precision, this method is not better than using historical price data. Liu (2019) uses a standard GARCH model which is limited in its ability to account for asymmetric effects. Thus, a hybrid GARCH model which is a combination of some or all the GARCH type models is more useful to capture a number of volatility characteristics at a time (He *et al.,* 2018). More complex models can also be tailored, such as ADCC-EGARCH, which forms part of the multivariate GARCH family to detect transmissions of volatility from one market or sector to another (Sultan, 2018). However, the use of hybrid and/or complex models may be time consuming, computationally intensive and complicated (Chakraborty and Lozano, 2019).

According to the study by Sultan (2018), a univariate EGARCH and multivariate ADCC-EGARCH model are applied to the Finnish market. Sultan (2018) specifically investigates the asymmetric nature of volatility in the context of volatility feedback for twenty-four shares listed on the Finnish market. The study analyses an updated sample period where the 2008 financial crisis and any newly listed companies within the time period are excluded. Data sets of daily market price data are obtained for the sample 1 January 2009 - 31 December 2017, an approximate period of 9 years. The variables analysed are returns from the OMX Helsinki 25 stock index against conditional volatility and covariance. It is found that the presence of the leverage effect is high and that of volatility feedback is low. Moreover, that negative returns affect the risk premium more than positive returns, in line with the asymmetric volatility phenomenon. The study concluded that it could only be comparable to emerging markets similar to the Finnish market (Sultan, 2018).

According to Sultan (2018), the presence of volatility feedback is more pronounced when using daily data. The choice of daily data is in contrast to the theory of Maneemaroj *et al.,* (2019), who states it is a source of unnecessary noise. In addition, Savva and Theodossiou (2018), show a stronger risk-return relationship when using monthly data. The use of monthly instead of daily data reduces the effects of time lags in price movements according to Khan *et al.,* (2016). However, this study centres on the risk estimation of a sensitive measure of volatility which is stochastic and persistent in nature (Harris *et al.,* 2019). According to Inkaya and Okur (2014), the use of high frequency data has become popular given the rise in high frequency trading. Thus, daily data provides a more precise estimate of variables (Jin, 2017; Inkaya and Okur, 2014). Additionally, the study by Liu (2019), finds daily data as the robust choice relative to the other frequencies.

Kim and Kim (2018) investigates volatility feedback by employing a unified framework which is a generalisation of a number of sub models to the US market. The data sets analysed are monthly for a sample period January 1959 to May 2014. The variables of interest are excess returns and macroeconomic fundamentals to account for risk. The excess returns are measured as the difference of the market return from the CRSP value-weighted portfolio and 1-month US T-bill rate over the sample period. The unified framework consists of expected returns which are modelled as a linear function of expected variance, a probit model and a tailored factor called “volatility feedback news”. The study explicitly takes into account that volatility feedback is driven by news as a result of changes in macroeconomic fundamentals (Kim and Kim, 2018). However, triggers of volatility feedback also include good and bad news which leads to the persistence of actual volatility (Umutlu, 2019). Kim and Kim (2018) conclude a positive risk-return relationship for their study. With respect to their model, the main advantage of a unified model is the complete generalisation of a number of sub models (Kim and Kim, 2018).

However, the main problem in the study by Kim and Kim (2018), is the linear function of volatility. It presents an issue which is similar to one of the main drawbacks of the parametric GARCH approach where the parameters are subject to a constraint of nonnegativity (Jin, 2017). Thus, if the parameters do not meet this restriction some adjustment has to be imposed to the data (Demirer *et al.,* 2019). Returns cannot be a linear function of volatility because empirically, both volatility and returns are not linear in nature (Gyldberg and Bark, 2019). The phenomenon asymmetric volatility describes the asymmetric nature of volatility where volatility has the tendency to increase more for negative returns than positive returns, or vice versa, for the same magnitude (Yu *et al.,* 2018). “Asymmetric returns exposure” describes the risk that arises from the asymmetric nature of returns. The asymmetric nature of returns is due to the dynamic nature of price data which constantly changes over time (Harris, 2017). Hence, it follows that a return distribution is asymmetric since returns are derived from price data (Gyldberg and Bark, 2019).

This is further in line with Maneemaroj *et al.,* (2019), who states that returns follow an asymmetric and heavy tailed distribution. The heavy tailed distribution is a characteristic of an emerging market due to being subject to higher levels of volatility (Herbert *et al.,* 2018). In parametric approaches, it is common practice to impose assumptions and constraints on data (Apergis *et al.,* 2017). In contrast, a Bayesian approach is where parameters are treated as random variables with no constraints imposed when introduced into the model (Agilan and Umamahesh, 2017). This means that the parameters are treated in accordance to the probability of an outcome based on the method used and not adjusted to fit a certain or fixed result (Kim and Kim, 2018). Random sampling methods are the most effective in producing unbiased estimates since the outcome is based on equal chance (Alvi, 2016). Essentially, a model free approach allows for more flexibility in the estimation of complex data with nonlinear and asymmetric properties (Demirer *et al.,* 2019).

Jensen and Maheu (2018) apply a nonparametric Bayesian approach to the US market for a period January 1885 – December 2011, a sample of 126 years. In contrast, Maneemaroj *et al.,* (2019) advocates a sample of at least 200 years to represent the expected returns variable when, to the best of the authors knowledge, the longest sample used in a study of this nature has been the data set of 126 years used by Jensen and Maheu (2018). Further, the choice of sample size is also dependent on the availability of data, which can be overly restrictive in most cases, especially for international studies. Thus, the 200-year sample advocated by Maneemaroj *et al.,* (2019), can be considered impractical in reality.

Jensen and Maheu (2018) analyse monthly excess returns, calculated as the difference between the S&P500 returns and risk-free rate for a specified sample of forty years followed by a 1-month US T-bill rate. A bias adjusted realised variance is calculated following Hansen and Lunde (2006), where the bias adjustment accounts for microstructure noise. Microstructure noise refers to the micro price movements in the market due to changes in supply and demand, and stale prices which are when prices do not update to recent information (Hansen and Lunde, 2006). Jensen and Maheu (2018) account for volatility feedback which is considered as an important source of asymmetry which affects risk estimation. Like Demirer *et al.,* (2019), the study recommends moving away from linearity to include densities with higher moment properties such as skewness, kurtosis and multiple modes.

According to the study by Jensen and Maheu (2018), results are graphically presented by plots of density estimation over a 90% credible interval. Volatility feedback is found to get stronger over time and shift outwards and rightwards. Once it is taken into account, the study finds a positive and nonlinear risk-return relationship. Results are further supported by quantile regression and contour plots, which confirms the asymmetric properties being effectively captured. Finally, the study by Jensen and Maheu (2018), recommends using high frequency for future research purposes.

2.3.2 Summary of International Literature

From the review of international empirical literature, it can be seen that the risk-return relationship is investigated widely by a number of studies (Savva and Theodossiou, 2018). However, the results vary a great deal since many studies show a positive or negative, nonlinear or linear, significant or insignificant relationship (Chari *et al.,* 2018). This confirms the inconclusive empirical backing to the theoretical risk-return relationship (Maneemaroj *et al.,* 2019). It can further be seen that despite the twenty-year gap, from 1988 to 2018, the same line of conventional quantitative finance methods and econometric models have been typically used in the investigation of the risk-return relationship (Savva and Theodossiou, 2018). However, recently there has been an inclination toward more nonlinear and nonparametric approaches, particularly an inclination to more mathematical and statistical based models according to the studies by Demirer *et al.,* (2019), Jensen and Maheu (2018) and Kim and Kim (2018). This is in line with Zitske (2019), who states that the fundamentals of mathematics and statistics remain the same but the application changes.

That is, complex mathematical and statistical theories are transformed into relatively simple and practical computational methods where one can easily obtain results (Bartlett and Keogh, 2016). This is made possible as a result of technological advancements and relevant up to date software (Karabatsos, 2016). Take the Bayesian method which originated by Bayes (1763) for example; this method consists of intense mathematical integration which refers to the process of averaging out the uncertainty surrounding a variable. In nonparametric Bayesian modelling, this is considered as a golden standard method and is increasingly being used in different fields, mainly because of computational ease (Karabatsos, 2016). During the investigation of the risk-return relationship, it is noted that there is always some source of variability to take into account as this affects risk estimation (Cenesizoglu and Reeves, 2018). Hence, this study has identified volatility feedback as an important source of asymmetry to take into account when investigating the risk-return relationship following Jensen and Maheu (2018), Kim and Kim (2018) and Harris *et al.,* (2019).

2.3.3 Local Evidence

Mandimika and Chinzara (2012) highlights the fact that although South Africa is the largest market in Africa, yet studies on the topic of the risk-return relationship are limited. The first relevant study is by Mangani (2008), who investigates the weekly data sets of forty-two shares of the All Share Index (ALSI) and the corresponding equally weighted portfolios. Mangani (2008) employs the GARCH (1, 1), EGARCH and

DGARCH (GJR-GARCH) models to the sample 23 February 1973 - 5 April 2002. Results reveal that risk is an unpriced factor, meaning, there is no risk-return relationship, as well as weak evidence for asymmetric volatility and the leverage effect in the South African market. Mangani (2008) concludes the GARCH (1, 1) model is the most effective. However, the study finds that the innovations demonstrate random behaviour by a randomness test, suggesting that nonlinearities remain uncaptured within the innovations of the model (Mangani, 2008).

Similarly, Ilupeju (2016) investigates a number of GARCH models and innovation distributions to the daily data of the JSE for the sample 20 May 2005 – 31 May 2016. The GARCH type models include GARCH (1, 1), EGARCH, TGARCH (GJR-GARCH) and APARCH. The innovation distributions are a skewed student-t distribution, Pearson Type IV distribution (PIVD), Generalised Pareto distribution (GPD), Generalised Extreme Value distribution (GEVD) and stable distribution. Sign and size bias tests reveal the presence of volatility clustering and the leverage effect in the South African market. For the best model, the study identifies the APARCH model along with PIVD and GPD for the short run while with the stable innovation distribution for the long run. However, a number of randomness tests applied to the APARCH model find nonlinearities remains uncaptured, in line with Mangani (2008).

Following Mangani (2008), Mandimika and Chinzara (2012) employ three innovation distributions with respect to the EGARCH-M, TARCH-M (GJR-GARCH-M) and GARCH-M models to improve results. The three innovation distributions are the normal, student-t and the generalised error distribution (GED). The analysis is made from the JSE daily data for the sample 1995 to 2009. Dummy variables are employed to account for extreme events such as the 2007/2008 financial crisis and the 9/11 September political shock. Results reveal the trending pattern of volatility as asymmetric, strongly persistent over time and unpriced, hence no relation between risk and return. The study concludes the TARCH-M to be the most robust model and GED to be the best innovation distribution from the diagnostic testing of information criteria. The TARCH-M model is in contrast to Mangani (2008), who finds GARCH (1, 1) to be the most effective and Ilupeju (2016), who finds APARCH to be the best. Mandimika and Chinzara (2012) concludes by the recommendation of taking into account skewness and kurtosis when results are inconclusive regarding the risk-return relationship.

The GARCH type models can account for high levels of volatility and heavy tails through model specification (Adu *et al.,* 2015). However, a certain level of risk is still left behind uncaptured in the innovations as highlighted by Mangani (2008) and Ilupeju (2016). It is important to account for the innovation distribution as it may affect parameter estimation and final results (Mandimika and Chinzara, 2012). This is in contrast to Spierdijk (2016), who states that the distribution of model innovations does not affect parameter estimation. With respect to model implementation in the EVIEWS software, according to Brooks (2014), there exists a coefficient covariance option to specify whether innovations are assumed to follow nonnormality. However, this has no impact on the parameter estimates (Brooks, 2014). Focusing on the study by Mandimika and Chinzara (2012), a normal innovation distribution is unrealistic as financial data is volatile in nature. According to Feng and Shi (2017), when fitting the nonnormal innovation distributions, results are consistent but inefficient. If fitted incorrectly, results are then biased (Feng and Shi, 2017).

On the other hand, although the innovations follow asymmetry for a GED, the distribution itself is still symmetric according to Brooks (2014). For the student-t distribution, the magnitude of the heavy tails is captured in the parameter containing the degrees of freedom which remains constant. Thus, both the innovation distributions governed by GED and student-t, suggests that the parameters are limited to change (Brooks, 2014). According to Feng and Shi (2017), both the innovation distributions governed by GED and student-t lack stability under aggregation. This means that the combination of two variables, such as for the risk-return relationship, results in a distribution that is not in line with what was originally used. Hence, the underlying problem regarding the innovation distribution can lead to inefficient risk estimation since the parameters are limited to change and fitting distributions with higher moment properties (Feng and Shi, 2016, 2017).

Due to the developments of the South African market over time, du Toit (2015) revisits a study by Brummer and Wolmarans (1995). Du Toit (2015) applies multiple regression of the CAPM and an analysis of *t*-tests, *p*-values and *R* squared. The risk-return relationship is investigated by the analysis of 107 JSE-listed firms for a sample 2002 - 2012, a period of 10 years. The study focuses on whether a positive relationship exists between returns and a number of risk factors such as levered beta, unlevered beta, the debt to equity ratio and in particular, leverage. Results reveal no normality, linearity or any relationship between the return and risk variables, in line with Brummer and Wolmarans (1995). As a result, this finding indicates that basic regression analysis is limited and not a feasible method in establishing the risk-return relationship according to du Toit (2015). Thus, in support of the theory of the inadequacy of basic regression analysis by Jensen and Maheu (2018). According to Brummer and Wolmarans (1995), the outcome of the study by du Toit (2015) is twofold. Firstly, market participants are too risk averse and secondly, that there may be too many factors affecting the variable returns in the JSE (du Toit, 2015).

The latter is in line with Herbert *et al.,* (2018), who states that the return characteristics of an emerging market shows higher levels of volatility. It is further in line with the theory by Maneemaroj *et al.,* (2019), who states the return distribution is asymmetric and heavy tailed. A BRICS study by Adu *et al.,* (2015), confirms the market return characteristics of an emerging market being heavy and longer tailed. The study by Adu *et al.,* (2015), analyses the MSCI index for the sample January 1995 - May 2014 at a daily, weekly and monthly frequency. However, focus is mainly placed on the latter two because the normality assumption is more likely to be in violation for daily data. The models applied are an AR (1), MA (1) and ARMA-EGARCH-M model. The AR (1) and MA (1) model results suggest that past shocks can forecast future returns allowing for opportunities of arbitrage (Adu *et al.,* 2015). Hence, given this information, it is possible that volatility feedback could be present since it follows the theory of Pindyck (1984).

However, the AR (1) and MA (1) models are essentially linear parametric models which are not designed to effectively fit data with higher moment properties (Brooks, 2014; Jin, 2017). Therefore, the study employs a more complex GARCH model, ARMA-EGARCH-M. Accordingly, results reveal a leverage effect for all the emerging markets except South Africa. The ARMA-EGARCH-M model indicates no risk-return relationship for South Africa, in line with the results of the previous South African studies by Mangani (2008), Mandimika and Chinzara (2012) and du Toit (2015). However, despite the model specifications for asymmetric effects offered by the hybrid EGARCH-M, a certain level of risk remains uncaptured in the innovations (Feng and Shi, 2017). In other words, the GARCH approach does not provide the best estimation of risk for the investigation of the risk-return relationship (Jensen and Maheu, 2018).

For an international study, with respect to South Africa, Bekiros *et al.,* (2017), investigates the risk-return relationship for eleven developed and nine developing global markets. Bekiros *et al.,* (2017) analyses the sample 2000 to 2014, a period of 14 years. The indices used are a US implied variance measure, as well as implied variance and market prices for the respective markets based on access and availability (Bekiros *et al.,* 2017). A quantile regression approach is employed due to its ability to capture asymmetry and extreme values which are often missed by simple OLS and GARCH-M (Chakrabarti and Kumar, 2017). The results reveal that the magnitude of the nonlinear risk-return relationship differs in level of significance and size (Bekiros *et al.,* 2017).

The return coefficient maintains a negative and insignificant value at a 1% level of significance, especially at the ends of the distribution (Bekiros *et al.,* 2017). In particular, for South Africa, Bekiros *et al.,* (2017) finds a negative and significant risk-return relationship across the global markets. Overall, the study concludes that behavioural theory is foremost, in comparison to volatility feedback and the leverage effect, in explaining the results of the risk-return relationship (Bekiros *et al.,* 2017). The behavioural theory result by quantile regression is in line with the study by Chakrabarti and Kumar (2017), for the Indian share market. However, using quantile regression without a nonparametric framework questions its validity in accounting for every possible risk-return relationship that can hold as well as adjusting for any model misspecifications (Demirer *et al.,* 2019).

In an international study by Jin (2017), a negative risk-return relationship is found for the majority of markets in its sample, including the JSE of South Africa. The study analyses the respective daily returns and variance for the sample January 2001 – October 2014 of sixteen stock markets. Jin (2017) highlights the GARCH approach as being prone to model misspecification as well as having a number of underlying assumptions and constraints due to being parametric. The study by Jin (2017) then makes use of a nonparametric approach to address the limitations of the parametric approach. The nonparametric approach is a “model free” approach that estimates data free from any predetermined assumptions. Thus, effectively allowing for and accounting for higher moment asymmetric properties. Jin (2017) applies the Hurst exponent to measure the long-term memory which refers to the persistence of risk over a long period of time. A detrended cross-correlation coefficient (DCCA) is then applied which has the ability to account for data on the verge of nonstationarity as well as different scales (Jin, 2017).

According to Jin (2017), results reveal a strong presence of long-term volatility and a negative risk-return relationship that gets stronger over time. With respect to the 2008 financial crisis, it has a significant impact on the negative risk-return relationship of the majority of the markets, except South Africa, China, France and the USA. The study further investigates whether the result of the negative risk-return relationship is due to the leverage effect or volatility feedback. The majority of the markets show very weak evidence of volatility feedback with the exception of South Africa, Brazil, India and Indonesia which are all emerging markets. Although both volatility feedback and the leverage effect are present, the negative risk-return relationship is found to be more return driven by the leverage effect for the majority of markets, including South Africa. This finding is in contrast to the result of the BRICS study by Adu *et al.,* (2015), who found the leverage effect present for all the markets except South Africa.

In line with the result of a negative risk-return relationship by Bekiros *et al.,* (2017) and Jin (2017), is a more recent study by Steyn and Theart (2019). A negative risk-return relationship is found where high risk earns low returns in the South African market. Steyn and Theart (2019) apply basic regression analysis of Jensen's alpha, the Sharpe ratio and paired *t*-tests. The aim of their study is to investigate whether South African investors are being compensated for taking on a higher level of risk. All the shares on the JSE are analysed for the sample July 2004 to September 2018, an approximate period of 14 years. Two risk measures are constructed, standard deviation a metric of total risk and beta which is a risk metric of the market (Charles and Okoro, 2019). With respect to the model, regression analysis consists of basic statistical tests based on parametric models, which are inadequate in fitting financial data that is volatile in nature (Jensen and Maheu, 2018).

To clarify, according to Karabatsos (2016), parametric methods are often based on a number of assumptions. In the context of basic regression analysis, most research problems and questions are framed in terms of the dependent variable as a linear function of the independent variable/s. Additionally, the innovations are assumed to follow a normal distribution. If the data properties are in violation of such assumptions then the parameter estimates are going to be misleading. In the context of time series analysis, this violation of assumptions is most likely to occur because financial data has a volatile, asymmetric and nonlinear nature (Jensen and Maheu, 2018; Karabatsos, 2016). As a result, the regression analysis method is unreliable, in line with du Toit (2015) and Jensen and Maheu (2018), in estimating risk and contributes to inconclusive results regarding the risk-return relationship. Consequently, this should lead researchers to use more robust methods that are available to effectively model complex data such as Bayesian nonparametric models (Karabatsos, 2016).

2.3.4 Summary of Local Literature

From the review of local empirical evidence, although South Africa is the largest market in Africa, the investigation of the risk-return relationship is limited (Savva and Theodossiou, 2018). This is in terms of volume over the years as highlighted by Mandimika and Chinzara (2012), and the methods employed by Steyn and Theart (2019). In contrast, the international empirical literature, particularly for the developed countries, have more literature as documented in the study by Savva and Theodossiou (2018). Additionally, recent studies use more unconventional sophisticated methods such as the unified framework by Kim and Kim (2018) and the nonparametric Bayesian approach by Jensen and Maheu (2018). The risk-return relationship topic is popular in the US due to the size of the market and the fact that investors have been seeking a superior return from as early as the 1950’s (Liu, 2019). In contrast, according to Mandimika and Chinzara (2012), the first study relevant to South Africa that investigated the risk-return relationship was conducted far later in the publication by Mangani (2008).

There is no risk-return relationship according to the early South African studies such as Mangani (2008), Mandimika and Chinzara (2012), du Toit (2015) and Adu *et al.,* (2015). However, recent studies by Bekiros *et al.,* (2017), Jin (2017) and Steyn and Theart (2019), find a negative risk-return relationship. This is in contrast to the recent studies by international literature which reveal a positive risk-return relationship (Jensen and Maheu, 2018; Kim and Kim, 2018; Harris *et al.,* 2019). A reason for this could be due to the failure to account for volatility feedback when investigating the risk-return relationship or the choice of model (Kim and Kim, 2018). Given the results of the studies by Mangani (2008), Mandimika and Chinzara (2012), Adu *et al.,* (2015) and Jin (2017), this suggests the presence of volatility feedback. Further, given the unique market return characteristics of an emerging market by Herbert *et al.,* (2018), the risk-return relationship and volatility feedback are worth investigating in South Africa.

2.4 Critical Analysis

According to Savva and Theodossiou (2018), the magnitude of the empirical risk-return relationship is as a result of different choices of data frequency, sample period and model specification. However, this is a given and comes from a broad perspective. To narrow it down, the theories by Maneemaroj *et al.,* (2019) and Jensen and Maheu (2018), are highlighted to the extent of relevance on the area of concern in the empirical review. The limitations of the models in conjunction with the two theories allow for a single problem to be highlighted which is "asymmetric returns exposure”.

Volatility arises from changes in price data as a result of the reaction and response of investors to news (Hussain *et al.,* 2019). Price movements usually occur due to volatility feedback, the leverage effect, inefficient information, behavioural biases and different investor sentiment (Yu *et al.,* 2018). Essentially, resulting in a nonlinear and asymmetric return distribution since returns are derived from the price data which deviate from their fundamental value (Gyldberg and Bark, 2019). Since the return distribution is strongly linked to risk, the variability found in price data can lead to misestimating risk. Therefore, although various studies investigate various sources of risk arising from macroeconomic and financial factors, the “asymmetric returns exposure” may be overlooked. That is, the risk that arises from the asymmetric nature of returns. This can be a major contributor to misestimating risk, contributing to the inconclusive results of the empirical risk-return relationship. Figure 7 illustrates the asymmetric nature of a return’s distribution.

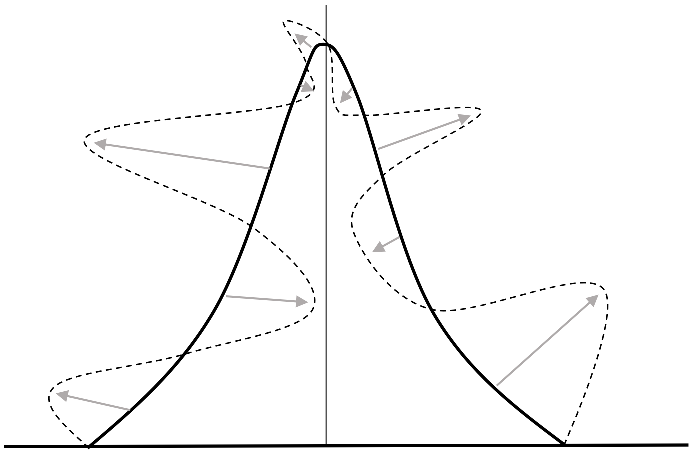

Figure 7: Asymmetric nature of returns

Source: Authors own

From Figure 7, the symmetric and bell-shaped curve represents the fundamental values of price data. The arrows represent the respective increase and decrease movements in price data as a result of various factors. This includes volatility, volatility feedback, the leverage effect, inefficient information, behavioural biases and different investor sentiment (Yu *et al.,* 2018). Consequently, this results in an asymmetric distribution of price data as shown by the broken grey line due to the random price movements. Hence, an asymmetric return distribution since returns is derived from price data (Gyldberg and Bark, 2019). By accounting for this risk, due to the asymmetric nature of returns, this provides a more efficient measure of risk and fundamentally addresses the omitted variable bias by Kim and Kim (2018).

The relationship between a return distribution and risk distribution may be linear or nonlinear as well as positive or negative, in relation to one another (Aboura and Chevallier, 2018). However, the very nature of a return distribution on its own is nonlinear, more so when taking on a higher level of risk which yields greater price movements (Hussain *et al.,* 2019). Therefore, a major contrast in this study is the investigation of the conditional mean of excess returns, instead of the traditional conditional variance typically used by the GARCH approach (Jensen and Maheu, 2018). Although excess returns are a source of risk, emphasis is placed on the asymmetric nature of returns. That is, the "asymmetric returns exposure" independent or dependent, due to taking on a higher level of risk. A greater risk, by the excess returns measure, simply lends greater exposure due to greater price movements (Hussain *et al.,* 2019).

Most importantly, this measure is latent and stochastic or random in nature and cannot be observed directly (Harris, 2017). Thus, a certain level of uncertainty is attached to this measure (Jensen and Maheu, 2018). Consequently, certain models that do not have the appropriate model specification/s heavily contributes to misleading results due to not accounting for this measure of risk (Jin, 2017). Specifically, regression analysis, the VAR framework, causality tests and GARCH approach are regarded as irrelevant in estimating the risk-return relationship (Jensen and Maheu, 2018). Due to these methods shortcomings, limitations and model misspecifications, they are prone to misestimating risk, heavily contributing to misleading results (Savva and Theodossiou, 2018). The ongoing debate regarding the risk-return relationship can benefit by recognising that these methods are limited and that advancements have been made in literature to deal with such issues. At the same time, encourage unconventional Bayesian and nonparametric methods that are robust, efficient and effective in risk estimation (Demirer *et al.,* 2019; Jensen and Maheu, 2018; Jin, 2017; Karabatsos, 2016).

This Bayesian approach has the ability to average out uncertainty affecting parameters and the nonparametric approach has the ability to account for every possible risk-return relationship (Waldmann, 2018). This provides improved risk estimation which ensures a credible estimation of risk and risk-return relationship (Demirer *et al.,* 2019). Further, in a nonparametric framework, the robustness of any model is enhanced as model misspecifications are corrected (Apergis *et al.,* 2017). As a result, there is no need for model extensions, specifications and accounting for different sources of variability to address the omitted variable bias (Kim and Kim, 2018). The actual nature of the data is modelled, thus, allowing "the data to speak for itself" and state the relationship held (Jensen and Maheu, 2018). This is in line with Bekiros *et al.,* (2017), who finds that the "actual returns are the most important factors" in the context of investigating the risk-return relationship. Thus, to account for these risks, the present study applies the Bayesian approach by Jensen and Maheu (2018), with emphasis on the nonparametric approach.

2.5 Conclusion of Literature Review

The Bayesian model has the ability to effectively account for "asymmetric returns exposure" from excess returns and risk exposure from realised variance. In the nonparametric framework, the Bayesian approach effectively accounts for nonlinear and asymmetric properties but also latent stochastic measures (Jin, 2017). Variance is often the choice of risk measure when the return distribution is normal which empirically does not hold (Gyldberg and Bark, 2019). By using realised variance which has a random and stochastic nature, this further enhances risk estimation (Maneemaroj *et al.,* 2019). The realised variance measure is more empirically in line with the volatile nature of financial price data and returns (Harris, 2017). The nonparametric framework enhances the Bayesian model by taking into account every possible relationship that can hold as well as asymmetric properties such as skewness, kurtosis and multiple modes (Demirer *et al.,* 2019). Hence, the nonparametric Bayesian approach has the ability to effectively account for higher moment properties (Jin, 2017).

This is in line with the recommendation by Mandimika and Chinzara (2012), to account for higher moment asymmetric properties when results are inconclusive with regards to the risk-return relationship. In conclusion, the gap in literature is highlighted which is the inconclusive empirical backing of the theoretical risk-return relationship (Maneemaroj *et al.,* 2019). Many studies show a positive or negative relationship or no relationship at all (Savva and Theodossiou, 2018). The theories from the theoretical framework by Maneemaroj *et al.,* (2019) and Jensen and Maheu (2018), are critiqued from the results and the limitations of the models. Thus, the limitations of the models in conjunction with the response to the above theories allow for a single problem to be highlighted which is "asymmetric returns exposure". The gap in empirical literature is highlighted by demonstrating the superiority of the nonparametric Bayesian approach in addressing the other models' limitations, shortcomings and misspecifications (Waldmann, 2018). This provides a strong foundation for the data and methodology as follows in the subsequent chapter.

CHAPTER 3

3. Data and Methodology

The data and methodology show how the aims of this study are met by listing the various tests and describing the modelling techniques to be implemented. It consists of five parts, each of which is summarised in the overview. First, the dataset information is discussed followed by second, the standard preliminary tests that the data undergoes before analysis. Third, is a review of the GARCH school of models and fourth, the Bayesian approach is introduced. Finally, the chapter concludes the method and strategy of model implementation.

3.1 Overview

The data details the collection method and dataset information such as the sample period choice of ten years, frequency of daily data and variables of interest – excess returns and realised variance. The data then undergoes a number of preliminary tests, including tests for stationarity, normality, autocorrelation and heteroskedasticity. A review of the GARCH school of models begins with the basic univariate ARCH model then the GARCH type models – GARCH (1, 1), EGARCH (1, 1), GJR-GARCH (1, 1) and APARCH (1, 1) – ending with a brief explanation of the multivariate GARCH models.

The model's limitations are highlighted and the gap is addressed by the use of the Bayesian approach. The basic definitions of the Bayesian approach are introduced and explained in the research design. Thereafter, the Bayesian econometric model is outlined followed by the method procedure of a parametric Bayesian model and then a nonparametric Bayesian model. The parametric Bayesian model serves as a preliminary test and for the purpose of a comparative analysis with the nonparametric Bayesian model. The chapter concludes with information about model implementation as well as how to interpret the results given in the next chapter.

3.2 Data

The data outlines the collection method of the secondary price data, frequency choice of daily data and the sample period of ten years.

3.2.1 Dataset

This study focuses on the largest South African financial market, the JSE, following the documented local literature; therefore, the FTSE/JSE ALSI data is analysed. The secondary daily price data of the ALSI is obtained from the IRESS database for the sample 15 October 2009 - 15 October 2019, a period of ten years.

3.2.1.1 Frequency

Monthly data shows a stronger risk-return relationship, in comparison to daily and weekly data, as documented in the international study by Savva and Theodossiou (2018). In contrast, an emerging market study by Liu (2019), finds that daily data is the more robust choice. This study uses daily data in line with Sultan (2018), who states that the presence of volatility feedback is stronger when using daily data. Additionally, given the rise in high frequency trading, there has been an increase in using high frequency data analysis which provides a more precise estimate of variables (Inkaya and Okur, 2014; Jin, 2017).

