Investigation of the South African Public TVET Colleges' Engineering Official Mathematics Curriculum for Entry Level Artisans

By

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Supervisor's permission to submit

As the candidate's supervisor, I agree to the submission of this dissertation.

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Abstract

One of the main objectives of any mathematics curriculum is to equip students with the necessary thinking skills for real-world problems. As the world evolves every day of our lives, so do the people living in it. Hence, the same exceptional functioning curriculum used in previous years is highly possible to be dysfunctional in the current days. Therefore, time and again curriculum evaluation is essential for both Basic and Higher education. However, before the actual curriculum evaluation, one should identify or develop suitable evaluation tool/s. In that regard, this study focused on the evaluation of the public Technical and Vocational Education and Training (TVET) colleges' mathematics curriculum from N1 to N2. Initially, the intention of the current study was to collect data across all KwaZulu-Natal TVET Colleges, which was unsuccessful due to a lack of cooperation from some of the TVET colleges' gatekeepers. The study was only able to access the eMnambithi TVET College data set, where 47 students participated. Two aspects were evaluated, namely, the participating students' attainment of the curriculum objectives and the ability of the curriculum to equip students with high order thinking skills (HOTS). The Tyler's objective model was adopted to evaluate the effectiveness of the curriculum to train students for the attainment of the curriculum's objectives. That was done using the pre- and post-assessments method as stated by the pioneer of that model. The results indicated that the curriculum was most likely to be incapable of equipping the students for the attainment of its own objectives. Further on, this study used the Susceptible-Infected-Recovered (SIR) model to develop a new model called the Susceptible-Vaccinated-Healthy-Infected-Recovered (SVHIR) model. The SVHIR model was used to evaluate the effectiveness of the curriculum to equip students with HOTS. Also, the results obtained from the SVHIR model indicated that the curriculum was most likely to be incapable of equipping the students with HOTS. It was also found that the students' ability to attain the curriculum objectives and their HOTS have a strong linear relationship. The latter implied that fully equipping students with HOTS should enable them to better attain the curriculum objectives. The convenience sampling supports the need to conduct a future study that covers all the TVET colleges that did not respond to the researcher's request for access on time. Further pursuance will give more clarity and findings that may or may not differ that much with the ones of this reported study.

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List of Abbreviations

CAR	Classroom Action Research
CIPP	Context evaluation, Input evaluation, Process evaluation and Product evaluation
DHOTS	Degradation of High Order Thinking Skills
DLP	Dual Language Program
HOTS	High Order Thinking Skills
ICT	Information Communication Technology
KICD	Kanya’s Institute of Curriculum development
KZN	KwaZulu-Natal
MHOTS	Mathematical Higher-Order Thinking Skills
MPK	Mathematical Prior Knowledge
N1, N2, N3, N4, N5 and N6	NATED Levels 1, 2, 3, 4, 5 and 6
NATED	National Accredited Technical Education Diploma
OBE	Outcomes-Based Education
ODEs	Ordinary Differential Equations
OIDDE	Orientation, Identify, Discussion, Decision, and Engage
SIR	Susceptible-Infected-Recovered
SPSS	Statistical Package for the Social Science
SVHIR	Susceptible-Vaccinated-Healthy-Infected-Recovered
TIMSS	Trade in International Mathematics and Science Study
TVET	Technical and Vocational Education and Training
UN	Ujian Nasional in Indonesia
WIL	Work-Integrated Learning

Chapter 1: Background and Overview

1.1. Introduction

The idea of evaluation is not a modern approach, as early as 2200 B.C. this idea existed. The public officials of Chinese emperors were required to show proficiency in formal tests during that time, which was and still is a form of evaluation *[Guba and Lincoln, 1981]*. It was between the years 1930 and 1960 when curriculum evaluation was noticed to mostly involve a variety of evidence on student performance and program effectiveness *[Tyler, 1949; Stufflebeam, 1971; Scriven, 1972; Lewy, 1973; Parlett and Hamilton, 1977; Eisner, 1979; Wood, 2001; Youker, 2013; Dewantara, 2017 and Zurqoni et al, 2018]*. The current study adopted the same idea to evaluate the public South African TVETs (Technical and Vocational Education and Training) Colleges' curriculum by using TVET Colleges situated in KwaZulu-Natal (KZN). The study chose the province of KZN because it has the highest number of the public TVET Colleges than all the provinces in South Africa.

Heong et al (2011) mentioned that HOTS comes with the most important abilities for a student, namely, information transfer, critical thinking and problem solving. *Tanujaya et al (2017)* in their study found a strong linear relationship between High Order Thinking Skill (HOTS) and a student's academic achievement. This implies that ensuring students are equipped with HOTS increases their chances of academic achievement. There are many benefits that comes with HOTS such as enabling students to evaluate and reason in the context of decision making, to be creative, and to analyse, evaluate and create *[Ariyanto, 2019]*. In that regard, the current study views HOTS as one of the most important skills every student should acquire. Hence, one of the main components to be evaluated in this study in the context of the TVET curriculum for mathematics in South Africa, is its ability to equip students with HOTS. Therefore, as an introduction this chapter presents to the reader an overview of what the current study intended to accomplish, and it is structured as follows: problem statement, aim of the study, objectives of the study, research questions, thesis overview, conclusions, and publications.

1.2. Problem statement

During the 1980s and early 1990s in South Africa, the TVETs national accredited technical education diploma (NATED) programmes (in report 191) were traditionally linked to apprenticeship through the Manpower training Act (No. 56 of 1981) *[Government Gazette, 1990]*. Those programmes were offered as pre-grade 12 (N1 to N3) and post-grade 12 (N4 to N6) programmes; grade 12 is the exit grade for the formal school system in South Africa. In those N1 to N6 programmes, the engineering programmes were offered on a trimester basis and the business programmes were offered on a semester basis.

Towards the end of the old government's system (apartheid) in South Africa, the apprenticeship system fell into decline. All the programmes were opened to pre-employed students instead of only students in apprenticeships *[Kraak, 2004]*. Then since year 2007, the public South African TVET Colleges offered a variety of NATED programmes (N1 – N6) to both employed and pre-employed students. Their engineering programmes prepare artisans for disciplines such as construction, metalwork, electricity, and mechanics. However, after completing these programmes, students still need to undergo a trade-test preparation and testing before they become entry level artisans.

Previous studies focusing on the evaluation of the curriculum are done using the qualitative method of data collections. However, the current study made use of the qualitative method to explore a new approach of evaluating the curriculum using quantitative method. Normally, curriculum is evaluated by collecting a non-numerical data (quantitative method); whereas the current study evaluates the curriculum by collecting numerical data (qualitative method). In that regard, most of the studies in our literature review are quantitative studies.

1.3. Aim of the study

In as much as we had two critical questions in front of us, it was incredibly important to first devise the optimal tools to answer those two questions. Therefore, the current study aimed to find the two optimal models, which could be used to investigate the capability of the N1 to N2 mathematics curriculum to deliver the curriculum objectives and to equip students with HOTS in the TVET Colleges.

1.4. Objectives of the study

The objectives of this work were:

i) To modify the Tyler's objective Model so that it could be used to evaluate if the N1 to N2 TVET mathematics curriculum is capable enough to equip students for objectives attainment.

ii) To develop a mathematical model, which could be used to evaluate if the N1 to N2 TVET mathematics curriculum is capable enough to equip students with the necessary HOTS.

1.5. Research questions

Under the current South African government, students with only N1 to N2 TVET training are permitted to undergo trade-test preparation and testing to become entry level artisans. This presumes that the N1 to N2 curriculum is sufficient to equip students with necessary thinking skills required by the industry or real-world problems space. Among those required thinking skills is the high order thinking skill (HOTS), where mathematics is one of the contributing subjects/modules to those kinds of skills.

Therefore, in this study we intend to find out if the N1 to N2 mathematics curriculum is capable enough to equip students with curriculum objectives attainment abilities and HOTS. Hence, the main research question of the study is:

How can the TVET mathematics curriculum from N1 to N2 be evaluated with a focus on the students' attainment of the curriculum objectives and the equipping of students with the necessary HOTS?
The study answers the main question by focusing on the following two sub-questions:

i) Is the TVET mathematics curriculum from N1 to N2 capable enough to equip the students for the attainment of the following curriculum objectives?
   - Application of mathematical principles.
   - Use of the correct mathematical terminology and to identify the appropriate formulae.
   - Use of the correct SI units
   - Logical reasoning

ii) Is the TVET mathematics curriculum from N1 to N2 capable to equip students with the necessary HOTS?

The above two sub-questions (i and ii) are of an extremely critical nature, and suitable tools to answer them were required. In turn, the construction of those tools almost formed a separate study on its own. Hence, the most part of this current study focused on selecting and developing suitable models (tools) that could be used to answer the above two sub-questions, since the precision of the answers to those sub-questions highly depended on the accuracy of those models.

1.6. Thesis chapter outline

The current study thesis consists of 6 chapters, and this section gives a summary of all the chapters in this thesis.

Chapter 1 presents the introduction and overview of the current study. It presents the problem formulation, aim and objectives of the study, research question, thesis overview and summary of the chapter.

Chapter 2 presents the literature review. It begins by reviewing the study of curriculum evaluation, including some of the evaluation models, and continues by reviewing the literature on the TVET Colleges in and outside South Africa. Again, it reviewed HOTS studies and lastly presented the chapter summary.

Chapter 3 presents conceptual framework. It begins by explaining the two relevant parts of Tyler's model to the current study, which are selection and evaluation of the learning experiences. Further on, its details the scope of this work. Lastly, it presents the chapter summary.

Chapter 4 presents the methodology. It first narrates the application of the Tyler's model to the current study. Secondly, it shows the development and application of the new model (SVHIR) for HOTS evaluation. Lastly, it includes the chapter summary.

Chapter 5 presents the application of the Tyler's objective model and SVHIR model to the actual data, which is the investigation of the capabilities of the TVET mathematics curriculum. Also, the chapter presents the relationship between the students' ability to attain curriculum objective and their HOTS. Lastly, it gives the chapter summary.

Chapter 6 presents the introduction, overview conclusions of the study, limitations of the study, and recommended themes for further studies.

1.7. Conclusion

This chapter has introduced the problem statement, aim and objectives of the study. The goal of this chapter was to prepare the reader for the upcoming chapters, which requires a clear understanding of this chapter.

Chapter 2: Literature Review

2.1. Introduction

Teaching and learning should be one of the fields that research needs to focus on, since most of the other fields' participants are products of teaching and learning. Hence improving this field ultimately improves many other fields. Over the years, there was notable growth of curiosity and realization in curriculum studies; specifically in assessing the relevance, reliability, effectiveness, and up-to-datedness of the curriculum in operation. As a result, curriculum evaluation is gradually becoming a main part in the development of the curriculum to assure the effectiveness of the program in use *[Danju, 2017]*. There were tremendous contributions achieved by various researchers around curriculum evaluation in the past. Therefore, this chapter reviews and appreciates the curriculum evaluation work that was done over the years. It consists of subsections structured in the following manner: Firstly, its curriculum evaluation where some of the most relevant and dominant curriculum models are reviewed. Secondly its curriculum evaluation in the Technical and TVET Colleges, where the progress of the curriculum evaluation studies in TVETs is reviewed inside and outside of South Africa. Thirdly its HOTS, this is the visitation of the HOTS studies done in the past in South Africa and outside South Africa. Lastly, it's the conclusion of the chapter which is the summary of the work done and the proposed gap to be closed by the current study.

2.2. Curriculum Evaluation

Evaluation is a methodological process rather than content specific *[Fox and Hackerman, 2003]*. It can be seen as a continuous effort to investigate the effect of an operational content and procedures to achieve pre-determined goals. Some scholars view evaluation as a process that essentially depends on gathering and combining data as to allow people's judgements about pre-set goals or scales *[Hunkins and Ornstein, 1988]*. The way individuals process the collected data, is mainly determined by the extent of their ideological approach. Looking at the humanists, their view is that quantitative expression of learning outcomes is not sufficient to decide the quality of learning *[Manaf and Rahman, 2017]*. They have a notion that the learning environment on its own is very important; it should assist the students to improve their self-concept. On the other hand, the behaviourists approach the evaluation in a different perspective where goals and objectives are a central part of the curriculum implementation *[Tyler, 1949]*. They also normally follow a sequenced or orderly manner, where they erect procedures or principles in chronological steps to be followed *[Tyler, 1949; Stufflebeam, 1971; Scriven, 1972 and Sharma and Raval, 2019]*. Nonetheless whatever approach may be adopted, the concept of evaluation “models” remains applicable on both humanistic and behavioural approach. Therefore, this section will be reviewing some of the curriculum evaluation models that have contributed to the past.

2.2.1. Evaluation Models in the Years 1970 to 1980

In the past years, numerous researchers dedicated their time in curriculum research for the improvement of students’ learning experiences. As a result, we (the researcher and his supervisor) noted that many curriculum evaluation models were contributed to the field of curriculum studies over the years. We also noticed that some models gained more fame than others because of variation of the properties and uniqueness. A review of the literature indicated that one of the most noticeable decades where development of evaluation models emerged was between the years 1970 – 1980. Therefore, this sub-section will look at some of the most relevant models that emerged in that decade.

Earlier in the beginning of that decade in 1971, Danial Stufflebeam developed a model that emphasized the importance of producing evaluative data for decision making *[Stufflebeam, 1971]*. The generation of the data with Stufflebeam model is mainly based on the four stages of the operational program namely context evaluation, input evaluation, process evaluation and product evaluation (CIPP) *[Stufflebeam, 1971]*. Recently in year 2018, the Stufflebeam model was applied at various branches of Welfare school system in Rawalpindi. Learning experiences were found to be satisfactory; however, teachers were identified to be focusing more on theoretical work and rote learning. That had a negative effect on students, since it put pressure on them and limited their intellectual abilities *[Aziz et al, 2018]*.

A year later after Stufflebeam model, Michael Scriven presented his goal-free model *[Scriven, 1972]*. He was the one person to question the importance of goals or objectives in the evaluation process. The model mostly focuses on the effects of the program rather than on it goals and objectives, hence it is called Scriven’s goal-free model. He explicitly stated that his model does not substitute but rather compliments the goal-based models. Therefore, if used alone it will not provide sufficient information to make decisions *[Scriven, 1972]*. His contribution was well recognized; hence the concept “goal-free evaluation” was appreciated by some scholars to such an extent that they applied it on their own developments *[Worhen, 1990; Youker, 2013 and Zurqoni et al, 2018]*.

Halfway through the decade, another different evaluation model called Stake’s responsive model was introduced. This typical model assumes that the concerns of those evaluated should be the priority *[Stake, 1975]*. This is one of the models which is continuously receiving recognition up to these current years. *Wood (2001)* found the Stake model to be demonstrating its effectiveness, when it was applied during the evaluation of an environmental education professional development course. The model was successfully applied through examination of both qualitative and quantitative data during all phases of course execution. On the other hand, Dewantara found the model to be not applicable for the small sample sizes (315 sample sizes). Hence, he suggested a larger sample size to get a clear picture of evaluation *[Dewantara, 2017]*.

Right before the end of that decade connoisseurship model was introduced in the field of curriculum studies, where connoisseurship means art of appreciation *[Eisner, 1979]*. That model's evaluation approach is mostly based on raising awareness of the qualities that constitute some process or object and to grasp their significance. The model was built on two related constructs: connoisseurship and criticism. That introduced a new perspective than what was commonly known during those times. Further, there were many other contributions witnessed during that decade (1970 – 1980) not discussed in this current study *[Lewy, 1973; Walker and Schaffarzick, 1974; Parlett and Hamilton, 1977; Sage, 1978 and Apple, 1980]*. We recognize and appreciate all those researchers and their contributions in the field of curriculum studies.

A particular important model has not appeared in our review so far. That model existed long before the 1970's began, it is called Tyler objective-centred model. That model is one of the essential tools for this current study, hence it is necessary for this study to give a bit more background on it.

2.2.2. Tyler's Objective-Centred Model

One of the old models that remain famous in many assessment projects was presented by Ralph Tyler called objective-centred model *[Tyler, 1949]*. The model mostly analyses, interprets, and explains the curriculum and instructional program of an educational institution in four simple steps. Those steps are selection of objectives, selection of learning experiences, organization of learning experiences and evaluation of learning experiences as shown in *Figure 2.1.* This model was identified to have several advantages. It has been defined as the most understandable and easy applicable model. It is rational and systematic. It mainly focuses on curriculum strengths and weaknesses, rather than on the performance of students as individuals. It also prioritizes the importance of continuing cycle of analysis, assessment, and improvement *[Glatthorn et al, 2015]*. Nonetheless, there is one main disadvantage of this model which some researchers are concerned about, as great as it is; the model does not provide the solution but rather it exposes only problems *[Guba, 1981]*. For instance, if a curriculum is found to be incapable through Tyler's model; the model might not provide the solution in most cases *[Glatthorn et al, 2015]*.

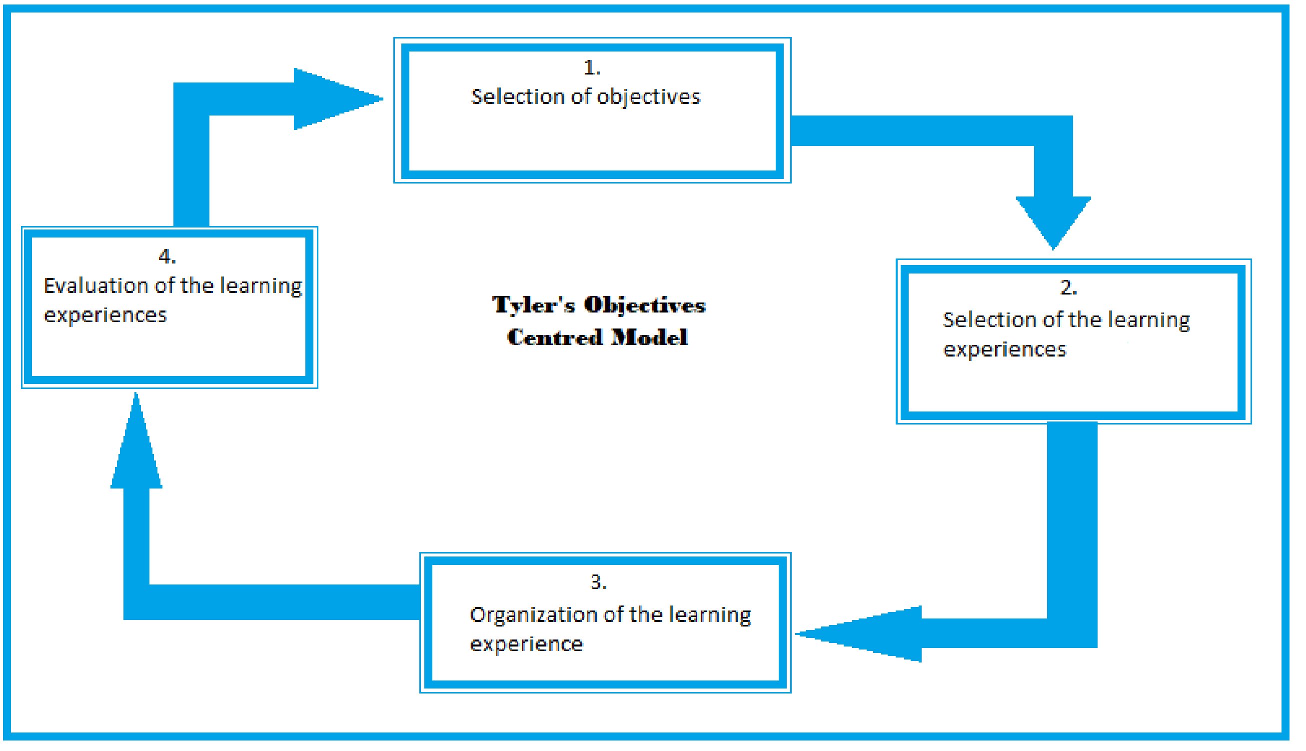

Figure 2.1: Illustration of Tyler's Objectives-Centred Model

The brilliance of the Tyler model attracted fame in the field of curriculum development, to such an extent that in some sense it has been considered as a standard procedure by many curriculum specialists and teachers *[Cruickshank, 2018]*. As Tyler mentioned in his book, he never intended to make a manual for curriculum construction*;* rather he was just constructing an alternative way for teachers and students to personally use to ensure the effectiveness of their curriculum at hand in their private spaces *[Tyler, 1949]*. Hence due to some of those reasons, the application of the model has been immense but without a proper due recognition in the space of publication. Nonetheless, there are some researchers who took it upon themselves to recognize the application of Tyler's model in their work. *Fundi (2015)* specifically focused on the evaluation component of Tyler's Model. He applied that component of the model on the Dunwoody High School student's physical science and biology end of course tests. In his analysis, he concluded that Dunwoody High School students met the physical science and biology curriculum objectives. Nevertheless, the current study holds a view that there is room for improvement in Fundi's application approach. The Tyler model emphasises on taking at least two appraisals for proper evaluation, the first one to be prior to the commencement of the program and second one at the end of the program *[Tyler, 1949]*. Fundi in his work never considered an appraisal prior to the commencement of the program. Again, it is not advisable to take the school test; given that under normal circumstances school tests don't cover all the objectives *[Tyler, 1949]*. Hence, they are not a good instrument for curriculum objectives evaluation. On the other hand, Fundi used the school tests for his evaluation.