3.2.1.2 Sample Period

With respect to the South African evidence from the empirical review, the study by Mangani (2008) includes both the financial crisis and apartheid era in the sample. However, the rest of the studies analyse the post-apartheid period and include the 2008 financial crisis. This study chooses its sample period along the lines of the recent emerging market study by Sultan (2018). An updated period of ten years is chosen, excluding both the financial crisis and apartheid era. The selection of this period is to limit the potential effects of structural breaks in the analysis (Herbert *et al.,* 2018). While it could be extended, it would become more susceptible to the influences of exogenous economic shocks (Wang and Tsay, 2018). Additionally, the sample includes only listed companies; thus, newly listed and delisted companies are excluded, in line with Sultan (2018).

3.2.2 Variables

The variables of interest are defined, motivated and then quantified. The choice of the risk and return variables are excess returns and realised variance, in line with the studies by Jensen and Maheu (2018) and Kim and Kim (2018).

3.2.2.1 Return

Excess returns are represented by r_t and are defined as the returns obtained due to taking on a higher level of risk by definition. Excess returns are synonymous with abnormal returns and the risk premium which refers to the risk-return relationship (He *et al.,* 2018). The choice of excess returns over standard returns is motivated by the application of Bayes (1763) theorem to the risk-return relationship. Equation 1 is statistically defined as the conditional mean probability of r given R, which is equal to the joint probability of r and R, divided by the unconditional probability of R:

$$\mathrm{P(r|R)} = \frac{\mathrm{P(r \cap R)}}{\mathrm{P(R)}} \tag{1}$$

where: $\mathrm{P(r|R)}$ is the probability of r conditional on R

$\mathrm{P(r \cap R)}$ is the joint probability of r and R

$\mathrm{P(R)}$ is the unconditional probability of R

Equation 2 is derived from the cross multiplication of Equation 1:

$$\mathrm{P(r \cap R)} = \mathrm{P(r|R)} * \mathrm{P(R)} \tag{2}$$

Given that r is defined as return and R is defined as risk, Equation 2 is defined as the probability estimation of the relationship of risk and return which is equal to the risk premium (return given risk) and risk. In the context of this study, the probability estimation of the risk-return relationship is equal to the relationship between excess returns and realised variance. The use of a risk-based measure of returns, emphasises capturing asymmetric returns exposure.

Excess returns refer to the returns earned due to taking on a higher level of risk (He *et al.,* 2018). However, asymmetric returns exposure is a return inherent risk that arises from the fundamental nature of returns, independent or dependent, due to taking on a higher level of risk. A greater risk simply lends greater exposure due to greater price movements (Hussain *et al.,* 2019). Essentially, in comparison to standard returns, excess returns lends greater return risk exposure to be captured, ultimately improving risk estimation, in line with Jensen and Maheu (2018).

The daily ALSI closing price data, obtained from IRESS, are converted to ALSI market returns by the log transformation of Equation 3:

$$R_m = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (3)$$

where R_m is market returns and P_t represents the share price for the current day t and P_{t-1} is the share price for the previous day $t-1$, where $t = 1, \dots, 2499$. This is a conventional method that follows a number of studies such as Mandimika and Chinzara (2012), Adu *et al.,* (2015) and Khan *et al.,* (2016).

Thereafter, the calculation of excess returns is shown by Equation 4:

$$r_t = R_m - R_f \quad (4)$$

where r_t is the market risk premium which is equal to the difference between the market return R_m and the risk-free rate R_f. This is by definition and follows the studies by Jensen and Maheu (2018), Kim and Kim (2018) and Demirer *et al.,* (2019). The South African T-bill is the proxy for the risk-free rate, primarily based on accessibility and availability, in line with Savva and Theodossiou (2018).

The annual risk-free rate is obtained from the South African Reserve Bank (SARB) and converted to a daily value from Equation 5 by Brooks (2014):

$$\text{Daily } R_f = (1 + \text{yearly } R_f)^{\left(\frac{1}{365}\right)} \quad (5)$$

This approach is taken due to the unavailability of a daily risk-free rate as highlighted in the international study by Savva and Theodossiou (2018).

3.2.2.2 Risk

Realised variance is represented by RV_t and is a data driven measure with a random and stochastic nature, in comparison to other measures of variance (Noguchi *et al.,* 2016). The conventional computation of realised variance is the summation of returns squared (Maneemaroj *et al.,* 2019). However, this study uses a bias adjusted realised variance measure, by Hansen and Lunde (2006), in line with Jensen and Maheu (2018). The bias adjustment accounts for the micro price movements in the market due to changes in supply and demand, and stale prices which are when prices do not update to recent information. This accounts for the unnecessary noise associated with the choice of daily data as pointed out by Maneemaroj *et al.,* (2019).

The calculation of the bias adjusted realised variance by Hansen and Lunde (2006), is shown by Equation 6:

$$\mathrm{RV}_{\mathrm{t}}^{\mathrm{q}} = \hat{\gamma}_0 + 2\sum_{\mathrm{j}=1}^{\mathrm{q}}\left(\frac{1-\mathrm{j}}{(\mathrm{q}+1)}\right) \hat{\gamma}_j, \text{where } \hat{\gamma}_j = \sum_{\mathrm{t}=1}^{\mathrm{N_{t-j}}} \mathrm{r_{t,i}r_{t,i+j}} \quad (6)$$

where $RV_{\mathrm{t}}^{\mathrm{q}}$ is equal to daily realised variance RV_t where the bias adjustment q is set to one by Hansen and Lunde (2006). Realised variance is equal to term one $\hat{\gamma}_0$ which is equal to the first order of autocorrelation. Term two is the product of the bias adjustment and $\hat{\gamma}_j$ which is realised variance by definition. The latter is equal to the summation of returns squared r_t^2 where N_t represents the number of daily returns in day t and $j = 0, \dots, q$. The addition of kernel weights, not shown in Equation 6, ensures a positive bias adjusted realised variance (Jensen and Maheu, 2018). In nonparametric Bayesian estimation, the application of kernels is often used which is a type of weighting function (Fouedjio, Desassis and Rivoirard, 2016). There are a number of different kernels; however, this study specifies the Bartlett kernel, in line with Hansen and Lunde (2006).

3.3 Preliminary Tests

The ALSI market returns undergo a number of preliminary tests to ensure they meet the necessary criteria for model estimation. These are standard methods, in line with Mandimika and Chinzara (2012), Adu *et al.,* (2015), Khan *et al.,* (2016) and Liu (2019).

3.3.1 Stationarity

Stationarity is an important requirement in financial modelling in order to justify the validity of time series analysis, in the context of certain methods such as GARCH modelling (Liu *et al.,* 2020). In order to investigate stationarity, a time series probability plot and stationarity tests are employed.

3.3.1.1 Probability Plot

Price data is dynamic because it is constantly changing over time; thus, a time series plot of ALSI price data is expected to show an irregular pattern over time because of the dynamic behaviour of the price data (Harris, 2017). There would also be an upward trend representing the explosive nature of the price data (Liu, 2019). This is because if there is a change in the variable, such as a volatility shock, it is expected to persist and not die out (Nahil and Lyhyaoui, 2018). The analysis of this type of data leads to what is known as a spurious regression, which is when the data appears statistically sound under a conventional model measure such as R^2, but provides meaningless results (Gulzar *et al.,* 2019).

As a result, price data is converted to market returns by the natural log transformation of the difference between the current and previous values of price data (Kim and Kim, 2018). A time series plot of ALSI returns is expected to reflect no visible trending pattern as it crosses its constant mean value of zero (Liu, 2019). As a result, the statistical properties of ALSI returns is valid for time series analysis, in the context of GARCH modelling (Gulzar *et al.,* 2019). To support whether the ALSI data are stationary, the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), Kwiatkowski, Phillips, Schmidt and Shin (KPSS) tests are employed. The ADF, PP and KPSS are unit root tests that determine the order of integration of the variables (Chakrabarti and Kumar, 2017). If a series contains a unit root, the process is nonstationary and the first difference would lead to stationary series (Liu, 2019).

3.3.1.2 Augmented Dickey-Fuller

For the Augmented Dickey-Fuller (ADF) test by Dickey and Fuller (1981), the regression equation is Equation 7:

$$\Delta y_t = \psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-1} + u_t \tag{7}$$

where for in a change Δ of the series y_t, the coefficient $\psi = \emptyset - 1$ is the difference of the unit root $\emptyset$ and one, α_i is a constant, Δy_{t-1} is the change of the lagged series and u_t is the innovation term for $t = 1, \dots, n$. In this case, the innovation term is assumed to be constant.

The ADF test statistic is Equation 8:

$$ADF = \frac{\hat{\psi}}{\widehat{SE}(\hat{\psi})} \tag{8}$$

where $\hat{\psi}$ is the estimated difference of the unit root and one divided by the estimated standard error of $\hat{\psi}$.

The calculated ADF test statistic is compared to the relevant critical value (CV). The CV is selected with respect to the sample size, level of significance as well as if it has an intercept and/or deterministic trend in the regression Equation 7. If the ADF test statistic is less than the CV, the null hypothesis that the ALSI series has a unit root ($\emptyset = 1$) can be rejected. Thus, it can be concluded the alternative hypothesis that the ALSI series is stationary (Dickey and Fuller, 1981).

3.3.1.3 Phillips-Perron

For the Phillips-Perron (PP) test by Phillips and Perron (1988), the regression equation is Equation 9:

$$y_t = \emptyset y_{t-1} + u_t \tag{9}$$

where the series y_t is equal to the sum of the product of the unit root $\emptyset$ and the lagged series y_{t-1}, and innovation term u_t for $t = 1, \ldots, n$. In this case, the innovation term is assumed to be inconstant and stationary.

The PP test statistics are Equations 10.1 and 10.2:

$$PP_t = \left(\frac{\hat{\sigma}_1^2}{\hat{\sigma}_2^2}\right)^{1/2} t_{\emptyset=1} - \frac{1}{2} = \left(\frac{\hat{\sigma}_2^2 - \hat{\sigma}_1^2}{\hat{\sigma}_2^2}\right)\left(\frac{T - SE(\hat{\emptyset}-1)}{\hat{\sigma}_1^2}\right) \tag{10.1}$$

$$PP_{\hat{\emptyset}} = T_{\hat{\emptyset}} - \frac{1}{2}\frac{T^2 SE(\hat{\emptyset}-1)}{\hat{\sigma}_1^2}(\hat{\sigma}_2^2 - \hat{\sigma}_1^2) \tag{10.2}$$

where T is the sample size and $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ are variance parameters defined, respectively as Equations 11.1 and 11.2:

$$\hat{\sigma}_1^2 = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} E(u_t^2) \tag{11.1}$$

$$\hat{\sigma}_2^2 = \sum_{t=1}^{T} E\left(\frac{S_T^2}{T}\right) \tag{11.2}$$

where S_t^2 is equal to the sum of innovations $S_t^2 = \sum_{i=1}^{t} u_t$.

The comparison of the calculated PP test statistic to the relevant CV follows the same procedure of the ADF test. If the PP test statistic is less than the CV, the null hypothesis that the ALSI series has a unit root can be rejected. Thus, it can be concluded the alternative hypothesis that the ALSI series is stationary (Phillips and Perron, 1988).

3.3.1.4 Kwiatkowski, Phillips, Schmidt and Shin

For the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test by Kwiatkowski, Phillips, Schmidt and Shin (1992), the regression equation is Equation 12:

$$y_t = \beta_t + \varepsilon_t + u_t \tag{12}$$

where β_t is a deterministic trend, ε_t is a random walk and u_t is an innovation term for $t = 1, \dots, n$. In this case, the innovation term is assumed to be inconstant and stationary.

$$\varepsilon_t = \varepsilon_{t-1} + v_t \tag{13}$$

From Equation 12, the random walk ε_t is defined as Equation 13, the sum of its lagged value ε_{t-1} and innovation term v_t that is assumed to be independent and identically distributed with a mean zero and variance $\hat{\sigma}_v^2$.

The KPSS test statistic is Equation 14:

$$KPSS = \sum_{i=1}^{T} \frac{S_t^2}{\hat{\sigma}_t^2} \tag{14}$$

where S_t^2 is equal to the sum of innovations and $\hat{\sigma}_t^2$ is the estimated variance.

The calculated KPSS test statistic is often one-sided and is compared to the relevant CV. The CV is selected with respect to the level of significance as well as if it has an intercept or linear trend. If the KPSS test statistic is greater than the CV, the null hypothesis that the ALSI series is stationary can be rejected. Thus, it can be concluded the alternative hypothesis that the ALSI series has a unit root (Kwiatkowski *et al.,* 1992).

3.3.2 Normality

In order to investigate normality, basic descriptive statistics, a quantile-quantile plot and normality tests are employed.

3.3.2.1 Basic Descriptive Statistics

The basic descriptive statistics provide a general overview of statistical properties of the ALSI returns distribution such as the mean, standard deviation, skewness and excess kurtosis.

By Gyldberg and Bark (2019), the average of the ALSI returns indicates a corresponding rate of return and is given as Equation 15:

$$\bar{r} = \frac{\sum_{i=1}^{N} r_i}{N} \tag{15}$$

A positive mean value indicates gains from a profitable trading strategy, whereas a negative mean value indicates losses (Gyldberg and Bark, 2019).

By Steyn and Theart (2019), standard deviation is a measure of total risk which comprises of both systematic and unsystematic risk and is given as Equation 16:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (r_i - \bar{r})^2}{N}} \tag{16}$$

The higher the standard deviation, the higher the risk which means the greater the deviation from an expected outcome (Charles and Okoro, 2019).

By Altinay (2016), skewness is the standardised third moment of a series which indicates whether or not a distribution is symmetrical around its mean value and is given as Equation 17:

$$S(r_t) = E\left[\left(\frac{r_t - \bar{r}}{\sigma}\right)^3\right] = \frac{E\left[(r_t - \bar{r})^3\right]}{(E[(r_t - \bar{r})^2])^{3/2}} = \frac{\mu^3}{\sigma^3} \tag{17}$$

A positive value for skewness indicates the distribution of returns are skewed to the left and a negative value indicates the distribution of returns are skewed to the right (Adu *et al.,* 2015). According to Li (2018), the skewness of a distribution indicates the payoff of an investor. A positive value indicates gains and a negative value indicates losses (Yao *et al.,* 2019).

By McAlevey and Stent (2017), kurtosis is the standardised fourth moment of a series and measures whether or not a distribution has heavy tails and is given as Equation 18:

$$K(r_t) = E\left[\left(\frac{r_t - \bar{r}}{\sigma}\right)^4\right] = \frac{E\left[(r_t - \bar{r})^4\right]}{(E[(r_t - \bar{r})^2])^2} = \frac{\mu^4}{\sigma^4} \tag{18}$$

A positive value for excess kurtosis indicates a distribution with heavy tails, known as a leptokurtic distribution, in line with the return characteristics of an emerging market by Herbert *et al.,* (2018). On the other hand, a negative value for excess kurtosis indicates a distribution with thin tails, known as platykurtic distribution (Adu *et al.,* 2015).

3.3.2.2 Quantile-Quantile Plot

According to Brooks (2014), a quantile-quantile (Q-Q) plot determines normality by the comparison of an empirical distribution and theoretical normal distribution. The theoretical distribution is represented by a bold dark diagonal line. A QQ-line is a line of reference and is 45 degrees. This line indicates normality by a perfect fit between the empirical and theoretical distribution. However, deviation indicates a mismatch between the empirical and theoretical distribution, indicating nonnormality (Brooks, 2014). To support whether the ALSI data are normally distributed, the Shapiro-Wilk (SW), Jarque-Bera (JB) and Anderson-Darling (AD) tests are employed.

3.3.2.3 Shapiro-Wilk

For the Shapiro-Wilk (SW) test by Shapiro and Wilk (1965), the SW test statistic is Equation 19:

$$SW = \frac{\left(\sum_{i=1}^{n} \alpha_i x_{(i)}\right)}{\sum_{i=1}^{n}(x_i - \bar{x})} \tag{19}$$

where $x_{(i)}$ are the sample statistics for order $i = 1, \dots, n$, of which the sample size n is normally distributed, the sample mean $\bar{x}$ and α_i are the constants. The latter is obtained from the statistical properties, such as the mean and variance, of the order statistics from the sample n.

The calculated SW test statistic follows a normal distribution and is compared to the relevant CV which is selected based on the sample size. If the SW test statistic is greater than the CV, the null hypothesis that the ALSI series is normally distributed can be rejected. Thus, it can be concluded the alternative hypothesis that the ALSI series is nonnormally distributed (Shapiro and Wilk, 1965).

3.3.2.4 Jarque-Bera

For the Jarque-Bera (JB) test by Jarque and Bera (1987), the JB test statistic is Equation 20:

$$JB = n\left[\frac{S^2}{6} + \frac{K^2}{24}\right] \tag{20}$$

where n is the sample size and S and K are parameters given and defined as skewness and kurtosis, respectively by Equations 21.1 and 21.2:

$$S = \frac{1}{n} \frac{\sum_{i=1}^{n}(r_t - \bar{r}_t)^3}{(\hat{\sigma}^2)^{3/2}} \tag{21.1}$$

$$K = \frac{1}{n} \frac{\sum_{i=1}^{n}(r_t - \bar{r}_t)^4}{(\hat{\sigma}^2)^2} - 3 \tag{21.2}$$

The calculated JB test statistic follows a chi-square distribution with two degrees of freedom and is compared against the relevant CV which is selected with respect to the level of significance α. If the JB test statistic is greater than the CV ($X^2_{\alpha,2}$), the null hypothesis that the ALSI series is normally distributed can be rejected. Thus, it can be concluded the alternative hypothesis that the ALSI series is nonnormally distributed (Jarque and Bera, 1987).

3.3.2.5 Anderson-Darling

Nonnormal behaviour is mainly shown at the tails of a distribution, especially for emerging markets since they are characterised as having longer and heavier tails (Herbert *et al.,* 2018). The Anderson-Darling (AD) test by Anderson and Darling (1954), is particularly useful since it is sensitive to the behaviour of the tails.

The AD test statistic is Equation 22:

$$AD^2 = -N - \frac{1}{N} \sum_{i=1}^{N}(2i-1)\{\ln F(Y_i) + \ln(1 - F(Y_{N+1-i}))\} \tag{22}$$

where F is the cumulative distribution function of the specified distribution of the ordered data Y_i for $i = 1, \dots, N$ of which N is the sample size.

The calculated AD test is a one-sided test and follows a specified distribution, and is compared to the relevant CV which is selected with respect to the level of significance. If the AD test statistic is greater than the CV, the null hypothesis that the ALSI series is normally distributed can be rejected. Thus, it can be concluded the alternative hypothesis that the ALSI the series is nonnormally distributed (Anderson and Darling, 1954).

3.3.3 Autocorrelation

Autocorrelation or serial correlation refers to the extent to which the current values of the ALSI return series is related to its previous values (Khan *et al.,* 2016). In order to investigate autocorrelation, a plot of an autocorrelation function and autocorrelation tests are employed.

3.3.3.1 Autocorrelation Function Plot

According to Chakrabarti and Kumar (2020), an ACF plot shows the lags from a horizontal zero reference band. If the majority of lags touch or pass over the 95% conditional interval, this indicates that the lags are significant and the presence of autocorrelation within the ALSI returns (Chakrabarti and Kumar, 202). To support whether the ALSI data are serially correlated, the Ljung-Box (LB) or Box Pierce and the Durbin Watson (DW) tests are employed.

3.3.3.2 Ljung-Box

For the Ljung-Box (LB) test by Ljung and Box (1978), the LB test statistic is Equation 23:

$$LB = N(N+2)\sum_{k=1}^{Q}\frac{\hat{\rho}_k^2(r_t)}{N-k} \quad (23)$$

where N is the sample size, Q is the maximum length of lag k and $\hat{\rho}_k^2$ is the correlation coefficients squared of the ALSI returns r_t.

The calculated test statistic LB follows a chi-square distribution with Q degrees of freedom and is compared to the relevant CV with respect to the level of significance. If LB is greater than the CV ($X_{1-\alpha}^2$) where $1-\alpha$ is the level of significance, the null hypothesis that autocorrelation is absent within the ALSI series can be rejected. Thus, it can be concluded the alternative hypothesis that autocorrelation is present within the ALSI series (Ljung and Box, 1978).

3.3.3.3 Durbin Watson

For the Durbin Watson (DW) test by Durbin and Watson (1951), the DW test statistic is given as Equation 24:

$$DW = \frac{\sum_{i=2}^{N}(r_i - r_{i-1})^2}{\sum_{i=1}^{N} r_i^2} \quad (24)$$

According to Brooks (2014), if the calculated DW test statistic lies between the values of 1.5 and 2.5, the null hypothesis that autocorrelation is absent within the ALSI series is not rejected. Thus, it can be concluded that autocorrelation is present within the ALSI series. This is a rule of thumb that is often used. If the calculated DW test statistic lies outside the interval of 1.5 and 2.5, the diagram below can be used. Figure 8 shows the regions of rejection and non-rejection for the DW test.

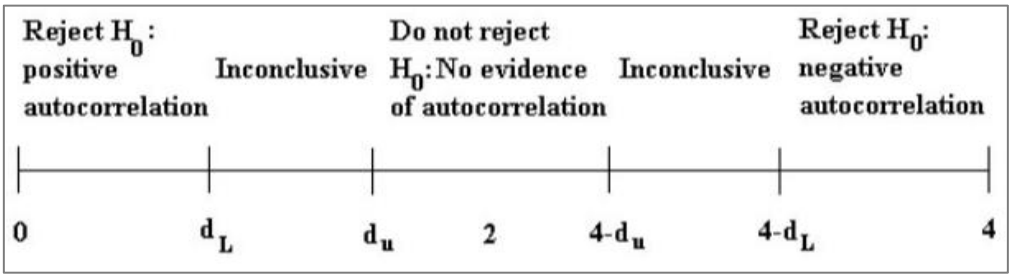

Figure 8: Regions of rejection and non-rejection for the DW test

Source: Brooks (2014)

The calculated DW test statistic is compared to the relevant CV. The DW tests have two CV's, an upper bound CV d_L and lower bound CV d_u. These are selected with respect to the sample size and number of independent variables excluding the constant of a regression equation. For exact values from Figure 8, the conclusions can be given as follows: If DW = 0, it can be concluded that there is a perfect positive serial correlation within the ASLI series. If DW = 2, it can be concluded that serial correlation is absent within the ASLI series. If DW = 4, it can be concluded that there is a perfect negative serial correlation within the ASLI series (Brooks, 2014).

3.3.4 Heteroskedasticity

Heteroskedasticity refers to the statistical property of variance being inconstant over time (Hung, 2019). Conditional heteroskedasticity refers to when the inconstant variance can be established during unidentified periods of low and high volatility (Khan *et al.,* 2016). In order to investigate conditional heteroskedasticity, a plot of an autocorrelation function and autocorrelation tests are employed to higher order autocorrelation where the ALSI data are squared. To support whether the ALSI data are heteroskedastic, the Ljung-Box (LB^2) and Autoregressive Conditional Heteroskedastic Lagrange multiplier (ARCH-LM) tests are employed.

3.3.4.1 Ljung-Box

For the Ljung-Box (LB^2) test by Ljung and Box (1978), the LB^2 test statistic is Equation 25:

$$LB^2 = N\,(N+2)\,\sum_{k=1}^{Q} \frac{\hat{\rho}_k^2(r_t^2)}{N-k} \tag{25}$$

where N is the sample size, Q is the maximum length of lag k and $\hat{\rho}_k^2$ is the correlation coefficients squared of the ALSI returns squared r_t^2.

The calculated test statistic LB^2 follows a chi-square distribution with Q degrees of freedom and is compared to the relevant critical value with respect to the level of significance. If LB^2 is greater than the CV ($X_{1-\alpha}^2$), the null hypothesis that the ARCH effect is absent can be rejected. Thus, it can be concluded the alternative hypothesis that the ARCH effect is present (Ljung and Box, 1978)

3.3.4.2 Autoregressive Conditional Heteroskedastic Lagrange Multiplier

The purpose of the Autoregressive Conditional Heteroskedastic Lagrange Multiplier (ARCH-LM) test by Engle (1982), is to detect the presence of heteroskedasticity within the ASLI data. The latter is also known as the ARCH effect and is based on the linear regression equation Equation 26:

$$y_t = \beta_0 + \beta_i x_{it} + u_t \tag{26}$$

where β_0 is a constant β is a coefficient for $i = 1, \dots, n$ and u_t is an innovation term.

The LM test statistic is Equation 27:

$$LM = TR^2 \tag{27}$$

where the product is the sample size T and the coefficient of multiple correlation R^2 obtained from the linear regression Equation 26.

The calculated test statistic LM follows a chi-square distribution with m degrees of freedom and is compared to the relevant critical value with respect to the level of significance. If LM is greater than the CV (X_m^2), the null hypothesis that the ARCH effect is absent can be rejected. Thus, it can be concluded the alternative hypothesis that the ARCH effect is present (Engle, 1982). The presence of the ARCH effect, hence, conditional heteroskedasticity motivates the use of the GARCH models (Khan *et al.,* 2016).

3.4 GARCH Approach

According to Savva and Theodossiou (2018), the GARCH approach is one of the foremost methods used in the investigation of the risk-return relationship. This school of models are briefly reviewed since these are standard methods in finance literature. While most documented studies highlight the advantages of the GARCH school of models, based on its conventional use, they fail to investigate the limitations and shortcomings.

Therefore, the aim of this section is to highlight the shortcomings and limitations of the GARCH type models, their inability to capture asymmetric return exposure and fully capture risk (Jin, 2017). Hence, making the GARCH approach an inefficient choice in estimating the risk-return relationship and contributing to inconclusive results (Jensen and Maheu, 2018). The employment of GARCH type models follow the studies by Mandimika and Chinzara (2012), Adu *et al.,* (2015), Khan *et al.,* (2016) and Savva and Theodossiou (2018).

3.4.1 Symmetric GARCH Models

The symmetric GARCH models are the ARCH and GARCH (1, 1) models.

3.4.1.1 ARCH (1, 1)

The ARCH model introduced by Engle (1982), is the first method to capture volatility dynamics, particularly the market return characteristics, volatility clustering and heavy tails. The ARCH model has the ability to allow the conditional variance to be a time varying function of past innovations. The conditional mean is given by Equation 28.1:

$$y_t = \beta_0 + \beta_i x_{i,t} + \cdots + \beta_n x_{nt} + u_t \tag{28.1}$$

From Equation 28.1 by Engle (1982), y_t is the conditional mean, β_i is a coefficient for $i = 0, \ldots, N$ and $x_{i,t}$ are the exogenous and endogenous variables for order i at time t.

$$u_t = \sigma_t . z_t, z_t \sim N\ (0,1) \tag{28.2}$$

From Equation 28.1, the innovation term u_t is equal to Equation 28.2 which is the product of the conditional standard deviation σ of the innovations and the standardised innovation z_t. The standardised innovation z_t is normally distributed with a mean of zero and unit variance so that the innovation u_t also follows a normal distribution.

$$\sigma_t{}^2 = \alpha_0 + \alpha_1 u^2{}_{t-1} \tag{28.3}$$

From Equation 28.3, $\sigma_t{}^2$ is the conditional variance where α_0 is a constant, α_1 is a co-efficient and $u^2{}_{t-1}$ is the lagged innovation term squared (Engle, 1982).

The nonnegativity constraint, $\alpha_0, \alpha_1 \geq 0$, is imposed to ensure a positive conditional variance; therefore, the model only accounts for the squared shocks (Hretski and Karachun, 2018). Consequently, it does not take into account negative shocks; hence, the asymmetric volatility phenomenon (Liu, Yao and Zhao, 2020). In this context, asymmetric volatility refers to when the conditional variance increases more for negative shocks than positive shocks, or vice versa, for the same magnitude (Savva and Theodossiou, 2018). Essentially, the ARCH model is a linear parametric model which cannot effectively account for asymmetric effects (Liu *et al.,* 2020).

According to Brooks (2014), determining the appropriate number of lags can pose a problem in two ways. Firstly, Equation 28.1 is an ARCH (1) model, but more lags can be added, as specified by *q* for an ARCH (*q*) model. While the optimal number of lags can be determined by means of a likelihood ratio test, this test has not been confirmed as being a credible choice. Secondly, the number of lags in Equation 28.3 can take on an infinite, or rather at least a very high number of parameters, in order to allow the conditional mean to fully capture the conditional variance. Consequently, in turn, this can result in the overparameterisation of the conditional variance. In other words, the conditional variance is being described by a very high number of parameters which can be computationally intensive (Al-Najjar, 2016; Brooks, 2014). Although this study does not include an empirical analysis of an ARCH model, it forms the basis upon which the other GARCH models are built on.

3.4.1.2 GARCH (1, 1)

Due to the limitations that arise from the ARCH model, Bollerslev (1986) introduced the standard GARCH (1, 1) model. GARCH is an extension of the ARCH model where the current conditional variance is a function of past volatility as well as the innovation terms (Hretski and Karachun, 2018).

$$\sigma_t{}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u^2{}_{t-1} + \sum_{j=1}^{p} \beta_j \sigma^2{}_{t-j} \tag{29}$$

From Equation 29 by Bollerslev (1986), the conditional variance ${\sigma_t}^2$ is time varying and dependent on the past lagged innovation terms squared ${u^2}_{t-1}$. The nonnegativity constraints, $\alpha_0 > 0$ and $\alpha_i \geq 0$, are applied to satisfy a positive conditional variance by Bollerslev (1986).

According to Al-Najjar (2016), the GARCH model is a parsimonious model, whereas the ARCH model is not. A parsimonious model refers to a model that can describe data precisely using the minimum number of parameters. As noted, the ARCH model can result in having an infinite number of parameters in order to fully capture conditional variance. However, the GARCH model, on the other hand, uses only two parameters α_1 and β_1 as shown in Equation 29 (Liu, 2019; Apergis *et al.,* 2018; Al-Najjar, 2016).

Volatility clustering describes the clustering nature of volatility, heavy tails and excess kurtosis (Adu *et al.,* 2015). This can be measured by their persistence in which the ARCH and GARCH effects are designed to capture (Savva and Theodossiou, 2018). The ARCH effect captured by α_1 which represents the persistence of risk over a short period of time and the GARCH effect is captured by β_1 which represents the persistence of risk over the long-term (Khan *et al.,* 2016).

According to Khan *et al.,* (2016), the conditional variance is subject to change but the unconditional variance of the innovation terms is constant. Therefore, the assumption of stationarity for the innovation terms only holds when $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$ (Khan *et al.,* 2016). If this constraint is satisfied, the assumption of stationarity holds and the long run mean variance converges to unconditional variance given as Equation 30 by Bollerslev (1986):

$$ {\sigma_t}^2 = var(u_t) = \frac{\alpha_0}{1-(\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j)} \tag{30} $$

From Equation 30, if $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$, the unconditional variance for the innovation terms are undefined and non-stationary (Khan *et al.,* 2016). Then the GARCH (*p*, *q*) model can be seen as a regular linear ARMA model which is essentially a linear parametric model (Brooks, 2014; Jin, 2017).

According to Khan *et al.,* (2016), for $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j = 1$, the unconditional variance is nonstationary and the above convergence does not occur. However, if $\sum_{i=1}^{q} \alpha_i +$

$\sum_{j=1}^{p} \beta_j > 1$, the forecast of conditional variance increases to infinity as the sample period increases (Khan *et al.,* 2016).