However, we appreciate all the good effort put in by the scholars in the application of Tyler's model; it was one of the inspirations for this current study.

This section reviewed different kinds of relevant evaluation models developed by Stufflebeam, Scriven, Stakes, Eisner and Tyler. In this study we hold the notion that these models did not get enough appreciation and respect they deserve in the space of publication. However, we don't nullify the fact that there is some appreciation and respect for those models. For some models, we witnessed their official application in the publication space such as Stufflebeam and Stakes models. On the other hand, Scriven goal-free model was indirectly appreciated where one scholar took the idea "goal-free" to further implement another model. The current study views the above models as underutilized powerful tools; hence we took it upon ourselves to take advantage of the Tyler's model.

2.3. Curriculum Evaluation in the TVET Colleges

TVET Colleges are identified by some studies to be one of the driving forces behind economic development *[Soyemi and Soyem, 2019]*. Hence, there was a focus on the TVET curriculum that has never stopped to be a centre of attention in many parts of the world *[Grubb, 1985; Akinseinde, 2004; Okorafor, 2009 and Howell, 2010]*. In this section, we will review some of the recent TVET curriculum evaluation studies in and outside South Africa.

2.3.1. TVET Curriculum Evaluation Studies in South African

Buthelezi (2017) mentioned that TVET Colleges in South Africa went through a major structural, institutional and curriculum change over the last two decades. That was the result of the country's exposure to an apartheid system that brought imbalance and inequalities in people's lives as well as their education. The South African TVET College's reform has since been a topic up to those current years and was well documented *[Gewer, 2001; Wedekind, 2008; Papier, 2009; Towani, 2010 and Kraak, 2016]*. The process of TVET reformation was a very tedious process, specifically on the curriculum point of view. That was because in the reformation process mutual connection between the curriculum, lecturers, students, and society was not always maintained, in some cases. That task is not only planning and financially demanding, but also pushed some researchers to further spend their time on the study of TVET curriculum in the recent years.

A study was conducted to investigate the challenges of the TVET College's lecturers in the post-apartheid era *[Buthelezi, 2017]*. It was conducted within a qualitative research paradigm and used interviews to find lecturer's challenges in the TVET education reform. The findings revealed that in the

TVET sector there is lack of compatibility between curriculum reforms, students, and lecturers' adaptability to the reform. That is a very concerning challenge, since without this bond between those three components the main objectives of the TVET sector will not be achieved. Hence this call for researchers to pay an undivided attention on this problem.

In 2018 two researchers investigated the need of TVET College's curriculum to be restructured to accommodate more industry requirements and TVET standards *[Terblanche and Bitzer, 2018]*. Their approach of the study was questionnaire survey and follow-up group interviews of different TVET's staff representatives, in the Western Cape Province in South Africa. The study confirmed that there is a necessity for curriculum reform in the TVET Colleges. They suggested that various cognitive and social competencies are necessary for curriculum change and to deal with the resulting challenges. Those competencies include getting industry knowledge and exposure, which seems important for TVET Colleges.

Another study occurred in 2019, work-integrated learning (WIL) curriculum of South African's TVET Colleges investigated to see if it does accommodate Fourth Industrial Revolution (Industry 4.0) *[George et al, 2019]*. Quantitative approach in the collection and analysis of the data was adopted for that study. Data was collected through a questionnaire completed by lecturers from different TVET Colleges. It was found that lecturers were excited about Fourth Industrial Revolution; nonetheless most of the lecturers (60%) were not convinced that TVET Colleges are ready for the industry 4.0.

2.3.2. TVET Curriculum Evaluation Studies outside South Africa

One study mentions that the main objective of the TVET is to provide for the immediate needs of community in which it exists *[Eze and Okorafor, 2012b]*. Hence in the study one African TVET curriculum was evaluated to test if it met the needs of the country it serves. The investigation revealed that the Euro-centric curriculum accepted by Africans, has not fully addressed the problems of the African community. A new approach was suggested for the development of the curriculum in the TVET sector. Amongst the suggestions was indigenous curriculum involvement, adaptation rather than adoption of foreign curriculum, attitude re-orientation toward TVET and information communication technology (ICT) skills in the TVET curriculum.

Two scholars in Nigeria investigated how TVET curriculum contributed to the issue of TVET graduates lacking employability skills required by industries *[Ismail, 2014]*. In their evaluation, they found that the curriculum focused less on practice-based courses. Also, the curriculum lacked content that instil good attitude and traits in students. That leads to lack of skills such as problem solving, decision making, lifelong learning and some other important competencies.

Maina et al (2017) mentioned that one of the challenges about TVET institutions was inflexible and out-dated curriculum that is unable to meet ICT changes. As a result, there is a mismatch between the industries' skills requirements and skills acquired from the TVET curriculum, since large part of the world of work requires ICT. Therefore, the mentioned scholars took it upon themselves to evaluate the relevancy of the curricula content in achieving integration of ICT in Kenya's TVET institutions. They approached the study by adopting quantitative research and probability sampling. Their study suggested that ICT should be integrated in the operating TVET curriculum in Kenya, to meet up with industries required skills. Further, it was suggested that Kenya's institute of curriculum development (KICD) and TVET authority together with stakeholders ought to promote access and relevance of TVET training courses *[Maina et al, 2017]*.

In 2019, a study was conducted between the two countries Malaysia and Nigeria *[Mohammed, 2019]*. Part of it was to evaluate the two TVET curriculums against each other; given the two countries almost have similar backgrounds in terms of their journey to get independence. Their approach of the investigation involved analytical method of study that included evaluation of different materials that contain various components of the information about the two countries. Their study revealed that the Malaysia TVET curriculum is well furnished with employability skills such as personal attributes, core skills and generic skills. Hence Malaysia graduates are more likely to be exposed to development and acquire full employment than Nigerians.

In the same year, the Nigeria TVET curriculum was evaluated through investigating the level of relevance of the employed graduates produced by the current TVET curriculum. The study was done through structured questionnaire distributed in various TVETs in Nigeria. Descriptive and inferential approach was used for the purpose of analysis. The findings revealed that their TVET curriculum was weak; it could not provide sufficient skills for career take-off. Hence the study suggested the revamping of that curriculum, to include all necessary inputs from the industries *[Soyemi and Soyem, 2019]*.

This section presented all the relevant TVET curriculum evaluation work the current study was able to find. It was evident from what we gathered that in the South African TVET not much was done in terms of curriculum evaluation. However, studies that have been done so far suggest that the following could be a few challenges about the South African TVET curriculum: a lack of mutual connection between curriculum reform, students, and lecturers. Also, the curriculum is found to not accommodating industry requirements and the fourth industrial revolution. When looking at the TVET curriculum outside South Africa, there was not much done too on the evaluation side except in Nigeria. The challenges found in Nigeria TVETs are almost like those of South African TVETs. Hence the main challenge of the TVET curriculum in Africa specifically, is that it fails to equip the students with necessary skills required by industries.

2.4. High Order Thinking Skills

According to *Heong et al (2011)* higher order thinking skills (HOTS) is the expanded use of the mind to meet different challenges and bearing in mind that thinking skills are associated with the learning process. The HOTS is the combination of the following three abilities: transfer, critical thinking and problem solving *[Brookhart, 2010 and Maharaj and Wagh, 2016]*. Transfer is the ability of a student to relate their learning to other elements, other than the ones they were taught to associate *[Brookhart, 2010 and Maharaj and Wagh, 2016]*. Critical thinking is the ability of a student to decide what to believe, reason, reflect and make sound decisions on their own and produce a reasoned argument *[Brookhart, 2010 and Maharaj and Wagh, 2016]*. Problem solving refers to an ability to solve problems using unfamiliar and new methods *[Brookhart, 2010 and Maharaj and Wagh, 2016]*. *Maharaj and Wagh, 2016* elaborated the Brookhart abilities as shown in *Table 2.1*. The following are some mathematical examples based on concepts that appear in the TVET engineering mathematics syllabus *[Department of high Education, 1994 and 1997]*. The concepts are turning point, operations, and mensuration (volumes of 3-dimensional spaces). These illustrative examples that are modifications of questions that appear in the prescribed textbook *[Van Rensburg N1, 2014 and Van Rensburg N2, 2014]*, indicate what is meant by transfer, critical thinking, and problem-solving skills respectively:

<u>Transfer</u>

Determine the turning point of the function defined by

$$f(x) = 2x^2 + 3x + 2 \tag{2.1}$$

Solution:

Normally students are taught to use the turning point formula,

$$x = \frac{-b}{2a}. \tag{2.2}$$

Table 2.1: Elaboration of the Brookhart abilities.

Brookhart Abilities	Elaboration
Transfer	• Identify possible applications of mathematics in their surroundings. • Translate a worded or graphically represented situation to relevant mathematical formalisms.
Critical Thinking	• Work systematically through cases in an exhaustive way. • Interpret and extend solutions of problems.
Problem Solving	• Use with reasonable skill available tools for mathematical exploration.

Hence the turning point coordinates will be,

$$\left(\frac{-b}{2a}; f\left(\frac{-b}{2a}\right)\right).$$

However, a student possessing a transfer skill can apply the concept of derivative as follows:

$$f'(x) = 4x + 3, \qquad (2.3)$$

where (2.3) describe the slope of (2.1). It is also known that (2.3) at the turning point is equal to zero, therefore:

$$4x + 3 = 0, \qquad (2.4)$$

hence

$$x = \frac{-3}{4}. \qquad (2.5)$$

Substituting (2.5) into (2.1) we get the turning point co-ordinates are:

$$\left(\frac{-3}{4}; \frac{7}{8}\right)$$

Critical Thinking

Given that $5 * 2 = 13;\ 6 * 5 = 17;\ 7 * 6 = 19\ and\ 8 * 9 = 23$. Then what will replace the question mark in the following:

$$9 * 11 =? \qquad (2.6)$$

In this problem there could be more than one answer. However, it all depends on the argument and the reasoning of a student. Therefore, to answer this question the students need to construct an argument supported by reasons.

Solution:

(i) When we look at each of the three given results based on the operation above, the first number on the left-hand side of each operation varies in ascending order (5, 6, 7 and 8).

(ii) Again, for each for the given operation, the second number on the right-hand side varies in ascending order (2, 5, 6 and 9).

(iii) Also, the result or output of each varies in ascending order (13, 17, 19 and 23).

(iv) Lastly, it seems the operation adds the two numbers on either side of it and increases the result by 6.

Therefore, from (iv) it seems $a * b = a + b + 6$, so we also conclude that it holds also for (2.6) and hence:

$$9 * 11 = 26 \tag{2.7}$$

<u>Problem Solving</u>

Three identical tanks are shown in *Figure 2.2*. The spheres in each tank are the same size and packed wall-to-wall. If the tanks are filled to the top with water, then which tank would contain the most water?

A solution to this type of problem requires problem solving skills, given at its current state is not constrained to any mathematical concept. It will be upon each student's problem-solving skills to decide which mathematical concept or method (new or unfamiliar) to use to solve this problem.

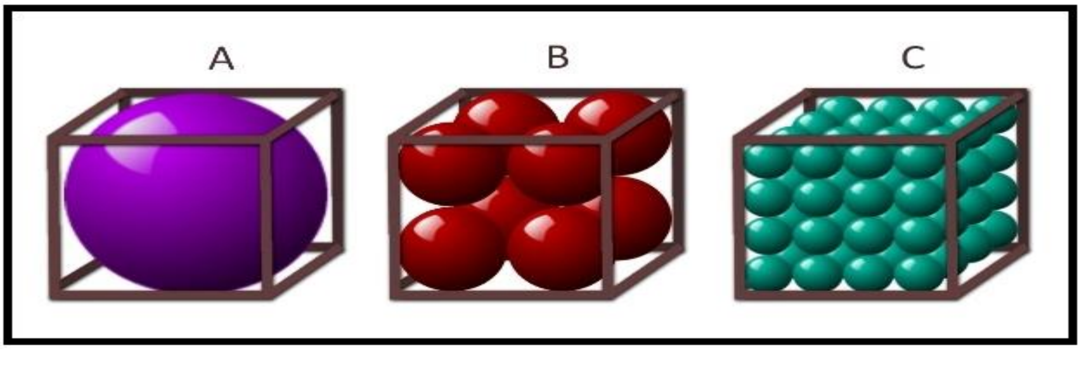

Figure 2.2: Three identical tanks with sphere(s) inside.

Solution:

One way to solve this problem is to relate it with the concept of volumes of 3-dimensional shapes. Since the tank(s) has a shape of a cube, therefore the volume of each tank in *Figure 2.2* is given by:

$$V_c = L^3, \tag{2.8}$$

where L correspond to the length of each side of the tank.

The volume of the sphere(s) inside each tank is given by,

$$V_s = k\frac{4}{3}\pi r^3, \tag{2.9}$$

where r correspond to the radius of the sphere and k is the number of spheres.

The volume of the empty space that will be filled with water is given by,

$$V_e = V_c - V_s = L^3 - \frac{4}{3}\pi r^3 \tag{2.10}$$

Therefore, if a side of each identical tank in *Figure 2.2* is the value d; we can find all the volumes we need in (2.8), (2.9) and (2.10).

Cube A:

The volume of the tank A from (2.8) is,

$$V_c = d^3. \tag{2.11}$$

The volume of the sphere inside tank A from (2.9) is,

$$V_s = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{\pi}{6}\ d^3. \tag{2.12}$$

The volume of the empty space inside tank A from (2.10) is,

$$V_e = V_c - V_s = d^3 - \frac{\pi}{6}\ d^3 \tag{2.13}$$

Cube B:

The volume of the tank B is,

$$V_c = d^3. \tag{2.14}$$

The volume of the 8 spheres in tank B is,

$$V_s = 8\left[\frac{4}{3}\pi\left(\frac{d}{4}\right)^3\right] = \frac{\pi}{6}\ d^3. \tag{2.15}$$

The volume of the empty space in tank B is,

$$V_e = V_c - V_s = d^3 - \frac{\pi}{6}\ d^3 \tag{2.16}$$

Cube C:

The volume of the tank C is,

$$V_c = d^3. \tag{2.17}$$

The volume of the 64 spheres in tank C is,

$$V_s = 64\left[\frac{4}{3}\pi\left(\frac{d}{8}\right)^3\right] = \frac{\pi}{6}\ d^3. \tag{2.18}$$

The volume of the empty space in tank C is,

$$V_e = V_c - V_s = d^3 - \frac{\pi}{6}\ d^3 \tag{2.19}$$

The empty volume (V_e) is the space that will contain or be filled with water in the tank/s. Therefore since (2.13), (2.16) and (2.19) shows that this free space is the same in all the tanks; hence all the tanks will contain the same amount of water.

In our view the concept of HOTS plays a very significant role in teaching and learning, given it is a concoction of components that are very vital for dealing with real world problems. In fact, in the absence of the components of HOTS a huge part of teaching and learning is meaningless. As expected for such a dominant concept, there have been numerous studies dedicated on HOTS *[Martin, 1989; Grossen, 1991; King et al, 1998; Kenimer and Morgan, 2003; Zohar and Dori, 2003; Nesbitt-Hawes, 2005; Polly and Ausband, 2009; Brookhart, 2010 and Collins, 2014]*. Therefore, this section will review *recent and relevant* studies on HOTS in South Africa. Also review studies that involve HOTS and models, given that is the interest of the current study. Then since the current study also focuses on mathematics curriculum, previous studies that relate HOTS and mathematics will be reviewed. Lastly the teaching of HOTS, since this is a broad and dominant concept that might be difficult to deliver, it is important to look at the discovered teaching methods and challenges surrounding the HOTS concept indicated in the literature.

2.4.1. HOTS studies in South Africa

Chabeli (2006) asked the question: What kinds of competencies the future working world expects from an outcomes-based education (OBE) graduate? The reason was since *Chabeli (2006)* had noticed that OBE had brought a huge shift in the teaching and learning context in South Africa. OBE shifted the system from teacher input to learner outcomes (transmission models to learner-centred model), where learning was emphasised as an active process. This also changed the role of teachers; they became facilitators and mediators of the process. Hence, they were responsible for the conduciveness of the environment and let learners explore their skills through interaction. The study of *Chabeli (2006)* found that OBE requires from learners HOTS competences such as critical thinking, reflective thinking, creative thinking, dialogic/dialectic thinking, decision making, problem solving and emotional intelligence. Ultimately the world can expect these skills from an OBE graduate. Where creative thinking can be defined as a thinking ability to generate authentic and new designs, produce different hypothesis, have multiple perspectives towards the problem and produce alternative solutions *[Glass, 2004; Young and Balli , 2014 and Birgili, 2015]*. However, in the above we do not disregard the extensive research done on the disadvantages of the OBE approach which led to its phasing out. *Mckernan (1993)* found that some teachers grading workload significantly increased in the OBE curriculum context. *Harden (1999)* found that the inclusion and emphasis on attitudes and values in stated outcomes was inappropriate. Moreover, numerous studies found that there were issues and challenges in OBE implementation in the education system *[Jennifer & Cheung, 2015; Guzman et al., 2017; Bakar et al., 2018; Erdem, 2019 and Evardo, 2020]*. Hence, the current study does not advocate for OBE, but rather only indicated some of the positive aspects of the OBE curriculum, as reported by the cited researchers.

Moodley (2013) suggested that types of questions teachers ask may promote high order thinking skills. That researcher's study focused on: (i) The need for a special programme structured for teachers to address gaps in pedagogical content knowledge. (ii) The need for learners to master/learn the art of thinking. The data was collected through observation notes and examination of 40 teachers from different public schools in Gauteng. Also, from three teachers through interviews; worksheets and a study of their lesson plans. The findings showed that it will be more beneficial for teachers if there is a structured programme dedicated to the curriculum and teachers' gap knowledge. They also suggested that teachers are the best candidate to promote high order thinking skills among their learners.

2.4.2. HOTS models

As the topic of HOTS became more dominant in teaching and learning, an effective valid and efficient learning model to enhance students' HOTS became a necessity. In Indonesia two scholars conducted research to develop a HOT learning model *[Anwar et al, 2017]*. Their research was a developmental study by using Richey and Klein methods of Type Model II conducted in two phases. The results showed that the developed Model passed the validity, effectiveness, and efficiency criteria. The application of the model shows that students' HOTS and electronics competences can be improved.

Husamah et al (2018) proposed orientation, identify, discussion, decision, and engage (OIDDE) as a tool that can stimulate HOTS in students. They suggested that this can be done by assessing three aspects namely: Critical thinking, self-regulated thinking, and creative thinking. Their study adopted a classroom action research (CAR) approach conducted in two cycles. The study was conducted on 45 biology education students during their fifth semester. The findings confirmed that OIDDE model improved the following three attributes of HOTS to a very good level: Critical thinking (81%), self-regulated thinking (82%) and creative thinking (80%). However, the study suggests that implementation of the model should be done in other subjects/courses during different semesters to support the conclusion that OIDDE model is applicable.

In Malaysia a particular study focused on examining the effectiveness of bar model method to understand and answer HOTS questions for primary school learners *[Ramasamy and Puteh, 2018]*. They collected their data from 35 learners from year 6 dual language program (DLP) class at SK Bukit Bandaraya. Findings were analysed by statistical package for the social science (SPSS) version 22, and they were based on the pre and post test result from the learners. The results revealed that the Bar model method showed HOTS effectiveness in those students. They found that with bar model kits learners pay more attention on the subject discussed. However, in earlier stages learners had challenges in estimating the size of bar in accordance with its value but later they were able to estimate the value.

Involving models in high order thinking skills, it's an approach that has been around for more than two decades *[Watson, 1995]*. Hence one would expect good research progress in this area during that period. However, according to the literature reviewed for the current study such an approach has not received much attention from researchers *[Anwar et al, 2017; Husamah et al, 2018 and Ramasamy and Puteh, 2018]*. Still, we are uncertain if the approach did receive more attention outside the publication space. Nevertheless, there is an unpredictable trend where now and again there will be those few researchers showing interest on the approach *[Nurkholik et al, 2021]*. This infers that there is a bigger room for scholars to explore their new skills and ideas in this approach.

2.4.3. HOTS in mathematics

A study which focused on developing high order thinking skills among first-year calculus students *[Maharaj and Wagh, 2016]*, was conducted in South Africa and India. The study adopted Brookhart's idea to identify the three core abilities that should be targeted. Those abilities are transfer of knowledge and skills across sections, critical thinking and problem solving. Hence from those abilities expected learning outcomes for the development of HOTS were documented. The resulted learning outcomes were used to structure sample tasks. The findings of the study indicated that there was significant lack of HOTS abilities from the students who were previously exposed to the concepts that the sample tasks were based on *[Maharaj and Wagh, 2016]*.

In 2017 research was conducted to investigate the relationship between HOTS and mathematics students' performance *[Tanujaya et al, 2017]*. The study adopted a correlation research method on 41 mathematics students. It was found that there is a strong linear relationship between HOTS and student's academic achievement. Students with high level of HOTS are expected to outperform in mathematics. That study suggested that HOTS appraisals can be used in new students' selection process as an indicator whether to accept or reject a student. Also, to improve academic performance of mathematics students, HOTS should be enhanced in students. One way to enhance HOTS, the study proposed revision of textbooks.

As the concept of HOTS emerged in many countries' mathematics education, it gained respect to such an extent that in Indonesia HOTS problems were used as a national examination. That national examination is called Ujian Nasional in Indonesia, commonly abbreviated as UN or UNAS *[Ariyanto, 2019]*. It's a standard evaluation system of primary and secondary education in Indonesia. The use of HOTS in UN prompted some scholars to be concerned about HOTS problems' levels and their characteristics. In that regard, a study was conducted to investigate the construction of HOTS problems levels and their characteristics. The results of that study were validated by investigator triangulation that involved two experts. By combination of cognitive level of trade in international mathematics and science study (TIMSS) and process of Bloom's taxonomy, the resulted levels of HOTS problems were four and each level has its own unique characteristic. The study further suggests that the results can be used as a guide by teachers to improve HOTS on students *[Ariyanto, 2019]*. An adaption of the four levels of HOTS problems and their characteristics is presented in *Table 2.2.*

Meryansumayeka et al (2019) endeavoured to describe mathematical problems that are categorized as HOTS problems. They used a descriptive qualitative approach, and the subjects were junior high school students in Palembang city. The study revealed that mathematical problems that belong to HOTS category are the ones that have instruction for evaluation, analysis, and creativity. Classifications of this HOTS related problem is based on Bloom's cognitive taxonomy. Presented in *Figure 2.3* is an adoption of classification of HOTS related problem and four levels of HOTS problems *[Meryansumayeka et al, 2019 and Ariyanto, 2019]*.

With the worldwide pandemic Covid-19, there was introduction of e-learning in most parts of the world in different schools and universities *[Alsoud and Harasis, 2021]*. A study was conducted in Ngawi Regency, East Java to describe HOTS of students using e-learning *[Setyowati, 2021]*. Data was collected using a HOTS development structured test which involved seven subjects, one of the subjects was mathematics. The results showed that students' scores ranged from 30 to 96.7. Within the period

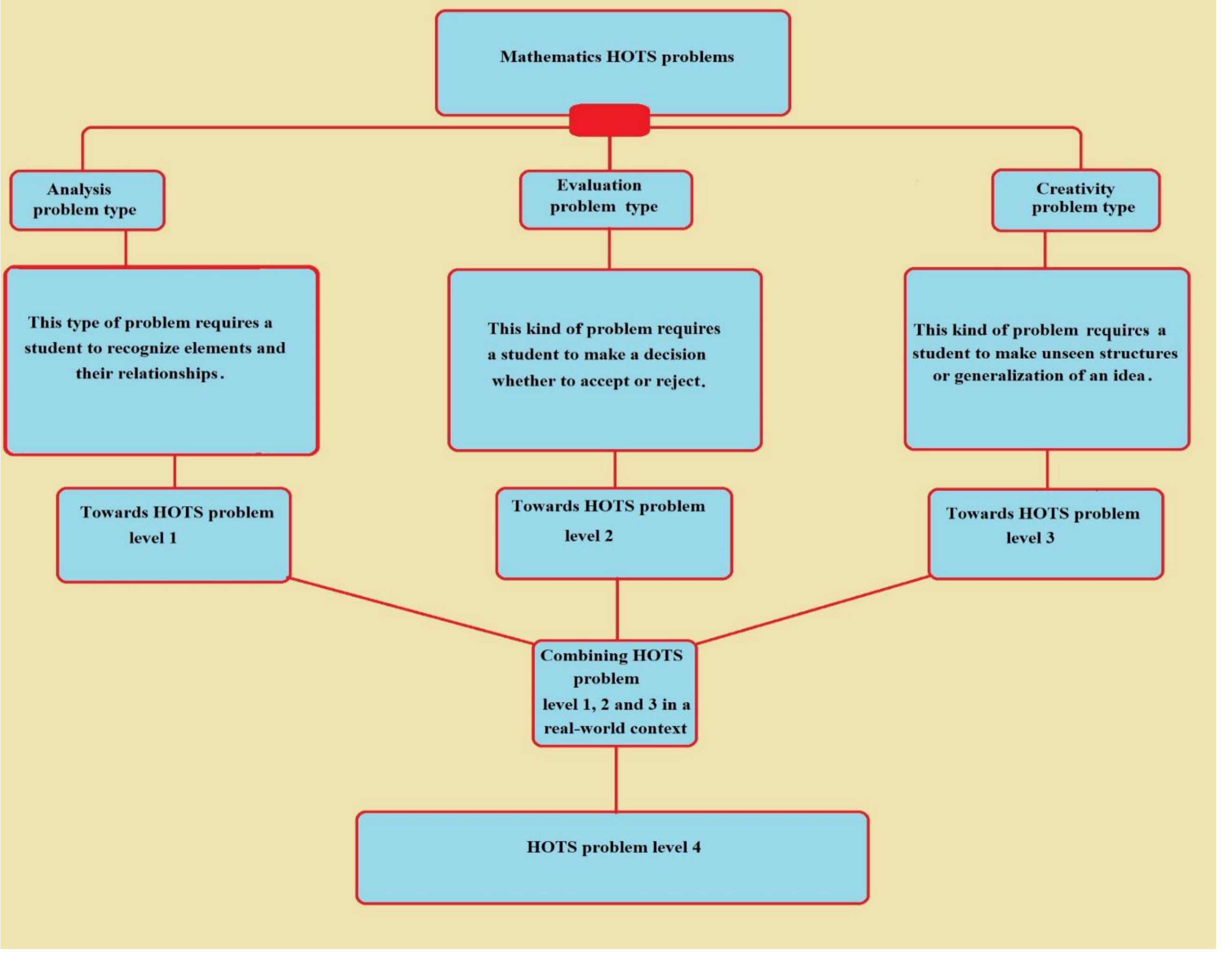

Figure 2.3: Mathematics HOTS problems categories.

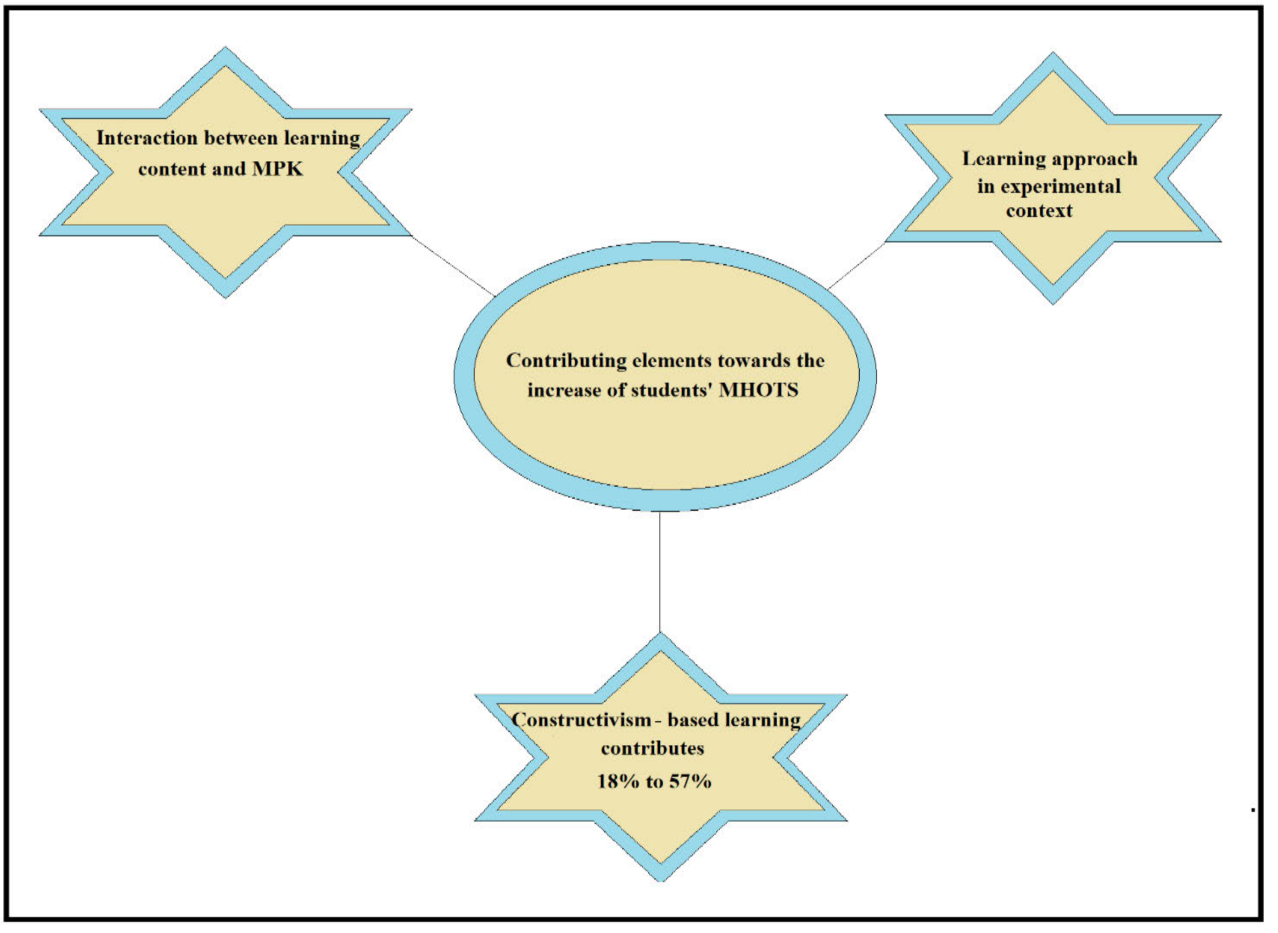

Figure 2.4: Contributing factors or elements towards students' MHOTS.

of eight months after e-learning was implemented due to the pandemic, only half of the sampled population were found to have acquired the expected abilities *[Setyowati, 2021]*.

Minarni and Napitupulu (2020) state that making students to comprehend mathematical higher-order thinking skills (MHOTS) is very challenging. However, this ability (MHOTS) is very vital for students as they move to higher levels of their studies, careers, and life in general especially in these days of industrial revolution. Some scholars suggest that one of the factors that affect students' thinking abilities is the learning context. Hence, they executed a study aiming to investigate the contribution of constructivism-based learning approaches and mathematical prior knowledge (MPK) to students achieving MHOTS *[Minarni and Napitupulu, 2020]*. The population of the study was primary junior high school (PJHS) students in Medan, Deli Serdang, Binjai, and Padang Sidempuan in the Province of North Sumatera, and Banda Aceh in the Province of Nanggroe Aceh Darussalam. Univariate and multivariate analysis was used as a statistical tool to analyze the contribution of treatment towards mathematical high order thinking skills (MHOTS) achievement, while t-Students was used as a statistical tool to determine the significant improvement of MHOTS (Glass & Hopkins, 1996). All analyses used a 0.05 level of significance. The role of the learning approach was elaborated through linking MHOTS achievements with the steps of the learning approach applied in the classroom based on the output of the regression analysis. The study found the elements that contributed to the increase of students' MHOTS. One of them was the contribution of constructivism-based learning to MHOTS, in the range of 18% to 57%, and the rest are presented in *Figure 2.4*. In addition, there were many more studies concerning HOTS and mathematics, accessible in the space of publication *[Nursalam et al, 2018; Maslihah et al, 2019; Hadi and Zaidah, 2020 and Makmuri, 2020].*

2.4.4. HOTS and pedagogical processes

The study by *Chinedu and Kamin (2015)* stated that HOTS should be a central part of teaching and learning specifically at higher levels of education. If students are to solve problems, thinking skills lessons ought to be focused on in the curriculum. On the other hand, educators should be familiar with techniques required for teaching HOTS. The aim of that study was to examine existing practices in teaching of HOTS in technology and design education in TVET classrooms. The study suggested that HOTS lessons in technology and design should focus on activities suggested by David, 2008 namely conceptual, technical, aesthetics, constructional and marketing *[Chinedu and Kamin, 2015].* Another study almost focused on the same goal *[Yen and HaliliYen and Halili, 2015].*

In Malaysia Pillay et al (2018) reviewed the implementation of HOTS in teaching and learning. Numerous studies have shown that implementation of HOTS is challenged where educators lack knowledge and understanding of HOTS strategies, have insufficient resources to teach HOTS and environmental settings that are not conducive for teaching HOTS *[Chinedu and Kamin, 2015; Collins, 2014; Kenimer, 2003; Retnawati, 2018 and Yen and Halili, 2015]*. Pillay et al (2018) study found that: (i) In Malaysia traditional pedagogical methods are used in teaching and learning, (ii) educators are anticipated to provide their own materials for the teaching and learning process, (iii) there is also a need for teachers to learn the art of thinking and teaching thinking skills. In summary it is still a challenge to implement HOTS in teaching and learning environments in Malaysia.

In India two scholars examined understandings that start from HOTS via thinking skills and teaching thinking skills in the academic landscape *[Gupta and Mishra, 2021].* Their study highlights some of the famous structured thinking skills programs. It suggests that the educational system needs to incorporate components of HOTS into the curriculum by framing the syllabus aimed to develop higher level cognitive functions. While focusing on HOTS, creative and critical thinking should also be offered the same attention. Since those two components play a vital role in the mind of a person as shown in

Table 2.2: HOTS problems levels and their characteristics.

Level	Characteristics
1	Enabling students to: a) Analyse pieces of information to recognize relationships or patterns. b) Structure pieces of information to recognize relationships or patterns. c) Recognize the causes and consequences of a complicated scenario. d) Formulate problems whose answering, or solution contributes to the development of knowledge. Summary: HOTS level 1 problems should enable individuals to analyse and reason.
2	Equipping students to: a) Assess possible solutions with the right criteria to ensure their effectiveness. b) Give a good assessment of an idea with the correct criteria to ensure its effectiveness. c) Provide an assessment of a methodology with the correct criteria to ensure its effectiveness. d) Make hypothesis. e) Provide constructive criticisms and judgments. f) Accept or reject according to the criteria set. Summary: HOTS level 2 problems should enable students to evaluate and reason in the context of decision making.
3	Training students to: a) Generalize ideas. b) To make a design to solve problems. c) Organize elements or parts into unfamiliar or new forms. Summary: HOTS level 3 problems should enable students' reasoning to be creative.
4	Enabling students in a real-world context to: a) Formulate problem questions. b) Explore experiences. c) Explore, discover, and create. Summary: HOTS level 4 problems in real-world contexts enable individuals to analyse, evaluate and create.

* Adapted from: *[Ariyanto, 2019]*

Table 2.3: Components of thinking structure and their functions

Component	Function
Creative thinking ➢ Authentic and new designs. ➢ Different hypothesis. ➢ Multiple perspective ➢ Alternative solutions	It raises curiosity in a person's mind.
Critical thinking ➢ Decision making ➢ Reasoning ➢ Producing reasoned argument	It stimulates reasoning in a person's mind.

Table 2.3. Therefore, the syllabus should also contain tasks based on convergent and divergent orientation *[Gupta and Mishra, 2021]*. Another study endeavouring to achieve a similar goal is available *[Retnawati et al, 2018]*.

Section 2.4 is a summary of some of the HOTS related studies done in the past. It is evident that the HOTS concept received a lot of attention from researchers; hence, tremendous work has been done on this concept. However, there is still a room for researchers to take the study further. The current study suggests that: (i) more new methods for students' HOTS assessment are needed; (ii) there is shortage of HOTS models which connect the curriculum and high thinking order skills; (iii) there is less research dedicated on TVET institutions concerning HOTS. Therefore, we encourage researchers to consider them since TVET institutions play a vital role in the economy.

2.5. Basic concept of mathematical modelling

According to *Bender (1978)*, model building involves imagination and skill, and the modelling process should follow the following steps: (i) **Problem Formulation –** What is it that you wish to know? The nature of the model you choose depends very much on what you want it to do. (ii) **Outline the Model** - At this stage you must separate the various parts of the universe into unimportant, exogenous, and endogenous. The interrelations among the variables must also be specified. (iii) **Is It Useful?** - Now stand back and look at what you have. Can you obtain the needed data and then use it in the model to make the predictions you want? If the answer is no, then you must reformulate the model (step 2) and perhaps even the problem (step 1). Note that "useful" does not mean reasonable or accurate, but it means the model fits the situation, or we will be able to use t. find a solution to the problem. (iv) **Test the Model -** Use the model to make predictions that can be checked against data or common sense. It is not advisable to rely entirely on common sense, because it may well be wrong. Start out with easy predictions, don't waste time on involved calculations with a model that may be no good. If these predictions are bad and there are mathematical errors, return to step 2 or step 1. If these predictions are acceptable, they should give you accuracy and range of applicability of the model. If they are less accurate than you anticipated, it is a good idea to try to understand why, since this may uncover implicit or false assumptions.

Mathematical modelling has been applied in different aspects of life over the years, such as in controlling disease spread, mathematical education problems, population growth, traffic flow and many more *[Bender, 1978; Arseven, 2015; Krutikhina et al, 2018; Tyagi et al, 2009; Mohamed and Dilip, 2020 and Diagne et al, 2021]*. In the current research, the application of mathematical modelling in controlling the spread of a disease became one of the important areas of study. That was because in that area *Kermack and McKendrick (1927)* developed an SIR model which was highly important for the current study. Further details of the SIR model are discussed in chapter 4.

2.6. Conclusion

This chapter presented a review of curriculum evaluation and HOTS studies for both inside and outside South Africa. Evaluation models were found to have not gained appreciation and respect they deserve in the space of publication. Looking at the study of curriculum evaluation in TVET Colleges inside and outside South Africa, not much has been achieved on the African continent except in Nigeria. However, the common challenge that seems to be dominant is that TVET Colleges fails to equip the students with necessary skills required by industries and the fourth industrial revolution technology in general.