According to Hretski and Karachun (2018), while both the ARCH and GARCH models can accommodate the time-varying nature of volatility, they have an inability to capture asymmetric properties. This is because of their underlying assumptions by Engle (1982). For example, both the ARCH and GARCH models are based on the assumption that volatility has a symmetric effect, meaning, all shocks on volatility are the same. This is because the conditional variance of the ARCH and GARCH models is determined by squared shocks and not their signs (Al-Najjar, 2016; Hretski and Karachun, 2018). As a result, the models do not take into account the market characteristic and empirical regularity asymmetric volatility (Yu *et al.,* 2018).

Consequently, other market characteristics that are not taken into account by the ARCH and GARCH models are volatility feedback and the leverage effect which can further explain asymmetric volatility (Yu *et al.,* 2018). Essentially, the ARCH and GARCH models are symmetric parametric models which makes them an inefficient choice to model ALSI returns which is subject to asymmetric returns exposure. To confirm that the GARCH (1, 1) model is an inefficient choice to model the ALSI returns, asymmetry tests are applied to determine whether the ALSI returns has been adequately captured following Park *et al.,* (2017) and Ilupeju (2016).

3.4.1.3 Asymmetry Tests

The asymmetry tests consist of the sign and size bias tests as well as joint effect test by Engle and Ng (1993). These tests are applied to the GARCH (1, 1) model to investigate the presence of asymmetry where the presence of asymmetry would indicate that the GARCH (1, 1) has not adequately captured the ALSI returns (Park *et al.,* 2017).

3.4.1.3.1 Sign Bias Test

For the sign and size bias test by Engle and Ng (1993), the regression equation is Equation 31.1:

$$\hat{u}_t^2 = \beta_0 + \beta_1 S_{t-1}^{-} + \beta_2 S_{t-1}^{-} + \beta_3 S_{t-1}^{+} + e_t \quad (31.1)$$

where the indicator variable $S_{t-1}^{+} = 1 - S_{t-1}^{-}$ and S_{t-1}^{-} is given as Equation 31.2:

$$S_{t-1}^{-} = \begin{cases} 1, & u_{t-1} < 0 \\ 0, & otherwise \end{cases} \tag{31.2}$$

From Equation 31.1, the coefficients β_i for $i = 1, 2, 3$ follow a student-t distribution (Engle and Ng, 1993).

According to Park *et al.,* (2017), if β_1 is found to be significant, it can be concluded that the sign bias is present. The sign bias test investigates the size of innovation which indicates the effect of the type of shock on future volatility. If β_2 or β_3 is found to be significant, it can be concluded that the size bias is present. The positive sign bias represents the impact of a positive shock and the negative sign bias represents the impact of a negative shock on future volatility (Ilupeju, 2016; Park *et al.,* 2017).

3.4.1.3.2 Joint Effect Test

The joint effect (JE) test statistic is given as Equation 32:

$$JE = TR^2 \tag{32}$$

where T is the sample size and R^2 is the coefficient of multiple correlation obtained from the linear regression Equation 31.1.

According to Park *et al.,* (2017), the joint effect test investigates the combined impact of both the sign and size bias on future volatility. If the JE test statistic is significant, it can be concluded that asymmetry is present and the GARCH (1, 1) model has not adequately captured asymmetric returns exposure. The presence of the joint effect, hence, asymmetry then motivates the use of the asymmetric GARCH models (Ilupeju, 2016; Park *et al.,* 2017).

According to Liu *et al.,* (2020), the ARCH model always has the least credible forecast ability, in comparison to the asymmetric GARCH type models such as GJR and EGARCH. Therefore, the accommodation of the asymmetry parameter in the asymmetric GARCH type models enhances the models forecast ability of empirical market return characteristics (Liu *et al.,* 2020). This is especially important for emerging markets which are characterised by heavy tails and higher levels of volatility by Herbert *et al.,* (2018).

While the ARCH and GARCH models can capture heavy tails and volatility clustering, they fail to capture asymmetric volatility as well as volatility feedback and the leverage effect (Hretski and Karachun, 2018). Therefore, various extensions have been made to the standard GARCH model to address this shortcoming and incorporate the asymmetric effects (Harris *et al.,* 2019). This refers to the addition of the asymmetric parameter to the standard GARCH model resulting in the asymmetric GARCH type models GJR-GARCH, EGARCH and APARCH (Savva and Theodossiou, 2018).

3.4.2 Asymmetric GARCH Models

In this section, the asymmetric GARCH type models reviewed are the GJR-GARCH, EGARCH and APARCH.

3.4.2.1 GJR-GARCH (1, 1)

The GJR model is a simple extension of the standard GARCH model by the addition of the term $\gamma\, u^2{}_{t-1} I_{t-1}$ by Glosten, Jagannathan and Runkle (1993). The GJR model accommodates both time varying and asymmetric volatility and is also known as DGARCH, TGARCH or TARCH (Mangani, 2008; Mandimika and Chinzara, 2012; Ilupeju, 2016).

$$\sigma_t{}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u^2{}_{t-1} + \sum_{j=1}^{p} \beta_j \sigma^2{}_{t-j} + \gamma\, u^2{}_{t-1} I_{t-1} \qquad (33.1)$$

From Equation 33.1 by Glosten *et al.,* (1993), α_0 is a constant term, α_1 is the parameter of the squared lagged innovation term $u^2{}_{t-1}$. The parameter α_1 and β_j capture short and long-term volatility persistence, respectively following the standard GARCH model. The asymmetry parameter of interest γ captures the asymmetric effects, namely, asymmetric volatility and the leverage effect.

$$I_{t-1} = \begin{cases} 1, & if\, u_{t-1} < 1 \\ 0, & otherwise \end{cases} \qquad (33.2)$$

From Equation 33.1, the interaction variable I_{t-1} is given by Equation 33.2 which distinguishes the effects of positive and negative volatility. The nonnegativity constraints are $\alpha_0,\ \alpha_i > 0,\ \gamma \geq 0$ and $\alpha_i + \gamma \geq 0$ and the model is still valid if $\gamma < 0$ provided $\alpha_i + \gamma \geq 0$ (Glosten *et al.,* 1993).

From Equation 33.1, for $\gamma \neq 0$, asymmetric volatility is present where positive and negative volatility shocks or news impact volatility differently (Adu *et al.,* 2015). According to Mandimika and Chinzara (2012), asymmetric volatility and the leverage effect are often treated synonymously by the asymmetric GARCH type models. The volatility shock or type of news is represented by the sign of α_1. For a positive sign of α_1, good news has a greater impact on volatility than bad news of the same magnitude in the South African market. If $\gamma > 0$, this indicates the presence of asymmetric volatility and the leverage effect. For a positive volatility shock, volatility decreases and results in an increase in prices, indicating the leverage effect. If $\gamma < 0$, this indicates the presence of asymmetric volatility but the absence of the leverage effect (Maneemaroj *et al.,* 2019; Adu *et al.,* 2015; Mandimika and Chinzara, 2012).

Like the rest of the asymmetric GARCH type models, the GJR-GARCH model can account for asymmetric volatility and the leverage effect, unlike the symmetric GARCH type models such as the ARCH and GARCH models (Hretski and Karachun, 2018). This is because of the asymmetry parameter and in the case of the GJR model, the interaction variable can take into account negative shocks (Glosten *et al.,* 1993). However, one of the main drawbacks of the GJR and other GARCH models is the nonnegativity constraint which is addressed by the EGARCH model (Adu *et al.,* 2015).

3.4.2.2 EGARCH (1, 1)

The exponential GARCH, EGARCH model by Nelson (1991), is specially designed to capture asymmetric volatility and the leverage effect by the asymmetry parameter γ.

$$\ln({\sigma_t}^2) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left[|z_{t-1}| - E|z_t|\right] + \sum_{j=1}^{p} \beta_j \ ({\sigma_t}_{t-j}^{2}) \ + \ \gamma \ z_{t-1} \qquad (34)$$

From Equation 34 by Nelson (1991), the terms α_1 and β_1 follow the standard GARCH model. Conditional variance is able to respond in a nonlinear manner to positive and negative returns by the magnitude of the innovation $\alpha_1 \left[|z_{t-1}| - E|z_t|\right]$ and the sign effect $\gamma \ z_{t-1}$. Like the GJR has the interaction variable, EGARCH has the sign effect to take into account negative shocks as well as positive shocks (Nelson, 1991). This property is effective since it allows for asymmetric effects - asymmetric volatility and the leverage effect (Hretski and Karachun, 2018).

Equation 34 contains a logarithmic transformation which means that there is no need to impose nonnegativity constraints (Nelson, 1991). In contrast, to all the other GARCH type models such as GARCH (1, 1) GJR-GARCH and APARCH among others (Savva and Theodossiou, 2018). The asymmetry parameter of interest γ indicates the asymmetric effects where the interpretation for γ follows the GJR-GARCH (Ilupeju, 2016).

3.4.3.3 APARCH (1, 1)

Following the asymmetric GARCH type models is the asymmetric power ARCH, APARCH model by Ding, Granger and Engle (1993), which includes the parameter γ for asymmetric effects.

$$\sigma_t^{\delta} = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left(|e_{t-i}| - \gamma_i\, e_{t-i}\right)^{\delta} + \sum_{j-1}^{q} \beta_j\, \sigma_{t-j}^{\delta} \tag{35}$$

From Equation 35 by Ding *et al.,* (1993), the terms α_1 and β_1 follow the standard GARCH (1, 1) model. The γ indicates the asymmetric effects where a positive value means that negative volatility shocks or news have a greater impact than positive (Ding *et al.,* 1993).

An underlying assumption for GARCH modelling by Engle (1982), is that asset returns follow a normal distribution. Since this does not hold true, as noted by asymmetric returns exposure, forecast performance is not statistically sound for GARCH type models that follow a standard normal innovation distribution. Therefore, in order to capture the heavy tails, a common characteristic by emerging markets, the innovation distributions student-t and skewed student-t was introduced. The APARCH model with the student-t and skewed student-t innovation distributions is noted to show superior model performance and is considered a credible choice when modelling asymmetry and heavy tails (Hretski and Karachun, 2018; Ilupeju, 2016).

The additional parameter of δ, shown as the exponent of Equation 35, is what makes the APARCH model unique (Ding *et al.,* 1993). It enhances the model, allowing for flexibility of the APARCH model, to the extent that it can take on a number of ARCH and GARCH models (Hretski and Karachun, 2018). Specifically, it has the ability to take on seven extensions of the GARCH type models. Table 1 shows the GARCH type models the APARCH model can be set to by the given conditions.

Table 1: GARCH type models APARCH can be set to by the following conditions

Model	By	Conditions
ARCH	Engle (1982)	$\delta \to 0$
Log-ARCH	Geweke (1986) and Pantula (1986)	$\delta \to 0$
NARCH	Higgins and Bera (1992)	$\gamma_i = 0$, where $i = 1, \dots, q$ $\beta_j = 0$, where $j = 1, \dots, q$
GARCH	Bollerslev (1986)	$\delta = 2$ $\gamma_i = 0$, where $i = 1, \dots, q$
TS-GARCH	Taylor (1986) and Schwert (1990)	$\delta = 1$ $\gamma_i = 0$, where $i = 1, \dots, q$
GJR-GARCH	Glosten *et al.,* (1993)	$\delta = 2$
TARCH	Zakoian (1994)	$\delta = 1$

This study implements GJR-GARCH by setting δ = 2 of the APARCH model (Tsay, 2013). Like the rest of the GARCH family models, this model is subject to a number of nonnegativity constraints. This includes $\alpha_0 > 0$, α_i, $\beta_j \geq 0$ and $0 \leq \sum_{i=1}^{p} \alpha_i + \sum_{j-1}^{q} \beta_j \, \sigma_{t-j}^{\delta} \leq 1$ (Ding *et al.,* 1993). Despite the flexibility of the APARCH model, the risk within the innovations of any GARCH type model still remains uncaptured (Feng and Shi, 2017; Ilupeju, 2016). Thus, this suggests that the GARCH approach is an efficient choice in the estimation of risk and the risk-return relationship (Jensen and Maheu, 2018).

3.4.3 GARCH-in-Mean

According to Al-Najjar (2016), due to the risk-return relationship being a fundamental concept in financial and economic theory, the risk premium parameter was introduced to quantify the pricing of risk - the relation between risk and return. Engle, Lilien and Robins (1987) introduced the ARCH-in-mean model which was then extended to the

GARCH-in-mean model since the GARCH type models became more popular. The 'in-mean' of the GARCH-in-mean or GARCH-M model refers to the fact that the conditional mean contains the conditional variance as well as the risk premium parameter. Like the asymmetry parameter in the asymmetric GARCH type models which captures asymmetric effects, the risk premium parameter is another useful feature in the GARCH type models which captures the risk-return relationship (Hretski and Karachun, 2018; Al-Najjar, 2016).

The 'in-mean' of a GARCH type model can further be understood by the implementation stage using the R software. According to Tsay (2013), during the implementation stage of the GARCH models, if the parameter estimates are insignificant, the process of demeaning is used in order to improve the significance of the parameter estimates. Meaning, if the parameter estimates were insignificant, the constant α_0 and/or mean μ can be dropped since it is not the parameter of interest or required for model adequacy. However, this procedure with respect to the mean cannot be followed for the GARCH (1, 1)-M and EGARCH (1, 1)-M models. This is because the risk premium parameter is embedded within the mean. Thus, the mean cannot be dropped because it contains the risk premium parameter of interest (Sultan, 2018; Tsay, 2013).

3.4.3.1 GARCH-M

GARCH-M is a popular model used to price risk and determine the risk-return relationship through the risk premium parameter (Savva and Theodossiou, 2018).

$$y_t = \mu + \sum_{j=1}^{p} \delta_j \sigma^2{}_{t-j} + u_t, u_t \sim N\ (0,1) \qquad (36.1)$$

From Equation 36.1 y_t is the conditional mean, μ is the constant, $\sigma^2{}_{t-j}$ is the variance and u_t is the innovation term that is normally distributed (Engle, 1982; Bollerslev, 1986). Specifically, Equation 36.1 contains an ARCH-M specification by Engle *et al.,* (1987), which is the risk premium δ_j.

$$\sigma^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u^2{}_{t-1} + \sum_{j=1}^{p} \beta_j \sigma^2{}_{t-j} \qquad (36.2)$$

Equation 36.2 is the conditional variance σ^2 which follows the standard GARCH model by Bollerslev (1986).

The risk premium parameter δ_j is of interest for the GARCH-M model, which is used to price risk and determine the risk-return relationship, following a number of previous studies such as Mandimika and Chinzara (2012), Adu *et al.,* (2015) and Khan *et al.,* (2016). Additionally, given that risk is priced, this signifies the presence of volatility feedback by Umutlu (2019) and Jensen and Maheu (2018).

However, analysing the risk premium alone to investigate the risk-return relationship is limited (Jin, 2017). The GARCH-M model cannot account for either volatility feedback or the leverage effect (Chakrabarti and Kumar, 2020). In other words, the GARCH-M model is limited in accounting for nonlinear and asymmetric properties (Maneemaroj *et al.,* 2019). The GARCH-M model can be used to price risk, simply state if there exists a relation between risk and return in the market, but cannot be used to estimate an actual risk-return relationship (Jensen and Maheu, 2018).

The GARCH-M model is heavily mispecified, according to Jin (2017), and unable to effectively account for risk and asymmetric returns exposure as previously discussed. Hence, the results are most likely to be misleading due to misestimating risk (Jensen and Maheu, 2018). The risk premium parameter can be used in conjunction with other GARCH type models, such as EGARCH or a hybrid GARCH model, which can account for asymmetry and other features (Park *et al.,* 2017). Thus, the asymmetric and hybrid GARCH type models are a more practical choice in determining the presence of the risk-return relationship and volatility feedback, respectively (Maneemaroj *et al.,* 2019).

3.4.3.2 EGARCH-M

In comparison, to the GARCH-M model, the EGARCH-M model is a more practical choice in the investigation of the risk-return relationship because EGARCH has the ability to account for asymmetric effects (Adu *et al.,* 2015; Hretski and Karachun, 2018).

The EGARCH-M model follows the mean model of the standard GARCH-M model shown by Equation 37.1:

$$y_t = \mu + \sum_{j=1}^{p} \delta_j \sigma^2{}_{t-j} + u_t, u_t \sim N\ (0,1) \tag{37.1}$$

and the conditional variance of EGARCH is shown by Equation 37.2:

$$\ln(\sigma_t{}^2) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left[|z_{t-1}| - E|z_t|\right] + \sum_{j=1}^{p} \beta_j \ \ln\left(\sigma^2{}_{t-j}\right) + \gamma \, z_{t-1} \qquad (37.2)$$

where the interpretation of the constants, coefficients and parameters are as above.

From the standard GARCH model, various further extensions, modifications and hybrid models have been introduced, such as the EGARCH-M (Park *et al.,* 2017). However, all the models arise from similar parametric assumptions and model constraints and are thus limited in fully capturing risk (Feng and Shi, 2016, 2017; Jin, 2017; Jensen and Maheu, 2018; Demirer *et al.,* 2019).

3.4.4 Multivariate GARCH (1, 1)

In order to estimate a high number of parameters, multivariate GARCH models were introduced to solve the problem of overparameterisation which is computationally intensive (Aboura and Chevallier, 2018). According to Savva and Theodossiou (2018), this family of GARCH type models are used to investigate a correlation between shares and/or markets. It is often used to detect transmissions or spillover effects of volatility. However, this method is no better than a univariate GARCH approach since the conditional mean of returns are modelled in linear relation to the current and past return variance (Sultan, 2018; Savva and Theodossiou, 2018). It is still a parametric approach where constraints are imposed on the data, limiting the possibility of a nonlinear risk-return relationship (Demirer *et al.,* 2019). Since this study is based on an aggregate market level, the multivariate GARCH family is not reviewed. Essentially, the GARCH school of models are considered an inefficient choice in risk estimation since it does not fully capture the asymmetric nature of risk (Mangani, 2008; Feng and Shi, 2016; Feng and Shi, 2017; Jin, 2017; Jensen and Maheu, 2018).

3.4.5 Information Criteria

According to Feng and Shi (2017), risk remains uncaptured within the innovations of the GARCH models. In order to confirm the uncaptured risk, the best fitting GARCH type model and probability distribution governing the innovations are first selected. This selection is made by information criteria which is a type of model testing used to determine if an econometric model specification/s and estimation is statistically sound (Harris *et al.,* 2019). This study employs Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC), following a number of studies such as Mandimika and Chinzara (2012), Adu *et al.,* (2015), Khan *et al.,* (2016) and Liu (2019).

3.4.5.1 Akaike Information Criterion

The choice of AIC is inconsistent but efficient and preferable for a small sample since it has the least strict penalty (Apergis *et al.,* 2018).

The Akaike Information Criterion (AIC) by Akaike (1973), is given as Equation 38:

$$AIC = -\frac{2}{k}\log(likelihood) + \frac{2}{k}(p+q) \tag{38}$$

where k is the sample size and $p+q$ is the number of parameters in the model.

The addition of more parameters improves the model fit but strengthens the penalty imposed on increasing the number of parameters (Pham, 2020).

3.4.5.2 Bayesian Information Criterion

The choice of BIC is consistent but inefficient and preferable for a large sample since it has a strict penalty (Apergis *et al.,* 2018).

The Bayesian Information Criterion (BIC) by Schwarz (1978), is given as Equation 39:

$$BIC = -2\log(likelihood) + [(p+q) + (p+q)\log(k)] \tag{39}$$

where k is the sample size and $p+q$ is the number of parameters in the model.

In contrast, to AIC which is not dependent on the number of observations, the penalty for BIC is affected by the sample size (Pham, 2020). By Apergis *et al.,* (2018), overall, there is no better information criterion; thus, both AIC and BIC are used in this study.

Hence, the best GARCH model is determined by the smallest AIC and BIC information criteria values following Mandimika and Chinzara (2012), Adu *et al.,* (2015), Khan *et al.,* (2016) and Liu (2019). Thereafter, the innovations of the selected GARCH model are investigated to determine if risk has been fully captured.

3.4.6 Uncaptured Risk Within Innovations

In order to confirm the uncaptured risk within the innovations by Feng and Shi (2017), the preliminary tests from section 3.3 are applied to the innovations. Additionally, so are randomness tests following Mangani (2008) and Ilupeju (2016). Drawn from Mangani (2008) and Feng and Shi (2017), the innovations are expected to show nonnormality, heteroskedasticity and random behaviour to prove that risk remains uncaptured.

3.4.6.1 Randomness Tests

The random behaviour of the innovations is investigated by independent and identically distributed (IID) tests. The Brock, Dechert and Scheinkman (BDS), Bartels rank and, Cox- Stuart tests are employed following the studies by Mangani (2008) and Ilupeju (2016). All three randomness tests are nonparametric tests which means that the innovations are tested free from any assumptions or knowledge (Luo, Bai, Zheng and Hui, 2020). The BDS test is not only a test for random behaviour but also detects for nonlinearities. Random behaviour is further investigated by a Bartels test by means of a ranking method, whereas the Cox and Stuart test analyses the shape of the trend exhibited by the innovations.

3.4.6.1.1 Brock, Dechert and Scheinkman

According to Luo *et al.,* (2020), the Brock, Dechert and Scheinkman (BDS) test is widely used in the field of economics and finance for a number of reasons. One of the popular reasons being, the time series sequence tested does not have to contain higher moment properties such as kurtosis or skewness. However, in the context of time series analysis, this can be viewed as a limitation for two reasons.

First, returns are subject to asymmetric returns exposure and second, asymmetry remains uncaptured within the innovations of GARCH type models according to Feng and Shi (2017). The limitation is addressed by the fact that another important feature of the BDS test is that it is not only a randomness test but also detects whether nonlinearities are present within the innovations. This is because the test is essentially a nonparametric test which means that it accounts for asymmetry whether it is known or unknown to be present within the series (Mangani, 2008; Luo *et al.,* 2020).

For the BDS test by Brock, Scheinkman, Dechert and LeBaron (1996), the BDS test statistic is Equation 40:

$$BDS_{m,e} = \frac{\sqrt{N}(C_{m,e} - \left|C_{1,e}\right|^{m})}{\sqrt{V_{m,e}}} \tag{40}$$

where N is the sample size, $C_{m,e}$ is a correlation integral and V is a consistent estimator of standard deviation of the numerator for dimensions m and distance e.

The calculated BDS test statistic follows a normal distribution with mean one and a unit variance and is compared to the relevant CV which is selected with respect to the level of significance. For the two-sided test and a 5% level of significance, if the BDS < -1.960 and BDS > 1.960, the null hypothesis of randomness in the innovations can be rejected. Thus, it can be concluded the alternative hypothesis of nonrandomness in the innovations (Brock *et al.,* 1996).

3.4.6.1.2 Bartels Rank

Like the BDS test, the test by Bartels (1982), is unaffected by the presence or absence of higher moment properties due to being nonparametric. Therefore, in the context of the probability distribution governing the innovations, this suggests that the test is unaffected by the choice of distribution being normal, student-t or skewed student-t (Luo *et al.,* 2020).

For the Bartels rank test by Bartels (1982), the RVN test statistic is Equation 41:

$$RVN = \frac{\sum_{i=1}^{n-1}[R(z_i)-R(z_{i+1})]^2}{n(n^2-1)/12} \tag{41}$$

where *R* is the rank of the standardised innovations z_i and z_{i+1} for $i = 1, \dots, n-1$ and n is the sample size

The calculated RVM test statistic follows a normal distribution with mean one and a unit variance. It is compared to the relevant CV which is selected with respect to the level of significance. For the two-sided test, if RVM < $z\alpha/_2$ and RVM > $z_{1-}\alpha/_2$ where α is the level of significance, the null hypothesis of randomness in the innovations can be rejected. Thus, it can be concluded the alternative hypothesis of nonrandomness in the innovations (Bartels 1982).

3.4.6.1.3 Cox and Stuart

The Cox-Stuart test by Cox and Stuart (1955), uses the process of detecting a trend of the standardised innovations. According to Rutkowska (2015), the test does not assume the shape of the trend. Hence, it is a credible test when asymmetry is present within the probability distribution governing the innovations such as a student-t or skewed student-t. Like the BDS and Bartels rank test, the Cox-Stuart test include not being influenced by the type of probability distribution as well as the size of the sample (Luo *et al.,* 2020; Rutkowska, 2015).

Given the set of innovations $z_1, \ldots, z_t$ they are arranged in grouped pairs as shown by Equation 42:

$$(z_1, z_{1+k}), (z_1, z_{2+k}), \ldots, (z_{t-k}, z_t) \tag{42}$$

where $k = \begin{cases} \frac{t}{2}, & if\ t\ is\ even \\ \frac{t+1}{2}, & t\ is\ odd \end{cases}$

Then a sign test from the grouped pairs in Equation 42 is given as Equation 43:

$$sign(\,z_i, z_{i+k}) = \begin{cases} +, if\ z_i < z_{i+k} \\ 0, if\ z_i = z_{i+k} \\ -, if\ z_i > z_{i+k} \end{cases} \tag{43}$$

where the test statistic is defined as *T* the number of positives and *N* is defined as the number of negatives.

The test statistic *T* is compared to the relevant CV for the selected level of significance α. If $T \leq \frac{1}{2} - Z_{\alpha/2}\sqrt{\frac{1}{4\,N}}$ or $T \geq \frac{1}{2} + Z_{\alpha/2}\sqrt{\frac{1}{4\,N}}$, the null hypothesis of no trend can be rejected. Thus, it can be concluded the alternative hypothesis of an upward or downward trending behaviour (Cox and Stuart, 1955).

3.4.7 Summary of the GARCH Approach

According to Hretski and Karachun (2018), the GARCH family shows a progression of model extensions to incorporate the market characteristics of financial data. The ARCH model is a function of past innovations, whereas the GARCH model is a function of past innovations and volatility. Both the ARCH and GARCH models are time varying, have the ability to capture volatility clustering and heavy tails. However, these models are essentially linear functions and cannot account for asymmetry, particularly the market characteristics, asymmetric volatility and the leverage effect. Extensions of the original GARCH model has led to the flexible APARCH and GJR-GARCH model which accounts for asymmetric effects. However, the EGARCH model accounts for both asymmetric volatility and the leverage effect more effectively by a sign and magnitude innovation without nonnegativity constraints (Adu *et al.,* 2015; Khan *et al.,* 2016; Hretski and Karachun, 2018).

The GARCH-M is used to price risk but this study limits the model from estimating and explaining the risk-return relationship due to its inability to account for asymmetric returns exposure. This return inherent risk is latent and stochastic in nature and cannot be observed directly; thus, it has a certain level of uncertainty attached to it (Jin, 2017). The GARCH family is ineffective in fully accounting for risk due to essentially being parametric and not fully capturing risk by their innovations (Feng and Shi, 2017). Despite this finding, studies still extensively apply the GARCH approach in the investigation of risk-return relationship as documented in the study by Savva and Theodossiou (2018). Theoretical and empirical literature indicates that the GARCH approach can be considered as irrelevant and obsolete given the existence of more robust methods such as the nonparametric Bayesian approach by Jensen and Maheu (2018).

3.5 Bayesian Approach

Bayesian statistics is introduced to give a brief background of this school of thinking. The research design provides a framework for which a problem, such as the risk-return relationship and volatility feedback topic, can be solved within the Bayesian approach. The econometric model then shows how the Bayesian school of thinking is applied to the investigation of the risk-return relationship and volatility feedback. This is followed by the method procedure of a parametric Bayesian model and then nonparametric Bayesian model which is the main method of investigation for this study.

3.5.1 Background

According to Herath (2019), classical or frequentist statistics is often used in the conventional approaches of quantitative finance where theory is either accepted or rejected based on the empirical results. On the other end of the spectrum, is Bayesian statistics which is used for estimation, inference and modelling of data where the theory and empirical model are closely related. This is made possible by accounting for prior information (Herath, 2019). Bayesian statistics is an extensive field of study built on Bayes (1763) theorem which is the probability estimation of a relationship given prior information. Although most researchers may not use Bayes theorem directly, the underlying idea of the concept is fundamental to aid one's understanding. That is, in terms of conditioning variables, how the probability of one variable, representing a relationship, theory or event, affects the probability of another

(Hatjispyros, Nicoleris and Walker, 2019). In addition, the updating of existing theories as additional data becomes available (Cai, 2018).

The inability of the classical or frequentist approach to take into account prior information suggests an inflexible approach (Herath, 2019). The empirical results of a classical approach are often presented in the form of *p*-values or confidence intervals, whereas the Bayesian approach presents a posterior parameter estimate (Wagenmakers *et al.,* 2018). According to Brooks (2014), a conventional 95% confidence interval defines a range of values that one can be 95% certain contains a parameter estimate. Wagenmakers *et al.,* (2018) notes that the confidence interval procedure is limited as one cannot specify the interval bounds and then find out the probability or confidence that the parameter estimate lies within that specified interval.

In contrast, to a regular confidence interval, is a Bayesian interval which is also known as a credible or density interval (Karabatsos, 2016; Jensen and Maheu, 2018). A credible interval has two advantages by Wagenmakers *et al.,* (2018). First, a credible interval accounts for conditional prior information. This leads to the second advantage, which means that the parameter estimate is a *posterior* parameter estimate where the data has updated to given information. To aid understanding this critical difference in data estimation, a classical approach can be thought of as “pre-data”, whereas a Bayesian approach is more of a “post-data” estimate due to taking into account prior information including uncertainty (Wagenmakers *et al.,* 2018).

One of the main advantages of a Bayesian approach over a frequentist approach is the ability to average out uncertainty surrounding a parameter (Waldmann, 2018). In the context of risk estimation, according to Aliu *et al.,* (2017), the probabilities of possible future outcomes can be estimated given prior information. Meaning, risk allows an individual to have some probability of knowledge, whereas in contrast, uncertainty does not (Aliu *et al.,* 2017). Therefore, a method that has the ability to account for uncertainty immediately suggests a more robust and informative measure of risk (Herath, 2019).

This is made possible by the fact that a Bayesian approach introduces parameters as random variables instead of a number or fixed value, such as returns as a linear function of volatility, in a parametric model (Kim and Kim, 2018). Structural breaks are treated the same way, allowing for the parameters to change in relation to these breaks in a quantitative rather than qualitative manner (Wang and Tsay, 2018). In contrast, the traditional approach by the GARCH school of modelling is to use dummy variables to account for extreme events (Mandimika and Chinzara, 2012). Thus, not only does the Bayesian approach has a greater ability to capture extreme events but accounts for the uncertainty associated with the random stochastic nature of the variables (Cai, 2018; Gong, Liu, Xiong and Zhuang, 2019).

Literature highlights the Bayesian approach as a novel approach used in a number of fields and real-life practical situations such as the medical field, psychology and economics (Karabatsos, 2016; Wagenmakers *et al.,* 2018; Herath, 2019). The fundamentals of a Bayesian approach remain the same whether the model is simple or complex because the common important feature is the posterior estimate (Wagenmakers *et al.,* 2018). When the posterior estimate cannot be determined analytically, it can be drawn from computational sampling techniques such as Markov Chain Monte Carlo (MCMC) methods (Herath, 2019). MCMC is often used to derive a probability estimation of a density given limited information about the distribution (Martino, Elvira and Camps-Valls, 2018; Gu, Zhang, Liu, Zhang and Ye, 2019; Griffin *et al.,* 2018).