Chapter 3: Conceptual Framework

3.1. Introduction

Learning experience is the interaction between the student(s) and external conditions existing in the learning environment *[Kapur, 2018 and Tyler, 1949].* It is one of the elements regarded as most important during the process of curriculum development. According to *Tyler (1949),* evaluation of the learning experience can possibly lead to the identification of the curriculum strengths and weaknesses. The first part of the evaluation to this study is to evaluate the TVET Colleges N1 to N2 mathematics curriculum by using Tyler's model. As indicated in chapter 2, the Tyler model consists of four stages namely: selection of objective(s), selection of learning experience(s), organization learning experience(s), and evaluation of the learning experience(s). In our case, given we are working with an already existing curriculum and objectives there is no need for the selection of objective(s) and organization of the learning experience(s) *[Department of high Education, 1994 and 1997]*. Therefore, the current study has only adopted the two stages from Tyler's model namely the selection of learning experience(s) and evaluation of the learning experience(s). Hence this chapter is structured as: selection of the learning experiences, evaluation of the learning experience and the scope of the current work.

3.2. Selection of the learning experiences

Choosing a suitable learning experience, is a highly significant measure since each learning experience produces typical characteristic on students *[Gravoso et al, 2002]*. Therefore, in any given teaching and learning space, what determines the learning experience to be used is the objective(s) aspired to be achieved in that space *[Khan et al, 2013]*. According to *Tyler (1949),* there are structured principles for selecting productive learning experience(s) according to the aspired objective(s). Hence this section attempts to explore those different principles of selecting typical learning experience(s).

3.2.1. General principle for selecting learning experiences.

According to *Tyler (1949),* even though the learning experiences vary according to the objective(s) aimed to be attained, there are general principles for selecting effective learning experiences regardless of the objective(s) desired to be attained. He states those general principles as follows:

- Firstly, for every given objective(s) to be attained, the student should be given a chance to practice a particular behaviour implied by the objective(s). For instance, if the objective is problem solving then to attain that objective the learning experience should be set-up in such a way that it allows students to solve problems. *Tyler (1949)* calls this a basic principle for selecting the learning experiences for all objective(s) according to their distinct characteristics.

- Secondly, the learning experience should be set in such a way that a student obtains satisfaction from carrying on the behaviour implied by the objective(s). This does not necessary mean that the objective(s) should satisfy the student, but the learning experience ought to, even if the objective(s) do not. For instance, if the objective(s) are to solve real world problems the learning experience should give the student both the opportunity to solve real world problems and satisfaction from effective solutions. Otherwise, if the experience is unsatisfying, it is more likely to obtain the opposite of the desired objective(s). Building up the satisfying learning experience requires one to have some information about the students such as: i) sufficient information about student's interests and needs, and ii) enough information about basic human satisfactions.

- The third general rule, learning experience should be set in such a way that the desired reaction for the experience is within the range of the possibilities for the student. That is to say, the learning experience should begin where the student is knowledge wise. For instance, if the objective(s) is to solve real world problems the learning experience should give a student an opportunity to solve real world problems beginning at the level of the knowledge they have. Again, creating this kind of learning experience requires someone to have enough information about students such as: i) the student's current attainment, ii) the student's present background, and iii) to know whether the current students mental set can achieve the desired behaviour or not.

- The fourth general rule, there are variety of learning experiences that can produce the same educational objective(s) on students. Hence this means learning institutions may create a wide range of learning experiences that are aiming at the same objective(s). However, the learning experiences should meet the whatever stipulated criteria for effective learning structured for that specific learning space, such as classroom assessment, instructional relevance, student engagement, etc.

All the above-mentioned general rules for selecting the learning experiences, attempt to achieve the same goal which is to influence the attainment of the desired objective(s) on students. However, mostly they differ according to the pre-information or data each rule requires. This is portrayed in *Figure 3.1*, where each general rule is displayed together with the required pre-information according to their stated order in the above paragraph (sub-section 3.2.1).

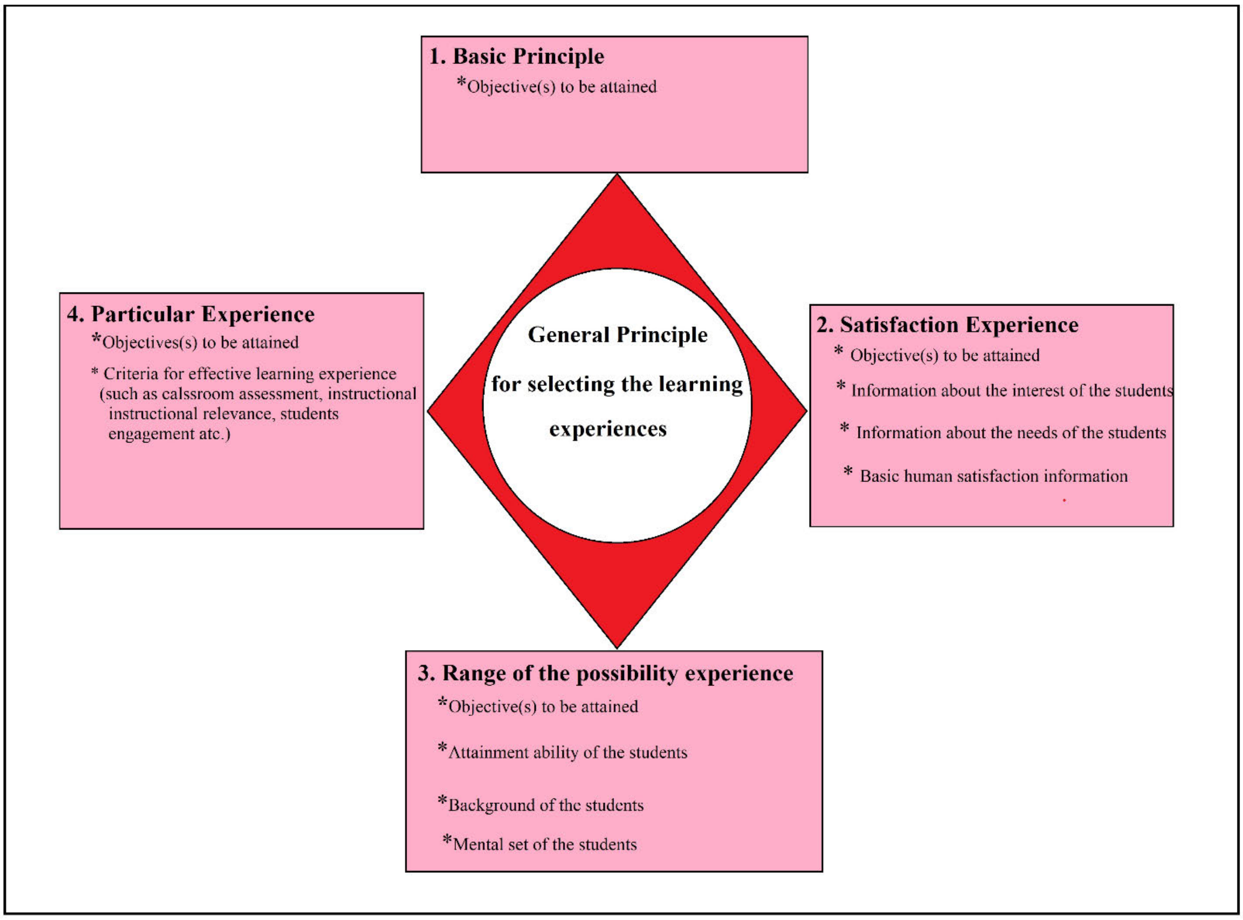

Figure 3.1: General principle for selecting the learning experiences with the information required prior for each rule.

3.2.2. Specific principle for selecting learning experiences.

Tyler (1949) also stated that even though there exist numerous possible objectives, but there are common types of them. Hence, he sampled some common objectives and noted the required learning experiences for their attainment. Out of those learning experiences mentioned by *Tyler (1949)*, we only chose three that are relevant to the subject (mathematics) under investigation in the current study. Hence on this sub-section we present the three objectives, and illustrative questions from the curriculum under investigation in the current study, that could assist in attaining the relevant objective(s).

- <u>Objective(s) focusing on thinking skills</u>: This category of objectives comprises of inductive, deductive, logical, creative thinking and many more kinds of thinking skills, where the first three skills are defined by *Tyler (1949)* as shown in *Table 3.1* and the others in the previous chapter. The suitable learning experience for this case should utilize various problems, where those problems should not be the kind of the problems that can be immediately solved by obtaining their solutions from textbooks or any other reference material. The problems should require a student to relate various facts and ideas to produce their solutions. Also, this kind of a learning experience should challenge students to see and follow the steps of thinking in their normal sequence when they solve problems. Some of those steps of thinking are displayed in *Figure 3.2 [Tyler, 1949]*. Example 3.1 presents one of the problems that could assist a student to attain the objective(s) focusing on thinking skills. This example is taken from the curriculum and its solution is produced using the steps of thinking in their normal sequence.

Table 3.1: Types of thinking skills and their characteristics.

Thinking skills	Characteristics
1) Inductive thinking	Involves drawing generalizations from several items of specific data.
2) Deductive thinking	Involves applying generalizations to specific cases.
3) Logical thinking	Involves the arrangement of assumptions and conclusions to develop a logical argument.

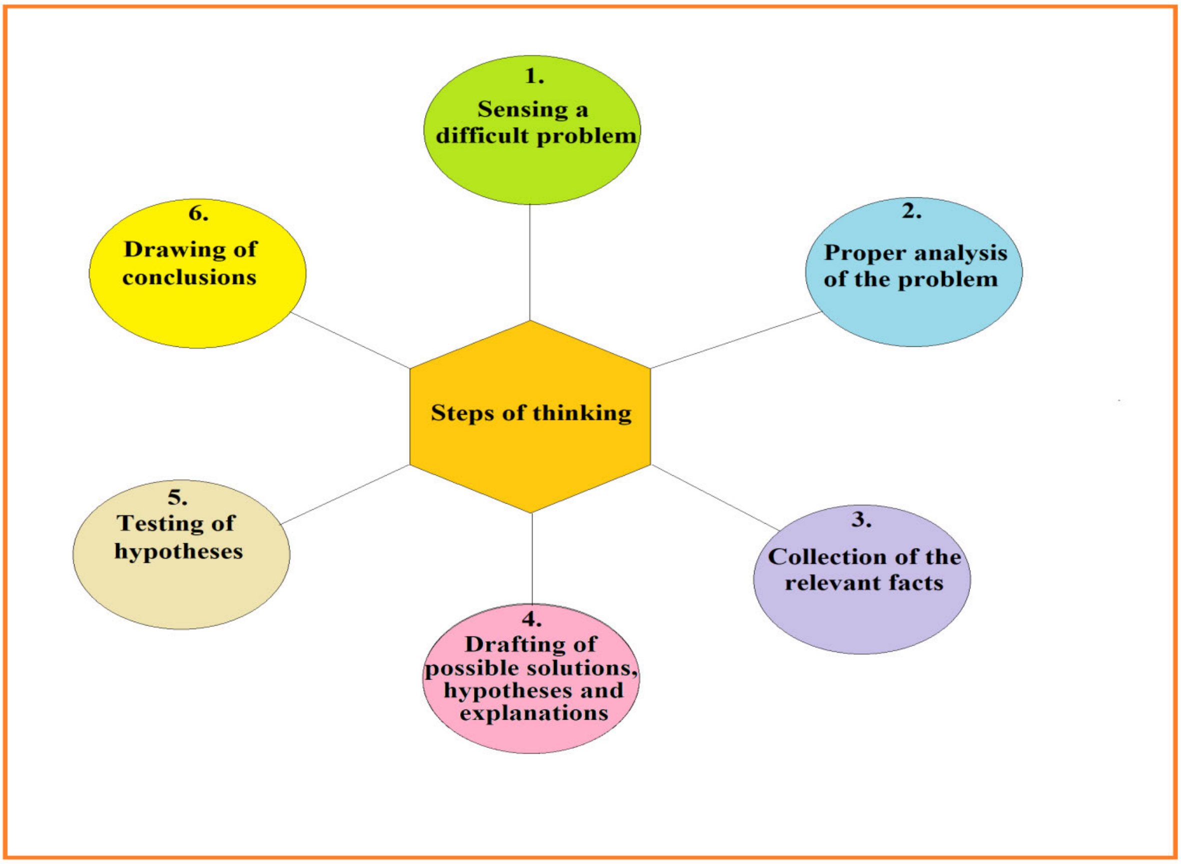

Figure 3.2: Some of the thinking sequence to follow when solving problems.

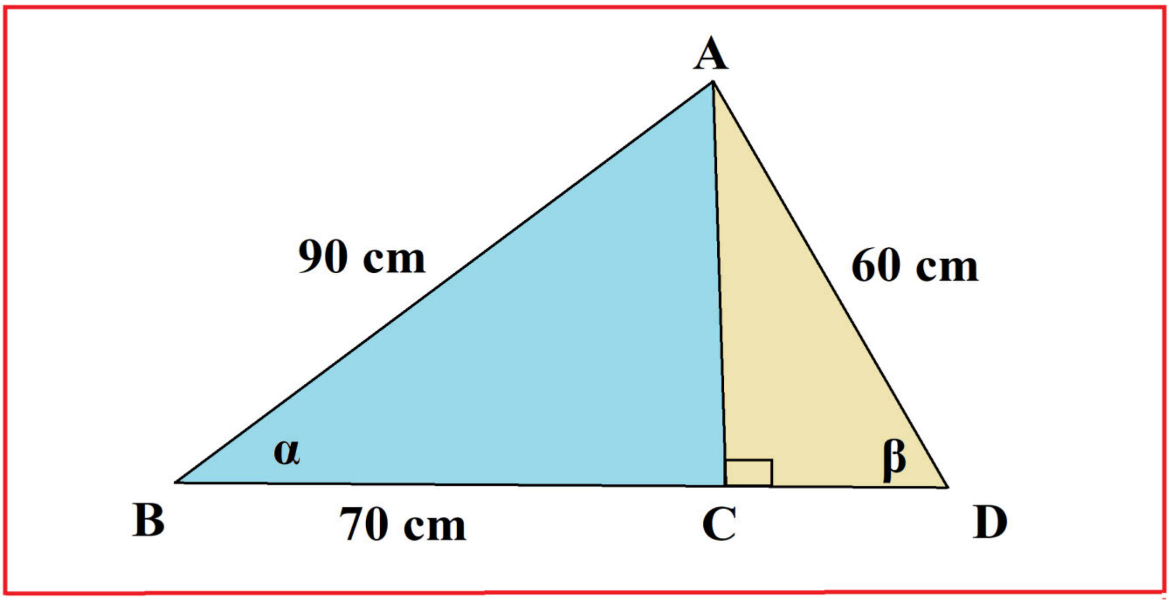

Figure 3.3: Two joined right-angle triangles that require steps of thinking to find the unknown angle.

Example 3.1

Figure 3.3 shows two joined right-angle triangles, where the first one is *ABC* and the other one is *ACD*. By using the given information, calculate the angle $B\hat{A}D$.

Solution:

Step 1: Sensing the dificult problem

This step is about identifying the avaliabity of all the necessary tools to solve the problem, in order to decide whether the problem can be solved at present.

In our case, we have all the required theoritical (knowledge) and physical (writing equipments) tool that we need to solve the problem. Therefore we decide to solve the problem at present.

Step 2: Proper analysis of the problem

This step is about identifying all the required small missing information that can help to solve the bigger problem.

In our case we need to find:

(i) the magnitude of the angle α,

(ii) the length of *AC*,

(iii) the magnitude of the angle β and

(iv) the length of *CD*.

Step 3: Collecting the relevent facts

As the caption of this step says, here we collect all the relavent facts applicable to our problem.

By relating trigonometry and triangle facts in our problem, we get that:

(i)

$$\alpha = Cos^{-1}\left(\frac{BC}{AB}\right), \qquad (3.1)$$

(ii)

$$AC = AB \times Sin\ \alpha, \quad (3.2)$$

(iii)

$$\beta = Sin^{-1}\left(\frac{AC}{AD}\right), \quad (3.3)$$

(iv)

$$B\hat{A}C = 180^0 - 90^0 - \alpha \quad (3.4)$$

(Applied triangle rule: Sum of the interior angles is equal to 180^0),

(v)

$$C\hat{A}D = 180^0 - 90^0 - \beta, \quad (3.5)$$

(Applied triangle rule: Sum of the interio angles is equal to 180^0),

(vii)$B\hat{A}D = B\hat{A}C + C\hat{A}D$ (3.6)

and

(viii)

$$B\hat{A}D = 180^0 - \beta - \alpha \quad (3.7)$$

(Applied triangle rule: Sum of the interior angles is equal to 180^0).

Step 4: Drafting of possible solutions

This step is based on organising the facts found in step 3 and decide upon a better and easy or possible solution.

In our case, from step 3 we found 2 possible ways to get to the solution.

Option 1

From step 3 apply: (i), (ii), (iii) and (viii)

Option 2

From step 3 apply: (i), (ii), (iii), (iv), (v) and (vii)

Therefore according to our analysis, option 1 is the best option for us due to it having minimal steps.

Step 5: Testing of the hypothesis

This step is application of step 2 and 3 to the problem.

In our case we chose option 1 in step 3, by applying all the necessary values displayed in Figure 3.3 we get that:

(i)

$$\alpha = 38{,}942^0 \tag{3.8}$$

(ii)

$$AC = 56{,}566^0 \tag{3.9}$$

(iii)

$$\beta = 70{,}521\ cm \tag{3.10}$$

(viii)

$$B\hat{A}D = 70{,}537^0 \tag{3.11}$$

Step 6: Drawing conclusions

This last stage it about justification of the final unswer.

In our case, even when we use option 2 we get the same answer. Therefore our final answer is:

$$B\hat{A}D = 70{,}537^0 \tag{3.12}$$

- <u>Objective(s) focusing on acquiring information</u>: This class of objectives involves developing understanding of things, expanding knowledge about various things, and the likes. Normally the type of information to be acquired includes laws, principles, facts, ideas, theories, terms and many more. During acquiring information, there are commonly identified challenges such as: i) students normally memorize instead of acquiring the real understanding, ii) students turn to forget the acquired information and have a huge number of inaccuracies in what they can remember and iii) students lack adequate organization, they are unable to relate the information in any organized or systematic fashion. Therefore, the suitable learning experiences for acquiring information would be the ones that can counteract the mentioned challenges and the likes. The first suggested learning experience is to acquire information in a form of problem solving. This kind of the learning experience is said to be less likely to produce a rote memorization but rather produce understanding in students. Secondly is the one with only important selected information worthy to be remembered. Instead of having numerous technical terms which some of them are more important to the next level or class, rather the number of terms chosen should be minimal and frequently used for students to acquire information with accuracy. Also, the kind of suitable experience is where intensity and variety of expressions is the priority. This combination is said to increase the likelihood of remembering in students. Thirdly, it is a learning experience that will present different schemes of information organization. This experience enables a student to organize the same material in two or more ways effectively for use. Example 3.2 below is one of those that could assist in attaining the objective(s) focusing on acquiring information, taken from the curriculum. It pushes someone to remember several branches of acquired information such as: logarithmic laws, exponential laws, knowledge of quadratic equations and factorization *[Tyler, 1949]*.

Example 3.2

Solve for x in the following equation:

$$log_2(x^2 - 2x) = 3. \tag{3.13}$$

Solution

By applying the logarithmic law (3.13) becomes,

$$x^2 - 2x = 2^3. \tag{3.14}$$

Changing the exponential number and rearrange (3.14) we get,

$$x^2 - 2x - 8 = 0. \qquad (3.15)$$

Factorizing (3.15) we get,

$$(x + 2)(x - 4) = 0 \qquad (3.15)$$

and

$$x = -2 \quad or \quad x = 4 \tag{3.16}$$

Substituting (3.16) into (3.14), it is found that both solutions satisfy (3.14).