This development of MCMC methods has been made possible as a result of technological advancements and relevant up to date software (Herath, 2019). According to Karabatsos (2016), MCMC has been specifically designed to fit Bayesian models which are uniquely beneficial from conventional quantitative finance methods which address a number of shortcomings. This includes parametric models such as regression analysis, the VAR model, causality tests and the GARCH approach, as highlighted in the empirical review. The application of Bayesian and MCMC methods in the fields of psychology and medicine demonstrates its level of usefulness in the real world due to its practicality and effectiveness (Jung *et al.,* 2020; van Doremalen *et al.,* 2020; Wagenmakers *et al.,* 2018; Karabatsos, 2016).

In conclusion, the Bayesian approach is suitable for models that understand the complexity of financial data, especially the nature of returns which has a nonlinear, asymmetric, volatile, stochastic and latent nature (Wagenmakers *et al.,* 2018). Thus, it is only fair to apply this method to the field of finance to improve conclusive findings.

3.5.2 Research Design

There are three specific reasons by Ferson (2005), where one can use the Bayesian method for a scientific analysis which is applicable to the context of this study. Firstly, it provides a framework to structure a problem where there could exist the following three sub challenges:

- First, there is a lack of existing literature regarding the subject of interest (Ferson, 2005). In this case, it is applicable to volatility feedback and the risk-return relationship topic in South Africa relative to other countries, respectively (Savva and Theodossiou, 2018).
- Second, there may be a need to incorporate a probabilistic approach rather than a deterministic one (Ferson, 2005). Asymmetric returns exposure has a stochastic and latent nature which means that it can be statistically analysed but not necessarily forecasted with certain precision (Harris, 2017; Jin, 2017).
- Third, there exists a substantial amount of uncertainty surrounding the parameters and model (Ferson, 2005). With respect to the parameters, due to the nature of asymmetric returns exposure, there is a certain level of uncertainty attached to the variable returns. In terms of the model, this can be shown by previous risk-return empirical studies that use methods that do not effectively account for asymmetric returns exposure. Specifically, the conventional parametric models that are not designed to handle the asymmetric nature of returns and the uncertainty associated to the variable (Jin, 2017). Thus, affecting the estimation of risk and contributing to inconclusive results (Jensen and Maheu, 2018).

Secondly, the ability to estimate probability distributions which are made up of two parts, namely, priors and posteriors (Hatjispyros *et al.,* 2019). A prior is an initial probability estimation based on existing information (Goudarzi, Jafari and Khazaei, 2019). A prior has the ability to update given the availability of more data by means of a likelihood function which consists of new observed data (Karabatsos, 2016). The combination of a prior and likelihood by means of model estimation results in the posterior which is an updated probability estimation (Cai, 2018).

With respect to the nonparametric Bayesian approach in this study, the prior is estimated by the Bayesian Dirichlet Process by Ferguson (1973), derived by the stick-breaking process by Sethuram (1994). The posterior is estimated by a slice sampler by Kalli, Griffin and Walker (2011) and a Gibbs sampling technique. This follows Jensen and Maheu (2018), which is the first and only study to apply the nonparametric Bayesian approach to the risk-return relationship and volatility feedback topic, to the best of the authors knowledge. These are golden standard nonparametric Bayesian methods (Dirichlet Process, slice and Gibbs sampler) which involve random sampling methods whereby every distribution has an equal chance of being drawn (Alvi, 2016). These methods suggest low levels of bias and systematic error, and a high level of reliability, validity and viability (Etikan and Bala, 2017). Thus, ensuring accurate estimates and reliable results (Karabatsos, 2016).

Thirdly, choosing and estimating the parametric or nonparametric approach to accompany the Bayesian model (Demirer *et al.,* 2019). By definition, a parametric model refers to a set number of parameters with respect to the sample size (Jin, 2017). This is in contrast to a nonparametric model where the number of parameters increases as the sample size increases (Apergis *et al.,* 2018). In other words, this means that as more data becomes available, the number of parameters increases, allowing for a greater number of possibilities (Demirer *et al.,* 2019). Further, the likelihood function of new observations can be captured due to the access or availability of additional data (Cai, 2018). Essentially, the nonparametric approach implies a model free approach, where this study highlights the normality assumption being relaxed, allowing for an array of asymmetric properties (Jensen and Maheu, 2018).

In the context of this study, according to Karabatsos (2016), a nonparametric Bayesian model is often referred to as being an infinite-mixture model. An infinite-mixture model describes a model that takes into account an infinite number of clusters. The cluster is a component of a mixture of, in this case, weights and parameters. The nonparametric Bayesian model assumes an infinite number of clusters, whereas the parametric Bayesian model assumes a finite number of clusters. As a result, the nonparametric Bayesian model is the more robust model, due to having greater flexibility in effectively accounting for higher moment asymmetric forms of the risk-return relationship, in an infinite sample space. A sample space refers to the number of possible outcomes of a random variable. The nonparametric approach is designed to effectively account for an infinite sample space, whereas a parametric approach is limited to a finite sample space by definition (Jensen and Maheu, 2018; Karabatsos, 2016). The property of the clustering mixture of parameters is further highlighted in the method procedure, after the motivation for the econometric model, as follows.

3.5.3 Motivation for Econometric Model

The risk premium is a source of risk and is synonymous with excess returns, investors risk compensation and is often used to investigate the risk-return relationship (He *et al.,* 2018). The prediction of excess returns is of wide interest to academics and market participants (Herbert *et al.,* 2018). From a finance perspective, forecasting power of returns is determined by an appropriate measure of a firm's prospects and systematic risk (Kim and Kim, 2018). In contrast, from an economic perspective, macroeconomic factors such as credit spreads, term spreads and exchange rates are more closely related (Park *et al.,* 2017).

However, according to Kim and Kim (2018), volatility feedback can be described as a predictor of unanticipated errors, as a result of macroeconomic variables, which affects returns. Consequently, by definition this mechanism suggests the ability to capture volatility which arises from an economic and financial perspective. Additionally, this allows volatility feedback to be treated as is and not tailor it to a factor such as "volatility feedback news" to explicitly state that this mechanism is driven by information or macroeconomic fundamentals (Kim and Kim, 2018). Volatility feedback is an important factor to take into account when predicting excess returns which can be mathematically shown.

According to Jensen and Maheu (2018), there are two specific opposing effects that act on excess returns. These effects are shown by the trade-off theory by Markowitz (1952), and the volatility feedback effect by Pindyck (1984). The econometric model by French *et al.,* (1987), shows how volatility feedback obscures any risk-return relationship. The application of Bayes (1763) theorem, in terms of conditioning variables, shows how excess returns, the risk-return relationship and volatility feedback are all related. The Bayesian econometric model by French *et al.,* (1987), is given as Equation 44. Note, throughout this study, the information set I_{t-1} refers to all the possible values made up from the dataset, excess returns r_t and realised variance RV_T, and is represented by $I_{t-1} = \{r_1, RV_1, \dots, r_t, RV_t\}$ where $t = 1, \dots, T$.

$$r_t = E[r_t | I_{t-1}] + \alpha_1(RV_t - E[RV_t | I_{t-1}]) + e_t \tag{44}$$

From Equation 44 by Jensen and Maheu (2018), $E[r_t | I_{t-1}]$ is defined as the expected conditional mean of excess returns, given the information set which is made up of excess returns and realised variance. This is an ex ante (future) term and captures the positive risk-return relationship. The coefficient α_1 of $\alpha_1(RV_t - E[RV_t | I_{t-1}])$ is a measure of the persistence of risk or a volatility shock on $(RV_t - E[RV_t | I_{t-1}])$. The latter is the difference between observed realised variance and expected realised variance. This is an ex post (actual) term and captures the volatility feedback effect. Finally, e_t is the innovation term of the model which captures the possible deviations between the observed and expected values (Jegadeesh, Noh, Pukthuanthong, Roll and Wang, 2019; Jensen and Maheu, 2018).

Provided volatility is a priced risk factor by Jensen and Maheu (2018), for a positive volatility shock α_1, that is for a strong persistence in risk; actual volatility persists and signals future volatility on expected returns. This results in a positive risk-return relationship by the trade-off theory which is captured by $E[r_t | I_{t-1}]$ (Markowitz, 1952). In turn, there is a demand for higher expected returns and in order to raise the expected returns; share prices decrease by the volatility feedback effect which is captured by the volatility innovation $\alpha_1(RV_t - E[RV_t | I_{t-1}])$ (Pindyck, 1984). In summary, for a positive volatility shock, there is an increase in $E[r_t | I_{t-1}]$ capturing the risk-return relationship and a decrease in $\alpha_1(RV_t - E[RV_t | I_{t-1}])$ capturing volatility feedback (Jensen and Maheu, 2018).

According to Jensen and Maheu (2018), this model mathematically shows the two opposing effects that act on excess returns. This is captured by a nonparametric joint distribution between excess returns and realised variance. Once volatility feedback is established, by its presence or absence in the market based on its magnitude, the risk-return relationship is investigated. Note, this only occurs when the volatility shock is zero as shown by Equation 45 which shows a pure trade-off between risk and return, once the presence of volatility feedback has been taken into account (Jensen and Maheu, 2018).

$$RV_t = E[RV_t | I_{t-1}] \tag{45}$$

To clarify, from Equation 45 by Jensen and Maheu (2018), in order for the volatility shock α_1 of $\alpha_1(\mathrm{RV_t} - \mathrm{E}[\mathrm{RV_t}|\ \mathrm{I_{t-1}}])$ to be zero, the observed realised variance $\mathrm{RV_t}$ needs to be equivalent to the expected realised variance $\mathrm{E}[\mathrm{RV_t}|\mathrm{I_{t-1}}]$. To aid understanding, this can be graphically represented as the intersection of the density lines of the observed and expected realised variance. Thereafter, to account for the presence of volatility feedback, a method of interpolation is applied which refers to where the specified points of intersection are constructed within all the existing data points (Jensen and Maheu, 2018).

3.6 Method Procedure

The econometric model follows the method procedure outlined below from which the parametric Bayesian model and then nonparametric Bayesian model are derived. Both models are under the Bayesian approach since they both have the ability to account for the uncertainty associated with the nature of asymmetric returns exposure (Agilan and Umamahesh, 2017). However, note that the nonparametric Bayesian model is the main method of investigation in this study since it has the greater ability to account for asymmetry (Jensen and Maheu, 2018).

The parametric Bayesian model is outlined first because it follows the order of the study by Jensen and Maheu (2018), where its purpose is twofold. In this case, first to serve as a preliminary test to provide an overall idea of the effects of volatility feedback and the risk-return relationship. Second, to allow for a meaningful comparative analysis with the final results of the nonparametric Bayesian model. The two variables of interest, excess returns and realised variance, are treated and modelled as continuous random variables due to their stochastic nature and ability to take on an infinite number of values (Demirer *et al.,* 2019)

3.6.1 Parametric Bayesian Model

The parametric Bayesian model follows the method procedure of the nonparametric Bayesian model with one exception - the model becomes a finite model and is further simplified by the author for computational ease. To recap, according to Karabatsos (2016), a nonparametric Bayesian model is often referred to being an infinite-mixture model. Essentially, a nonparametric Bayesian model assumes an infinite number of clusters, whereas a parametric Bayesian model assumes a finite number of clusters (Jensen and Maheu, 2018; Karabatsos, 2016). The parametric Bayesian model is a finite model because by definition a parametric model has a set number of parameters with respect to its sample size (Jin, 2017).

Therefore, although the parametric Bayesian model takes into account asymmetric properties, it does so to a limited extent because the model is essentially parametric (Jin, 2017). Hence, the parametric model does not have the capacity to take into account an infinite number of possibilities and account for every possible risk-return relationship that could exist (Demirer *et al.,* 2019). Consequently, there is no need to apply a prior to reduce an infinite model to a finite model since it is already finite (Karabatsos, 2016). However, in order to derive the posterior parameter estimates, the posterior methods, the slice sampler by Kalli *et al.,* (2011) and Gibbs sampler are still applied.

3.6.1.1 Joint

Following Jensen and Maheu (2018), in this studies procedure in uncovering the risk-return relationship, the first step is to model a joint distribution of excess returns and realised variance. This results in a number of bivariate density functions which are the consequent possible densities of the joint model of the two variables shown in Equation 46:

$$\mathrm{p}(\mathrm{r_t}, \log(\mathrm{RV_t})\,|\mathrm{I_{t-1}}, \Omega, \Theta) = \sum_{\mathrm{j}=1}^{\infty} \mathrm{w_j} * \mathrm{f}\,(\mathrm{r_t}, \log(\mathrm{RV_t})|\theta_\mathrm{j}, \mathrm{I_{t-1}}) \tag{46}$$

where the probability of excess returns and log realised variance is conditional on the following: The information set, the mixture weights $\Omega = w_j$ where $\sum_{j=1}^{\infty} w_j = 1$ and mixture parameters $\Theta = \theta_j$ where $j = 1, \dots, \infty$ which refers to the number of clusters of mixture parameters. This is equivalent to the sum of all the weights and functions of excess returns and log realised variance, given the parameters and information set.

The next step involves deriving a parametric version of the risk-return relationship by reducing the notation of Equation 46 to only the necessary components and rewriting Equation 46 as Equation 47:

$$\mathrm{f}\,(\mathrm{r_t}, \log(\mathrm{RV_t})\big|\theta_\mathrm{j}, \mathrm{I_{t-1}}\big) \equiv \mathrm{f}\,(\mathrm{r_t}|\log(\mathrm{RV_t}), \theta_\mathrm{j}, \mathrm{I_{t-1}}) * \mathrm{f}\,(\log(\mathrm{RV_t})\big|\theta_\mathrm{j}, \mathrm{I_{t-1}}\big) \tag{47}$$

where the latter is simply the product of the conditional distribution (term 1) and marginal distribution (term 2) by the law of total probability (Chan, Guo, Lee and Li, 2018; Jensen and Maheu, 2018). Since excess returns and log realised variance theoretically tends to a normal distribution, Equation 47 allows for the representation of Equation 48 and 49, respectively (Jensen and Maheu, 2018).

Equations 48 and 49 follow a functional form of normality like conventional parametric methods such as the GARCH approach (Madaleno and Vieira, 2018). However, the mixing of the finite number of cluster parameters allows for a wider array of joint densities, including asymmetric densities, in a finite sample space (Karabatsos, 2016). Therefore, suggesting that the parametric Bayesian model is more robust than other conventional parametric models such as the GARCH approach (Jensen and Maheu, 2018).

$$\mathrm{f}\left(\mathrm{r_t}\middle|\log(\mathrm{RV_t}),\theta_\mathrm{j},\mathrm{I_{t-1}}\right)=\mathrm{f_N}(\mathrm{r_t}|\alpha_0+\alpha_1\mathrm{RV_t},\eta_1^2\mathrm{RV_t}) \tag{48}$$

From Equation 48 by Jensen and Maheu (2018), the conditional mean function of excess returns given log realised variance, the parameter and information set, is equivalent to the normal conditional mean function of excess returns. The latter is conditional on the following: The coefficient α_1 on RV_t represents the persistence of risk on realised variance and this term represents volatility feedback. The η_1^2 on RV_t indicates the systematic error on realised variance. This term refers to the error surrounding the stochastic measure of realised variance which is unavoidable regardless of the number of times the model is run (Beyhaghi, Alimo and Bewley, 2018; Jensen and Maheu, 2018).

$$\mathrm{f}\left(\log(\mathrm{RV_t})\,\middle|\theta_\mathrm{j},\mathrm{I_{t-1}}\right) \tag{49}$$

$$=\mathrm{f_N}((\log(\mathrm{RV_t})\mid\gamma_0+\gamma_1\log(\mathrm{RV_{t-1}})+\gamma_2\log(\mathrm{RV_{t-i}})+\gamma_3\frac{\mathrm{r_{t-1}}}{\sqrt{\mathrm{RV_{t-1}}}}+\gamma_4\left|\frac{\mathrm{r_{t-1}}}{\sqrt{\mathrm{RV_{t-1}}}}\right|,\eta_2^2)$$

From Equation 49 by Jensen and Maheu (2018), the conditional mean function of log realised variance, given the parameters and information set, is equivalent to the normal conditional mean function of log realised variance. The latter is conditional on the following: The coefficients γ_1, γ_2, γ_3 and γ_4 which refer to the persistence of the variables. The first two terms cater for volatility feedback but the last two terms cater for the leverage effect. Although the latter two variables are taken into account, it is not within the scope of this study; thus, it is ignored (Jensen and Maheu, 2018).

Equations 48 and 49 are the main equations of interest for the parametric Bayesian model. Conditioning has been dropped for convenience following Jensen and Maheu (2018). The addition of the innovation terms ε_t and υ_t shown in both equations is to aid understanding. Further simplifications made by the author for computational ease are discussed.

$$r_t = \alpha_0 + \alpha_1 RV_t + \varepsilon_t \tag{50}$$

$$\log(RV_t) \tag{51}$$
$$= \gamma_0 + \gamma_1 \log(RV_{t-1}) + \gamma_2 \frac{1}{2488}\sum_{i=1}^{2488} \log(RV_{t+1-i}) + + \gamma_3 \frac{r_{t-1}}{\sqrt{RV_{t-1}}} + \gamma_4 \left|\frac{r_{t-1}}{\sqrt{RV_{t-1}}}\right| + \upsilon_t$$

In Equation 50, the realised variance measure is not introduced into the innovation term. This is because the error variance in the model accounts for all unexplained variance that arises from sources such as uncertainty and measurement errors (Chakraborty and Lozano, 2019). This includes the systematic error on the realised variance measure which refers to the error surrounding the stochastic nature of the risk measure (Beyhaghi *et al.,* 2018). Hence, both the innovation terms ε_t and υ_t of Equation 50 and 51 capture the possible deviations between the observed and expected values (Jegadeesh *et al.,* 2019).

It is accounted for and reflected through the σ_1^2 and σ_2^2 of both models (Karabatsos, 2016). Further, according to Jensen and Maheu (2018), in Equation 49, the coefficient γ_2 is supposed to cater for volatility feedback over a six-month period. However, it is not shown because in this study, the entire sample period is taken into account as shown by Equation 51. This is in order to essentially determine the presence and persistence of volatility over time in the South African market to provide an overall state and condition of the market (Jensen and Maheu, 2018).

3.6.1.2 Posterior

The posterior procedure consists of a number of steps. Initially, random samples are drawn from a joint distribution by means of a slice sampling technique by Kalli *et al.,* (2011). The slice sampler is applied to Equation 46, except an additional random variable represented by u_t is introduced, as shown in Equation 52 by Jensen and Maheu (2018):

$$p(r_t, \log(RV_t), u_t|\Omega, \Theta, I_{t-1}) = \sum_{j=1}^{\infty} \mathbf{1}\left(u_t < w_j\right) * f\left(r_t, \log(RV_t)|\theta_j, I_{t-1}\right) \quad (52)$$

where the aim of adding this variable u_t is to ensure that only positive weights are retained and all weights of zero are “sliced away” (Karabatsos, 2016; Liu and Luger, 2018; Jensen and Maheu, 2018). Thereafter, the following iteration method by Jensen and Maheu (2018), is applied which refers to the repetitive resampling process of a collection of steps.

Firstly, a Gibbs sampling technique is applied which is often used when the joint distribution is unknown and it is simpler to draw samples from the known conditional distribution (Merel, Shababo, Naka, Adesnik and Paninski, 2016). In this case, the conditional distribution contains the cluster mixture parameters and weights (Jensen and Maheu, 2018). Secondly, since the priors are strong, this allows for the formation of a conjugate conditional posterior, meaning, a conditional posterior that shares similar model properties to the prior (Gu *et al.,* 2019). Thirdly, consequently, each of the random variables tends to form a homogenous distribution provided the given weights and parametric space (Jensen and Maheu, 2018). Finally, if the cluster count is amended, there may be further prior draws (Merel *et al.,* 2016).

This procedure will continue; however, the Gibbs sampling process is subject to a burn-in period in which samples in the earlier stages that are no longer accurately representative of the required distribution are discarded (Merel *et al.,* 2016). The original base distribution is then updated to the posterior distribution (Cai, Mitzemacher and Adams, 2018). Hence, so are the coefficients and parameter estimates of Equation 50 and 51, from which conclusive results can be drawn with respect to the risk-return relationship and volatility feedback (Jensen and Maheu, 2018).

3.6.2 Nonparametric Bayesian Model

According to Jensen and Maheu (2018), the nonparametric Bayesian model follows the derivation of the joint model in 3.6.1.1, with one exception. The nonparametric model simply takes on a second subscript *j* which represents the cluster mixture parameters in an infinite sample space. It is this component *j*, the cluster mixture of parameters, which allows for the infinite asymmetric properties in the model. Hence, a greater number of possibilities, as in asymmetric densities, of the risk-return relationship in an infinite sample space. Without component *j*, the model is consequently limited and does not take into account the higher moment asymmetric properties such as skewness, kurtosis and multiple modes as effectively (Demirer *et al.,* 2019; Jensen and Maheu, 2018). While the derivation and interpretation of the variables are identical to the parametric Bayesian model, they are recapped for convenience.

3.6.2.1 Joint

Following Jensen and Maheu (2018), in this studies procedure in uncovering the risk-return relationship, the first step is to model a joint distribution of excess returns and realised variance. This results in a number of bivariate density functions which are the consequent possible densities of the joint model of the two variables shown in Equation 53:

$$\mathrm{p}(\mathrm{r_t}, \log(\mathrm{RV_t})\,|\mathrm{I_{t-1}}, \Omega, \Theta) = \sum_{\mathrm{j}=1}^{\infty} \mathrm{w_j} * \mathrm{f}\,(\mathrm{r_t}, \log(\mathrm{RV_t})|\theta_\mathrm{j}, \mathrm{I_{t-1}}) \tag{53}$$

where the probability of excess returns and log realised variance is conditional on the following: The information set, the mixture weights $\Omega = w_j$ where $\sum_{j=1}^{\infty} w_j = 1$ and mixture parameters $\Theta = \theta_j$ where $j = 1, \dots, \infty$ which refers to the number of clusters of mixture parameters. This is equivalent to the sum of all the weights and functions of excess returns and log realised variance, given the parameters and information set (Jensen and Maheu, 2018).

The next step involves deriving a nonparametric version of the risk-return relationship by reducing the notation of Equation 53 to only the necessary components and rewriting Equation 53 as Equation 54:

$$\mathrm{f}\,(\mathrm{r_t}, \log(\mathrm{RV_t})\big|\theta_\mathrm{j}, \mathrm{I_{t-1}}\big) \equiv \mathrm{f}\,(\mathrm{r_t}|\log(\mathrm{RV_t}), \theta_\mathrm{j}, \mathrm{I_{t-1}}) * \mathrm{f}\,(\log(\mathrm{RV_t})\big|\theta_\mathrm{j}, \mathrm{I_{t-1}}\big) \tag{54}$$

where the latter is simply the product of the conditional distribution (term 1) and marginal distribution (term 2) by the law of total probability (Chan *et al.,* 2018; Jensen and Maheu, 2018). Since excess returns and log realised variance theoretically tends to a normal distribution, Equation 54 allows for the representation of Equation 55 and 56, respectively (Jensen and Maheu, 2018).

Equation 55 and 56 follows a functional form of normality like conventional methods such as the GARCH approach (Madaleno and Vieira, 2018). However, the mixing of the infinite number of cluster parameters allows for a wide array of joint distributions, effectively including asymmetric densities, in an infinite sample space (Karabatsos, 2016).

Equations 55 and 56 are the main equations of interest for the nonparametric Bayesian model. As discussed, the only difference between the parametric and nonparametric Bayesian model specifications is that the nonparametric Bayesian model has component *j* as the second subscript of Equations 55 and 56. It is this component *j*, the cluster mixture of the parameters, which allows for an infinite number of asymmetric properties in an infinite sample space (Jensen and Maheu, 2018).

The component *j* is in contrast to the conventional parametric models, such as the GARCH approach, which is already limited to a range of asymmetric possibilities (Apergis *et al.,* 2018). The nonparametric approach allows for moving away from linearity and effectively includes densities with higher moment properties such as skewness, kurtosis and multiple modes in an infinite sample space (Demirer *et al.,* 2019). The thinking behind the *j* component and Equations 55 and 56 following a form of normality is further explained after the model specifications are noted.

$$f\left(r_t \middle| \log(RV_t), \theta_j, I_{t-1}\right) = f_N\left(r_t | \alpha_{0,j} + \alpha_{1,j} RV_t, \eta_{1,j}^2 RV_t\right) \quad (55)$$

From Equation 55 by Jensen and Maheu (2018), the conditional mean function of excess returns given log realised variance, the parameter and information set, is equivalent to the normal conditional mean function of excess returns. The latter is conditional on the following: The coefficient $\alpha_{1,j}$ on RV_t represents the persistence of risk on realised variance and this term represents volatility feedback. The $\eta_{1,j}^2$ on RV_t indicates the systematic error on realised variance (Jensen and Maheu, 2018). This term refers to the error surrounding the stochastic measure of realised variance which is unavoidable regardless of the number of times the model is run (Beyhaghi *et al.,* 2018).

$$f\left(\log(RV_t) \middle| \theta_j, I_{t-1}\right) \quad (56)$$

$$= f_N(\log(RV_t) | \gamma_{0,j} + \gamma_{1,j} \log(RV_{t-1}) + \gamma_{2,j} \log(RV_{t-i}) + \gamma_{3,j} \frac{r_{t-1}}{\sqrt{RV_{t-1}}}$$

$$+ \gamma_{4,j} \left| \frac{r_{t-1}}{\sqrt{RV_{t-1}}} \right|, \eta_{2,j}^2)$$

From Equation 56, by Jensen and Maheu (2018), the conditional mean function of log realised variance, given the parameters and information set, is equivalent to the normal conditional mean function of log realised variance. The latter is conditional on the following: The coefficients $\gamma_{1},_{j}$, $\gamma_{2},_{j}$, $\gamma_{3},_{j}$ and $\gamma_{4},_{j}$ which refers to the persistence of the variables. The first two terms cater for volatility feedback but the last two terms cater for the leverage effect. Following Jensen and Maheu (2018), although the latter two variables are taken into account, it is not within the scope of this study; thus, it is ignored. Further, with respect to model implementation, conditioning is dropped for convenience for the equations of interest - Equation 55 and 56, and realised variance is not introduced into the error term by the author for computational ease.

The thinking behind Equations 55 and 56 following a linear form of normality but allowing for asymmetric properties is illustrated. Figure 9 shows a density estimation which represents the form and shape of an unknown distribution.

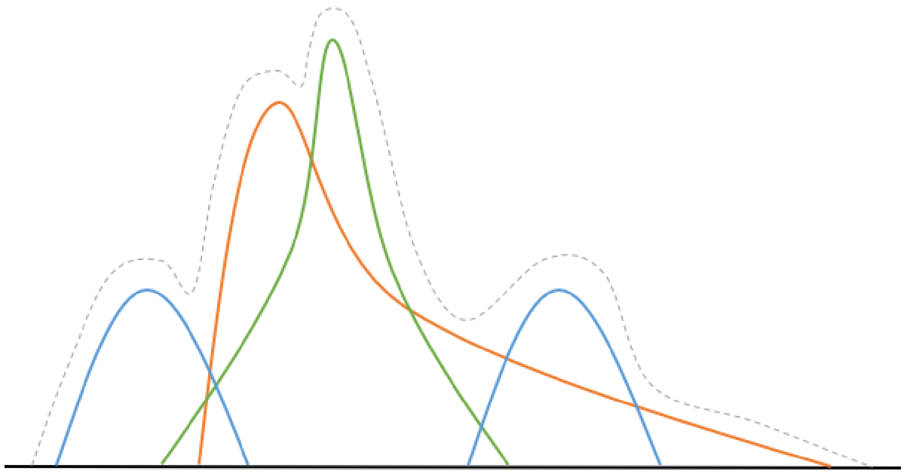

Figure 9: Mixture of normals

Source: Authors own

From Figure 9, the density estimation is represented by the broken grey lines. This is made up of a number of normal distributions with various cluster mixture parameters. The second subscript *j* in Equation 55 and 56 represent the cluster mixture parameters which allows for the infinite asymmetric properties (Jensen and Maheu, 2018). As a result, the distributions show a number of higher moment properties such as skewness (orange line), kurtosis (green line) and multiple modes (blue lines). Thus, for an infinite mixture model, it follows that every possible risk-return relationship that could hold is taken into account, allowing for an infinite number of possibilities (Demirer *et al.,* 2019). However, from an infinite number of possibilities arises a certain level of uncertainty attached to the variables due to their stochastic nature (Jensen and Maheu, 2018). Therefore, the Bayesian Dirichlet Process is applied to reduce the uncertainty associated with an infinite model by effectively reducing it to a finite means.

3.6.2.2 Prior

The Bayesian Dirichlet Process (DP) is defined as a probability estimation made up of a number of densities for a given sample space (Lawless and Arbel, 2019). According to Ferguson (1973), in the context of nonparametric Bayesian problems, there are two conditions that need to be met in order to ensure strong priors. One, the prior is large, provided an infinite sample space to allow for conjugacy, that is, to allow for the sharing of model properties. Two, the shape of the prior is adequate. This is because both conditions affect the form of the posterior since a sufficient prior in combination with the likelihood determines the posterior. The Bayesian DP prior model addresses both conditions (Goudarzi *et al.,* 2019; Cai, 2018; Ferguson, 1973).

Conjugacy is a property of the DP for the prior as well as one of the steps in the Gibbs sampling technique for the posterior (Jensen and Maheu, 2018). This property is highlighted as one of importance because conjugacy refers to the sharing of model properties which affects the density form of the posterior (Goudarzi *et al.,* 2019). Meaning, if a prior had a density with asymmetric higher moment properties, then by conjugacy the posterior is subject to those asymmetric higher moment properties while updating its parameters (Phadia, 2016). Without the property of conjugacy, this can affect the model's convergence which describes the model's tendency to a statistically sound result (Phadia, 2016).

This is in contrast to a limitation found in the innovations of the GARCH approach. According to Feng and Shi (2017), both the innovation distributions governed by the student-t and GED lack stability under aggregation. This means that the combination of two variables, such as for risk and return, results in a relationship represented by a density that is not in line with what was originally used. Therefore, a nonparametric Bayesian model is more effective in fitting an unknown distribution as well as automatically account for measurement errors and uncertainty (Chakraborty and Lozano, 2019; Feng and Shi, 2016, 2017).

The DP can be derived by a number of ways; however, the stick-breaking process is foremost for practical application (Lawless and Arbel, 2019). The stick-breaking process by Sethuram (1994), involves the mixture weights being randomly drawn (from the joint distribution in this case) by the successive breaking of intervals for a point of positive mass. Figure 10 shows a graphical representation of the stick-breaking process.