- <u>Objective(s) focusing on developing interests</u>: This is a class of objective(s) that enables students to derive interest from areas of experiences in which the interest is to be developed. Therefore, the suggested learning experience is the one which allows students to be exposed to areas in which interests are to be developed and have satisfaction from their explorations. If possible, the learning experience should also provide students with a chance to obtain fundamental satisfactions from the explorations, where some of the fundamental satisfactions are presented in *Figure 3.4*. In this case we wouldn't know a precise example, given a separate data is needed to know the students' interests. Also, students' interest will differ according to colleges and locations *[Tyler, 1949]*.

3.3. Evaluation of the learning experiences

The learning experience selection stage (pre-evaluation) provides various criteria to check the relation between the set-up objectives and learning experience *[Tyler, 1949]*. However, the strong correlation between the set-up objectives and learning experience do not assure that all

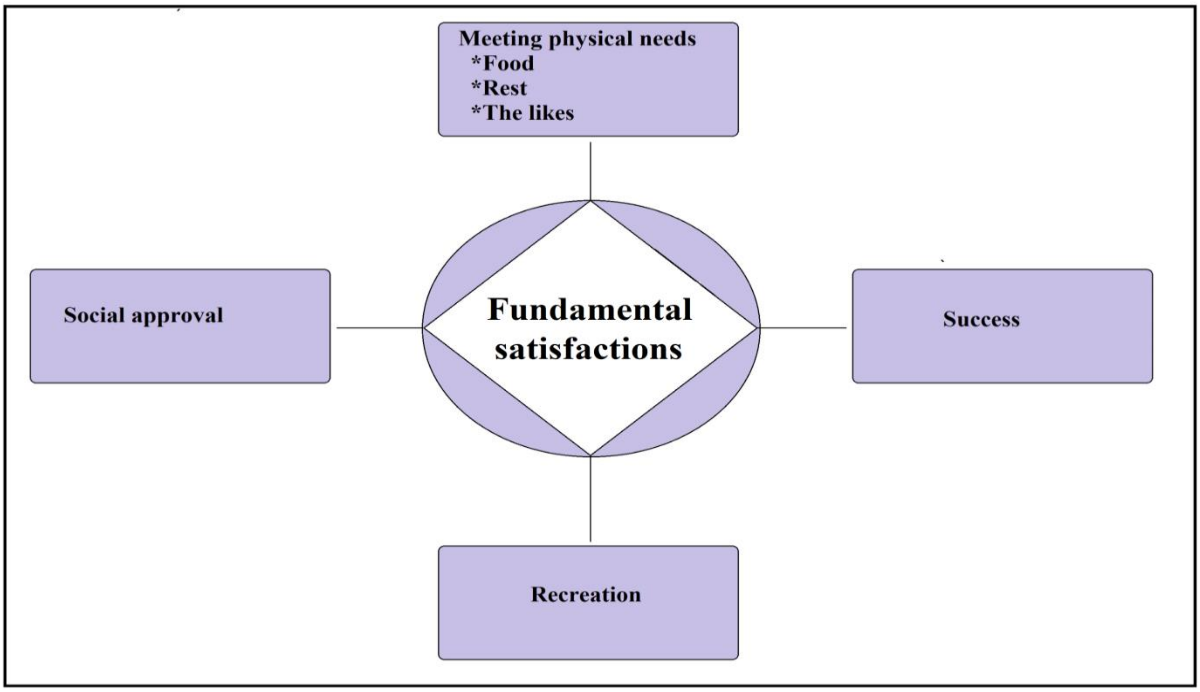

Figure 3. 4: Some of the human fundamental satisfaction.

the objectives will be achieved. According to *Tyler (1949),* after pre-evaluation we need to check how far the objectives are being realized or achieved, which is the role of the learning experience evaluation. Therefore, this section elaborates the *Tyler's (1949)* perspective on learning experience evaluation.

3.3.1. Basic notion of evaluation

According to *Tyler (1949),* there are two important things to note about the learning experience evaluation. Firstly, the evaluation ought to appraise the behaviour of the students, given that educational evaluation sought the change of behaviour such as: (i) knowledge students have, (ii) ability of students to analyse, (iii) students' problem-solving skills, (iv) students' social adjustment and the likes. He continues to state that some of these behaviours can be gotten through paper and pencil test, whereas others cannot except with other methods of appraisals such as interviewing and the likes. Therefore, the method of appraisal is determined by the characteristics of the behaviour being evaluated. Secondly, he states that the educational evaluation should at least involve two appraisals, since one cannot draw a conclusion (about the effectiveness of the program in use) by only assessing students at the end of the program without knowing where they were at beginning of the program. It is possible that a student had a good progress on (a) particular objective(s) before beginning the program, hence that cannot be identified if the appraisal is only done at the end of the program. Therefore, for proper evaluation at least two appraisals should be done, one at the beginning of the program and the other towards or at the end of the program.

3.3.2. Evaluation procedure

For learning experience evaluation, firstly objective(s) should be clearly stated; otherwise, it is not possible to find how far are they being realized without knowing them *[Tyler, 1949]*. After objective(s) has/have been stated, secondly the evaluator should identify the situation(s) (evaluation instruments) that will give students a chance to express the behaviour inferred by the objective(s) *[Fundi, 2015 and Tyler, 1949]*. This is where the evaluator will be at a position to identify the extent to which the objective(s) are being realized. Identifying evaluation instrument(s) includes examining them to see if they serve the evaluation purpose against the objective(s); given some evaluation instruments are widely recommended but it does not mean they will always automatically serve the purpose of all evaluation suggested for *[Tyler, 1949]*. Therefore, the evaluation instrument(s) need to be examined by checking them against objective(s) aimed at. Conversely, it is also possible to not find any evaluation instrument(s) that will serve the purpose of a particular evaluation; in that case the evaluation instrument(s) should be constructed. *Tyler (1949)* provides a guide for evaluation instrument(s) construction as follows:

- The first step is to find the situation(s) that will give students a chance to express the behaviour implied by the objective(s).

- The second step is to formulate the means of acquiring recordings of the targeted students' behaviour. Normally this is not a problem especially in the case where the targeted behaviour can be assessed through a pencil and paper test, given in such a case students make their own record in their writings. Conversely in cases where students' reaction is observed, the observer should devise means of getting recordings of the targeted students' behaviour.

- Thirdly, the evaluator should decide on the terms or units that will used to categorize the records of the behaviour obtained. Classification of the behaviour differs according to the objectives and the evaluators. For instance, when we look at a particular student's problem-solving behaviour; it can be rated as good in the perspective of creativity and rated poor when viewed in terms of analysis. Again, the same behaviour (problem solving) is possible to be not categorized, given its characteristics (creativity and analysis) are separate objectives themselves.

- Fourthly, the evaluation instrument should be objective. If two different individuals (presumably competent) would score a behaviour obtained from an evaluation instrument and their scores vary markedly or would not reach the same score, that means the objectivity of the evaluation instrument needs improvement. One of the simplest ways to fix that problem is to put specifications on the scorings.

- Fifth, the instrument reliability should be validated. The instrument should be investigated to determine if the questions in the instrument are within the capabilities of the participants.

3.4. Scope of the current work

As explained in chapter 1 section 1.2, N1 to N2 mathematics curriculum is being presumed by South African educational system as being capable enough to equip students with necessary thinking skills for industries and real-world problems. Therefore, the current study aims to investigate the curriculum' ability to equip students with the necessary thinking skills. That will be executed by performing the pre-evaluation of the learning experience, actual evaluation of the learning experience and evaluation of the ability of the curriculum to equip students with HOTS.

3.4.1. Pre-evaluation of the learning experience

The evaluation of the learning experience is ultimately the curriculum evaluation; given the learning experience is formulated upon the curriculum. Also, the beginning of the evaluation of the curriculum's ability to equip students with necessary thinking skills start by assessing the attainment of the curriculum objectives. Therefore, the first part of the study is to evaluate if the curriculum is capable enough to equip students for the attainment of its own objectives through evaluating the learning experience. However, prior to that evaluation, knowing the kind of learning experience being evaluated is highly important. That helps the evaluator to distinguish between a wrong experience and the right experience which do not produce results due to unrecognized experiences requirements. That identification of the experience is called pre-evaluation because some problems can be identified at this stage *[Tyler, 1949]*. As this current work attempted to evaluate the learning experience, it was vital to first identify the category and characteristics of the learning experience being evaluated.

According to *Tyler (1949)* it is possible for two students to experience two different learning experiences simultaneously in the same class. For example, suppose one student is focused and following during the time when the teacher is giving an explanation. While the other one is not focusing but thinking about his or her personal problems. Those two students at the end of the teaching lesson will leave with two different learning experiences. However, the fact that students do not respond the same way to a planned learning experience, it does not mean the lecturer or teacher should not set up a desired learning experience. Given the learning experience planned or presented by a lecturer or teacher is the one that defines the class learning experience, regardless of its level of effectiveness. In that regard in this study all our attention was directed to the lecturers' perception when we investigated the type of learning experience operating at the TVET Colleges. According to *Tyler (1949) and eGyankosh (2017)*, the learning experience depends on three components which are: selected learning situations, learning activities and students' interaction. Hence the formulated questionnaires (*Appendix* 1) to investigate the learning experience were based on the first two components, which are the selected learning situations and learning activities. Given these two components is where lecturers have the most power and authority to make decisions than students do. Whereas the third component (students' interaction), most power lies on the students than the lecturers. Therefore, since we investigate the learning experience planned by lecturers, then it was appropriate to utilize the two components for questionnaire formulation in *Appendix 1*.

3.4.2. Evaluation of the learning experience

Learning experience evaluation is one of the approaches that might be used to identify the strength and weaknesses of the curriculum *[Tyler, 1949]*. That can be achieved after analysing the results obtained from the evaluation instrument, since those results reflect students' abilities that are mostly influenced by the curriculum. Hence this study has taken that approach to investigate the TVET mathematics curriculum.

This current work did not find a recommended evaluation instrument, which could suitably assess the level of objectives attainment by the students in the N1 to N2 TVET College mathematics' curriculum. Hence it adopted the *Tyler's (1949)* idea to develop an evaluation instrument suitable for the TVET College students. Two almost similar tests based on the objectives were formulated (part 1 of *Appendix* 2 and *Appendix* 3), one was used to assess students at the beginning of N1 and the other towards the end of N2. In compering and combining N1 and N2 curriculum's objectives, we identified seven objectives *[Department of high Education, 1994 and 1997]*. Out of those seven, four were found to be most vital for the purpose of this study. Hence part 1 of *Appendix* 2 and *Appendix* 3 were formulated based on those four objectives, those objectives are explained below:

- <u>Application of mathematical principles mastered:</u> This objective is one of the significant elements in a career of an artesian, given failure to attain this objective can lead to the disempowerment of artisans during competency-based training programs that require atomised mathematics principles; since they will require that understanding to build upon *[Fitzsimons, 2013]*. Further on, this objective does not only deal with human (artesian) empowerment but it's also contributing to work productivity in the workplace. Given according to *Ahmad (2019)*: (i) communication becomes better and powerful due to the application of the mathematical principles, (ii) hazards and accidents can be avoided in workplaces through application of mathematical principles, and (iii) through this objective, reliable and dependable systems in industries can be developed. In that regard, investigating the attainment of this objective is vital in the space of artisans and South Africa as a whole.

- Use of the correct mathematical terminology and to identify the appropriate formulae: This objective contains two pairs of important elements for artisans. Firstly, it's the correct terminology usage, which is the key factor for common understanding in a workplace team. Given, according to *Sandrini (2014)* terminologies are just instruments used by experts to easily communicate concepts related to their fields. Secondly, the objective also includes the knowledge and identification of appropriate formulae. According to *Isik et al (2005),* formulae summarize relations between quantities through mathematical symbols. Hence, knowing the correct formulae in some extent can simplify the process of problem solving. Therefore, ensuring acquirement of the abilities implied by this objective is important in the career of artisans.

- Use of the correct SI units: *Osakue and Lewis (2013)* state that a number gives information around the size of the quantity, whereas the unit describes the type of the quantity. Hence the SI unit gives the number a context so that it can be better understood and interpreted. Therefore, it is crucial for artisans to have the correct knowledge of SI units for enhancement of the understanding and interpretation when communicating about different concepts in the workplace.

- Logical reasoning: This is an essential component, given that it acts as a brain behind any decision-making process in the workplace *[CC Ong, 2011]*. The career of an artisan demands decision making ability most of the time. Given now and again they should deal with complex problems that require unfamiliar solutions in most cases, hence, there is a need to investigate the attainment of this objective.

Therefore, the two almost similar tests (parts 1 of *Appendix* 2 and *Appendix* 3) for investigating the attainment of objectives consist of four questions, where the questions are based on each of the above objectives.

3.4.3. Evaluation of the HOTS

The other portion of this study was to investigate the contribution of the curriculum to improving the students' HOTS. The execution of this portion required a mathematical evaluation model and data. A mathematical evaluation model was developed, and the data collection instrument was developed by the application of the work of *Brookhart (2010) and Maharaj and Wagh (2016).* The data collection instruments are two similar tests in part 2 of both *Appendix 2* and *Appendix 3,* which consist of the five questions based on the following five HOTS components respectively:

- Working systematically through cases in an exhaustive way.
- Interpret and extend solutions of problems.
- Identify possible applications of mathematics in their surroundings.
- Translate a worded or graphically represented situation to relevant mathematical formalisms.
- Use with reasonable skill available tools for mathematical exploration.

HOTS questions were selected from the curriculum being investigated. After that, the developed mathematical evaluation model was applied on the data set to investigate the capability of the curriculum to equip students with HOTS.

3.4.4. Reliability of the data collection instruments.

There are three data collection instruments (*Appendices 1, 2 and 3*) in this study. *Appendix* 1 needed no reliability test, given it was not newly constructed but adopted from the Tyler's model; hence it is already reliable. The test for reliability should assure that the questions in the instruments are within the capabilities of the participants *[Tyler's, 1949]*. Then *Appendices 2 and 3* remain reliable, given questions used in our constructed instruments were taken from the N1 to N2 mathematics curriculum. Hence all our data collection instruments are reliable. However, question 10 of *Appendix* 3 is used to confirm if the data collection instrument is relatable to what has been taught according to students' perspective. That will indicate to us if the data collection instrument is within capabilities of the students.

Also, according to some scholars there are many variables that can contribute to students attaining HOTS, but the most critical are lecturer's content delivery ability and the curriculum *[Yen and Halili, 2015; Chinedu and Kamin, 2015; Tanujaya et al, 2017; Retnawati et al, 2018; Gupta and Mishra, 2021]*. Therefore prior to curriculum HOTS evaluation, the state of the lecturer's content delivery ability variable is investigated using question 10 of *Appendix 3*. The students' responses of question 10 in *Appendix 3* are used as a partial evaluation of the lecturers' content delivery ability.

3.5. Conclusion

This chapter presented the conceptual framework for the work that was covered in this current study, which focused on the evaluation of the curriculum capability to equip students for objectives attainment and HOTS. The evaluation of the objectives' attainment will be executed through learning experiences evaluation. The evaluation consists of the two components, namely, pre-evaluation and the actual evaluation, for these Tyler's objective model will be applied. On the other hand, the evaluation of HOTS in the TVET Curriculum requires a development of a suitable mathematical evaluation model. Both the Tyler's and HOTS evaluation model will be applied to execute the evaluation (use of *Appendix* 2 and *Appendix* 3).

Chapter 4: Methodology

4.1. Introduction

As mentioned in chapter 3, lecturers or teachers have more power to decide on which learning experience will be used. However, a major influence behind every contributing factor in the learning experience is the curriculum. Even for lecturers or teachers, every decision they take regarding the learning experience it should be in favour of the curriculum all the time. Hence, the evaluation of the learning experience is ultimately the evaluation of the curriculum. Therefore, this chapter presents the development of the mathematical approach taken by the current study to evaluate the curriculum. The first part is the modification of the Tyler's model, which will be used to evaluate the capability of the curriculum to make students attain its own objectives. The second part is the development of the SVHIR model, which will be used to evaluate the capability of the curriculum to equip students with the necessary HOTS. In that regard, this chapter is structured as follows: evaluation of the curriculum objectives attainment, evaluation of HOTS in the N1 to N2 Curriculum, data collection and participants, and chapter summary.

4.2. Evaluation of the curriculum objectives attainment

When we evaluate the objectives attainment, we are basically evaluating the learning experience; that is because the students' objective attainment highly depends on the type of the learning experience imposed on them. In that regard, before evaluation, it is important to first determine the type of the learning experience in practice. That guides the decisions that might have to be taken after the evaluation if need be. Hence, this section consists of the learning experience identification (pre-evaluation) and actual evaluation, which is achieved by the adoption of the Tyler's model.

4.2.1. Identification of the selected learning experience

As elaborated in chapter 3, according to *Tyler (1949)* there are general and specific learning experiences. Where the specific learning experiences are categorized into three categories, and given as follows:

1. Learning experiences that develop thinking skills,

2. Learning experiences focusing on acquiring information,
3. Learning experiences focusing on developing interests.

Therefore, our learning experience identification will be based on seeking to find out which of the above learning experiences are being selected for practice in the N1 and N2 TVET mathematics classes.

As mentioned in chapter 3, according to *Tyler (1949) and eGyankosh (2017)* the learning experience depends on three components which are: selected learning situations, learning activities and students' interaction. The first two of those components depend on the lecturer, hence he/she is the one with more control over deciding the type of learning experience to be practiced. Therefore, we directed our investigation to the lecturers' perception for the identification of the learning experience in operation. We formulated a questionnaire (*Appendix 1*) that was based on the first two components. In *Appendix 1* the first two questions represent selected learning situations, and the other two questions are for learning activities. Each of the four questions required the lecturers to choose options that were formulated from Tyler's model. Each of the options focused on a particular characteristic of a learning experience based on the Tyler model. Whatever the lecturer chose, was compared with the Tyler model to conclude on the type of the learning experience that was practised.

4.2.2. Learning experience evaluation

According to *Tyler (1949),* evaluation should be done through both pre- and post- assessments. Therefore, questionnaires in part 1 of both *Appendixes 2 and 3* are composed of the four questions. Those questions were formulated to focus on the four main objectives of the curriculum, indicated in chapter 3 sub-section 3.4.2. As explained also in chapter 3, students were assessed with *Appendix 2* on their first day of arrival on campus (pre-assessment). This was to investigate the level of knowledge and skills students already had. At the conclusion of the curriculum, students were assessed with *Appendix 3* (post-assessment). The percentage difference (x_i) between each student's pre- and post- assessment results, determined the impact of the learning experience on each student. Hence the average difference ($\bar{X}$) will determine the overall impact the learning experience had on students. Given the total number of students (N), the percentage difference (x_i) and average difference ($\bar{X}$) are mathematically expressed for n number of students as follows:

$$\left.\begin{aligned} x_i &= \left(\frac{b_i - a_i}{100 - a_i}\right) \times 100\,; \quad i \in \{1, 2, 3, \dots, n\},\ a_i \neq 100\ \ and\ \ x_i \geq 0. && (a) \\ and & \\ \bar{X} &= \frac{\sum_{i=1}^{i=n} x_i}{N}\,; \qquad N > 0 && (b) \end{aligned}\right\} \quad (4.1)$$

The percentage difference in (4.1)(a), originates from this study. It consists of a_i and b_i that are each student's pre- and post- assessment results, respectively. Its notion is to measure the improvement of a student between pre- and post- assessment in percentage. Therefore, the student's improvement is given by the difference between post- and pre- assessment results $(b_i - a_i)$. The percentage in general is given a fraction of a part, over a whole, multiply by 100. In our case to express percentage difference, we need to find the whole, since we've already had the part $(b_i - a_i)$. Therefore, given that the student started by obtaining a_i mark, then the room for improvement will be less than 100 if a_i is greater than zero, and if 100 is the maximum mark a student is striving to improve to. In that regard, the room for improvement (whole) will depend on a_i, hence our whole becomes $(100 - a_i)$. From applying the notion of percentage calculation, we produced (4.1)(a). We also applied the notion of averaging to produce (4.1)(b).

Two cases of the average difference $(\bar{X})$ were used in this study to conclude on the impact of the learning experience. The two cases were not adopted from the curriculum or Tyler model, but were standards chosen by the researchers for the study to base our judgement upon.