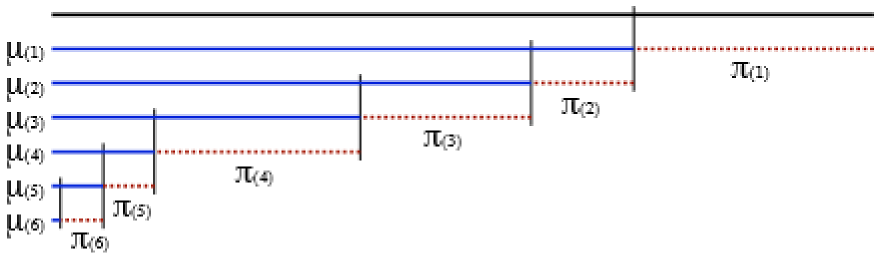

Figure 10: Stick-breaking process

Source: Ghahramani (2009)

From Figure 10, each weight is represented as π for every *jth* cluster mixture and the μ represents the corresponding mean values of the joint densities. This is shown by Equation 57 by Sethuram (1994):

$$\pi_j = w_j \prod_{i=1}^{j-1} (1 - w_j), \; w_j \sim \text{IID Beta}(1, k) \tag{57}$$

where the stick-breaking weights π_j is equal to the weights represented as w_j for every *jth* cluster mixture, where w_j is independent and identically distributed for a beta distribution with a mean of one and variance of k. The mean of one refers to the sum of cluster weights in order to represent the probability base distribution (Cai *et al.,* 2018). The precision parameter, also known as the concentration parameter, given as *k* determines the extent of clustering and variance, in relation to the base distribution (Karabatsos, 2016). As the concentration parameter increases, the spread of the positive cluster weights increases and there is no parameter mixing which leads to a continuous prior base distribution (Hennig, Meila, Murtagh and Rocci, 2016). On the other hand, if the concentration parameter decreases to zero, the spread decreases and the cluster weights concentrate at a single point. Therefore, a positive concentration parameter accompanied by a sufficient sample is desirable (Hennig *et al.,* 2016).

There are two noteworthy aspects of the DP method that need to be mentioned which are the mixture model and the hierarchical representation (Jensen and Maheu, 2018). The DP mixture model is able to analytically adapt and manage complex data such as the joint distribution of infinite mixture weights and parameters (Karabatsos, 2016). The hierarchical representation has the ability to represent a number of different distributions but most importantly, produces links across the distributions to induce a "shrinking" property (Camerlenghi, Lijoi, Orbanz and Prunster, 2019). This solves the many challenges associated with a nonparametric approach when analysing a large dataset which may affect the model fit, sensitivity and results (Chen and Yang, 2016).

Although price data has useful information and indicators, a large trading volume may contain some variables that are redundant (Nahil and Lyhyaoui, 2018). According to Chen and Yang (2016), there may be a high number of variables which may impact the model's density and thus affect its form. Moreover, this may induce model sensitivity and/or overfitting which is the fitting of a larger model than necessary to capture the data dynamics. Further, the problem of multicollinearity may arise which is when several independent variables become correlated to each other. In turn, this can affect model fit and results (Khan *et al.,* 2016; Chen and Yang, 2016). The application of the DP reduces the uncertainty attached to a high number of unknowns as well as asymmetric returns exposure; thus, ultimately improving the predictive ability of the model (Camerlenghi *et al.,* 2019).

3.6.3.3 Posterior

The posterior procedure consists of a number of steps. Initially, random samples are drawn from a joint distribution by means of a slice sampling technique by Kalli *et al.,* (2011). The slice sampler is applied to Equation 53, except an additional random variable represented by u_t is introduced, as shown in Equation 58 by Jensen and Maheu (2018):

$$\mathrm{p}(\mathrm{r_t}, \log(\mathrm{RV_t}), \mathrm{u_t}|\Omega, \Theta, \mathrm{I_{t-1}}) = \sum_{\mathrm{j}=1}^{\infty} \mathbf{1}\left(\mathrm{u_t} < \mathrm{w_j}\right) * \mathrm{f}\left(\mathrm{r_t}, \log(\mathrm{RV_t})|\theta_\mathrm{j}, \mathrm{I_{t-1}}\right) \qquad (58)$$

where the aim of adding this variable u_t is to ensure that only positive weights are retained and all weights of zero are "sliced away" (Karabatsos, 2016; Liu and Luger, 2018; Jensen and Maheu, 2018). Thereafter, the following iteration method by Jensen and Maheu (2018), is applied which refers to the repetitive resampling process of a collection of steps.

Firstly, a Gibbs sampling technique is applied which is often used when the joint distribution is unknown and it is simpler to draw samples from the known conditional distribution (Merel *et al.,* 2016). In this case, the conditional distribution contains the cluster mixture parameters and weights (Jensen and Maheu, 2018). Secondly, since the priors are strong, this allows for the formation of a conjugate conditional posterior, meaning, a conditional posterior that shares similar model properties to the prior (Gu *et al.,* 2019). Thirdly, consequently, each of the random variables tends to form a homogenous distribution provided the given weights and parametric space (Jensen and Maheu, 2018). Finally, if the cluster count is amended, there may be further prior draws (Merel *et al.,* 2016).

This procedure will continue; however, the Gibbs sampling process is subject to a burn-in period in which samples in the earlier stages that are no longer accurately representative of the required distribution are discarded (Merel *et al.,* 2016). The prior base distribution is then updated to the posterior distribution (Cai *et al.,* 2018). In this case, so are the coefficients and parameter estimates of Equation 55 and 56, from which conclusive results can be drawn with respect to the risk-return relationship and volatility feedback (Jensen and Maheu, 2018).

3.7 Summary of Bayesian Approach

Bayesian modelling is a popular and credible method, in a number of fields, mainly due to technological advancements by means of MCMC methods (Martino *et al.,* 2018). The modelling of the risk-return relationship and volatility feedback fits the Bayesian framework established by Ferson (2005). In turn, this confirms the choice and appropriateness of implementing Bayesian modelling, particularly the nonparametric Bayesian model, step by step to the topic on hand (Jensen and Maheu, 2018). The econometric model demonstrates the application of Bayes (1763) fundamental theorem which shows the link between excess returns, realised variance, the trade-off theory and volatility feedback. It highlights the effect of volatility feedback and how it has the ability to obscure any risk-return relationship (Jensen and Maheu, 2018). Due to the ability of the nonparametric Bayesian model to account for asymmetric returns exposure and asymmetric properties, the result of the risk-return relationship is more reliable (Jin, 2017). This is in line with improving risk estimation by Mandimika and Chinzara (2012) and Jensen and Maheu (2018).

3.8 Model Implementation

The methodology this study follows is reintroduced in order to briefly outline the method and strategy of model implementation.

3.8.1 Preliminary Tests and GARCH Approach

The conventional preliminary tests describing the data dynamics and GARCH approach draw on the methodology of a number of studies such as Mangani (2008), Mandimika and Chinzara (2012), Adu *et al.,* (2015), Khan *et al.,* (2016), Ilupeju (2016), Savva and Theodossiou (2018) and Liu (2019).

3.8.1.1 Software

The test results for the preliminary tests and the parameter estimates for the GARCH approach are determined by the R software.

3.8.1.2 Lag Order

The lag order for *p* and *q* are both set to one as this is considered appropriate to capture volatility dynamics based on information criteria. During model implementation, this choice was confirmed for all the GARCH type models.

3.8.1.3 Estimation Method

The parameters are estimated by the maximum likelihood (ML) method due to its ability to account for nonlinearity, in comparison to OLS, following the early and recent recommendation by Chou (1988) and Madaleno and Vieira (2018).

3.8.1.4 Innovation Distributions

Following the study by Mandimika and Chinzara (2012), all the GARCH (1, 1) type models are estimated for four probability distributions governing the innovations - standard normal (NORM), student-t (Std-t), skewed student-t (Skew-t) and the generalised error distribution (GED).

3.8.1.5 Interpretation of Results

For the preliminary tests and GARCH approach, the results are drawn from the *p*-value which is the conventional approach with respect to the R software and majority of studies reviewed studies such as Mangani (2008), Mandimika and Chinzara (2012), Adu *et al.,* (2015), Ilupeju (2016), Savva and Theodossiou (2018) and Liu (2019). If the *p*-value is less than the standard 5% level of significance, the null hypothesis can be rejected; thus, the alternative hypothesis is concluded. If the p-value is greater than the standard 5% level of significance, the null hypothesis is not rejected and the null hypothesis is concluded.

For the preliminary tests, this approach is in contrast to the EVIEWS software which provides in its output, the *p*-value, the calculated test statistic as well as the relevant critical values (Brooks, 2014). However, the EVIEWS software is considered to be limited in its functionality, whereas R is the leading software by econometricians due to the high level of flexibility it has to offer (Liashenko, Kravets and Krytsun, 2018).

According to Trapletti, Hornik and LeBarron (2019), the *p*-value in R is determined by the method of interpolation with respect to the relevant table of critical values. Interpolation refers to an estimation made within a set of data and in this case, the set of data refers to the relevant table of critical values. Hence, the conclusions of the *p*-values are drawn from the comparison between the test statistics and relevant critical values. Thus, whether the test statistic is compared with the relevant critical value or by the *p*-value, the same conclusion is reached. In other words, if the null hypothesis is rejected, based on the decision from the comparison of the test statistic and critical value at a five percent level of significance, the null hypothesis would be rejected, based on the *p*-value at a five percent level of significance (Trapletti *et al.,* 2019).

3.8.2 Bayesian Approach

The parametric and nonparametric Bayesian model follows the methodology by Jensen and Maheu (2018), in terms of the choice of the prior and posteriors implemented in this study. The Dirichlet prior process by Ferguson (1973), is derived by the stick-breaking process by Sethuram (1994). The posterior MCMC methods are based on the slice sampler by Kalli *et al.,* (2011) and Gibbs sampling technique.

3.8.2.1 Software

The test results of both the parametric and nonparametric Bayesian models are estimated by a specialised MATLAB software, '*Bayesian Regression: Nonparametric and Parametric Models*' designed by George Karabatsos. This menu-driven software is used mainly due to its computational ease and efficiency (Karabatsos, 2016).

3.8.2.2 Density Estimation

According to the study by Jensen and Maheu (2018), the results of the nonparametric Bayesian approach are presented by graphical plots of density estimation over a 90% credible interval. A density estimation refers to a probability estimation of an unknown distribution based on the given data (Damnjanovic and Reinschmidt, 2020). A credible or density interval is a Bayesian interval that accounts for conditional prior information, in contrast to a regular confidence interval (Karabatsos, 2016). This study performs density estimation by means of a probability density function (pdf) plot over a 95% credible interval (Karabatsos, 2016). The densities are plotted against the mean to essentially give the average form of the distribution (Karabatsos, 2016).

Graphically, the peak of the density represents the mean value since it is a measure of central tendency (Gulzar *et al.,* 2019). The magnitude and form of the density estimation are what is important and is further supported by the posterior parameter estimates (Karabatsos, 2016). These posterior parameter estimates are determined over a 75% and 95% credible interval in the form of numerical analysis (Karabatsos, 2016). With respect to the investigation of volatility feedback, a line graph is also used to make a comparative analysis with the actual and expected result drawn from the study by Jensen and Maheu (2018) and Harris *et al.,* (2019).

3.8.2.3 Interpretation of Results

For both the parametric and nonparametric Bayesian models, the posterior point estimates of the parameters are analysed over a 75% and 95% credible interval. Additionally, their robustness is determined by the Monte Carlo mixing value.

According to Karabatsos (2016), for a 75% credible interval, if zero is excluded then the parameter estimate is statistically significant. If zero is included then the parameter estimate is statistically insignificant (Karabatsos, 2016).

According to Jensen and Maheu (2018), for a 95% credible interval, if the point estimate lies within the interval, it is concluded as statistically significant. The negative or positive signs indicate the direction in which the variable moves. If the point estimate does not lie within the interval, it is concluded as statistically insignificant (Jensen and Maheu, 2018).

According to Karabatsos (2016), the Monte Carlo (MC) mixing value refers to how well all the parameters mix where the value lies between an interval of zero and one. A small value of 0.1 or 0.01 confirms adequate mixing, whereas an approximate value of 0.5 indicates optimal mixing. If the MC mixing value is inadequate, this indicates the number of iterations, that is the number of repetitive sampling, is to be increased (Karabatsos, 2016).

3.8.2.4 Model Specifications

Although not within the scope of this study, both the parametric and nonparametric Bayesian models require a number of prior and posterior specifications in order for model implementation (Karabatsos, 2016). The specification details of the parametric and nonparametric Bayesian models are briefly listed with their respective references. Table 2 shows the prior and posterior model specifications for model implementation.

Table 2: Model specifications

Prior specifications			
Model	Specification	Value	Author
Parametric Bayesian	Prior variance of the slope parameters	1000	Karabatsos (2016)
	Prior gamma distribution of the error variance	$\frac{0.001}{2}$	Karabatsos (2016)
Nonparametric Bayesian	Prior gamma distribution of the error variance	$\frac{5}{2}$	Jensen and Maheu (2018)
	Intercept variance of the base distribution for the Dirichlet Process	5	Karabatsos (2016)
	Prior gamma distribution, shape and rate, of the concentration parameter	1	Karabatsos (2016)
Posterior specifications			
Parametric & Nonparametric Bayesian	MCMC	20 000	Jensen and Maheu (2018)
	Burn-in	5 000	Jensen and Maheu (2018)
	Thin	5	Karabatsos (2016)

From Table 2, values were modified in accordance with the procedure of Jensen and Maheu (2018), in combination with model optimisation tests. Specifically, for the parametric Bayesian model, the choice of the selected values was considered optimal due to resulting in a zero error variance. The error variance refers to the unexplained variance that arises from sources such as uncertainty and measurement errors of which a Bayesian model can automatically adjust and take into account (Chakraborty and Lozano, 2019).

For both the parametric and nonparametric Bayesian models, with respect to the posterior MCMC methods, slice sampling technique and Gibbs sampler are employed.

First, the slice sampler ensures that only positive weights are retained and all weights of zero are “sliced away” (Jensen and Maheu, 2018). Then, the Gibbs sampler ensures the property of conjugacy, that is the sharing of properties between the prior and posterior, for a more robust posterior (Gu *et al.,* 2019).

The posterior estimates are determined by following Jensen and Maheu (2018), 20 000 MCMC sampling iterations and a burn-in period of 5000. The MCMC sampling iterations refers to the repetitive resampling process used to determine the posterior parameter estimates of the parametric and nonparametric Bayesian model (Waldmann, 2018). The burn-in period refers to samples in the earlier stages that are discarded when no longer accurately representative of the required distribution (Merel *et al.,* 2016). Additionally, a thin number of 5 is used which is based on the default value by Karabatsos (2016). In this case, it means that every fifth sampling iterate of the total 20 000 MCMC sampling iterations is collected or retained to determine the posterior parameter estimates (Agilan and Umamahesh, 2017).

3.9 Chapter Summary

The GARCH approach can contribute as a preliminary test, particularly to the basic pricing of risk which refers to whether return has a relation to risk. However, not on the actual basis of estimating the risk-return relationship (Jensen and Maheu, 2018). The nonparametric Bayesian model is a combination of the two most recommended methods found in existing literature, respectively for robust parameter estimation by Kang (2014), Chang *et al.,* (2017), Demirer *et al.,* (2019) and Waldmann (2018). The nonparametric framework has the ability to effectively account for every possible risk-return relationship, allowing for an infinite number of possibilities according to Demirer *et al.,* (2019). Consequently, this allows for higher moment asymmetric properties which cannot be effectively accounted for by parametric models. Further, it has the ability to adjust for model misspecifications (Apergis *et al.,* 2018; Demirer *et al.,* 2019).

Similarly, the Bayesian method has the ability to adjust to uncertainty and measurement errors surrounding parameters (Walmann, 2018; Chakraborty and Lozano, 2019). Whereas in contrast, the GARCH approach caters more for the confirmation of the presence of market characteristics such as volatility clustering, heavy tails and pricing of risk. The nonparametric Bayesian methods (Bayesian Dirichlet Process, slice sampler and Gibbs sampler) fundamentally involves random sampling methods whereby every distribution has an equal chance of being drawn (Alvi, 2016). These methods suggest low levels of bias and systematic error, and a high level of reliability, validity and viability (Etikan and Bala, 2017). Thus, ensuring robust results, the best possible estimation of parameters and drawing of conclusions (Alvi, 2016).

CHAPTER 4

4. Empirical Results and Discussion

The analysis for the graphical and numerical test results are presented in this section which consists of three parts. First, the basic data analysis of the JSE ALSI data is shown then the nature and properties of the ALSI returns are described by the preliminary tests. Second, the volatility dynamics and pricing of risk in the South African market is determined by the GARCH type models. Third, is the test results of volatility feedback and the risk-return relationship from the parametric Bayesian model followed by the nonparametric Bayesian model. Finally, the chapter concludes the main findings and implications of the results.

4.1 Data Dynamics

The dataset used in this study is recapped and then tested for stationarity which forms the basis of time series analysis for further investigation. The ALSI returns are then described by basic descriptive statistics to provide an overview of its distribution in the South African market. Thereafter, the nature of the ALSI returns is investigated by means of normality, autocorrelation and heteroskedasticity tests. These tests are undertaken in order to motivate the use of nonlinear models, the GARCH approach, for further analysis.

4.1.1 Stationarity

Stationarity is investigated in order to determine if the ALSI returns form a valid time series for further analysis to substantiate the use of the GARCH approach.

4.1.1.1 Index Price Data

The daily closing prices of the ALSI index are obtained from IRESS for the sample 15 October 2009 to 15 October 2019, a period of ten years. The stationarity and nature of the ALSI price data are investigated by the analysis of a time series plot and stationarity tests. Figure 11 shows a time series plot of the ALSI price data.

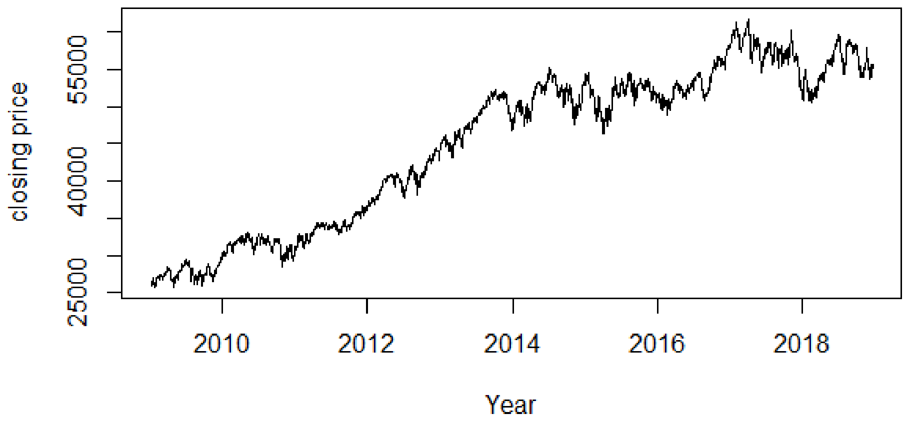

Figure 11: Time series plot of the ALSI closing prices

From Figure 11, the time series plot of the ALSI price data shows an upward trend over time and nonstationarity due to the inconstant mean. The ALSI price data shows an asymmetric and nonlinear pattern; however, the market characteristic volatility clustering is not shown in Figure 11. This refers to the tendency of volatility to appear in clusters where high returns follow high returns and low returns follow low returns (Yu *et al.,* 2018). To support the presence of nonstationarity, the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), Kwiatkowski, Phillips, Schmidt and Shin (KPSS) tests are employed. Table 3 shows the results for the stationarity tests.

Table 3: Stationarity tests for the ALSI price data

Test	**Test Statistics**	***p*-value**
ADF	-2.151	0.514
PP	-16.433	0.193
KPSS	25.949	0.010

From Table 3, since the *p*-values of both the ADF and PP tests are greater than 0.05, the null hypothesis that the ALSI price data has a unit root is not rejected at a 5% level of significance. Since the *p*-value for the KPSS test is less than 0.05, the null hypothesis that the ALSI price data is stationary is rejected at a 5% level of significance. Thus, the ADF, PP and KPSS tests confirm that the ALSI price data are nonstationary. Hence, from the time series plot and stationarity tests, it can be concluded that the ALSI price data has an asymmetric nature and is nonstationary.

4.1.1.2 Index Returns

The closing ALSI price data is then transformed into market returns by taking the natural log form of the difference between the current and previous closing prices. The market returns are then subtracted by the T-bill risk-free rate obtained from the SARB to compute the ALSI excess returns (ALSI returns). These steps are in line with the studies by Jensen and Maheu (2018), Demirer *et al.,* (2019) and Kim and Kim (2018). Figure 12 shows the time series plot of the ALSI returns.

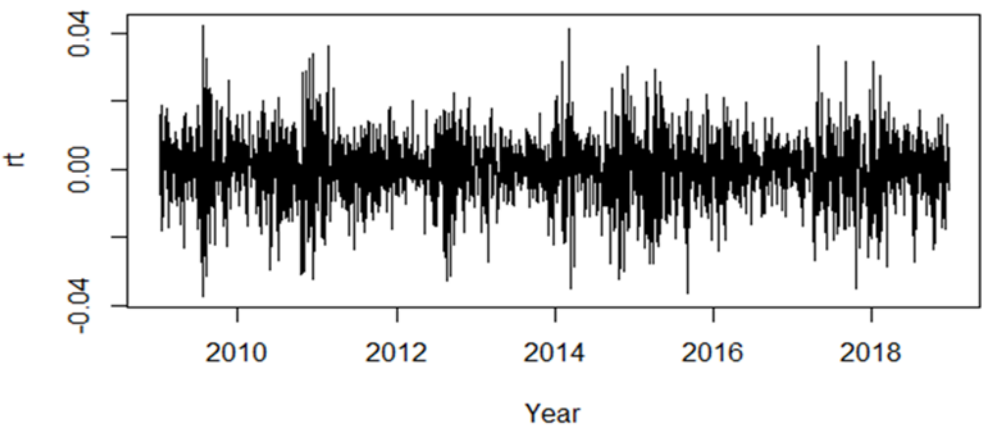

Figure 12: Time series plot of the ALSI returns

Figure 12 shows a constant mean where there is no distinctive trending behaviour such as upward as in the case of Figure 11. However, since the movements are inconstant and nonlinear, this suggests the presence of heteroskedasticity. Heteroskedasticity refers to the inconstant nature of variance over time (Hung, 2019). Additionally, volatility clustering is now noted, where high returns follow high returns and low returns follow low returns. In particular, volatility clustering is shown just after 2010 and before 2015 and 2018, where there are relatively higher than lower price movements. To support the presence of stationarity, the Augmented Dickey-Fuller

(ADF), Phillips-Perron (PP), Kwiatkowski, Phillips, Schmidt and Shin (KPSS) tests are employed. Table 4 shows the results for the stationarity tests.

Table 4: Stationarity tests for the ALSI returns

Test	Test Statistics	*p*-value
ADF	-14.561	0.010
PP	-50.775	0.010
KPSS	0.140	0.100

From Table 4, since the *p*-values of both the ADF and PP tests are less than 0.05, the null hypothesis that the ALSI returns have a unit root is rejected at a 5% level of significance. Since the *p*-value for the KPSS test is greater than 0.05, the null hypothesis that ALSI returns are stationary is not rejected at a 5% level of significance. Thus, the ADF, PP and KPSS tests confirm that the time series of the ALSI returns are stationary. Hence, from the time series plot and stationarity tests, it can be concluded that the ALSI returns are valid in forming a time series for further analysis.

4.1.2 Descriptive Statistics

The distribution of the ALSI returns in the South African market is investigated by basic descriptive statistics. Table 5 shows the basic descriptive statistics of the ALSI returns.

Table 5: Descriptive statistics of the ALSI returns

Number of observations	2498
Minimum	-0.037
Maximum	0.042
Mean	0.000
Standard deviation	0.010
Skewness	-0.167
Excess kurtosis	1.277

From Table 5, there are a total of 2498 observations from a range of -0.037 to 0.042 and has an average value of 0.000. The average indicates a corresponding rate of return according to Gyldberg and Bark (2019). In this case, there is no profitable trading strategy for an investor in the South African market. The standard deviation of 0.010 is relatively low, in comparison to the studies that reported a standard deviation value for standard returns. For example, Mandimika and Chinzara (2012) reported a value of 0.568, whereas Adu *et al.,* (2015) reported a value of 1.253. This is unexpected since ALSI excess returns are supposed to reflect a riskier value as a result of a higher level of risk undertaken. The negative skew value of -0.167 describes a distribution that tends to the left. This indicates losses for investors since the left tail is associated to a negative payoff according to Yao *et al.,* (2019). Excess kurtosis of 1.277 is a positive value which means that the ALSI returns follow a leptokurtic distribution which is a peaked distribution that has heavy tails. This finding is in line with the return characteristics of an emerging market by Herbert *et al.,* (2018) and Adu *et al.,* (2015). Thus, the results from the basic descriptive statistics indicate that the ALSI returns follow an asymmetric distribution.

4.1.3 Normality

The normality of ALSI returns is investigated by the analysis of a normal Q-Q plot and normality tests. Figure 13 shows a Q-Q plot for the ALSI returns.

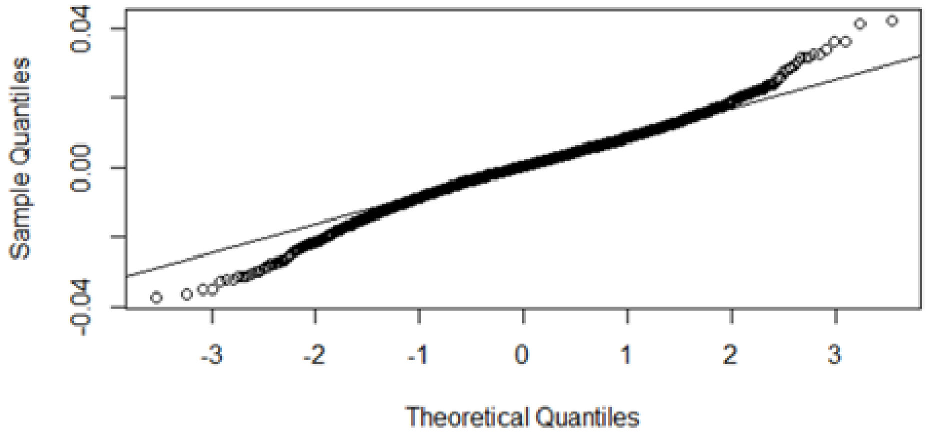

Figure 13: Q-Q plot for ALSI returns

From Figure 13, the deviation from the 45-degree line of reference indicates a mismatch between the empirical and theoretical distribution, indicating nonnormality. To support the presence of nonnormality, the Shapiro-Wilk (SW), Jarque-Bera (JB) and Anderson-Darling (AD) tests are employed. Table 6 shows the results for the normality tests.

Table 6: Normality tests for the ALSI returns

Test	**Test Statistics**	***p*-value**
SW	0.986	< 0.0001
JB	182.260	< 0.0001
AD	8.006	< 0.0001

From Table 6, since the *p*-values of the SW, JB and AD tests are all less than 0.05, the null hypothesis that ALSI returns are normally distributed is rejected at a 5% level of significance. Thus, from the basic descriptive statistics, the Q-Q plot and normality tests, ALSI returns follow a nonnormal and asymmetric distribution. This finding is in contrast to the theory of Li (2018), where financial price data assumes a symmetric and normal distribution. However, it is in line with the concept of asymmetric returns exposure which refers to the fundamental nature of returns which is asymmetric and nonnormal. It is further supported by Herbert *et al.,* (2018) and Maneemaroj *et al.,* (2019), who state that the return distribution is asymmetric and heavy tailed.

4.1.4 Autocorrelation

The autocorrelation or serial correlation of ALSI returns is investigated by the analysis of an ACF plot and autocorrelation tests. Figure 14 shows an ACF plot of ALSI returns.

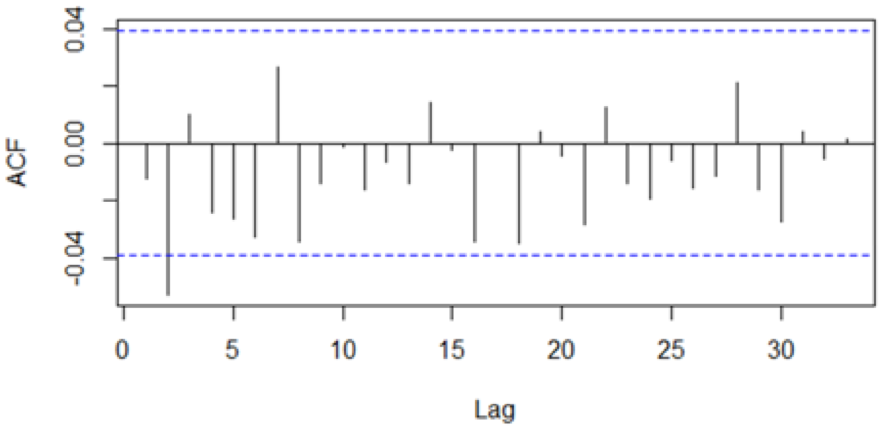

Figure 14: ACF plot of the ALSI returns

From Figure 14, since the majority of lags do not touch or pass over the 95% conditional interval, this indicates the lags are insignificant. This means that autocorrelation within the ALSI returns is absent. To support the absence of autocorrelation, the Ljung-Box (LB) and Durbin Watson (DW) tests are employed. Table 7 shows the results for the autocorrelation tests.

Table 7: Autocorrelation tests for the ALSI returns

Test	**Test Statistics**	***p*-value**
LB	26.463	0.151
DW	2.024	0.727

From Table 7, since the *p*-values of both the LB and DW tests are greater than 0.05, the null hypothesis that autocorrelation is absent within the ASLI return series is not rejected at a 5% level of significance. According to Khan *et al.,* (2016), the absence of autocorrelation means that the current ALSI returns does not impact future returns.

4.1.5 Heteroskedasticity

Heteroskedasticity of the ALSI returns is investigated by the analysis of an ACF plot and heteroskedasticity tests. Figure 15 shows an ACF plot of the ALSI returns squared.

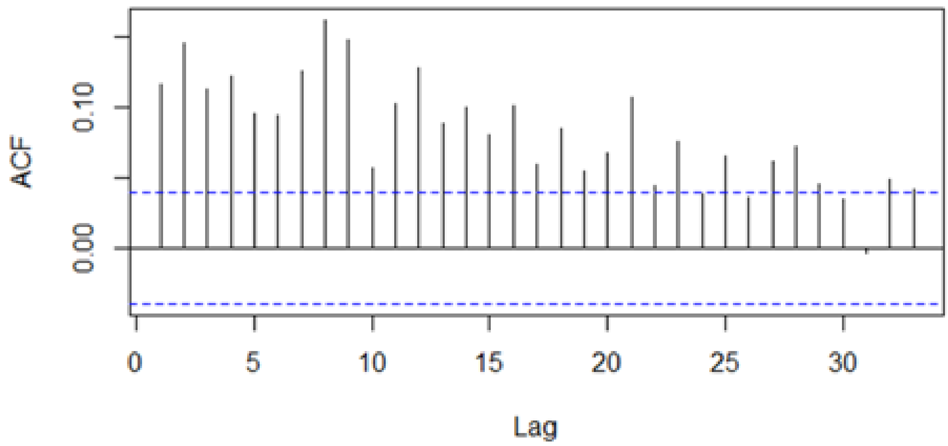

Figure 15: ACF plot of the ALSI returns squared

From Figure 15, since the majority of lags touch or pass over the 95% conditional interval, this indicates the lags are significant. This means that autocorrelation within the ALSI returns is present. To investigate the presence of heteroskedasticity, the Ljung-Box (LB^2) and Autoregressive Conditional Heteroskedastic Lagrange Multiplier (ARCH-LM) tests are employed. Table 8 shows the results for the heteroskedasticity tests.

Table 8: Heteroskedasticity tests for the ALSI returns squared

Test	**Test Statistics**	***p*-value**
LB^2	567.920	< 0.0001
ARCH-LM	213.370	< 0.0001

From Table 8, since the *p*-values of both the LB^2 and ARCH-LM tests are less than 0.05, the null hypothesis that the ARCH effect is absent within the ALSI return series is rejected at a 5% level of significance. Thus, it can be concluded that heteroskedasticity is present within the ALSI returns series.

4.1.6 Summary of the Data Dynamics

From the preliminary tests based on the data dynamics, the ALSI closing price data is found to be nonstationary over time and asymmetric in nature. Hence, the ALSI price data is then transformed to returns and is confirmed to be stationary in order to form a valid time series for further analysis. From the basic descriptive statistics and normality tests, the ALSI returns follow an asymmetric and nonnormal distribution. The presence of heteroskedasticity, inconstant variance, confirms the volatile nature of the data since variance is a measure of volatility. The nonnormal, asymmetric and volatile nature of the ALSI data is in line with the return inherent risk, asymmetric returns exposure. Since the data shows the market characteristic volatility clustering, the clustering nature of volatility, this motivates the employment of the GARCH approach for further investigation by Khan *et al.,* (2016).

4.2 GARCH Approach

The GARCH approach contains a number of GARCH (1, 1) type models that serve as preliminary tests to confirm the existence of market return characteristics in the South African market. This includes volatility clustering, heavy tails, asymmetric effects and the pricing of risk which refers to whether return has a relation to risk.

The GARCH type model parameter estimates are determined by the maximum likelihood (ML) method for the four innovation distributions, standard normal (NORM), student-t (Std-t), skewed student-t (Skew-t) and the generalised error distribution (GED). The results are presented and then discussed in summary.

4.2.1 Volatility Dynamics

The volatility dynamics of the South African market is first investigated by a simple GARCH (1, 1) model then by the asymmetric GARCH type models.

4.2.1.1 Symmetric GARCH Model Test Results

Table 9 shows the significant ML parameter estimates of GARCH (1, 1) with different probability distributions governing the innovations.

Table 9: ML parameter estimates of GARCH (1, 1)

Parameter Estimates	**NORM**	**Std-t**	**Skew-t**	**GED**
$\hat{\mu}$	-	0.021 **	-	0.000 ***
$\hat{\alpha}_1$	0.057 ***	0.059 ***	0.060 ***	0.059 ***
$\hat{\beta}_1$	0.942 ***	0.940 ***	0.939 ***	0.940 ***
$\hat{\alpha}_1 + \hat{\beta}_1$	0,999	0,999	0,999	0,999
AIC	-6.557	-6.575	-6.583	-6.573
BIC	-6.552	-6.565	-6.574	-6.564

*NOTE: *, **, *** means the p-value is significant at a 10%, 5% and 1% level of significance, respectively*

The parameter estimates of the GARCH (1, 1) model are discussed in summary with the rest of the results under discussion with respect to volatility dynamics. To investigate whether the GARCH (1, 1) model has adequately captured the ALSI returns, the sign and size bias tests are employed as recommended by Park *et al.,* (2017). Table 10 shows the results of the sign and size bias tests.

Table 10: Sign and size bias tests of GARCH (1, 1)

Innovation Distributions	**Sign Bias**	**Negative**	**Positive**	**Joint Effect**
NORM	1.243 ***	1.066 ***	2.793 ***	34.524 ***
Std-t	0.985 ***	1.062 ***	2.932 ***	32.760 ***
Skew-t	1.276 ***	0.931 ***	2.867 ***	34.694 ***
GED	1.006 ***	1.069 ***	2.909 ***	32.841 ***

*NOTE: *, **, *** means the p-value is significant at a 10%, 5% and 1% level of significance, respectively*

From Table 10, for the sign bias test, since the *p*-values are less than 5%, the null hypothesis of the sign bias being absent is rejected at a 5% level of significance. Thus, it can be concluded that positive and negative shocks have a different impact on future volatility. For the negative sign bias test, since the *p*-values are less than 5%, the null hypothesis of the negative sign bias being absent is rejected at a 5% level of significance. Similarly, for the positive sign bias test, since the *p*-values are less than 5%, the null hypothesis of the positive sign bias being absent is also rejected at a 5% level of significance. Thus, from both the positive and negative sign bias tests, it can be concluded that the magnitude of the positive and negative shocks affects volatility asymmetrically. This is in line with the phenomenon of asymmetric volatility, which has been systematically proven over time, according to Yu *et al.,* (2018).

For the joint effect test, since the *p*-values are less than 5%, the null hypothesis of asymmetry being absent is rejected at a 5% level of significance. Thus, it can be concluded that asymmetry is present and has not been adequately captured by the GARCH (1, 1) model. This finding is consistent with studies by Mangani (2008), Mandimika and Chinzara (2012), Ilupeju (2016) and Park *et al.,* (2017). In order to capture further characteristics of the ALSI returns, such as asymmetric effects, there have been a number of extensions of the GARCH (1, 1) model. This includes the GJR-GARCH (1, 1), EGARCH (1, 1) and APARCH (1, 1) models as recommended by Savva and Theodossiou (2018).

4.2.1.2 Summary of GARCH (1, 1)

From the sign and size bias tests, it can be concluded that there exists an asymmetric impact on volatility where positive shocks affect volatility more than negative shocks of the same magnitude. The GARCH (1, 1) model does not adequately capture the asymmetric effects in the ALSI returns since the joint effect confirms the presence of asymmetry. As a result, this motivates further testing by the use of asymmetric GARCH type models such as the GJR-GARCH (1, 1), EGARCH (1, 1) and APARCH (1, 1). These models are more credible in capturing the emerging markets return characteristics and follow a number of studies such as Adu *et al.,* (2015), Khan *et al.,* (2016), Savva and Theodossiou (2018) and Maneemaroj *et al.,* (2019).

4.2.2 Asymmetric Effects

The asymmetric GARCH type models have model specifications to capture further market characteristics such as asymmetric volatility and the leverage effect. The results are presented and then discussed in summary.

4.2.2.1 Asymmetric GARCH Models Test Results

Table 11 to 14 shows the results for the asymmetric GARCH (1, 1) type models governing the four innovation distributions. The results are discussed in summary thereafter.

Table 11: ML parameter estimates of the asymmetric GARCH (1, 1) type models for the NORM probability distribution governing the innovations

Parameter Estimates	**GJR-GARCH (1, 1)**	**EGARCH (1, 1)**	**APARCH (1, 1)**
$\hat{\alpha}_0$	-	-0.234 ***	-
$\hat{\alpha}_1$	0.038 ***	-0.127 ***	0.060 ***
$\hat{\beta}_1$	0.951 ***	0.975 ***	0.951 ***
$\hat{\gamma}$	0.517 ***	0.0780 ***	0.622 ***
$\hat{\delta}$	2	-	1
$\hat{\alpha}_1 + \hat{\beta}_1$	0.989	0.848	1.011
AIC	-6.581	-6.618	-6.576
BIC	-6.574	-6.608	-6.569

*NOTE: *, **, *** means the p-value is significant at a 10%, 5% and 1% level of significance, respectively*

Table 12: ML parameter estimates of the asymmetric GARCH (1, 1) type models for the Std-t probability distribution governing the innovations

Parameter Estimates	**GJR-GARCH (1, 1)**	**EGARCH (1, 1)**	**APARCH (1, 1)**
$\hat{\alpha}_0$	-	-0.227 ***	0.000 ***
$\hat{\alpha}_1$	0.0348 ***	-0.131 ***	0.064 ***
$\hat{\beta}_1$	0.952 ***	0.976 ***	0.926 ***
$\hat{\gamma}$	0.603 ***	0.077 ***	1.000 ***
$\hat{\delta}$	2	-	1
$\hat{\alpha}_1 + \hat{\beta}_1$	0.986	0.845	0.990
AIC	-6.595	-6.624	-6.621
BIC	-6.586	-6.613	-6.610

*NOTE: *, **, *** means the p-value is significant at a 10%, 5% and 1% level of significance, respectively*

Table 13: ML parameter estimates of the asymmetric GARCH (1, 1) type models for the Skew-t probability distribution governing the innovations

Parameter Estimates	**GJR-GARCH (1, 1)**	**EGARCH (1, 1)**	**APARCH (1, 1)**
$\hat{\alpha}_0$	-	-0.219 ***	0.000 ***
$\hat{\alpha}_1$	0.038 ***	-0.132 ***	0.064 ***
$\hat{\beta}_1$	0.950 ***	0.977 ***	0.927 ***
$\hat{\gamma}$	0.482 ***	0.074***	1.000 ***
$\hat{\delta}$	2	-	1
$\hat{\alpha}_1 + \hat{\beta}_1$	0.988	0.845	0.991
AIC	-6.602	-6.637	-6.634
BIC	-6.590	-6.623	-6.620

*NOTE: *, **, *** means the p-value significant at a 10%, 5% and 1% level of significance, respectively*

Table 14: ML parameter estimates of the asymmetric GARCH (1, 1) type models for the GED probability distribution governing the innovations

Parameter Estimates	GJR-GARCH (1, 1)	EGARCH (1, 1)	APARCH (1, 1)
$\hat{\alpha}_0$	-	-0.235 ***	0.000 ***
$\hat{\alpha}_1$	0.037 ***	-0.129 ***	0.063 ***
$\hat{\beta}_1$	0.951 ***	0.975 ***	0.926 ***
$\hat{\gamma}$	0.546 ***	0.079 ***	1.000 ***
$\hat{\delta}$	2	-	1
$\hat{\alpha}_1 + \hat{\beta}_1$	0.988	0.846	0.989
AIC	-6.592	-6.623	-6.620
BIC	-6.583	-6.611	-6.608

*NOTE: *, **, *** means the p-value is significant at a 10%, 5% and 1% level of significance, respectively*

4.2.2.2 Discussion of GARCH Test Results

The ARCH effect $\hat{\alpha}_1$ represents the persistence of risk over a short period of time and the GARCH effect $\hat{\beta}_1$ over the long-term (Khan *et al.,* 2016). The ARCH and GARCH effects are statistically significant for all the GARCH (1, 1) type models – GARCH (1, 1), GJR-GARCH (1, 1), EGARCH (1, 1) and APARCH (1, 1) – at the conventional 5% level of significance. However, it is noteworthy that volatility is mainly driven by the GARCH effect which dominates the ARCH effect for all the models. This means that long-term volatility is more persistent in the South African market than on a short-term basis. The volatility can be further described by the sum of the ARCH and GARCH effects ($\hat{\alpha}_1 + \hat{\beta}_1$). All the GARCH type models have a sum of the ARCH and GARCH effects being close but less than one, with the exception of the standard APARCH (1, 1) model. The former finding is twofold: Firstly, the assumption of stationarity holds and secondly, the volatility present in the South African market can be further described.

According to Khan *et al.,* (2016), if the assumption of stationarity holds, this implies that the long run mean variance converges to unconditional variance. If the unconditional variance for the innovation terms is less than one, as in this case for the majority of the results, then the innovation terms are undefined and nonstationary. Consequently, the GARCH (1, 1) models can be seen as a regular linear ARMA model, which is essentially a simpler linear parametric model, according to Brooks (2014) and Jin (2017). Thus, limited in its ability to fully capture asymmetric properties. Secondly, volatility is present and persistent over a short and long period of time. For the majority of the GARCH (1, 1) models, the sum of the ARCH and GARCH effects is equal to 0.9 which is close to one, confirming the presence and persistence of long-term volatility.

The only exception is for the APARCH (1, 1) model for a normal innovation distribution where the sum is greater than one. In this case, the forecast of unconditional variance increases to infinity as the number of observations increases according to Khan *et al.,* (2018). The ARMA, standard GARCH and the asymmetric GARCH type models which are essentially parametric models has a set number of parameters with respect to the finite sample size. Consequently, the models design cannot make an appropriate forecast of an infinite number of observations, in line with Brooks (2014) and Jin (2017). On the other hand, for a nonparametric Bayesian approach, the conditional variance can be reduced by a prior process into a finite and a quantifiable value (Karabatsos, 2016). Hence, there is a greater ease in updating a probability estimation as additional data becomes available (Cai, 2018).

Thus, the parameter estimates from the above analysis confirm the presence of high and persistent levels of volatility mainly in the long run, volatility clustering and heavy tails in the ASLI returns. This finding is in line with previous studies on the South African market by Mangani (2008), Mandimika and Chinzara (2012), Adu *et al.,* (2015), Ilupeju (2016), Bekiros *et al.,* (2017) and Jin (2017). Additionally, the high persistence of volatility, where shocks are less likely to die out over time, suggests the presence of volatility feedback in the South African market by Harris *et al.,* (2019). Specifically, for the asymmetric GARCH (1, 1) type models - GJR-GARCH (1, 1), EGARCH (1, 1) and APARCH (1, 1) – the asymmetry parameter $\hat{\gamma}$ is of main interest. The asymmetry parameter captures asymmetric effects according to Khan *et al.,* (2016).

For all the asymmetric GARCH (1, 1) type models, $\hat{\gamma}$ is statistically significant at a 5% level of significance and positive. This finding is twofold: Firstly, for a positive volatility shock, volatility decreases and results in an increase in prices, indicating the leverage effect by Adu *et al.,* (2015). The presence of the leverage effect is in contrast to the BRICS study by Adu *et al.,* (2015), who found an insignificant asymmetry parameter and consequent absent leverage effect for the South African market. However, the evidence of the leverage effect in the South African market is in line with the studies by Mangani (2008), Mandimika and Chinzara (2012), Ilupeju (2016) and Jin (2017). Similarly, the leverage effect is present in the emerging Finnish market by Sultan (2018) as well as in the US market by Harris *et al.,* (2019).

Secondly, due to the presence of asymmetric volatility, this suggests that the type of volatility shocks impacts volatility differently. That is, positive and negative shocks have an asymmetric effect on volatility. In contrast, to the BRICS study by Adu *et al.,* (2015), who found an insignificant asymmetry parameter and absence of asymmetric volatility for the South African market. However, evidence of asymmetric volatility is in line with the South African studies by Mangani (2008), Mandimika and Chinzara (2012) and Ilupeju (2016). For the GJR-GARCH (1, 1) and EGARCH (1, 1) models, positive volatility shocks have a greater impact on volatility than negative shocks of the same magnitude in the South African market. However, in direct contrast for the APARCH model, the negative volatility shocks have a greater impact than positive. In order to settle this inconsistency, the best fitting GARCH model is determined by model testing.

Taking into account all the GARCH models, the best fitting GARCH model and innovation distribution is selected by information criteria following Mandimika and Chinzara (2012). The model testing is performed by the analysis of AIC and BIC where the minimum values indicate the best fitting model. From the above analysis, the best fitting GARCH type model is EGARCH (1, 1) followed by APARCH (1, 1) and then GJR-GARCH (1, 1) model. The result of the EGARCH (1, 1) being foremost is in contrast to previous South African studies. This includes the standard GARCH (1, 1) by Mangani (2008), GJR-GARCH (1, 1) by Mandimika and Chinzrara (2012) and APARCH by Ilupeju (2016). Given this finding, the result regarding which type of volatility shocks have a greater impact is not statistically sound. This is because the two foremost models, EGARCH (1, 1) and APARCH (1, 1), give contradictory results to one another. A possible reason for this is that a certain level of risk remains uncaptured in the innovations according to Feng and Shi (2017).

From information criteria for the best fitting innovation distribution, first is the skewed student-t (Skew-t) followed by student-t (Std-t), the generalised error distribution (GED) and lastly the normal innovation distribution (NORM). Since the normal innovation distribution is the least good fit, this confirms the inadequacy of fitting a normal innovation distribution to volatile financial data. The fitting of a normal or linear model to nonlinear and asymmetric data is not adequate. This is because it is not essentially designed to capture the higher moment properties, in line with Jin (2017) and Jensen and Maheu (2018). To clarify, according to Karabatsos (2016), parametric methods are often based on a number of assumptions. In this case, the innovations are assumed to follow a normal distribution.

If the data properties are in violation of such assumptions then the parameter estimates are going to be misleading (Karabatsos, 2016). In the context of time series analysis, this violation of assumptions is most likely to occur because financial data has a volatile, asymmetric and nonlinear nature (Jensen and Maheu, 2018). As a result, the use of the NORM innovation distribution is unreliable in the estimation of risk and contributes to inconclusive results regarding the risk-return relationship, in line with Jensen and Maheu (2018). Thus, the model adequacy of the selected best fitting asymmetric GARCH model, the standard EGARCH (1, 1) models standardised innovation terms are assessed following Mangani (2008).

4.2.3 EGARCH (1, 1) standardised innovation terms

The standardised innovations of the EGARCH (1, 1) model are investigated by normality, autocorrelation and randomness tests to establish if they have adequately captured risk.

4.2.3.1 Descriptive Statistics

The distribution of the innovations of the EGARCH (1, 1) model is investigated by basic descriptive statistics. Table 15 shows the basic descriptive statistics of the innovations.

Table 15: Descriptive statistics of the innovations

Minimum	-3.976
Maximum	3.987
Median	-0.047
Mean	0.003
Standard deviation	1.001
Skewness	0.310
Excess kurtosis	0.505

From Table 15, the standard deviation of 1.001 is relatively high which reflects the high level of risk present in the innovations. The positive skew value of 0.310 describes a distribution that tends to the right which is not symmetrically bell-shaped. Excess kurtosis is 0.505 which is positive means that the standardised innovations follow a leptokurtic distribution which refers to a heavy tailed distribution. Thus, from the standard deviation, skewness and kurtosis values, it can be concluded that the innovations follow a distribution that is heavy tailed and asymmetric in nature. This means that the innovations still contain uncaptured risk as suggested by Feng and Shi (2017) and Mangani (2008).

4.2.3.2 Normality

The normality of the innovations is investigated by a Q-Q plot and normality tests. Figure 16 shows a Q-Q plot for the innovations.

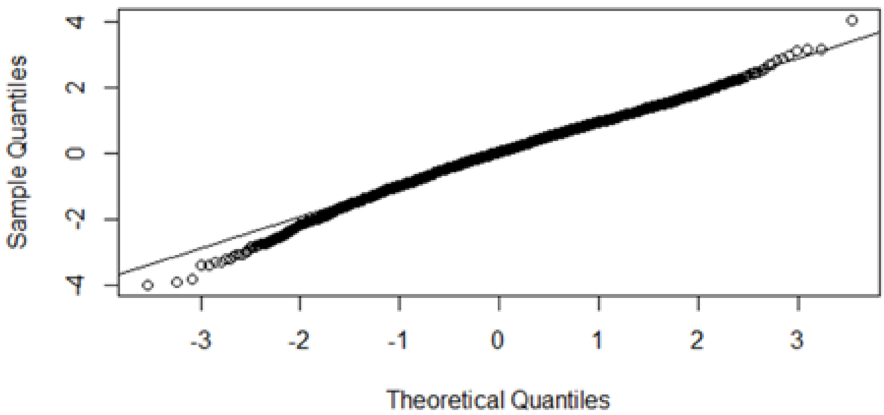

Figure 16: Q-Q plot of innovations

From Figure 16, the deviation from the 45-degree line of reference indicates a mismatch between the empirical and theoretical distribution, indicating nonnormality. To support the presence of nonnormality, the Shapiro-Wilk (SW), Jarque-Bera (JB) and Anderson-Darling (AD) tests are employed. Table 16 shows the results of the normality tests.

Table 16: Normality tests for the innovations

Test	**Test Statistics**	***p*-value**
SW	0.993	< 0,0001
JB	66.782	< 0,0001
AD	4.118	< 0,0001

From Table 16, since the *p*-values of the SW, JB and AD tests are all less than 0.05, the null hypothesis that the innovations are normally distributed is rejected at a 5% level of significance. Thus, the results of the basic descriptive statistics, Q-Q plot and normality tests confirm that the innovations follow an asymmetric and nonnormal distribution.

4.2.3.3 Autocorrelation

The autocorrelation of the innovations is investigated by the analysis of an ACF plot and autocorrelation tests. Figure 17 shows an ACF plot for the innovations.

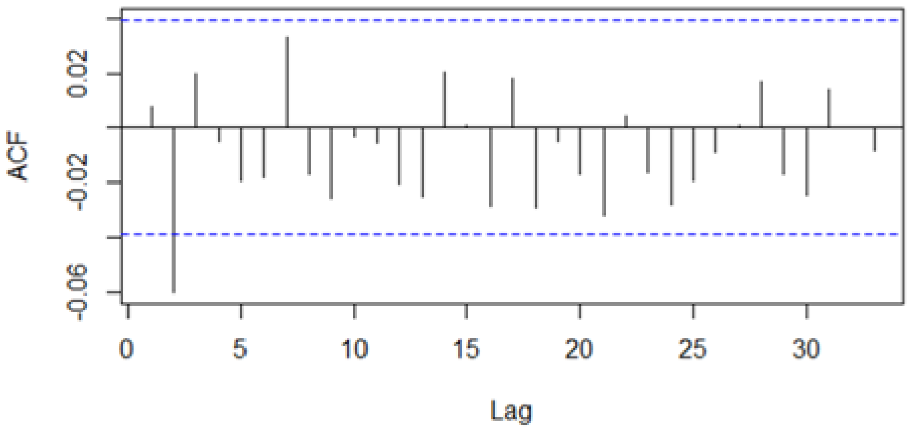

Figure 17: ACF plot for the innovations

From Figure 17, since the majority of lags do not touch or pass over the 95% conditional interval, this indicates the lags are insignificant. This means that autocorrelation within the innovations is absent. To support the absence of autocorrelation, the Ljung-Box (LB) and Durbin Watson (DW) tests are employed. Table 17 shows the results for the autocorrelation tests.

Table 17: Autocorrelation tests for the innovations

Test	**Test Statistics**	***p*-value**
LB	26.336	0.155
DW	1.984	0.346

From Table 17, since the *p*-values of both the LB and DW tests are greater than 0.05, the null hypothesis that autocorrelation is absent within the innovation is not rejected at a 5% level of significance. According to Khan *et al.,* (2016), the absence of autocorrelation means that there is no relationship between the current and future innovations.

4.2.3.4 Heteroskedasticity

Heteroskedasticity of the innovations is investigated by the analysis of an ACF plot and heteroskedasticity tests. Figure 18 shows an ACF plot of the innovations squared.

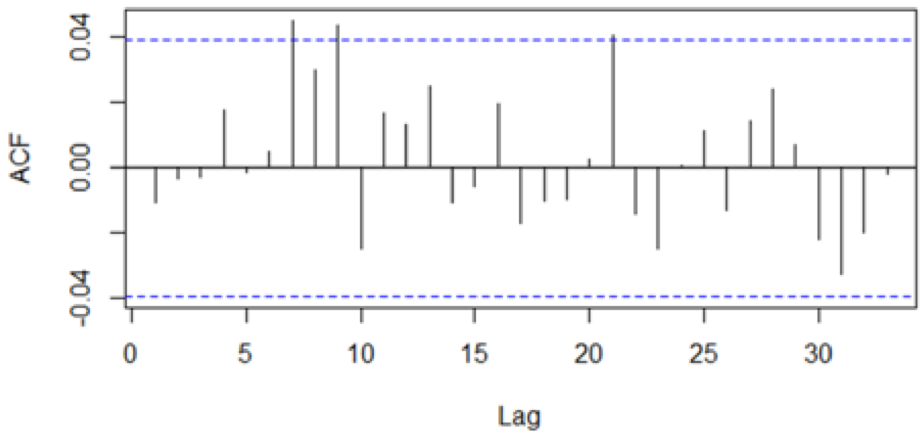

Figure 18: ACF plot for the innovations squared

From Figure 18, since the majority of lags do not touch or pass over the 95% conditional interval, this indicates the lags are insignificant. This means that autocorrelation within the innovations is absent. To investigate the presence of heteroskedasticity, the Ljung-Box (LB^2) and Autoregressive Conditional Heteroskedastic Lagrange Multiplier (ARCH-LM) tests are employed. Table 18 shows the results for the heteroskedasticity tests.

Table 18: Autocorrelation tests for the squared innovations

Test	**Test Statistics**	***p*-value**
LB^2	19.084	0.516
ARCH-LM	19.062	0.518

From Table 18, since the *p*-values of both the LB^2 and ARCH-LM tests are greater than 0.05, the null hypothesis that the ARCH effect is absent in the innovations is not rejected at a 5% level of significance. Thus, it can be concluded that heteroskedasticity is absent within the innovations. This means that the EGARCH model has adequately captured the volatile nature of the innovations.

4.2.3.5 Randomness

The random behaviour of the innovations is investigated by independent and identically distributed (IID) tests, namely, Bartels rank, Cox-Stuart and Brock, Dechert and Scheinkman (BDS). Table 19 shows the results for the random behaviour tests.

Table 19: Random behaviour tests of innovations

Test	Test Statistics	*p*-value
Bartels rank	-0.095	0.925
Cox-Stuart	602.000	0.213
BDS	2.000	0.500

From Table 19, since the *p*-values of all three IID tests are greater than 0.05, the null hypothesis that the innovations are IID is not rejected at a 5% level of significance. Thus, the innovations of the EGARCH (1, 1) model show random behaviour. Hence, from the descriptive statistics, normality and IID tests, it can be concluded that the innovations have a nonnormal, asymmetric and random nature. These findings are in line with Mangani (2008) and Ilupeju (2016), confirming the uncaptured risk within the innovations by Feng and Shi (2017).

4.2.4 Summary of GARCH Approach

A noteworthy property of the GARCH model is that the probability distribution governing the model innovations does not affect parameter estimation according to Spierdijk (2016). From the analysis of the GARCH type models discussed in 4.2.2.2, this theory can be supported. This is because the majority of the parameter estimates remain unaffected by the innovation distribution and asymmetric GARCH type model. However, although the asymmetric GARCH (1, 1) type models can account for a certain level of volatility, heavy tails and asymmetric properties, a certain level of risk remains uncaptured in the innovations. This is because of an inconsistency found between the EGARCH and APARCH model with regards to which type of shocks has a greater impact in the South African market. Since EGARCH is selected to be the best fitting model by information criteria, the standardised innovations are investigated to determine if they can fully capture risk.

From the above results, the innovations support the presence of nonnormality, asymmetry and random behaviour within the innovations of the EGARCH (1, 1) model. The absence of heteroskedasticity means that EGARCH has adequately captured the volatile nature of the innovations, in line with Ilupeju (2016). However, the model has failed to capture asymmetry and nonnormality which is the fundamental nature of returns, in line with the novel concept of asymmetric returns exposure. Thus, the GARCH approach is inadequate in fully capturing risk, making it an inefficient choice in estimating the risk-return relationship. This is in line with recent studies by Mangani (2008), Mandimika and Chinzara (2012), Feng and Shi (2017), Jin (2017), Apergis *et al.,* (2018), Demirer *et al.,* (2019) and Jensen and Maheu (2018). All of which advocate the use of nonlinear and nonparametric methods.

4.2.5 Risk-Return Relationship

The risk-return relationship is investigated by the symmetric GARCH (1, 1)-M and asymmetric EGARCH (1, 1)-M model.

4.2.5.1 GARCH (1, 1)-M and EGARCH (1, 1)-M

Table 20 and 21 shows the results for the GARCH (1, 1)-M and EGARCH (1, 1)-M models with different probability distributions governing the innovations.

Table 20: ML parameter estimates for GARCH (1, 1)-M

Parameter Estimates	**NORM**	**Std-t**	**Skew-t**	**GED**
$\hat{\mu}$	-0.000	-0.000	-0.000	-0.000
$\hat{\alpha}_1$	0.059 ***	0.060 ***	0.062 ***	0.060 ***
$\hat{\beta}_1$	0.940 ***	0.939 ***	0.937 ***	0.939 ***
$\hat{\alpha}_1 + \hat{\beta}_1$	0.999	0.999	0.999	0.999
$\hat{\delta}$	0.143 *	0.133 *	0.091	0.149 *
AIC	-6.557	-6.575	-6.596	-6.574
BIC	-6.548	-6.564	-6.580	-6.562

*NOTE: *, **, *** means the p-value is significant at a 10%, 5% and 1% level of significance, respectively*

Table 21: ML parameter estimates for EGARCH (1, 1)-M

Parameter Estimates	NORM	Std-t	Skew-t	GED
$\hat{\mu}$	-0.002 ***	0.002 ***	0.002 ***	-0.001***
$\hat{\alpha}_0$	-	-	-	-0.345 ***
$\hat{\alpha}_1$	-0.119 ***	-0.119 ***	-0.126 ***	-0.127 ***
$\hat{\beta}_1$	1.000 ***	1.000 ***	1.000 ***	0.964 ***
$\hat{\alpha}_1 + \hat{\beta}_1$	0.880	0.881	0.874	0.836
$\hat{\delta}$	-0.242 ***	-0.206 ***	-0.204 ***	0.184 ***
$\hat{\gamma}$	0.063 ***	0.060 ***	0.062 ***	0.080 ***
AIC	-6.603	-6.610	-6.624	-6.623
BIC	-6.591	-6.596	-6.608	-6.607

*NOTE: *, **, *** means the p-value is significant at a 10%, 5% and 1% level of significance, respectively*

4.2.5.2 Discussion of GARCH-M Test Results

The ARCH and GARCH effects represented by $\hat{\alpha}_1$ and $\hat{\beta}_1$, respectively are significant at a 5% level of significance for both the GARCH (1, 1)-M and EGARCH (1, 1)-M models. Given the GARCH-M models are adequate for analysis by the significance of the ARCH and GARCH effects, focus is placed on the prime parameter of interest, the risk premium parameter $\hat{\delta}$. From Table 20, the risk premium parameter is statistically insignificant at a 5% level of significance for all the innovation distributions. However, the risk premium is statistically significant at a 10% level of significance for the remaining three innovation distributions (NORM, Std-t and GED) except Skew-t. The latter finding is in contrast to the model testing based on SIC and BIC by the prior findings of the GARCH type models where the Skew-t innovation distribution was determined to be the best fitting and in capturing risk. This point emphasises the importance of efficient risk estimation in order to estimate a robust risk-return relationship by Demier *et al.,* (2018) and Jensen and Maheu (2018). Specifically, the choice of probability distribution governing the innovations, in line with Mandimika and Chinzara (2012).

From Table 21, all the risk premium parameters are statistically significant at a 5% level of significance for all the innovation distributions. This is in direct contrast to the GARCH (1, 1)-M model, particularly for the Skew-t innovation distribution. The EGARCH (1, 1) model is asymmetric and Skew-t assumes the innovations are asymmetric, implying a more robust combination in estimating the risk-return relationship. However, the combination of the GARCH (1, 1)-M with Skew-t results in an insignificant risk premium. This is because the GARCH (1, 1) model is not designed to account for the asymmetry and is mispecified, in line with Jin (2017). Since EGARCH (1, 1) provides a more robust risk estimation than GARCH (1, 1) due to being asymmetric, the overall EGARCH (1, 1)-M results are favoured. Thus, according to the innovation distributions, NORM, Skew-t, Std-t and GED, the presence of risk-return relationship and volatility feedback is confirmed by the EGARCH (1, 1)-M model.

The risk premium parameter is significant, indicating the presence of the risk-return relationship. The significant risk premium parameter is in contrast with the early South African studies by Mangani (2008), Mandimika and Chinzara (2012), du Toit (2015) and Adu *et al.,* (2015). The positive risk premium parameter means that an investor is being compensated for taking on a higher level of risk in the South African market. The significant risk premium parameter of this study is in contrast to the recent studies by Bekiros *et al.,* (2017), Jin (2017) and Steyn and Theart (2019), who find a negative risk-return relationship in the South African market. That is, where an investor is being negatively compensated, by earning low returns for taking on a higher level of risk. The positive risk-return relationship is more in line with recent studies from international literature such as Madaleno and Vieira (2018), Kim and Kim (2018), Jensen and Maheu (2018) and Harris *et al.,* (2019).