Case 1: $\bar{X} \geq 50\%$

This means most of the students' improvement is at least 50%, which we interpret as an indication of a functional learning experience and curriculum.

Case 2: $\bar{X} < 50\%$

This means most of the students' improvement is less than 50%, which we interpret as an indication of a dysfunctional learning experience and curriculum.

Note, the subject examination is different from the curriculum objective assessment. Examination sometimes only focuses on specific curriculum objectives, whereas the curriculum objective assessment focuses on all the objectives. Hence our view is that students who passed the subject does not ultimately mean they had mastered the necessary curriculum objectives. In that regard, our curriculum objective attainment pass mark is at least 50%;

whereas the subject pass mark stipulated in the curriculum is at least 30% *[Department of high Education, 1994]*.

4.3. Evaluation of HOTS in the N1 to N2 Curriculum

This section focuses on the development of the new mathematical model that can be used to evaluate the capability of the TVET curriculum to equip students with HOTS. Hence it is structured as: review of the SIR (Susceptible-Infected and Recovered) model; new model development for the current study, determination of the new model's parameters; validation of the new model; derivation of the basic reproduction ratio, association of the actual data with the new model; and the model application instructions.

4.3.1. Review of the SIR Model

A major assumption of many mathematical models of pandemics is that the population can be divided into a set of distinct compartments. These compartments are defined with respect to disease status. The simplest model is the SIR Model described by *Kermack and McKendrick (1927)* that consists of three compartments which are susceptible (S), infected (I) and recovered (R); where β is the transmission rate and γ is the recovery rate as shown in *Figure 4.1*.

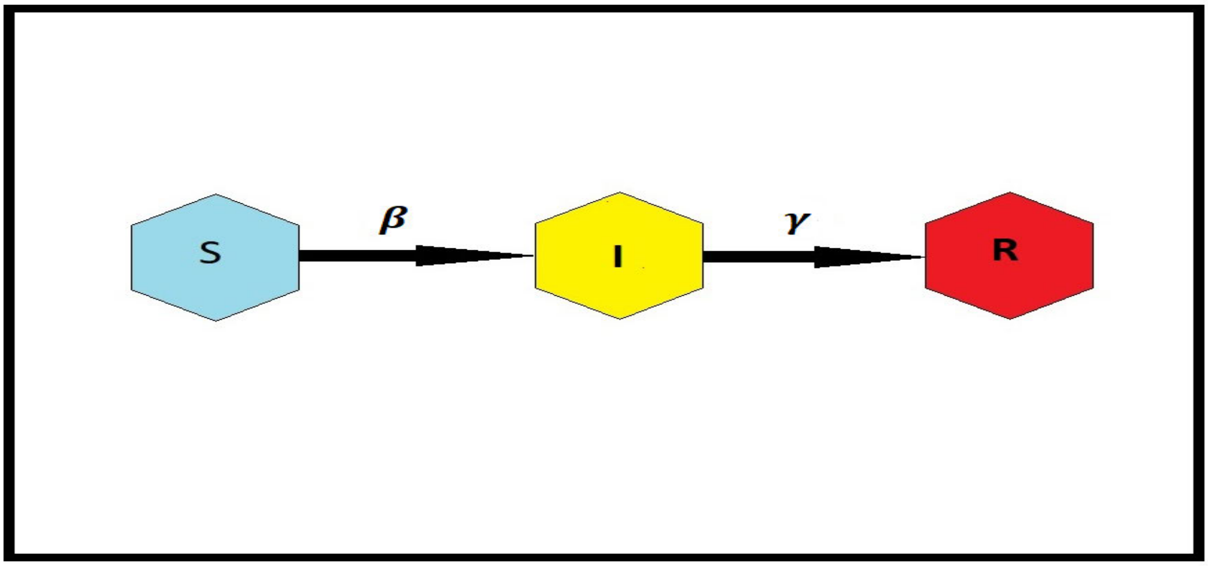

Figure 4.1: Susceptible, Infected and Recovered Model (SIR Model).

Susceptible: means individuals that were never infected, and they can catch the disease. Once they have it, they move into the Infected compartment.

Infected: means individuals that can spread the disease to susceptible individuals. The time they spend in the infected compartment is the infectious period, after which they enter the recovered compartment.

Recovered: means individuals in the recovered compartment are assumed to be immune for life.

The SIR model is easily written using ordinary differential equations (ODEs), which implies a deterministic model (no randomness is involved, the same starting conditions give the same output). Analogous to the principles of reaction kinetics, the model assumes that encounters between infected and susceptible individuals occur at a rate proportional to their respective numbers in the population. The SIR model is given as follows *[Kermack and McKendrick, 1927]*:

$$\left.\begin{aligned} \frac{dS}{dt} &= -\beta SI \\ \frac{dI}{dt} &= \beta SI - \gamma I \\ \frac{dR}{dt} &= \gamma I \end{aligned}\right\} \tag{4.2}$$

The SIR Model possess the following quantities, parameters, and rates of change *[Kermack and McKendrick, 1927; Mohamed and Dilip, 2020 and Diagne et al, 2021]*:

S(t): number of susceptible individuals

S'(t): rate of change of *S*

I(t): number of infected individuals

I'(t): rate of change of *I*

R(t): number of recovered individuals

R'(t): rate of change of *R*

β: disease transmission rate

γ: recovery rate

The SIR Model also has the following assumptions about the nature of the disease *[Kermack and McKendrick, 1927; Mohamed and Dilip, 2020 and Diagne et al, 2021]*:

- The duration of infection is the same for everyone.

- Once recovered, you are immune, and can no longer infect anyone.
- Only a fraction of contacts with the disease cause infection.
- The units of S, I, and R are persons.
- The units of time are days.
- The units of S', I', and R' are persons per day, written person/day.
- The system is closed; this simply means that the total size of the population, which equals the sum $S + I + R$, does not change ($S + I + R = N$).

One of the most vital parameters in epidemiology is the basic reproductive ratio (R_0). It is defined as the average number of secondary cases transmitted by a single infected individual that is placed into a fully susceptible population. If $R_0 > 1$ there will be a pandemic or the pandemic will continue, and if $R_0 < 1$ the introduced infected people will recover (or die). Ultimately there will be no pandemic, or the pandemic will stop. The basic reproductive ratio is given as *[Rodrigues, 2016 and Mohamed and Dilip, 2020]*:

$$R_0 = \frac{\text{Disease transmission rate}}{\text{Recovery rate}} = \frac{\beta}{\gamma}\,. \tag{4.3}$$

The concept of the SIR model has been a centre of attention for many scholars especially after the breakout of the covid-19. As a result, some went as far as to improve the model to best suit the current situation of Covid-19. For instance, *Diagne et al (2021)* extended the concept of SIR model by adding another four compartments. They divided the total population *N(t)* into several epidemiological states, depending on individuals' health status as follows: susceptible *S(t)*, vaccinated *V(t)*, exposed *E(t)*, symptomatic infected individuals *I(t)*, infected asymptomatic *A(t)*, hospitalized *H(t)*, and recovered *R(t)*.

4.3.2. Model Development for this study

Some studies revealed that study peer groups have an influence on students, and such influence could be positive or negative on their academic achievement *[Filade et al, 2019* and *Monyamane and Keletsositse, 2021]*. On the other hand, *Mirani (2015)* focused on the mixed ability classroom; where there is low-ability, average-ability, and high-ability students. One of the findings is that division of students on basis of academic performances, would cause the quick learners to think that they are very intelligent and thus stop putting in greater effort. In other words, the division can cause low-ability students to negatively impact high-ability students to some extent. Given division will always exist whether physically or mentally, for instance scaling of marks or scores (A+, A, B etc.) that is a form of division which could be necessary, but unintentional. Therefore low-ability students could always negatively affect high-ability students to some extent. In that regard, in this study we assume that thinking skills of high-ability students can be influenced negatively when they interact or share the same environment with low-ability students. By thinking skills, we also refer to HOTS. In epidemiological terms, in this study we assume that degradation of high order thinking skills (DHOTS) in some extent can be contagious through interaction between high-ability and low ability students.

To investigate the ability of the N1 to N2 mathematics curriculum to equip with HOTS, we will use the above assumption to develop a mathematical model. The curriculum is taught in six months (180 days), in which it is also expected to equip students with HOTS during that period. During this period, we will assume there is a pandemic called DHOTS within students enrolled for the N1 to N2 curriculum, and the curriculum acts as a vaccine against the pandemic. In that context we use the help of *Kermack and McKendrick (1927)* and *Diagne et al (2021)* to model the behaviour of the DHOTS pandemic. Our model called SVHIR has five compartments which are susceptible *S(t),* vaccinated *V(t),* healthy *H(t),* infected *I(t)* and recovered *R(t)* as shown in *Figure 4.2.* Where parameters such as μ, θ, β and γ are vaccination rate, healthy individuals' discovery rate, disease transmission rate and recovery rate respectively. The first compartment is susceptible *S(t),* which is the stage where all the students first arrive at N1 level before being exposed to the curriculum. At this compartment, the fact is certain percentage of students might be already infected by DHOTS and the other portion is not infected (is healthy). That might also be shown by the scores/marks of the first HOTS test (*Appendix 2* part 2) that students obtained, some students passed the test without being exposed to the curriculum. However according to some researchers, one assessment is not enough to conclude on the state of the student being assessed *[Tyler, 1949; Caris, 2004 and Shivaraju et al, 2017]*. Hence in this study, we assume all the students at this stage are neither healthy nor infected but are at risk of being infected. The first HOTS test student's marks do not indicate the student's status or model compartment but indicate the symptoms of the students in the susceptible compartment. Secondly it is at the vaccinated compartment *V(t),* the stage where students begin to be taught the curriculum. The third compartment is healthy *H(t),* these are students who had never been infected by DHOTS and are not at risk of being infected during

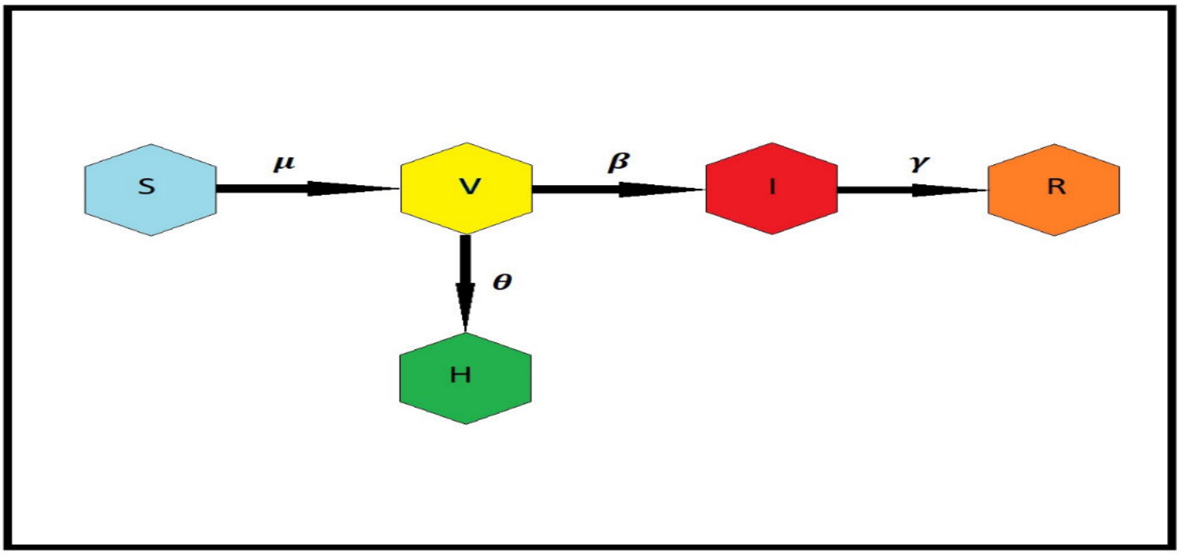

Figure 4.2: Susceptible S(t), vaccinated V(t), healthy H(t), infected I(t) and recovered R(t) Model (SVHIR Model)

the time span of N1 and N2. The fourth compartment is infected *I(t);* these are students who are infected by DHOTS during the time span of N1 and N2. The last compartment is the recovered *R(t),* which are students who have recovered from the DHOTS infection through the vaccine (curriculum). There is a criterion that will be explained later, which is used to determine the compartment of each student at time *t* between the five mentioned compartments.

With the help of *Kermack and McKendrick (1927)* and *Diagne et al (2021)* the SVHIR model is easily written using ordinary differential equations (ODEs) as follows:

$$\left.\begin{aligned}
\frac{dS}{dt} &= -\mu S(t), && (a)\\
\frac{dV}{dt} &= \mu S(t) - \theta H(t) - \beta V(t), && (b)\\
\frac{dH}{dt} &= \theta V(t), && (c)\\
\frac{dI}{dt} &= \beta V(t) - \gamma I(t), && (d)\\
\frac{dR}{dt} &= \gamma I(t). && (e)
\end{aligned}\right\} \qquad (4.4)$$

Where,

$$V(t) = H(t) + I(t) + R(t), \qquad (4.5)$$

and

$$N(t) = S(t) + V(t); \qquad N(0) = S(0). \tag{4.6}$$

All the parameters in the SVHIR model which have not been described before, are described in *Table 4.1*.

4.3.3. Determination of the SVHIR Model parameters

Since we know the time interval (1 to 180 days), we can determine the finite parameters as follows:

from (4.4)(a) we get,

$$\frac{dS}{dt} = -\mu S(t),$$

$$\frac{dS}{S} = -\mu dt,$$

$$\int_{S_0}^{S_f} \frac{dS}{S} = -\mu \int_{t_0}^{t_f} dt\ ,$$

$$\ln(S_f) - \ln(S_0) = -\mu(t_f - t_0)$$

$$\mu = \frac{\ln(S_f) - \ln(S_0)}{t_0 - t_f}; \qquad t_0 \neq t_f \tag{4.7}$$

Dividing (4.4)(c) by (4.4)(a), we get:

$$\frac{dH/_{dt}}{dS/_{dt}} = \frac{\theta V}{-\mu S},$$

$$\frac{dH}{dS} = \left(\frac{\theta V}{-\mu}\right)\frac{1}{S},$$

$$\int_{H_0}^{H_f} dH = \left(\frac{\theta V}{-\mu}\right)\int_{S_0}^{S_f} \frac{1}{S} dS,$$

Table 4.1: SVHIR model parameters and their descriptions.

Parameters	Description
μ	Vaccination rate
θ	Healthy individuals' discovery rate
β	Disease transmission rate
γ	Recovery rate
t_0	Initial or 1st day $t_0 = 1$
t_f	Final or 180th day $(t_f = 180)$.
S_0	Susceptible individuals on the 1st day/ Initial Susceptible individuals
S_f	Susceptible individuals on the 180th day/ Final Susceptible individuals
I_0	Infected individuals on the 1st day/ Initial Infected individuals
I_f	Infected individuals on the 180th day/ Final Infected individuals
H_0	Healthy individuals on the 1st day/ Initial Healthy individuals
H_f	Healthy individuals on the 180th day/ Final Healthy individuals
R_0	Recovered individuals on the 1st day/ Initial Recovered individuals
R_f	Recovered individuals on the 180th day/ Final Recovered individuals
V_f	Vaccinated individuals on the 180th day/ Final Vaccinated individuals
N_0	Total number of individuals on the 1st day/ Initial Total number of individuals

$$H_f - H_0 = \frac{-\theta V}{\mu}\left(\ln(S_f) - \ln(S_0)\right),$$

$$\theta = -\left[\frac{H_f - H_0}{\left(\ln(S_f) - \ln(S_0)\right)}\right]\frac{\mu}{V}.$$

Substituting (4.7) in the above we get:

$$\theta = \frac{H_f - H_0}{V(t_f - t_0)}; \qquad V \neq 0 \quad and \quad t_f \neq t_0 \tag{4.8}$$

Dividing (4.4)(e) by (4.4)(a), we get:

$$\frac{dR}{dS} = \frac{\gamma I}{-\mu S},$$

$$\int_{R_0}^{R_f} dR = \frac{\gamma I}{-\mu} \int_{S_0}^{S_f} \frac{1}{S} dS,$$

$$R_f - R_0 = \frac{-\gamma I}{\mu} \left[ln(S_f) - ln(S_0) \right],$$

$$\gamma = \frac{\mu}{I} \left[\frac{R_0 - R_f}{ln(S_f) - ln(S_0)} \right]$$

Substitute (4.7) in the above and solving we get:

$$\gamma = \left[\frac{\ln(S_f) - \ln(S_0)}{I(t_0 - t_f)} \right] \left[\frac{R_0 - R_f}{ln(S_f) - ln(S_0)} \right]$$

$$\gamma = \frac{R_f - R_0}{I(t_f - t_0)}; \qquad I \neq 0 \quad and \quad t_f \neq t_0 \qquad (4.9)$$

From (4.4)(c),

$$\frac{dH}{dt} = \theta V,$$

$$\int_{H_0}^{H_f} dH = \theta \int_{t_0}^{t_f} V dt,$$

$$\int_{t_0}^{t_f} V dt = \frac{H_f - H_0}{\theta}. \qquad (4.10)$$

Substituting (4.8) into (4.10), we get:

$$\int_{t_0}^{t_f} V dt = V(t_f - t_0). \qquad (4.11)$$

Add (4.4)(d) and (4.4)(e) we get,

$$\frac{dI}{dt}+\frac{dR}{dt}=\beta V\ ,$$

$$\int_{I_0}^{I_f} dI+\int_{R_0}^{R_f} dR=\beta\int_{t_0}^{t_f} Vdt\ , \qquad (4.12)$$

Substituting (4.11) into (4.12), we get:

$$I_f-I_0+R_f-R_0=\beta V(t_f-t_0)$$

$$\beta=\frac{I_f+R_f-(I_0+R_0)}{V(t_f-t_0)};\quad V\neq 0\quad and\quad t_f\neq t_0 \qquad (4.13)$$

4.3.4. Validation of the SVHIR model

It is part of model development procedure to validate the model before it first application. The general notion to validate any model is to compare it with the actual data. Unfortunately, in this study we have a very limited data in terms of data points of different times. The current study's data will not allow us to apply that notion into our model in (4.4), given we only have data at two points ($t=1$ and $t=180$). Nonetheless, that does not mean we cannot validate the model; it only means we cannot use the general notion. But nonetheless, this study only needs the model to be valid at $t=180$; given this is the point where we want to apply the model to investigate whether students benefited from the curriculum or not. Therefore, in our view the resulted functions after integrating (4.4) are predictions of the respective compartments at time t. Hence if those prediction functions are valid at $t=180=t_f$, we conclude that (4.4) is also valid at t_f.

4.3.4.1. Development of the general prediction functions and four parameters (μ,θ,γ and β)

Since we intend to find general prediction functions, indefinite integrals will be applied; hence the four parameters will be also general in this subsection.

From (4.4)(a), we get:

$$\frac{dS}{dt} = -\mu S,$$

$$\frac{dS}{S} = -\mu dt,$$

$$\int \frac{dS}{S} = -\mu \int dt,$$

$$\ln(S) = -\mu t + c_1,$$

$$S_p = S_0 e^{-\mu t}; \qquad S_0 = e^{c_1}. \tag{4.14}$$

Then solving (4.14) we get,

$$\mu = \frac{ln\left(\frac{S_0}{S_p}\right)}{t}; \qquad t \neq 0 \quad and \quad S_0, S_p > 0 \tag{4.15}$$

Where $S(t) = S,\ S(1) = S_0,\ S_p(t) = S_p$ and c_1 are respectively defined as the susceptible individuals at time t, susceptible individuals at $t = 1$, SVHIR model predicted number of susceptible individuals at time t and the first constant of integration.