From Table 20 and 21, according to AIC and BIC for the GARCH (1, 1)-M and EGARCH (1, 1)-M model, the best innovation distribution is Skew-t in line with findings for the linear and asymmetric GARCH (1, 1) type models. This is followed by the Std-t, GED and NORM innovation distribution. Once again, it can be can be concluded that the NORM innovation distribution is the least best fitting innovation distribution. This is because a NORM innovation distribution is unrealistic, given financial data is volatile and nonlinear in nature, in line with Jensen and Maheu (2018). This is further supported by the insignificant risk premium parameter found by the GARCH (1, 1)-M and Skew-t innovation distribution.

Although the Skew-t is the best fitting innovation distribution, the finding of GARCH (1, 1)-M and EGARCH (1, 1)-M reveal an inconsistency in results. This could essentially be because GARCH (1, 1)-M is symmetric in nature and EGARCH (1, 1)-M is asymmetric. The underlying problem regarding the innovation distribution with respect to the GARCH approach confirms inefficient risk estimation in the investigation of the risk-return relationship. The inefficiency of GARCH (1, 1) misestimating risk is in line with Mangani (2008), Ilupeju (2016) and Feng and Shi (2017). Nonetheless, the presence of risk-return relationship and volatility feedback, respectively is confirmed by the EGARCH (1, 1)-M model. Since risk remains uncaptured within the innovations of the EGARCH (1, 1) model, in turn, this makes the EGARCH-M results not statistically sound. Hence, this motivates further investigation of the risk-return relationship.

4.2.6 Summary of Risk-Return Relationship

The GARCH (1, 1)-M model finds an insignificant risk premium parameter only for the Skew-t innovation distribution. In contrast, the EGARCH (1, 1)-M model finds the opposite result for all four innovation distributions, at all three levels of significance. The theory by Spierdijk (2016), where the distribution of model innovations does not affect parameter estimation can now be unsupported in the context of estimating the risk-return relationship. This is because the significance of the parameter estimates changes for the innovation distribution Skew-t for the asymmetric EGARCH (1, 1) model. Due to this inconsistency, it is reaffirmed that the results of the GARCH approach are not strictly adhered to and simply regarded as preliminary tests to motivate further testing.

The finding of the presence of the risk-return relationship is not statistically sound by the GARCH approach and this is essentially due to three reasons. First, the GARCH (1, 1) model is a symmetric parametric model which cannot effectively account for asymmetric volatility (Hretski and Karachun, 2018). In this study, the sign and size bias tests showed that asymmetry remains uncaptured by the GARCH (1, 1) model, in line with Park *et al.,* (2017) and Ilupeju (2016). Thus, according to He *et al.,* (2018), the risk premium remains constant for a specified period of time. In order to address this limitation, various extensions have been made to the standard GARCH model in order to accommodate the market characteristics - asymmetric volatility, the leverage effect and volatility feedback (Harris *et al.,* 2019). While these extensions have been made, resulting in the EGARCH, GJR and APARCH, the model is still not free from being limited (Jin, 2017).

Second, the GARCH type models are essentially parametric which limits its ability to account for higher moment asymmetric forms of the risk-return relationship (Demirer *et al.,* 2019). Therefore, the use of a nonparametric approach, in order to effectively account for asymmetry as well as account for model misspecifications, is highlighted in a number of studies such as Jin (2017), Apergis *et al.,* (2018), Jensen and Maheu (2018) and Demirer *et al.,* (2019).

Third, risk remains uncaptured by the probability distributions governing the innovations of the GARCH type models (Feng and Shi, 2017). While the innovations have shown to capture volatility in this study, nonlinearities and asymmetries; however, still remain uncaptured in line with Ilupeju (2016) and Mangani (2008). Hence, asymmetric returns exposure remains uncaptured, leading to misestimating risk and contributing to inconclusive results regarding the risk-return relationship. Thus, the GARCH approach is not an efficient choice for estimating risk when investigating the risk-return relationship, in line with Jensen and Maheu (2018); Feng and Shi (2017) and Jin (2017).

Ultimately, while GARCH type models can account for volatility, more robust predictions can be made by models that can effectively account for measures with nonlinearity, asymmetry as well as latent and stochastic properties (Karabatsos, 2016; Jin, 2017; Jensen and Maheu, 2018; Wagenmakers *et al.,* 2018). Therefore, this study applies the Bayesian approach, which accounts for the uncertainty associated to asymmetric returns exposure due to being latent and stochastic in nature.

The Bayesian approach consists of a parametric Bayesian model and a nonparametric Bayesian model. The parametric approach takes on a finite number of possibilities, in terms of higher moment asymmetric forms of the risk-return relationship (Jin; 2017). On the other hand, the nonparametric approach takes on an infinite number of possibilities, being more effective in accommodating higher moment asymmetric properties such as skewness, kurtosis and multiple modes (Demirer *et al.,* 2019). Essentially, a parametric Bayesian model or nonparametric Bayesian model both provide more robust results, in comparison to the conventional quantitative finance methods such as regression analysis, the VAR model, causality tests and the GARCH approach. This is because the Bayesian approach, as a parametric or nonparametric model, can account for uncertainty and asymmetry more effectively (Karabatsos, 2016). However, the nonparametric Bayesian model by Jensen and Maheu (2018), is more robust than any parametric model, including a parametric Bayesian model, mainly because by definition, it can account for an infinite number of possibilities. In other words, it is more effective in accounting for higher moment asymmetric forms of the risk-return relationship in an infinite sample space (Demirer *et al.,* 2019; Jensen and Maheu, 2018; Karabatsos, 2016; Jin, 2017).

4.3 Bayesian Approach

The Bayesian approach consists of four parts. Firstly, the data exploration gives a brief overview and comparative analysis to substantiate the choice of the selected risk and return variables. Secondly, the model specifications for the priors and posteriors are noted. Thirdly, the test results of the parametric Bayesian model are presented followed by the test results of the nonparametric Bayesian model. Finally, the Bayesian test results and their implications are discussed in summary.

4.3.1 Data Exploration

Table 22 shows the basic descriptive statistics of the risk and return variables.

Table 22: Basic descriptive statistics of excess returns and realised variance

Variables	Mean	Variance	Kurtosis	Skewness	Minimum	Maximum
r_t	0.0001	0.0001	1.277	-0.167	-0.037	0.042
r_t^2	0.118	0.004	-1.131	-0.043	0.000	0.227
RV_t	0.981	0.000	3.185	-1.567	0.946	0.988
$\log(RV_t)$	-0.019	0.000	3.323	-1.593	-0.056	-0.012
$z = \frac{r_t}{\sqrt{RV_t}}$	0.000	0.000	1.350	-0.168	-0.038	0.043

From Table 22, the summation of returns squared r_t^2 is realised variance by definition (Maneemaroj *et al.,* 2019). The RV_t is a bias adjusted measure of realised variance where the realised variance RV_t^q is equivalent to RV_t since *q* is set to one by Hansen and Lunde (2006). From the variance column, the realised variance by definition r_t^2 contains more risk than the bias adjusted realised variance RV_t. Additionally, the sum of squared returns r_t^2 has negative excess kurtosis which indicates a thin tailed distribution. In contrast, to the characteristics of an emerging market which is heavy tailed and has higher levels of volatility (Herbert *et al.,* 2018).

The bias adjusted realised variance RV_t is heavy tailed, as indicated by the positive excess kurtosis, implying that it contains more risk. However, it has a lower variance, in comparison to the realised variance by definition. This means that the bias adjusted realised variance RV_t is less risky because it has the ability to account for microstructure noise. The bias adjustment accounts for the micro price movements in the market due to changes in supply and demand, and stale prices which are when prices do not update to recent information (Hansen and Lunde, 2006). Thus, the bias adjusted realised variance RV_t provides a better estimate of risk, in line with Jensen and Maheu (2018).

On the other hand, returns r_t has a positive excess kurtosis indicating a heavy tailed distribution. This finding is in line with the return characteristics of an emerging market by Herbert *et al.,* (2018) and Adu *et al.,* (2015). With respect to skewness, all the variables reflect a negative value where for returns, this indicates losses for investors. This is because negative skewness is associated to a negative payoff by Yao *et al.,* (2019). It can be concluded that returns have an asymmetric nature since it follows an asymmetric distribution, in line with the concept of asymmetric returns exposure. Figure 19 shows a general relationship between excess returns and log realised variance by a basic scatter plot.

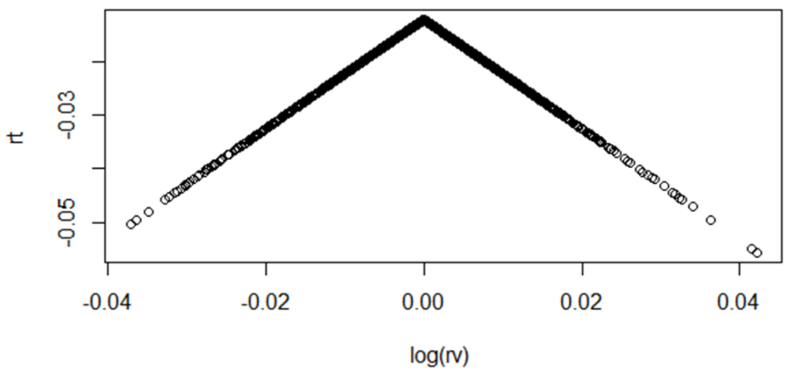

Figure 19: Scatter plot for excess returns and log realised variance

From Figure 19, the data of excess returns and log realised variance shows a nonlinear and asymmetric pattern. This result is in line with the South African studies by du Toit (2015) and Steyn and Theart (2019). The ends, in particular, show a number of outliers, in line with Jensen and Maheu (2018). Thus, in conclusion from Figure 19, the asymmetric properties and outliers shown by the data can effectively be accounted for by an asymmetric model. This is in line with the recommendations by Demirer *et al.,* (2019) and Jensen and Maheu (2018).

4.3.2 Model Specifications

Volatility feedback and the risk-return relationship is investigated by a parametric and nonparametric Bayesian model guided by the methodology of Jensen and Maheu (2018). The majority of the prior specifications follow the values suggested by Karabatsos (2016), which are noninformative. A noninformative prior is an objective prior since it is not guided by a source of subjectivity which overcomes the main limitation of the Bayesian approach (Bartlett and Keogh, 2016).

For the parametric Bayesian model, the prior variance of the slope parameters is specified as 1000 and the prior inverse gamma distribution of the error variance is specified as $\frac{0.001}{2}$.

For the nonparametric Bayesian model, the prior is the Dirichlet Process by Ferguson (1973), derived by the stick-breaking process by Sethuram (1994). The prior inverse gamma distribution of the error variance is specified as $\frac{5}{2}$. For the Dirichlet Process, the intercept variance of the base distribution is specified as five. The prior gamma distribution, shape and rate, of the concentration parameter are both specified as one.

For both the parametric and nonparametric Bayesian model, the MCMC methods used to compute the posterior parameter estimates are the slice sampler by Kalli *et al.,* (2011) and the Gibbs sampling technique. The posterior parameter estimates are determined by 20 000 MCMC sampling iterations, a burn-in period of 5000 and a thin number of 5.

4.3.3 Parametric Bayesian Test Results

The test results of the parametric Bayesian model serve as a preliminary test with regards to the presence of asymmetric effects in the South African market. It also allows for a comparative analysis between the final test results of the parametric Bayesian model and nonparametric Bayesian model.

4.3.3.1 Summary Statistics

The summary statistics serve as a preliminary test to provide an overview of the parametric Bayesian model. Table 23 shows the results of the parameter estimates.

Table 23: Posterior parameter estimates

Parameter Estimates	75% Credible Interval	Mean Value	95% Credible Interval	MC Mixing Value
$\hat{\alpha}_0$	(-0.118, -0.077)	-0.098	(-0.156, -0.037)	0.499
$\hat{\alpha}_1$	(0.078, 0.121)	0.100	(0.038, 0.159)	0.499
$\hat{\sigma}_1^2$	(0.000, 0.000)	0.000	(0.000, 0.000)	0.492
$\hat{\gamma}_0$	(-0.019, -0.018)	-0.019	(-0.019, -0.018)	0.518
$\hat{\gamma}_1$	(-0.000, 0.000)	0.000	(-0.000, 0.000)	0.512
$\hat{\gamma}_2$	(-0.096, 0.186)	0.047	(-0.370, 0.460)	0.523
$\hat{\gamma}_3$	(0.062, 0.080)	0.070	(0.044, 0.098)	0.490
$\hat{\gamma}_4$	(-0.126, -0.099)	-0.113	(-0.152, -0.073)	0.499
$\hat{\sigma}_2^2$	(0.000, 0.000)	0.000	(0.000, 0.000)	0.500

From Table 23, the coefficient $\hat{\alpha}_1$ on RV_t represents the persistence of risk on realised variance and this term represents volatility feedback (Jensen and Maheu, 2018). The value is positive and statistically significant, confirming the presence of volatility feedback in the South African market, by both the 75% and 95% credible intervals. On the other hand, for the 95% credible interval, $\hat{\gamma}_1$ has a mean value of zero which suggests an absence of volatility feedback. Further, $\hat{\gamma}_2$ which in this case accounts for volatility feedback over the entire sample period is positive, has a high mean value and is statistically significant. However, for the 75% credible interval, both $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are statistically insignificant, indicating no volatility feedback. Overall, these results indicate a weak presence of volatility feedback in the South African market.

According to the 75% and 95% credible intervals, the leverage effect is present in the South African market as indicated by the significance of $\hat{\gamma}_3$ and $\hat{\gamma}_4$. The presence of the leverage effect in the South African market is in line with Mandimika and Chinzara (2012), Ilupeju (2016) and Jin (2017) but in contrast to Adu *et al.,* (2015). However, their mean values of 0.070 and -0.113 are weak, in comparison to volatility feedback which has a mean value of 0.100 as shown by $\hat{\alpha}_1$. The weak presence of the leverage effect in the South African market is in line with Mangani (2008). It can be concluded that volatility is weakly persistent, resulting in volatility feedback being weakly present or close to being absent, in the South African market. This result is in contrast to the international study by Jin (2017), who found the presence of volatility feedback in the South African market. As a result, this motivates further testing of the volatility feedback mechanism.

The values of $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ are both zero which indicates that there is no error variance. The error variance refers to the unexplained variance that arises from sources such as uncertainty and measurement errors of which a Bayesian model can automatically adjust to (Chakraborty and Lozano, 2019). This includes the systematic error on the realised variance measure according to Jensen and Maheu (2018). The systematic error refers to the error surrounding the stochastic measure of realised variance which is unavoidable regardless of the number of times the model is run (Beyhaghi *et al.,* 2018). Finally, since all the MC mixing values are an approximate value of 0.5, this indicates optimal mixing of the parameters. Hence, this confirms the robustness of the posterior parameter estimates as suggested by Karabatsos (2016).

4.3.3.2 Volatility Feedback

Following Jensen and Maheu (2018), in order to truly capture the volatility feedback mechanism, log realised variance is determined in relation to three levels, namely, low, average and high volatility. Since this study analyses daily data, the three periods are daily low, average and high volatility. The three periods are defined as 10 May 2010, 24 February 2011 and 16 March 2012, which are calculated as the minimum, average and maximum log realised variance values, respectively. Drawn from theory, log realised is expected to get stronger over time in relation to the three levels of volatility (Harris *et al.,* 2019; Jensen and Maheu, 2018). Table 24 shows the results of the parametric Bayesian model.

Table 24: Posterior parameter estimates

Parameter Estimates	75% Credible Interval	Mean Value	95% Credible Interval	MC Mixing Value
$\hat{\beta}_0$	(-0.028, -0.012)	-0.020	(-0.044, 0.004)	0.484
$\hat{\beta}_{low}$	(-0.137, 0.127)	-0.005	(-0.412, 0.381)	0.489
$\hat{\beta}_{avg}$	(-0.144, 0.142)	0.001	(-0.394, 0.410)	0.497
$\hat{\beta}_{high}$	(-0.140, 0.140)	-0.004	(-0.409, 0.387)	0.502
$\hat{\sigma}^2$	(0.000, 0.000)	0.000	(0.000, 0.000)	0.506

From Table 24, for a low level of volatility, the mean value is negative and statistically significant. The mean value then increases and reflects a positive value for an average level volatility. However, although the mean value is significant for a high level of volatility, it decreases from an average level of volatility, indicating a dissipation of volatility. This finding is in direct contrast to its expected magnitude of volatility feedback where it is characterised by tendencies to get stronger over time by Jensen and Maheu (2018) and Harris *et al.,* (2019). Further, for the 75% credible interval, all the mean values are found to be statistically insignificant. Thus, from the overall results of the parametric Bayesian model, it can be concluded that volatility feedback is absent in the South African market.

4.3.3.3 Risk-Return Relationship

Since there is no significant evidence of volatility feedback in the market, the risk-return relationship is analysed free from empirical distortions that result from volatility feedback. If only the risk premium is captured, it should theoretically be positive and linear over time, as suggested by traditional theoretical expectations. Table 25 shows the results of the parametric Bayesian model.

Table 25: Posterior parameter estimates

Parameter Estimates	75% Credible Interval	Mean Value	95% Credible Interval	MC Mixing Value
$\hat{\beta}_0$	(0.002, 0.002)	0.002	(0.001, 0.003)	0.494
$\hat{\beta}_{\log(rv)}$	(0.078, 0.117)	0.098	(0.040, 0.157)	0.498
$\hat{\sigma}^2$	(0.000, 0.000)	0.000	(0.000, 0.000)	0.494

From Table 25, for the 75% credible interval, the relationship between risk and return is significant. This result is supported by the 95% credible interval which indicates a positive and significant risk-return relationship. Since all the MC mixing values are an approximate value of 0.5, this indicates optimal mixing of the parameters, hence, robust results as suggested by Karabatsos (2016). Figure 20 is a plot that shows the mean (M) and 95% quantiles of returns as a function of log realised variance.

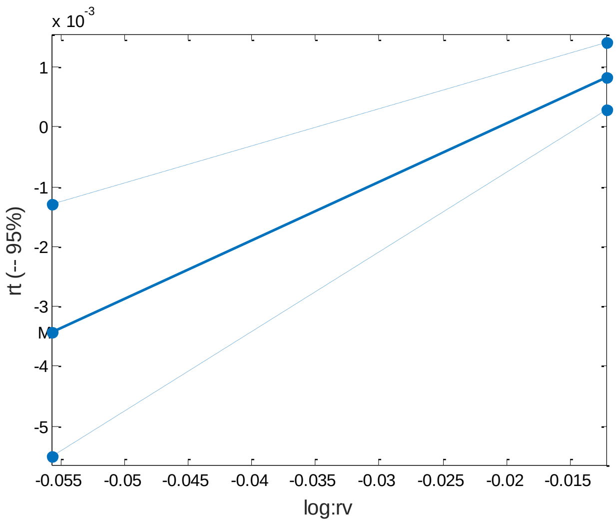

Figure 20: Relationship between risk and return

From Figure 20, a positive and linear relationship between risk and return is shown over time, in line with expectations from conventional theory by Markowitz (1952), Sharpe (1964), Lintner (1965) and Mossin (1966). However, this result is in contrast to the majority of the South African studies who found a negative or no risk-return relationship at all. The positive and linear risk-return relationship result is more in line with recent studies from international literature by Madaleno and Vieira (2018), Kim and Kim (2018), Jensen and Maheu (2018) and Harris *et al.,* (2019).

However, the positive and linear test result of the risk-return relationship is from the parametric Bayesian model. Meaning, a model that is essentially a parametric finite model that is limited to accounting for every possible higher moment asymmetric form of the risk-return relationship (Karabatsos, 2016). Therefore, a nonparametric Bayesian model would be more effective in capturing the risk-return relationship, in line with recommendations from literature by Mandimika and Chinzara (2012), Kang (2014), Chang *et al.,* (2017), Jin (2017), Waldmann (2018), Jensen and Maheu (2018) Demirer *et al.,* (2019).

4.3.4 Nonparametric Bayesian Test Results

The nonparametric Bayesian model is the main method of investigation for this study with respect to the investigation risk-return relationship and volatility feedback in the South African market.

4.3.4.1 Volatility Feedback

The analysis of volatility feedback for the nonparametric Bayesian approach consists of a comparative analysis of how the form of density estimation with respect to the different levels of volatility changes. The graphical analysis is then concluded by a brief comparative analysis of the expected and actual result.

To recap, following Jensen and Maheu (2018), in order to truly capture the volatility feedback mechanism, log realised variance is determined in relation to three levels of volatility which are low, average and high volatility. Since this study analyses daily data, the three periods are daily low, average and high volatility. The three periods are defined as 10 May 2010, 24 February 2011 and 16 March 2012, which are calculated as the minimum, average and maximum log realised variance values, respectively. Drawn from theory, log realised is expected to get stronger over time in relation to the three levels of volatility (Harris *et al.,* 2019; Jensen and Maheu, 2018).

In the context of this study, for the probability density functions (pdfs), the plots display the spread and probability of occurrence of the log realised variance in relation to the specified level of volatility. Graphically, the peak of the density represents the mean value since it is a measure of central tendency (Gulzar *et al.,* 2019). Figure 21, 22 and 23 are density estimations of log realised variance for low, average and high levels of log realised variance, respectively. All three figures show a distinct form for each plot. Figure 21 shows the density estimation for a period of low volatility.

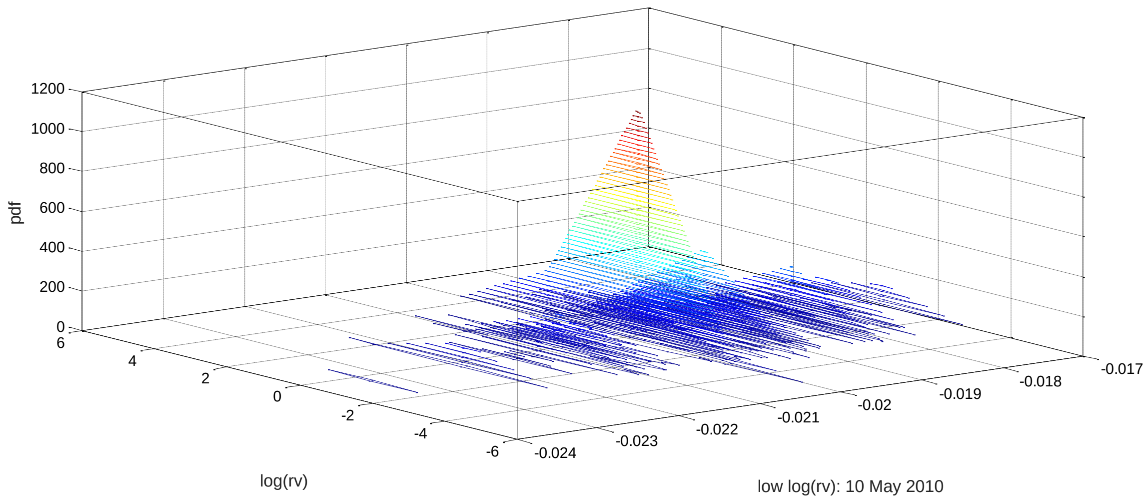

Figure 21: Density estimation of log(rv) in relation to a low level of log(rv)

From Figure 21, the spread is relatively high and the peak of the pdf is the highest, in comparison to the plots for average and high levels of volatility. Specifically, the peak of the pdf is approximately 900. Meaning, the probability of occurrence of log realised variance is exceptionally high within the market conditions of low volatility. Thus, the South African market is subject to relatively low volatility conditions. This finding is in contrast to the high levels of volatility characterised by an emerging market by Herbert *et al.,* (2018).

This is further in contrast to the studies that find persistent levels of volatility in the South African market by Mangani (2008), Mandimika and Chinzara (2012), Adu *et al.,* (2015), Ilupeju (2016) and Jin (2017). As a result, suggesting the presence of a stronger form of volatility, volatility feedback (Harris *et al.,* 2019). This period of low volatility is representative of unusually stable conditions that does not necessarily facilitate an investor to achieve a higher return from taking on a higher level of risk. However, the result of low volatility is in line with the recent South African study by

Steyn and Theart (2019). Figure 22 shows the density estimation for a period of average volatility.

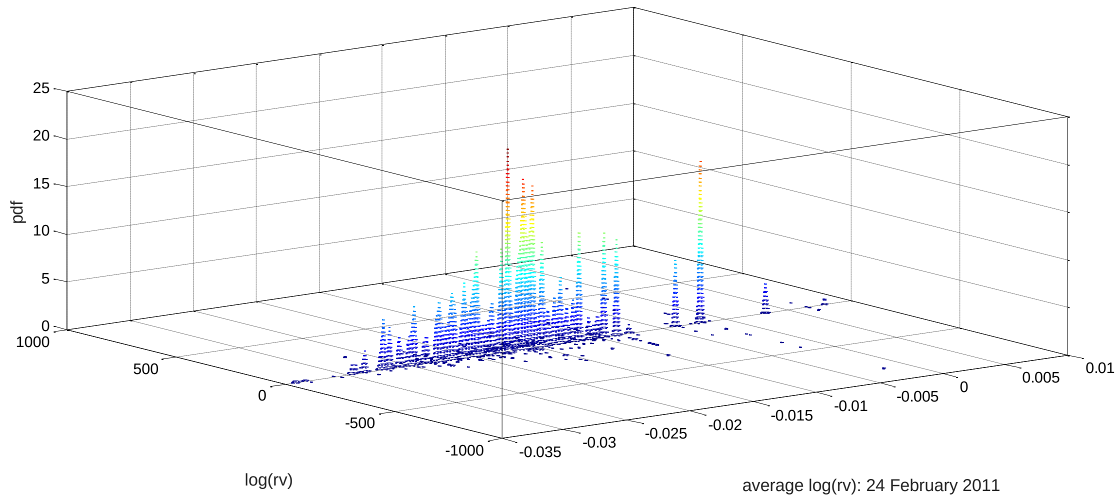

Figure 22: Density estimation of log(rv) in relation to an average level of log(rv)

In direct contrast to Figure 21, Figure 22 reflects a density estimation that has a very constricted spread. The form of Figure 22 strongly reflects a horizontal and uniform shape along the y-axis. This relatively flat form suggests that volatility in relation to an average level of volatility is linear. The positive uniform and linear shape can be interpreted as a reflection of the trade-off theory by Markowitz (1952), where an investor only takes on a high level of risk if compensated by a high level of return. This is in contrast to the recent documented South African studies which finds a negative relationship and would expect a nonlinear shape (Bekiros *et al.,* 2017; Jin, 2017; Steyn and Theart, 2019). On that note, if the earlier South African studies were taken into account, which found an insignificant relationship, one would expect no particular shape or form for the density estimation (Mangani, 2008; Mandimika and Chinzara, 2012; Adu *et al.,* 2015).

The probability estimation for the average level of volatility 13, is drastically low, in comparison to a low level of volatility 900 in Figure 21. The result for the average level of volatility is unexpected as one would expect a higher probability estimation within this period. This is because it resembles the conditions of a typical market that promotes investment activity. That is, where investors are being compensated for their investment ventures. Hence, the low probability estimation is in contrast to the expectation of recent studies by Madaleno and Vieira (2018), Kim and Kim (2018), Jensen and Maheu (2018) and Harris *et al.,* (2019). All of which found investors are being compensated in their respective markets. Figure 23 shows the density estimation for a period of high volatility.

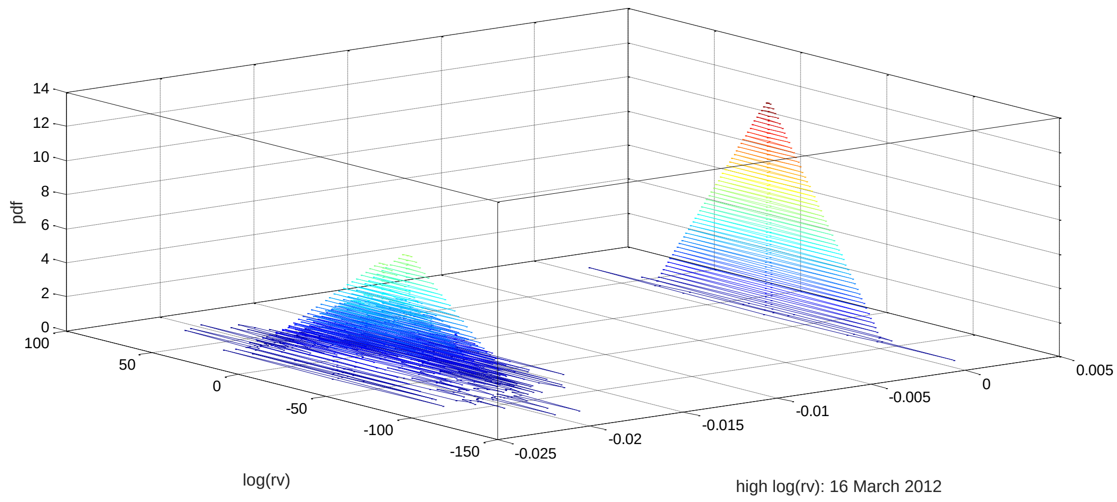

Figure 23: Density estimation of log(rv) in relation to a high level of log(rv)

Figure 23 is the most unique density estimation as it reflects the asymmetric property of multiple modes of which a nonparametric model can capture according to Jensen and Maheu (2018) and Karabatsos (2016). The spread is constricted in two distinct areas which could be as a result of the high level of volatility, thus, causing a distinct dispersion in the spread of the data. The probability estimation for log-realised variance in relation to the highest level of volatility has the lowest value 11. Meaning, the probability of log realised variance in relation to a high level of volatility has the lowest chance of occurring in the South African market.

Specifically, its value is approximately about eighty times smaller than the pdf of the low specified level of volatility of 900. This means that the probability of occurrence of log realised variance within a period of high volatility is very small, smaller than that for the low volatility period. Thus, log realised variance is strongly dissipating with respect to an increase in the level of volatility over time. The finding is threefold. Firstly, it is in contrast to the high levels of volatility characterised by an emerging market by Herbert *et al.,* (2018). Secondly, it is further in contrast to South African studies who find persistent levels of volatility in the South African market by Mangani (2008), Mandimika and Chinzara (2012) Adu *et al.,* (2015) and Jin (2017). Thirdly, the result is in contrast to the expected theoretical result of volatility feedback which is characterised by tendencies to get stronger over time and take longer to die out by Harris *et al.,* (2019) and Jensen and Maheu (2018).

Looking at the data alone, focus is placed on the probability estimation indicated by the peak of the pdf on the vertical axis which represents the mean value. The mean value represents the probability estimation of obtaining the value of log realised variance in relation to the specified level of volatility. The approximate values are derived from Figure 21, 22 and 23, and are 900, 13 and 11, respectively. These values are plotted as a simple line graph for the purpose of a comparative analysis, with the expected theoretical and empirical result, drawn from Harris *et al.,* (2019) and Jensen and Maheu (2018). Figure 24 shows the point estimates of the pdfs of log-realised variance in relation to low, average and high levels of volatility.

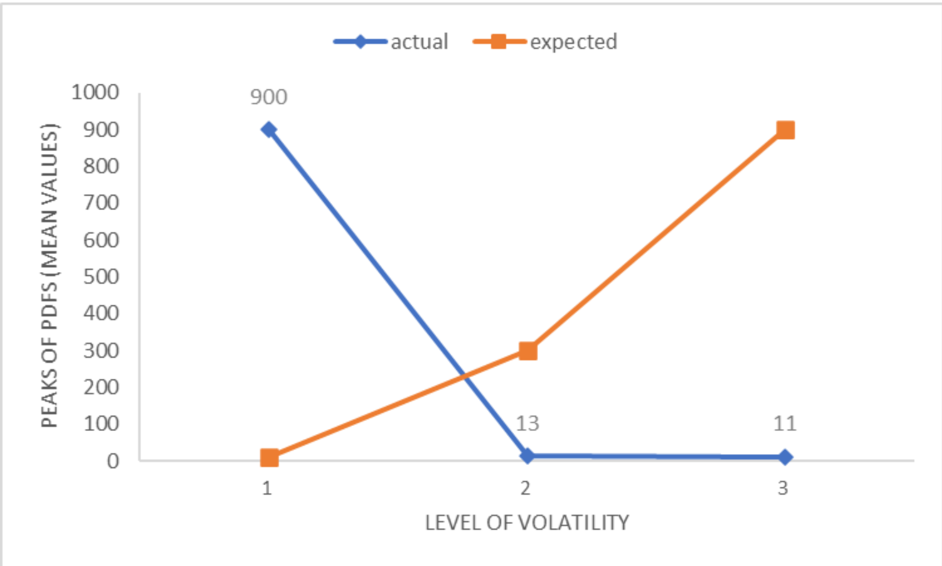

Figure 24: Line graphs of the mean values of log-realised variance

From Figure 24, the blue line is the actual finding and the orange line is an expected result drawn from the study by Jensen and Maheu (2018). According to the expected result (orange line), log realised is expected to shift rightwards and upwards in relation to the three levels of volatility, whereas the actual finding is the opposite result. More importantly, the slope of the actual result (blue line) distinctly flattens over the latter period, reaffirming the dissipation of volatility over time. Table 26 shows the results of the nonparametric Bayesian model.

Table 26: Posterior parameter estimates

Parameter Estimates	75% Credible Interval	Mean Value	95% Credible Interval	MC Mixing Value
$\hat{d}_{low}$	(-0.093, 0.066)	-0.311	(-24.503, 26.842)	0.496
$\hat{d}_{avg}$	(-0.144, 0.127)	0.406	(-28.389, 37.115)	0.499
$\hat{d}_{high}$	(-0.091, 0.064)	4.480	(-35.850, 28.768)	0.487

From Table 26, the mean values of the density estimations increase from a low, to average, to a high level of volatility. This finding is in line to the volatility feedback definition by Harris *et al.,* (2019) and expected result by Jensen and Maheu (2018). However, the density estimation of log realised variance in relation to the three levels of volatility are all insignificant for the 75% credible interval, but significant for the 95% credible interval. Overall, the test results suggest an absence of the volatility feedback mechanism, in line with the results of the parametric Bayesian model. Thus, from the graphical analysis as well as numerical analysis of both the parametric and nonparametric Bayesian models, it can be concluded that volatility feedback is absent in the South African market.

This finding is in contrast to a number of studies. First, Jin (2017) who found the presence of volatility feedback for South Africa and the emerging markets Brazil, India and Indonesia. Further, studies who found a strong presence of volatility feedback such as Kim and Kim (2018) and Jensen and Maheu (2018) for the US market and Harris *et al.,* (2019) for the UK. One would expect volatility feedback to be strongly prevalent in emerging markets since they are characterised by high levels of volatility (Herbert *et al.,* 2018). However, the study by Sultan (2018), found a weak presence of volatility feedback in the emerging Finnish market. This study follows the sample period of Sultan (2018), which does not account for the 2008 financial crisis. As a result, this choice could be the main reason for the absence of volatility feedback in the South African market.

According to theory, volatility feedback is a useful tool in understanding market conditions by acting as an indicator of market stability (Mancino and Sanfelici, 2019). The presence of strong volatility feedback suggests recessionary market conditions, whereas a weak presence indicates expansionary market conditions (Inkaya and Okur, 2014). In this case, the absence of volatility feedback suggests the market conditions of an economic recovery. Hence, the result of absent volatility feedback is theoretically and empirically expected since this study analyses the post 2008 financial crisis period. The actual risk-return relationship of South Africa was also unaffected by the 2008 financial crisis in the study by Jin (2017), in support of its exclusion in this study. Thus, from the results of the parametric and nonparametric Bayesian models it can be concluded that the volatility feedback mechanism has a negligible effect in the investigation of the risk-return relationship in the South African market.

4.3.4.2 Risk-Return Relationship

According to literature, the nonparametric approach has been found to be effective, in accounting for asymmetric and nonlinear properties, in the investigation of the risk-return relationship (Demirer *et al.,* 2019; Apergis *et al.,* 2018; Jensen and Maheu, 2018). Thus, the risk-return relationship is determined nonparametrically following Jensen and Maheu (2018), by means of density estimation in the form of a graphical and numerical analysis as follows. The density of the risk-return relationship is plotted against the mean to essentially give the average form of the distribution. Figure 25 is a plot of the density estimation for the risk-return relationship.

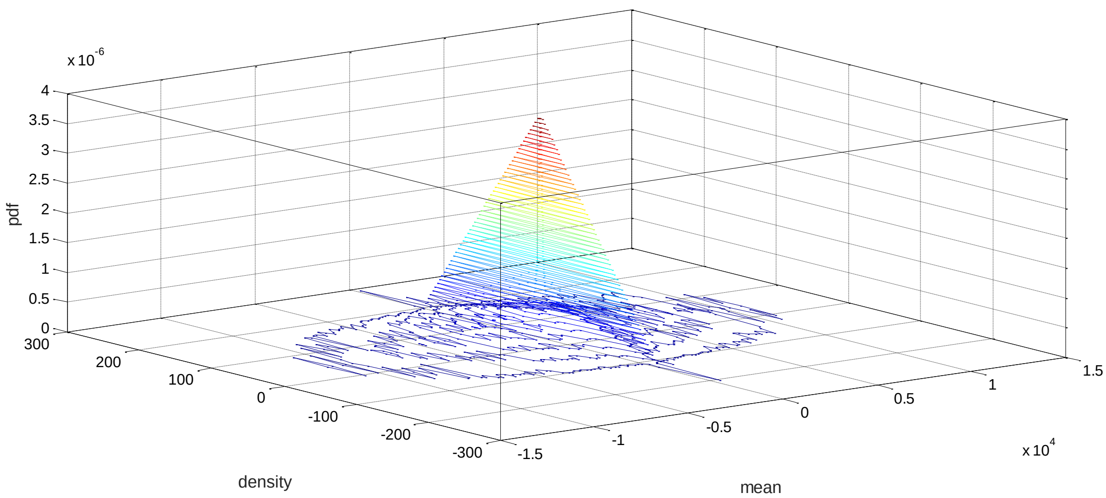

Figure 25: Density estimation of return in relation to risk

On the flat surface of Figure 25 encompassing the x and y-axes, a distinct nonlinear relationship between risk and return is exhibited by the pdf. In distinct contrast to the linear positive straight line from the parametric Bayesian model. A reason for this difference could be because the nonparametric approach is more robust in accounting for asymmetric properties as suggested by Jensen and Maheu (2018), Apergis *et al.,* (2018) and Demirer *et al.,* (2019). Thus, this suggests the relationship between risk and return is nonlinear in the South African market. However, according to the pdf, the probability estimation of this occurrence is exceptionally low, specifically 0.000004 ≈ 0. Meaning, the risk-return relationship is insignificant as it has a zero probability which translates to an event that would never happen. The numerical result of the density estimation provides more insight to this result. Table 27 shows the results for the nonparametric Bayesian model.

Table 27: Posterior parameter estimates

Parameter Estimates	**75% Credible Interval**	**Mean Value**	**95% Credible Interval**	**MC Mixing Value**
$\hat{d}_{rr}$	(-0.229, 0.215)	3.540	(-1.892, 1.371)	0.510

From Table 27, according to the 75% and 95% credible interval, the risk-return relationship is insignificant in the South African market. Hence, the final result is that there is no relationship between risk and return in the South African market. The insignificant risk-return relationship is in direct contrast to the results of the parametric Bayesian model which shows a significant positive and linear risk-return relationship. However, it does support the pdf plot in Figure 25 which has a probability estimation of zero which means that the event of a nonlinear risk-return relationship has no chance of occurrence.

Therefore, an investor does not earn a higher compensation, in the form of return, when taking on any level of risk in the South African market. This finding is in line with the early South African studies by Mangani (2008), Mandimika and Chinzara (2012), du Toit (2015) and Adu *et al.,* (2015). This includes recent international studies such as Savva and Theodossiou (2018), who applies the GARCH approach, Umutlu (2019) and Apergis *et al.,* (2018), who apply the nonparametric approach to VAR and Granger causality tests, respectively. However, this is in contrast to the significant positive risk-return relationship found in the recent studies from international literature by Madaleno and Vieira (2018), Kim and Kim (2018), Jensen and Maheu (2018) and Harris *et al.,* (2019).

4.3.5 Discussion of Bayesian Test Results

The absence of volatility feedback from the Bayesian test results of both the parametric and nonparametric models is possibly expected since this study analyses the post 2008 financial crisis period. In turn, this result does not significantly impact the rising or falling of market prices, in line with Aboura and Wagner (2016). Since volatility feedback does not affect the dynamics of the South African financial market, it has a lesser impact with respect to an investor's decision making and investment strategies. That is, in terms of identifying and capitalising from arbitrage opportunities and mispricing's in the South African market.

In the context of this study, volatility feedback has been identified as a source of asymmetry that needs to be taken into account when investigating the risk-return relationship. This is guided by the studies of Jensen and Maheu (2018), Kim and Kim (2018) and Harris *et al.,* (2019). Since emerging markets such as South Africa are characterised by higher levels of volatility, this suggests that the presence of volatility feedback is expected to be more pronounced. However, the Bayesian test results found the effects of volatility feedback to get weaker over time as well as have a statistically insignificant effect. Hence, it was confirmed that that volatility feedback mechanism is absent in the South African market. As a result, the risk-return relationship was investigated free from empirical distortions that result from volatility feedback.

The result of the parametric Bayesian model was a statistically significant positive and linear relationship between risk and return, in line with the theoretical expectations by Markowitz (1952), Sharpe (1964), Lintner (1965) and Mossin (1966). In contrast, the result of the nonparametric Bayesian model was found to be statistically insignificant. Hence, the final result of no relationship between risk and return, in line with the early South African studies by Mangani (2008), Mandimika and Chinzara (2012) and Adu *et al.,* (2015). All of which applied the GARCH approach to the South African market.

The absence of volatility feedback and no relationship between risk and return in the South African market is still a source of useful information. This is with respect to investors, policy makers and researchers. Although investors and arbitrageurs cannot capitalise on mispricing's, they can still be guided in terms of their investment strategies and decisions (Hussain *et al.,* 2019). An investor can include South Africa in their investment portfolio with higher risk countries in order to spread their risk and derive diversification benefits. Risk averse investors can find a safe environment within the market of South Africa and earn a return in accordance to their risk tolerance (Huang and Startz, 2019).

Policy makers are able to understand the behaviour of their market participants and create policies that promote economic growth on a macroeconomic level (Liu, 2019). Although volatility feedback is absent, volatility itself is present as guided by the GARCH results which motivated the investigation of volatility feedback. This is reinforced by the analysis of density estimation where the highest probability estimation is with respect to low volatility levels. The low persistent levels of volatility can still pose a challenge for policy makers (Vo *et al.,* 2019). It has the potential to slowly deteriorate market stability along with macroeconomic and political shocks, in line with Marozva (2019). Given global interconnectedness, this makes markets prone to volatility spillover effects which can lead to economic instability, major cash outflows and a potential financial crisis (Gulzar *et al.,* 2019). This can be avoided by export diversification and the maintenance of macroeconomic and political regulatory frameworks according to Mandimika and Chinzara (2012).

While the risk-return relationship varies from study to study, a common theme of differences arises from data frequency, sample period and model specification (Savva and Theodossiou, 2018). However, given that a parametric model such as the GARCH approach is subject to a number of drawbacks, it is still extensively used (Maneemaroj *et al.,* 2019). The drawbacks include the nonnegativity constraints, model misspecifications, measurement errors and choice of the underlying innovation distribution (Feng and Shi, 2017). All of which contribute to misestimating risk and inconclusive results (Apergis *et al.,* 2018). Instead of focusing on the inconclusive results of the risk-return relationship, focus should be extended to more robust methods in existing literature to help solve the problem (Demirer *et al.,* 2019). By providing a practical means to solve the ongoing debate, progression can be made on a local and international level (Jensen and Maheu, 2018).

4.3.6 Summary of Bayesian Approach

According to the parametric Bayesian model, volatility feedback is suggested to be absent in the South African market. However, this finding is regarded as a preliminary test since volatility feedback is further investigated. This is by means of the relationship between log realised variance in relation to three levels of volatility following Jensen and Maheu (2018). The numerical and graphical output, from the parametric and nonparametric Bayesian test results, found volatility feedback to be statistically insignificant. Thus, the risk-return relationship is investigated free from empirical distortions that result from the volatility feedback mechanism. The parametric Bayesian test result finds a significant risk-return relationship that is positive and linear in nature. In contrast, the nonparametric Bayesian test result reveals the risk-return relationship is insignificant. Hence, the final result of this study is that there is no relationship between risk and return in the South African market.

4.4 Chapter Summary

According to Li (2018), financial price data of an entire financial system assumes a symmetric and normal distribution. However, financial data is intuitively and empirically known to have a volatile nature, in line with Harris (2017). From the empirical test results that describe the data dynamics, it is confirmed that the ALSI returns has an asymmetric and volatile nature by the basic descriptive statistics, normality and heteroskedasticity tests. Given the nature of the ALSI returns and the presence of volatility clustering, the application of the GARCH approach is motivated, in line with the recommendation by Khan *et al.,* (2016).

According to Spierdijk (2016), the probability distribution governing the model innovations does not affect parameter estimation. Thus, the GARCH (1, 1) models are estimated for four innovation distributions, NORM, Std-t, Skew-t and GED, following Mandimika and Chinzara (2012). The GARCH (1, 1) model has an inability to capture asymmetric volatility (Park *et al.,* 2017; Hretski and Karachun, 2018). From the sign and size bias tests, the presence of asymmetric volatility is confirmed. Further, the GARCH (1, 1) model does not adequately capture asymmetric effects within the ALSI returns as shown by the joint effect test. Given the nature of the data and the inability of GARCH (1, 1) capturing the asymmetric effects, the employment of asymmetric GARCH type models are motivated. The asymmetric GARCH type models are GJR-GARCH (1, 1), EGARCH (1, 1) and APARCH (1, 1), in line with a number of studies such as Mandimika and Chinzara (2012), Adu *et al.,* (2015), Khan *et al.,* (2016) and Savva and Theodossiou (2018).

The GJR-GARCH (1, 1), EGARCH (1, 1) and APARCH (1, 1) models confirm the presence and persistence of volatility by the ARCH and GARCH effects. From the model testing, information criteria, the EGARCH (1, 1) model is the best fitting asymmetric GARCH type model. However, a certain level of risk is left behind in the innovations and remains uncaptured (Feng and Shi, 2017). From the normality, randomness and heteroskedasticity tests, it is found that asymmetry remains uncaptured within the innovations of the EGARCH (1, 1) model, in line with Managni (2008), Ilupeju (2016) and Feng and Shi (2017). Consequently, asymmetric returns exposure is not being effectively captured. Thus, the GARCH approach can be concluded as an inefficient choice in estimating the risk-return relationship, in line with Jensen and Maheu (2018) and Jin (2017).

The GARCH (1, 1)-M and EGARCH (1, 1)-M model is used to price risk to establish the presence of the risk-return relationship and volatility feedback, respectively. The GARCH (1, 1)-M model finds an insignificant risk premium for the Skew-t innovation distributions at all three levels of significance. On the other hand, the risk premium parameter is significant for NORM, Std-t and GED. In contrast, the EGARCH (1, 1)-M model finds a significant risk premium for all the innovation distributions. Thus, due to the mixed results, the theory by Spierdijk (2016), where the distribution of model innovations does not affect parameter estimation is unsupported, in the context of the risk-return relationship. Overall, the GARCH approach does not provide statistically sound results regarding the presence of the risk-return which motivates further testing.

Therefore, this study applies the Bayesian approach by Jensen and Maheu (2018). This is in line with the early recommendation of effectively accounting for asymmetric and nonlinear properties when the results of the risk-return relationship are inconclusive by Mandimika and Chinzara (2012). Volatility feedback is investigated by determining the relationship between log realised variance in relation to three levels of volatility. The Bayesian test results, consisting of both the parametric and nonparametric Bayesian models, indicate that volatility feedback is insignificant in the South African market. In contrast, to a number of studies such as Jin (2017), Kim and Kim (2018), Jensen and Maheu (2018) and Harris *et al.,* (2019), who found a strong presence of volatility feedback in other markets. However, the study by Sultan (2018), found a weak presence of volatility feedback in the emerging Finnish market. This study follows the sample period of Sultan (2018), which does not account for the 2008 financial crisis. As a result, this choice could be the main reason for the absence of volatility feedback in the South African market.

The risk-return relationship is investigated free from empirical distortions that result from volatility feedback. The test results of the parametric Bayesian model found a significant positive and linear risk-return relationship. This finding is in line with recent studies from international literature such as Madaleno and Vieira (2018), Kim and Kim (2018), Jensen and Maheu (2018) and Harris *et al.,* (2019). In contrast, the result of the nonparametric Bayesian model found no relationship between risk and return in the South African market, in line with early South African studies by Mangani (2008), Mandimika and Chinzara (2012), du Toit (2015) and Adu *et al.,* (2015).

CHAPTER 5

5. Conclusion

This chapter sets out to conclude this study with a discussion of the findings and final test results in the context of the research objectives. First, is a summarised overview then second, are the limitations and finally, a number of recommendations are made for future research purposes.

5.1 Summary of Study

The risk-return relationship holds fundamental importance to the fields of finance and economics as well as useful information to various market participants. Due to conflicting results over the years, this has caused an ongoing local and international debate to arise. There are a number of factors and theories that attempt to explain the magnitude of varying results which motivated the pursuit of this research. From a broad analysis, results can easily vary, from study to study, as a result of different choices such as data frequency, sample period and model specification as noted by Savva and Theodossiou (2018). However, this study identified a trend in the use of conventional methods over a twenty-year gap, despite the drawbacks of the models being highlighted in literature. The foremost being the parametric GARCH approach which is subject to a number of nonnegativity constraints, limited in its ability to capture asymmetric properties and fully capture risk. This heavily contributes to the problem of inconclusive results regarding the risk-return relationship, thus, offering no conclusive solution to the ongoing debate.

With respect to South African literature, the earliest studies found an insignificant risk-return relationship, whereas recent studies found a negative result. More importantly, the methods employed showed no real progression over the years, in comparison to international literature. Most of the studies applied the GARCH approach such as Mangani (2008), Mandimika and Chizara (2012), Adu *et al.,* (2015) and Ilupeju (2016). On the other hand, the most recent study by Steyn and Theart (2019), applied regression analysis which is a basic parametric method that is limited in its ability to capture asymmetric properties. In contrast, although international literature had significantly more conflicting results, recent studies had two distinct differences relative to local literature.

Firstly, there was a progression in the methods applied to the risk-return relationship. At first, the majority of studies applied the GARCH approach based on its conventional use such as Chou (1988), Park *et al.,* (2017) Sultan (2018) and Savva and Theodossiou (2018). Then in recent years, a number of studies began using the nonparametric approach in conjunction with conventional methods. This was in order to derive the benefits of a nonparametric approach such as accounting for asymmetry and model misspecifications (Apergis *et al.,* 2018; Demirer *et al.,* 2019). The nonparametric approach was applied to methods of interest such as the VAR model by Umutlu (2019) and causality tests by Apergis *et al.,* (2018). Additionally, more unconventional methods were introduced such as the unified framework by Kim and Kim (2018) and the nonparametric Bayesian approach by Jensen and Maheu (2018).

Secondly, following these studies that used the more robust unconventional methods, volatility feedback was taken into account and a positive risk-return relationship was found, in line with theoretical expectations. Volatility feedback, which is a stronger measure of volatility, was treated as an important source of asymmetry by Jensen and Maheu (2018), Kim and Kim (2018) and Harris *et al.,* (2019). This ultimately led to the main aim of this study which was to investigate the risk-return relationship provided volatility feedback was taken into account by its magnitude.

In the build up to this research's objective, the limitations of conventional methods such as regression analysis, VAR, causality tests and the GARCH approach was highlighted. It was noted that a number of studies individually recommended the nonparametric approach and Bayesian approach, respectively for more robust data estimation (Karabatsos, 2016; Jin 2017; Chang *et al.,* 2017; Waldmann, 2018; Wagenmakers *et al.,* 2018; Apergis *et al.,* 2018; Jensen and Maheu, 2018; Demirer *et al.,* 2019).

The Bayesian approach has the ability to automatically adjust for sources of uncertainty and measurement errors surrounding parameters; thus, ensuring an efficient estimation of risk. The nonparametric approach has the ability to effectively account for asymmetric properties such as skewness, kurtosis and multiple modes in an infinite sample space. In direct contrast, to the design of the conventional parametric approach where the number of parameters is restricted to the sample size. Hence, the parametric model has an inability to account for every possible risk-return relationship that can hold, particularly higher moment asymmetric forms of the risk-return relationship.

The nonparametric framework is a “model free” approach where there are no assumptions or constraints imposed on the data. Model misspecifications are adjusted for and as a result, there is no need for model extensions, specifications and accounting for various sources of asymmetry. In contrast, to the GARCH family where a number of modifications have been made over the years to the standard GARCH (1, 1) model. However, despite these modifications, the drawbacks of the parametric approach essentially still hold such as the assumptions and nonnegativity constraints imposed on the data as well as the risk that remains uncaptured within the innovations. This is because data analysis of real world data often requires a method that relaxes parametric assumptions. Thus, allow for flexibility that enables the actual fundamental nature of data to be captured.

A model that satisfies these conditions is the nonparametric Bayesian approach by Jensen and Maheu (2018). The nonparametric Bayesian approach is a combination of the two most robust methods recommended by literature, respectively in the estimation of data with nonlinear, asymmetric, latent and stochastic properties (Karabatsos, 2016; Wagenmakers *et al.,* 2018). Consequently, this produced a powerful method for the estimation of the risk-return relationship. Hence, the methodology of this study followed Jensen and Maheu (2018), who made use of golden standard nonparametric Bayesian methods, namely, the Dirichlet Process, the slice sampler and Gibbs sampling technique. The study by Jensen and Maheu (2018), was the first and only study to apply the nonparametric Bayesian approach to the risk-return relationship and volatility feedback topic, to the best of the authors knowledge.

In order for the main aim to be addressed, the presence of the risk-return relationship and volatility feedback were both investigated by the GARCH approach. The results of the GARCH approach were not statistically sound as they were inconsistent due to the risk being uncaptured by the innovations, in line with Mangani (2008), Ilupeju (2016) and Feng and Shi (2017). The inconsistent results were also in contrast to Spierdijk (2016), who stated that the probability distributions governing the model innovations does not affect parameter estimation. As a result, this reaffirmed the decision of regarding the GARCH approach as a preliminary test. Since the GARCH approach was an inefficient choice to estimate the actual risk-return relationship, the application of the novel Bayesian approach was motivated.

In this study, the Bayesian approach consisted of a parametric Bayesian model and nonparametric Bayesian model. Both models have the ability to effectively account for the uncertainty associated to “asymmetric returns exposure”. However, the nonparametric Bayesian model is more robust because it has the ability to effectively account for every possible asymmetric higher moment form of the risk-return relationship in an infinite sample space.

Given that South Africa is an emerging market which is subject to higher levels of volatility, the presence of volatility feedback was expected to be more pronounced. However, contrary to expectations, the test results from both the parametric and nonparametric Bayesian model showed that volatility feedback had an insignificant effect in the South African market. The result of absent volatility feedback was in contrast to a number of studies that found volatility feedback present in emerging and developed markets such as Jin (2017), Harris *et al.,* (2019), Kim and Kim (2018) and Jensen and Maheu (2018). However, the result was in line with theory drawn from literature by Inkaya and Okur (2014), where the absence of volatility feedback suggests the market conditions of an economic recovery. This was further in line with the sample period analysed in this study - 2009 to 2019 - which was the post 2008 financial crisis period.

The risk-return relationship was investigated free from empirical distortions that resulted from volatility feedback. The test results of the parametric Bayesian model found a significant and positive risk-return relationship, in line with traditional theoretical expectations as well as recent studies by Madaleno and Vieira (2018), Kim and Kim (2018) and Harris *et al.,* (2019). In the context of the nonparametric Bayesian model, the absence of volatility feedback enhanced the approach as not only was the risk-return relationship estimated free from empirical distortions but the parametric assumptions were relaxed. The estimation of the actual fundamental nature of the data essentially allowed “the data to speak for itself” and model its own robust result free from any predetermined assumptions or bias. This was in line with Bekiros *et al.,* (2017), who found that the “actual returns are the most important factors” in the context of investigating the risk-return relationship.

In contrast, to the test results of the parametric Bayesian model, the nonparametric Bayesian model found an insignificant risk-return relationship. The insignificant risk-return relationship was in line with the early South African studies that applied the GARCH approach such as Mangani (2008), Mandimika and Chinzara (2012) and Adu *et al.,* (2015), as well as international studies by Savva and Theodossiou (2018), Umutlu (2019) and Apergis *et al.,* (2018).

In summary, the absence of volatility feedback and no relationship between risk and return in the South African market is still a source of useful information with respect to investors, policy makers and researchers. Although investors and arbitrageurs cannot capitalise on mispricing's, they can still be guided in terms of their investment strategies and decisions (Hussain *et al.,* 2019).

An investor can include South Africa in their investment portfolio with higher risk countries in order to spread their risk and derive diversification benefits. Policy makers are able to understand the behaviour of their market participants and create policies that promote economic growth on a macroeconomic level (Liu, 2019). Although volatility feedback is absent, particularly high levels of volatility, volatility itself is still present at low persistence levels. Thus, pose a challenge for policy makers (Vo *et al.,* 2019). It has the potential to slowly deteriorate market stability along with macroeconomic and political shocks, in line with Marozva (2019). Given global interconnectedness, this makes markets prone to volatility spillover effects of which can lead to economic instability, major cash outflows and a potential financial crisis (Gulzar *et al.,* 2019). This can be avoided by export diversification and the maintenance of macroeconomic and political regulatory frameworks according to Mandimika and Chinzara (2012).

To conclude, if a model can effectively estimate risk, there is no need for model extensions, specifications and omitted variables biases. This includes accounting for sources of asymmetry that seem manifold, considering there are so many factors and theories. This includes volatility feedback, the leverage effect, skewness, macroeconomic fundamentals, inefficient information, behavioural biases and different investor sentiment (Yu *et al.,* 2018). Moreover, a model designed to capture nonlinear and asymmetric properties is more likely to effectively capture these properties and estimate a nonlinear risk-return relationship. Given the magnitude of international literature, the importance and ongoing debate regarding the risk-return relationship, this study offers a contribution from a South African market perspective.

In order to make a meaningful contribution, a study should not employ methods that can be considered irrelevant and obsolete, given the existence of more robust methods such as the nonparametric Bayesian approach by Jensen and Maheu (2018). According to Thomson (1994), “Experience with real world data, however, soon convinces one that both stationarity and Gaussianity are fairy tales invented for the amusement of undergraduates”. Thus, sophisticated and unconventional methods are encouraged as it can inspire a new perspective, a way of thinking and an approach to a problem. Additionally, a robust method is more likely to give a reliable result paving the way for progression in any field and topic.

5.2 Limitations of Study

Although the nonparametric and parametric Bayesian model accounts for the leverage effect, it was not the focus of this study and was thus ignored (Jensen and Maheu, 2018). This is because volatility feedback is empirically favoured, in comparison to the leverage effect for a number of reasons briefly given: First, it reflects the risk-return relationship based on its assumptions by Umutlu (2019) and second, it is in line with the theoretical risk-return relationship by Jensen and Maheu (2018). Third, the main reason is that it is not associated to the amount of debt a firm has (Cao *et al.,* 2018). The amount of debt a firm has is associated to capital structure which may have a negligible effect on volatility as opposed to a negative effect (Horpestad *et al.,* 2019; Aboura and Chevallier, 2018).

In the Bayesian approach, the theory and empirical model are closely related by means of prior information (Herath, 2019). This is also the main drawback of the Bayesian approach because prior information can be modelled from a source of subjectivity such as prior beliefs or experience (Bartlett and Keogh, 2016). However, this is addressed by incorporating a relatively mathematically convenient mechanism in this study - volatility feedback as well as objective prior specifications in the model implementation stage (Karabatsos, 2016).

5.3 Extensions for Future Research

According to Inkaya and Okur (2014), the use of high frequency data has become popular given the rise in high frequency trading. Thus, daily data provides a more precise estimate of variables (Liu, 2019; Jin, 2017; Inkaya and Okur, 2014). Further, the presence of volatility feedback is more pronounced when using daily data according to Sultan (2018). Consequently, this may increase the magnitude of the volatility feedback mechanism and improve the estimation of the risk-return relationship once it is taken into account. Therefore, this study recommends using higher frequency data such as tick data, in line with Jensen and Maheu (2018), provided its availability and accessibility.

According to Inkaya and Okur (2014), the presence of strong volatility feedback suggests recessionary market conditions, whereas a weak presence indicates expansionary market conditions. In this study, the absence of volatility feedback indicates the conditions of an economic recovery which makes sense because the sample period analysed was the post financial crisis period. Therefore, this study recommends analysing a sample period before, including and excluding the 2007/2008 financial crisis because it can provide useful information to researchers and investors alike. First, to investigate if the result of volatility feedback lines up with theory drawn from Inkaya and Okur (2014) as it did in this study. Second, the different levels of volatility in each period is likely to affect risk estimation; thus, the empirical result of the risk-return relationship. Third, information on the magnitude volatility feedback alone or in relation to the risk-return relationship can assist an investor in strategising their investment to improve their probability of realising a superior return (Huang and Startz, 2019).

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ETHICAL CLEARANCE REPORT

UNIVERSITY OF
KWAZULU-NATAL
INYUVESI
YAKWAZULU-NATALI

Miss Nitesha Dwarika (214545974)
School Of Acc Economics&Fin
Westville

Dear Miss Nitesha Dwarika,

Protocol reference number: 00003164
Project title: The risk-return relationship and volatility feedback in South Africa: a nonparametric Bayesian approach

Exemption from Ethics Review

In response to your application received on 20 August 2019, your school has indicated that the protocol has been granted **EXEMPTION FROM ETHICS REVIEW.**

Any alteration/s to the exempted research protocol, e.g., Title of the Project, Location of the Study, Research Approach and Methods must be reviewed and approved through an amendment/modification prior to its implementation. The original exemption number must be cited.

For any changes that could result in potential risk, an ethics application including the proposed amendments must be submitted to the relevant UKZN Research Ethics Committee. The original exemption number must be cited.

In case you have further queries, please quote the above reference number.

PLEASE NOTE:
Research data should be securely stored in the discipline/department for a period of 5 years.

I take this opportunity of wishing you everything of the best with your study.

Yours sincerely,

11 October 2019

Prof Josue Mbonigaba
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