Dividing (4.4)(c) by (4.4)(a), we get:

$$\frac{dH/dt}{dS/dt} = \frac{\theta V}{-\mu S},$$

$$\frac{dH}{dS} = \left(\frac{\theta V}{-\mu}\right)\frac{1}{S},$$

$$\int dH = \left(\frac{\theta V}{-\mu}\right)\int \frac{1}{S} dS,$$

$$H_p = \frac{-\theta V}{\mu}\ln(S_p) + c_2; \quad \mu \neq 0 \quad and \quad S_p > 0 \tag{4.16}$$

Substituting (4.15) into (4.16) and solve we get:

$$\theta = \frac{ln\left(\frac{S_0}{S_p}\right)(c_2 - H_p)}{t\,V\ln(S_p)}; \quad \{c_2 \geq H_p \mid S_0, S_p, t, V > 0 \ \ and\ \theta \geq 0\}. \tag{4.17}$$

In the above $V(t) = V,\ H_p(t) = H_p$ and c_2 are respectively defined as the number of the vaccinated individuals at time t (given by (4.5)), SVHIR model predicted number of healthy individuals at time t and second constant of integration. Each resulted lower or up bound of a constant of integration produced in each equation, will be retained throughout the study for simplicity. Then, starting by the constant of integration c_2, its lower bound found in (2.17) will be retained throughout the study.

Dividing (4.4)(e) by (4.4)(a), we get:

$$\frac{dR}{dS} = \frac{\gamma I}{-\mu S},$$

$$\int dR = \frac{\gamma I}{-\mu} \int \frac{1}{S} dS,$$

$$R_p = \frac{-\gamma I}{\mu} \ln(S_p) + c_3. \tag{4.18}$$

Substituting (4.15) into (4.18) and solve we get:

$$\gamma = \frac{ln\left(\frac{S_0}{S_p}\right)(c_3 - R_p)}{t\, ln(S_p)\, I}; \quad \{c_3 \geq R_p \,|\, I, S_0, S_p, t > 0 \ \ and\ \gamma \geq 0\}. \tag{4.19}$$

Then $I(t) = I$, $R_p(t) = R_p$ and c_3 are respectively defined as the number of the infected individuals at time t, SVHIR model predicted number of recovered individuals at time t and third constant of integration.

From (4.4)(c),

$$\frac{dH}{dt} = \theta V,$$

$$\int dH = \theta \int V dt,$$

$$\int V dt = \frac{H_p + c_4}{\theta}, \tag{4.20}$$

When adding (4.4)(d) and (4.4)(e) we get,

$$\frac{dI}{dt} + \frac{dR}{dt} = \beta V,$$

$$\int dI + \int dR = \beta \int V dt,$$

$$I_p + R_p + c_5 = \beta \int V dt \;. \tag{4.21}$$

Substitute (4.20) into (4.21) we get,

$$I_p + R_p + c_5 = \beta \left(\frac{H_p + c_4}{\theta} \right),$$

$$I_p = \beta \left(\frac{H_p + c_4}{\theta} \right) - \left(R_p + c_5 \right); \qquad \theta \neq 0 \tag{4.22}$$

$$\beta = \frac{\theta (I_p + R_p + c_5)}{H_p + c_4}; \tag{4.23}$$

Substitute (4.17) into (4.23) we get,

$$\beta = \frac{ln\left(\frac{S_0}{S_p}\right)\left(c_2 - H_p\right)(I_p + R_p + c_5)}{t\, V \ln\left(S_p\right)\left(H_p + c_4\right)} \;. \tag{4.24}$$

In the above $I_p(t) = I_p$, c_4 and c_5 are respectively defined as SVHIR model predicted number of the infected individuals at time t, fourth and fifth constant of integration. The constant c_5 is one of the most influential parameters in (4.24), since as it increases it brings a drastic change that will require more attention and in our view that is a separate study on its own. Therefore, since this study is more focused on the development side of the new approach (SVHIR model) than advancing it, we will keep $c_5 = 0$ for this current study and investigate other possibilities of this constant in the future if need be. Therefore, by retaining the lower bound of c_2 as mentioned in (4.17), we get (4.24) to be:

$$\beta = \frac{ln\left(\frac{S_0}{S_p}\right)\left(c_2 - H_p\right)(I_p + R_p)}{t\, V \ln\left(S_p\right)\left(H_p + c_4\right)} \;; \;\; S_0, S_p, t > 0 \;\; and \;\; \left\{c_4 > -H_p, c_2 \geq H_p \,|\, \beta \geq 0\right\} \tag{4.25}$$

4.3.4.2. Determination of the constants of integration (c_2, c_3 *and* c_4)

In the previous sub-section, we arrived at four different constants of integration but three of them remain partially unknown, given we only know their lower limits. Determining these constants was crucial for this study since they play a very important role towards achieving the goal of the study on HOTS, as it will be seen towards the end of this chapter.

From equation (4.17) to (4.25), we have been able to find the lower bounds or limits of the constants given as follows:

$$\left.\begin{array}{ll} c_2 \geq H_p & (a) \\ c_3 \geq R_p & (b) \\ c_4 > -H_p & (c) \end{array}\right\} \tag{4.26}$$

If we refer to the above sub-sections, S', H', I' and R' are rate of change according to their respective compartments, where their units are persons per day. It is true that each compartment rate of change cannot be greater than the total number of the population (N given by (4.6)), hence:

$$\left.\begin{array}{ll} \frac{dH}{dt} \leq N\,, & (a) \\ \frac{dR}{dt} \leq N\,, & (b) \\ \frac{dI}{dt} + \frac{dR}{dt} \leq N. & (c) \end{array}\right\} \tag{4.27}$$

In (4.27)(a) we have,

$$\frac{dH}{dt} \leq N. \tag{4.28}$$

Substituting (4.4)(c) into (4.28) we get,

$$\theta V \leq N,$$

$$\theta \leq \frac{N}{V}; \qquad V > 0 \tag{4.29}$$

Substituting (4.17) into (4.29) we get,

$$\frac{ln\left(\frac{S_0}{S_p}\right)\left(c_2 - H_p\right)}{t\,V \ln\left(S_p\right)} \leq \frac{N}{V}\,,$$

$$c_2 - H_p \leq \frac{t\,N \ln\left(S_p\right)}{ln\left(\frac{S_0}{S_p}\right)}\,,$$

$$c_2 \leq \frac{t\,N \ln\left(S_p\right)}{ln\left(\frac{S_0}{S_p}\right)} + H_p; \qquad S_0, S_p > 0 \tag{4.30}$$

Therefore, combining (4.26)(a) and (4.30) we get:

$$\left\{H_p \le c_2 \le \frac{t\,N\ln(S_p)}{ln\left(\frac{S_0}{S_p}\right)} + H_p\right\}. \tag{4.31}$$

Again, in (4.27)(b) we also have,

$$\frac{dR}{dt} \le N. \tag{4.32}$$

Substituting (4.4)(e) into (4.32) we get,

$$\gamma I \le N,$$

$$\gamma \le \frac{N}{I}. \qquad I \ne 0 \tag{4.33}$$

Substituting (4.19) into (4.33) we get,

$$\frac{ln\left(\frac{S_0}{S_p}\right)(c_3 - R_p)}{t\,ln(S_p)\,I} \le \frac{N}{I},$$

$$c_3 \le \frac{Nt\,ln(S_p)}{ln\left(\frac{S_0}{S_p}\right)} + R_p. \qquad S_0, S_p > 0 \tag{4.34}$$

Therefore, combining what (4.26)(b) and (4.34) we get:

$$\left\{R_p \le c_3 \le \frac{Nt\,ln(S_p)}{ln\left(\frac{S_0}{S_p}\right)} + R_p\right\}. \tag{4.35}$$

Where we differentiate (4.5), we get:

$$\frac{dV}{dt} = \frac{dH}{dt} + \frac{dI}{dt} + \frac{dR}{dt}. \tag{4.36}$$

Substituting (4.4)(c), (4.4)(d) and (4.4)(e) into (4.36) we get,

$$\frac{dV}{dt} = \theta V + \beta V. \tag{4.37}$$

Equating (4.4)(b) and (4.37) we get,

$$2\beta V = \mu S - \theta(H + V). \tag{4.38}$$

Substituting (4.15), (4.17) and (4.25) into (4.38) we get,

$$\frac{2\,ln\left(\frac{S_0}{S_p}\right)\left(c_2 - H_p\right)(I_p + R_p)V}{t\,\mathrm{V}\ln\left(S_p\right)\left(H_p + c_4\right)} = \frac{S\,ln\left(\frac{S_0}{S_p}\right)}{t} - \frac{ln\left(\frac{S_0}{S_p}\right)\left(c_2 - H_p\right)(H + V)}{t\,V\ln\left(S_p\right)}\,. \tag{4.39}$$

Solving (4.39) we get,

$$c_4 = \frac{2(I_p + R_p)}{\frac{S\,ln\left(S_p\right)}{c_2 - H_p} - \left(\frac{H + V}{V}\right)} - H_p. \tag{4.40}$$

Substituting (4.30) into (4.40) we get,

$$c_4 \le \frac{2(I_p + R_p)}{\frac{S\,ln\left(\frac{S_0}{S_p}\right)}{tN} - \left(\frac{H + V}{V}\right)} - H_p\,. \tag{4.41}$$

Hence,

$$c_4 \le \frac{2tNV\left(I_p + R_p\right)}{S\,ln\left(\frac{S_0}{S_p}\right)V - tN(H + V)} - H_p; \qquad S\,ln\left(\frac{S_0}{S_p}\right)V \ne tN(H + V) \tag{4.42}$$

Therefore from (4.26) – (4.42), we get the range of the constants of integrations as:

$$\left.\begin{aligned} H_p \le c_2 \le \frac{t\,N\ln\left(S_p\right)}{ln\left(\frac{S_0}{S_p}\right)} + H_p \\ R_p \le c_3 \le \frac{Nt\,ln\left(S_p\right)}{ln\left(\frac{S_0}{S_p}\right)} + R_p \\ -H_p < c_4 \le \frac{2NtV\left(I_p + R_p\right)}{S\,ln\left(\frac{S_0}{S_p}\right)V - Nt(H + V)} - H_p \end{aligned}\right\} \tag{4.43}$$

4.3.4.3. Validation

Note that from (4.14) to (4.22), it can be observed that we only produced four compartment prediction functions, whereas we have 5 compartments. We purposefully left out the vaccinated prediction functions (V_p), because of the following reason:

It has no influence at all in the final conclusions of this study. If all other prediction functions are valid except the vaccinated prediction function, the SVHIR model will still be effective for this current study. However, it should be clear that only the prediction function that is insignificant not the compartment itself. Given the vaccinated compartment plays a vital role to other compartments more specially on healthy and infected.

Therefore, the 4 resulted prediction functions from (4.14) to (4.22) that represent our SVHIR model are as follows,

$$\left.\begin{aligned} &S_p = S_0 e^{-\mu t}\ , \\ &H_p = \frac{-\theta V}{\mu}\ln\left(S_p\right) + c_2\ , \\ &R_p = \frac{-\gamma I}{\mu}\ln\left(S_p\right) + c_3, \\ &I_p = \beta\left(\frac{H_p + c_4}{\theta}\right) - R_p, \quad where\ c_5 = 0\ in\ (4.25). \end{aligned}\right\} \tag{4.44}$$

Let's consider the model (4.44) to be valid at $t\ = 180 = t_f$. In that case it will accurately predict all the compartments from the actual data. Hence $S_p = S_f$, $H_p = H_f$, $R_p = R_f$ and $I_p = I_f$ at $t = t_f$; where the S_f, H_f, R_f and I_f are from the actual data at time $t = t_f$. Since the validation is done at $t = t_f$ implies $N = N_f$, $V = V_f$ and $S = S_f$. Then the constants of integration from (4.43) ranges as follows:

$$\left.\begin{aligned} H_f \le c_2 &\le \frac{t_f\ N_f \ln\left(S_f\right)}{ln\left(\frac{S_0}{S_f}\right)} + H_f\ , \\ R_f \le c_3 &\le \frac{N_f t_f\ ln\left(S_f\right)}{ln\left(\frac{S_0}{S_f}\right)} + R_f\ , \\ -H_f < c_4 &\le \frac{2N_f t_f V_f\left(I_f + R_f\right)}{S_f ln\left(\frac{S_0}{S_f}\right)V_f - N_f t_f\left(H_f + V_f\right)} - H_f\ . \end{aligned}\right\} \tag{4.45}$$

Note, there are two types of the four parameters derived in this chapter. Firstly, the indefinite type found from (4.15) to (4.25), which was used to estimate the lower limits of the constants of integration. Secondly, the finite type found from (4.7) to (4.13). Both those types serve the same purpose when the model is valid, hence either one of them will be suitable. In our case, we choose the finite type given it is less complex; hence at $t = 180$ the four parameters in (4.7), (4.8), (4.9) and (4.13) become:

$$\left.\begin{aligned} \mu &= \frac{\ln(S_f) - \ln(S_0)}{t_0 - t_f}\,, && (a) \\ \theta &= \frac{H_f - H_0}{V_f(t_f - t_0)}\,, && (b) \\ \gamma &= \frac{R_f - R_0}{I_f(t_f - t_0)}\,, && (c) \\ \beta &= \frac{I_f + R_f - (I_0 + R_0)}{V_f(t_f - t_0)}. && (d) \end{aligned}\right\} \tag{4.46}$$

If we can accurately predict the respective compartments in the actual data by using (4.44), (4.45) and (4.46), then we conclude that the SVHIR model is valid at $t = 180$.

4.3.5. Basic reproductive ratio

The basic reproductive ratio of the SVHIR model is given as follows:

$$R_0 = \frac{\text{Disease transmission rate}}{\text{Recovery rate}} = \frac{\beta}{\gamma}\,. \tag{4.47}$$

The basic reproductive ratio at $t = 180 = t_f$, is found by substituting (4.46)(c) and (4.46)(d) into (4.46), hence:

$$R_0 = \frac{\dfrac{I_f + R_f - (I_0 + R_0)}{V_f(t_f - t_0)}}{\dfrac{R_f - R_0}{I_f(t_f - t_0)}}\,,$$

$$R_0 = \frac{I_f\left[I_f + R_f - (I_0 + R_0)\right]}{V_f(R_f - R_0)}\,. \tag{4.48}$$

According to our model, at $t = 1$ all students are just arriving and susceptible. None is infected or recovered, hence $I_0 = 0$ and $R_0 = 0$. Then (4.48) become,

$$R_0 = \frac{I_f(I_f + R_f)}{V_f R_f}; \qquad V_f = I_f + H_f + R_f. \tag{4.49}$$

4.3.6. Association of the actual data with the SVHIR model

In this study we have two HOTS tests (*Appendix 2 and 3* part 2) and we use their test scores or marks (M_i) to categorise students according to the SVHIR model compartments. However, before detailing the compartments, we first define the tests scores ranges respectively as shown in *Table 4.2*. Note the following when interpreting the table:

1. A score less than or equal to 5% cannot be used to define the status of a student, it's a nil. Given this score is highly possible to be obtained by a person who guessed the answers without being exposed to the curriculum. Therefore, we equivalate this person as someone who never took the test, hence this score is associated with susceptible compartment.
2. A student with a score between 5% and 50% counts as a failed, hence this score is associated with infection compartment.
3. A student with a score at 50% and above counts as a pass, hence this score is associated with healthy or recovery compartment.

There are 15 possible combination outcomes if a student takes the two HOTS tests (*Appendix 2 and 3*, part 2), and each outcome defines the SVHIR model compartment as shown in *Table 4.3*. Those outcomes are explained as follows:

1. A student who got nil in the first test and nil in the second test is considered susceptible. The first test shows symptoms of susceptibility (neither infected nor healthy but at risk of infection), towards the end of the curriculum the second test confirms the symptoms remained the same, that means the student did not move to the vaccine compartment. Hence the student stays in the susceptible compartment. Nonetheless, this does not mean the curriculum was not presented to the student but rather means it was presented and did not make any significant impact or sink to the student. Therefore, the student is the same as the time of arrival, which happens at the susceptible stage.
2. A student who got nil in the first test and failed in the second test is considered infected. The first test show symptoms of susceptibility, towards the end of the curriculum the second test confirms that the student is infected. Hence the student will move from susceptible *S(t)*, vaccinated *V(t)* and to infected *I(t)* compartment. In this case, the curriculum was presented and did make an impact to the student but not enough.

Table 4.2: Description and compartmental categorization of students based on HOTS tests scores range.

Order	Scores	Description	Compartment
1.	$0 \leq M_i \leq 5\%$	Nil	susceptible
2.	$5\% < M_i < 50\%$	fail	infection
3.	$M_i \geq 50\%$	pass	healthy or recovery

Table 4.3: Compartmental categorization of students based on HOTS tests scores.

Outcome	Test 1 Marks $(t = 0)$	Test 2 Marks $(t = 180)$	Resultant Compartment $(t = 180)$
1	$0 \leq M_i \leq 5\%$	$0 \leq M_i \leq 5\%$	Susceptible
2	$0 \leq M_i \leq 5\%$	$5\% < M_i < 50\%$	Infected
3	$0 \leq M_i \leq 5\%$	$M_i \geq 50\%$	Recovered
4	$5\% < M_i < 50\%$	$0 \leq M_i \leq 5\%$	Infected
5	$5\% < M_i < 50\%$	$5\% < M_i < 50\%$	Infected
6	$5\% < M_i < 50\%$	$M_i \geq 50\%$	Recovered
7	$M_i \geq 50\%$	$0 \leq M_i \leq 5\%$	Infected
8	$M_i \geq 50\%$	$5\% < M_i < 50\%$	Infected
9	$M_i \geq 50\%$	$M_i \geq 50\%$	Healthy
10	$0 \leq M_i \leq 5\%$	None	Excluded $N_f < S_0$
11	$5\% < M_i < 50\%$	None	Excluded $N_f < S_0$
12	$M_i \geq 50\%$	None	Excluded $N_f < S_0$
13	None	$0 \leq M_i \leq 5\%$	Excluded $N_f > S_0$
14	None	$5\% < M_i < 50\%$	Infected $N_f > S_0$
15	None	$M_i \geq 50\%$	Excluded $N_f > S_0$

$*M_i$ – Student's HOTS Test Marks/score **t – Days*

3. A student who got nil in the first test and pass in the second test is considered recovered. The first test show symptoms of susceptibility, towards the end of the curriculum the second test confirms the symptoms has improved. Hence the student will move from susceptible *S(t),* vaccinated *V(t)* and to healthy *I(t)* compartment. When the curriculum is presented to this student is much impactful.
4. A student who got fail in the first test and nil in the second test is considered infected. The first test show symptoms of infection, towards the end of the curriculum the second test confirms the symptoms of at risk of infection. This student is considered infected. In the model this student will move from susceptible *S(t),* vaccinated *V(t)* and to infected *I(t)* compartment. In this case, the curriculum was presented and did make impact to the student but not enough.
5. A student who got fail in the first test and fail in the second test is considered infected. The first test show symptoms of infection, towards the end of the curriculum the second test confirms the symptoms remained the same. Hence the student will move from susceptible *S(t),* vaccinated *V(t)* and to the infected *I(t)* compartment. In this case, the curriculum was presented and did make impact to the student but not enough.
6. A student who got fail in the first test and pass in the second test is considered recovery. The first test show symptoms of infection, towards the end of the curriculum the second test confirms the symptoms has gotten better. Hence the student will move from susceptible *S(t),* vaccinated *V(t),* infected *I(t)* and to the recovered *R(t)* compartment. In this case, the curriculum was presented and did make impact to the student.
7. A student who got pass in the first test and nil in the second test is considered infected. The first test shows symptoms of being healthy, towards the end of the curriculum the second test confirms the symptoms susceptible. For a student to be from healthy to susceptible, it is the indication of degradation of the skill; and that can only happen when someone is infected. Hence the student will move from susceptible *S(t),* vaccinated *V(t)* and to infected *I(t)* compartment. In this case, the curriculum was presented and did make impact to the student but not enough.
8. A student who got pass in the first test and fail in the second test is considered infected. The first test show symptoms of being healthy, towards the end of the curriculum the second test confirms the symptoms infection. For a student to be from healthy to susceptible, it is the indication of degradation of the skill; and that can only happen when someone is infected. Hence the student will move from susceptible *S(t),*

vaccinated *V(t)* and to infected *I(t)* compartment. In this case, the curriculum was presented and did make impact to the student but not enough.

9. A student who got pass in the first test and pass in the second test is considered healthy. The first test show symptoms of being healthy, towards the end of the curriculum the second test confirms the symptoms has remained the same. Hence the student will move from susceptible *S(t),* vaccinated *V(t)* and to healthy *H(t)* compartment. This student is presumed to have arrived already equipped with HOTS, hence when the curriculum is presented to them is much impactful.
10. A student who got nil in the first test and did not get a chance to participate in the second test, is excluded in the current study. The reason is, with the 3 possible scores (Nil, fail and pass) the student could have obtained in the second test, the student could either be outcome 1 or 2 or 3 in *Table 4.3*. Which are 3 different compartments (Susceptible, infected, and recovered) the student could possibly belong to, and the study is unable to conclude about the student' compartment between the 3 in the absence of the second test score. Hence, the student is excluded.
11. A student who got fail in the first test and did not get a chance to participate in the second test, is excluded in the current study. The reason is, with the 3 possible scores (Nil, fail and pass) the student could have obtained in the second test, the student could either be outcome 4 or 5 or 6 in *Table 4.3*. Which are 2 different compartments (Infected and recovered) the student could possibly belong to, and the study is unable to conclude about the student' compartment between the 2 in the absence of the second test score in that case. Hence, the student is excluded.
12. A student who got pass in the first test and did not get a chance to participate in the second test, is excluded in the current study. The reason is, with the 3 possible scores (Nil, fail and pass) the student could have obtained in the second test, the student could either be outcome 7 or 8 or 9 in *Table 4.3*. Which are 2 different compartments (Infected and healthy) the student could possibly belong to, and the study is unable to conclude about the student' compartment between the 2 in the absence of the second test score in that case. Hence, the student is excluded.
13. A student who did not participate in the first test and got nil in the second test, is excluded in the current study. The reason is, with the 3 possible scores (Nil, fail and pass) the student could have obtained in the first test, the student could either be outcome 1 or 4 or 7 in *Table 4.3*. Which are 2 different compartments (Susceptible and infected) the student could possibly belong to, and the study is unable to conclude about

the student’ compartment between the 2 in the absence of the first test score in that case. Hence, the student is excluded.

14. A student who did not participate in the first test and get fail in the second test, is considered infected. The reason is, with the 3 possible scores (Nil, fail and pass) the student could have obtained in the first test, the student could either be outcome 2 or 5 or 8 in *Table 4.3*. Which are all the infected compartments.
15. Lastly, this is a student who only participated in the second test and passed. This student will also be excluded in the current study. The reason is, with the 3 possible scores (Nil, fail and pass) the student could have obtained in the first test, the student could either be outcome 3 or 6 or 9 in *Table 4.3*. Which are 2 different compartments (Recovered and healthy) the student could possibly belong to, and the study is unable to conclude about the student’ compartment between the 2 in the absence of the first test score in that case. Hence, the student is excluded.

4.3.7. Application of SVHIR model instruction

From the SVHIR model, we produced the basic reproduction in (4.49) expressed as:

$$R_0 = \frac{I_f\left(I_f + R_f\right)}{V_f R_f}; \qquad V_f = I_f + H_f + R_f \neq 0 \quad and \quad R_f \neq 0 \qquad (4.50)$$

For this study, the data was collected at a TVET College and categorized according to *Table 4.3*. In chapter 5, (4.50) is applied into the data to determine which of the two following two cases result:

Case 1 ($R_0 > 1$)

This mean the DHOTS will continue, the curriculum has failed to equip students with HOTS.

Case 2 ($R_0 < 1$)

This mean the DHOTS will or is dying out, the curriculum has equipped students with HOTS.

4.4. Data collection and Participants

Ethical clearance was received from our university (UKZN) to conduct this study. Our study was based on TVET Colleges in KwaZulu-Natal (KZN), that in the year 2022 were found to be 9 in total. Those KZN TVET Colleges were eLangeni, eThekwini, eMnambithi, eMgungundlovu, eMajuba, eMfolozi, eSayidi, Mthashane and Coastal TVET College. Contacting the gate keepers of the mentioned KZN TVET Colleges for data collection was not successful as the study intended. We first contacted the Department of Higher education; they co-operated and even gave us the permission letter to carry out the study in the TVET Colleges in KZN. The first College we contacted was Elangeni TVET College, most of the campus managers were more than willing to let us collect data but only after the principal's permission was granted. We communicated with the principal's office for some time, they requested all the necessary paperwork, after that they discontinued the communication without bringing any reason to our attention. For six months we tried to communicate with them, they did not respond. Secondly, we tried to contact Ethekwini, Costal and eMgungudlovu TVET Colleges; every person we could find in the three Colleges directed us to the principals and the principals were always not found in their offices up to the time of this thesis compilation. We also contacted eMajuba TVET College and submitted all the necessary paperwork, they asked for some time to go through the submitted paperwork. After 6 months, they gave use the permission but unfortunately it was towards the end of N1 trimester. For our study we needed students at the beginning of N1 and at the end of N2, hence they couldn't be a part of the current study. However, we have agreed to visit them in the year of 2023 when we advance the current study. EMfolozi, eSayidi and Mthashane TVET Colleges were not contacted due to their location, we could not afford to arrive at those TVETs due to financial limitations at the time of data collection. Only eMnambithi TVET College gave us permission to collect data in due time. Hence our data is collected from 47 students and 2 lecturers of the N1 and N2 mathematics classes, at Ezakheni E-section campus.

As the aim of the study indicated, this study focused more on the two models' development than on their applications. Therefore, in that case the limited data of 47 mathematics students is sufficient to some extent, given it is there for the testing of the models. There will be a separate study in the future, where the models in the current study are applied in a larger-scale data. Nonetheless, conclusions drawn from the current limited data still have a substantial contribution to some extent; since it depends on how data sampling is categorized. There are four famous sampling strategies namely convenience, simple random, stratified random and cluster *[Majid, 2018]*. Convenience is focusing on the most accessible or available participants. That was the category of our sample and where our data should make a substantial contribution; given it was the only accessible data we had access to. The second sampling strategy is simple random, this is where everyone in the population has an equal chance to be selected. This is where all the KZN TVETs have equal opportunity, hence our data in this case have slight contribution. The limited data has a slight disadvantage in this case if the given population is large. Thirdly, it is a stratified random, this is a simple random but within a pre-defined subgroup. This is another extent where the limited data has a contribution chance. For instance, in our case, pre-defining our population as uThukela district, eMnambithi TVET College is the only one in that district. Hence our limited data has a very substantial contribution in that instance. Lastly it a cluster, this is also a simple random but within a natural occurring sub-group. This is also where our limited data might get a chance to make a substantial contribution, if it happens that the natural occurrence sub-group criteria is in favour of eMnambithi TVET College. In that regard, even though our limited data was used for testing the models, but also, the conclusions drawn from it are taken into consideration for further investigation.

Where we collected our data (Ezakheni E-section campus), at the time of data collection, there were 2 lecturers responsible for lecturing N1 and N2 mathematics curriculum. Therefore, for the identification of the learning experience, *Appendix 1* was used as a data collection instrument administered to those mathematics lecturers. The data for the curriculum objective attainment and HOTS evaluation, was collected from the 47 students using both *Appendices 2 and 3; parts 1 and 2*. The first part of data collection took place in March 2022 and the second part in October 2022. Normally, N1 classes start in January but due to covid-19; classes were disturbed. N1 ended up starting in March 2022. Nonetheless students were given an equal opportunity of 180 days to do the N1 to N2 curriculum.

See *Appendix 4* for what the questions in *Appendices 2 and 3* focused on and the mark distribution for those questions.

4.5. Conclusion

Two models were discussed in this chapter, the first one is the Tyler model which was adapted in this study for application. The Tyler model has 4 stages, namely, selection of objective(s), selection of learning experience(s), organization learning experience(s), and evaluation of the learning experience(s). However, the current study only adapted the last 2 stages, given we were dealing with an existing curriculum. The second one was the SVHIR model, which originated from this study. The SVHIR model was developed for the purpose of HOTS evaluation in the context of the TVET college curriculum. Both models at this stage were developed but had not been applied. The reporting on their application is done in chapter 5. The majority of the KZN TVET colleges did not participate in the study, and that deprived us the opportunity to observe the behaviour of our models in a larger scale data. However, the available data was sufficient to make necessary conclusions, to some extent.

Chapter 5: Application of the Models

5.1. Introduction

This chapter present the application of the two models namely Tyler's objective model and SVHIR model. It starts with the pre-evaluation of the learning experience, where the participants are N1 and N2 lecturers. It continues with the learning experience evaluation which consist of the instrument reliability test and the actual learning experience evaluation. Further on, it presents the application of the SVHIR model to investigate HOTS. That consists of the SVHIR model validation and application. Lastly the chapter presents the relationship between the students' Objectives attainment and HOTS scores.

5.2. Pre – evaluation of the learning experience

As mentioned in chapter 3 and 4 that prior to the actual learning experience evaluation, identification of the learning experience is very crucial. *Table 5.1* presents the lecturers' perspective of the type of the learning experience practiced at eMnambithi TVET College in N1 and N2 mathematics classes. The results were obtained after assessing N1 and N2 mathematics lecturers separately using *Appendix 1.* Further on, each lecturer's response was categorized according to the learning experience it reflects in chapter 3 section 3.2.2.

According to the N1 lecturer's response in *Table 5.1*, the operating learning experience in N1 mathematics class is characterised with 50% of acquiring information and 50% of thinking skills. On the other hand, the same class when it proceeds to N2 it encounters a bit different learning experience. According to the N2 lecturer's response, in the two responses we couldn't extract the information about the learning experience. However, the two clear responses shows that the most dominant learning experience is the acquiring of information. Then when combining N1 and N2, the learning experience that is highly possible to be in operation is the acquiring of information and thinking skills type of learning experience.

In general, the acquiring of information and thinking skills type of learning experience seems to be suitable for a subject like mathematics, however that is not enough. What determines the suitability of a learning experience is its effectiveness on students. Hence at this stage we can't conclude on the learning experience in operation.

Table 5.1: Pre - evaluation results from Appendix 1.

N1 MATHEMATICS LECTURER		
Question	**Response**	**Learning experience**
1) When you give examples, scenarios or explanations during your teaching sessions; which of the following is mostly your priority? [*see Appendix 1*]	Using different schemes of information organization is my priority.	Focusing on acquiring information
2) If opportunities allow you, what kinds of equipment or instruments do you or would you mostly use for demonstration and explanation during your teaching sessions?	Equipment or instruments that will develop the student's thinking skills.	Focusing on thinking skills
3) What kinds of questions do you mostly ask your students during your class sessions or in a form of homework, tests, activities, and examinations?	I ask questions that require a student to use the acquired information.	Focusing on acquiring information
4) If you get a chance, what kinds of questions do you or would you influence your students to ask you during your teaching session or outside your teaching session?	Questions that cannot be immediately found in the textbook or any other publication.	Focusing on thinking skills
N2 MATHEMATICS LECTURER		
Question	**Response**	**Learning experience**
1) When you give examples, scenarios or explanations during your teaching sessions; which of the following is mostly your priority?	Using different schemes of information organization is my priority.	Focusing on acquiring information
2) If opportunities allow you, what kinds of equipment or instruments do you or would you mostly use for demonstration and explanation during your teaching sessions?	I'm not using or would not use any equipment or instruments for demonstration and explanation purposes.	No information extracted

3) What kinds of questions do you mostly ask your students during your class sessions or in a form of homework, tests, Activities, and examinations?	I ask unfamiliar questions that require a student to relate various facts and ideas.	Focusing on acquiring information
4) If you get a chance, what kinds of questions do you or would you influence your students to ask you during your teaching session or outside your teaching session?	Other kinds of questions not mentioned above.	No information extracted

5.3. Evaluation of the learning experience

After identifying the acquiring of information and thinking skills learning experience operating at eMnambithi TVET College, the arising question is around its effectiveness to equip students for the attainment of curriculum objectives. Hence this section focuses on the actual evaluation of the learning experience. However, prior to the actual evaluation it presents the reliability test of the evaluation instrument.

5.3.1. Reliability of the evaluation instrument

As mentioned in chapter 3 sub-section 3.4.4, *Appendix 3* question 10 was used to confirm the reliability of both data collection instruments (*Appendix 2 and 3*). Given both *Appendices 2 and 3* are the same except their numerical values, the reliability of *Appendix 3* implies the reliability of *Appendix 2*. The testing of the reliability was to make sure that the instrument was within the capabilities of the students, even though we have taken the questions from their curriculum. *Table 5.2* presents the responses of students when they were asked if the data collection instrument is relevant to their curriculum? The results indicate that the first portion of 55.6% responded yes, the second portion of 4.3% responded no, and the last portion of 40.4% did not answer the question. The last portion (40.4%) will be excluded since their stand is unclear. Therefore, when we only consider the first and second portion; we find that majority (55.6%) of the students considered the questions in *Appendix 1 and 2* to be relevant to their curriculum. Hence the data collection instruments were reliable, from both the perspectives of the researcher and the participants.

Table 5.2.: Validation of the evaluation instrument with Appendix 3 question 10.

Students' response	Number of students	Percentage
Yes	26	55.6%
No	2	4.3%
No comment	19	40.4%

On the other hand, the 55.6% majority also indicate a possibility of satisfactory N1 and N2 mathematics lecturers' content delivery abilities. In that regard, the current work establishes an assumption that the lecturer's content delivery ability was satisfactory at eMnambithi TVET College. As mentioned, according to some scholars there are many variables that can contribute to students attaining HOTS; but the most critical are lecturer's content delivery abilities and the curriculum *[Yen and Halili, 2015; Chinedu and Kamin, 2015; Tanujaya et al, 2017; Retnawati et al, 2018; Gupta and Mishra, 2021]*. The 55.6% majority results of the current study, leaves the curriculum as the only major contributing factor in the occasion where students are found to have a poor HOTS. Hence, in that case the impact of the curriculum remains the only variable to be evaluated in *Appendix 1 and 2* students' responses.

5.3.2. Actual evaluation of the learning experience

The actual learning experience evaluation was executed using the average difference ($\bar{X}$) developed in chapter 4, see sub-section 4.2.2. The average difference does not necessary measure the actual quantity of the curriculum objectives attained, but rather it measures the improvement of student's objectives attainment from the pre-assessment to post assessment. Hence a higher average difference does not necessarily reflect the high number of objectives attained, but rather it reflects a good impact of the learning experience on students. For instance, student A obtained 2% on the pre-assessment and 60% on the post assessment. Student B obtained a 90% on the pre-assessment and 90% on the post assessment. Student B will have a lower average difference (improvement) but higher objectives attainment than student A.

In our case, all the necessary parameters for the calculation of average difference were taken from the actual data summarised in *Appendix 5* and the results are presented in *Table 5.3*. The average difference was found to be 15.34%, excluding student number 29 in *Appendix 5*. The

Table 5.3: Parameters for learning experience evaluation extracted from Appendix 5.

Parameters	Actual Data Values
Summation of percentage difference	$\sum_{i=1}^{47} x_i = 705.44$
Total number of students	$N = 47$
Average difference	$\bar{X} = 15.34\%$

Table 5.4: SVHIR compartment values from the actual data in Appendix 5.

Compartment Parameters	Actual Data Values
Initial time (in days)	$t_0 = 1$
Final time (in days)	$t_f = 180$
Initial Susceptible individuals	$S_0 = 47$
Final Susceptible individuals	$S_f = 2$
Initial Infected individuals	$I_0 = 0$
Final Infected individuals	$I_f = 36$
Initial Healthy individuals	$H_0 = 0$
Final Healthy individuals	$H_f = 6$
Initial Recovered individuals	$R_0 = 0$
Final Recovered individuals	$R_f = 3$
Final Vaccinated individuals	$V_f = H_f + I_f + R_f = 45$
Initial Total number of individuals	$N_0 = 47$
Final Total number of individuals	$N_f = 47$

* $V_f = H_f + I_f + R_f$ *from equation 5 in chapter 4.*

student obtained 100% in both assessments of the objectives' attainment; hence improvement cannot be measured since there was no room for improvement for that student. The obtained 15.34% average difference is far less than 50%. This means on average the learning experience in operation only improves the students' Objectives attainment by 15.34% from the pre-assessment. Hence, we deduce that the current learning experience in operation is not effective enough to equip students for the curriculum objectives attainment.

As elaborated in chapter 4 sub-section 4.2.1, the three components of the learning experience are selected learning situations, learning activities and students' interaction. However, the foundation of the three components is the curriculum, given the curriculum has a major influence to determining the nature of each component. Therefore, whatever is reflected by each component relates to the effectiveness of the curriculum. In that regard, we deduce that the N1 to N2 TVET College's mathematics curriculum is most likely to be incapable of equipping students for the attainment of its own objectives at eMnambithi TVET College.

5.4. Evaluation of HOTS

This section presents the investigation of the capability of the curriculum to equip the students with HOTS. The evaluation was executed using the SVHIR model developed in chapter 4. Hence this section specifically presents the validation and application of the SVHIR model by using the actual data in *Appendix 5*. Note, the categorization of students according to the SVHIR model compartments in *Appendix 5* was accomplished by applying *Table 4.3* in chapter 4. From those categorizations, *Table 5.4* was produced.

5.4.1. Validation of SVHIR Model

In chapter 4 sub-section 4.3.4.3, our validation notion deduces that the SVHIR model is valid if and only if (4.44) can predict the compartments with the integration constants satisfying (4.45). Substituting all the necessary values in (4.45), the integration constants' intervals for the current study's actual data are as follows:

$$\left.\begin{aligned} 6 \leq c_2 &\leq 1\,863.46\,, \\ 3 \leq c_3 &\leq 1\,860.47\,, \\ -6 < c_4 &\leq -0.03\,. \end{aligned}\right\} \tag{5.1}$$

The compartment rates calculated using (4.46) and the actual data are presented in *Table 5.5*. Those rates together with the actual data were used to predict the SVHIR model compartments in *Table 5.6*, where the integration constants satisfy the stipulated intervals. Therefore, this study concludes that the SVHIR model is valid at $t = t_f = 180$.

Table 5.5: SVHIR compartment rates calculated from the actual data.

Compartment rate names	Calculated values
Vaccination rate	$\mu = 0.0180$
Healthy individuals' discovery rate	$\theta = 0.0007$
Disease transmission rate	$\gamma = 0.0005$
Recovery rate	$\beta = 0.0050$

Table 5.6: Predicted versus actual SVHIR compartments.

Integration constant	**Predicted compartment at t_f**	**Actual compartment at t_f**
---	$S_p = 1.84 \approx 2$	$S_f = 2$
$c_2 = 7.21$	$H_p = 6$	$H_f = 6$
$c_3 = 3.69$	$R_p = 2.71 \approx 3$	$R_f = 3$
$c_4 = -0.54$	$I_p = 36$	$I_f = 36$

5.4.2. Application of SVHR Model

As mentioned, in chapter 4 sub-section 4.3.7, the HOTS was investigated by applying the extension of the SVHIR model called basic reproductive ratio. Substituting all necessary variables taken from *Table 5.4* into (4.50) in chapter 4 we get,

$$R_0 = 10.4$$

This basic reproductive ratio relates to the case 1 ($R_0 > 1$) according to chapter 4 sub-section 4.3.7, which means the students were not cured of the DHOTS during the period of 6 months that they were exposed to the N1 to N2 mathematics curriculum. In other words, the SVHIR model indicates that the curriculum was found to be most likely incapable of equipping students with HOTS at eMnambithi TVET College, since the lecturers' content delivery abilities are satisfactory.

The current study identifies an satisfactory level of accuracy of the SVHIR model. Looking at *Appendix 5* or *Table 4.4*, at the end of the curriculum about 77% of students remain DHOTS infected. That is a huge portion of the population, hence the basic reproductive ratio is expected to be far beyond 1. Indeed, the basic reproductive ratio from the SVHIR model was as anticipated.

5.5. Relationship between the students' Objectives attainment and HOTS.

In this section we are focusing on one of the HOTS aspects, which is to influence students' curriculum objectives attainment. We wanted to know the impact of HOTS on the curriculum objectives attainment. Hence, we used *Appendix 5* to compare the objectives attainment and

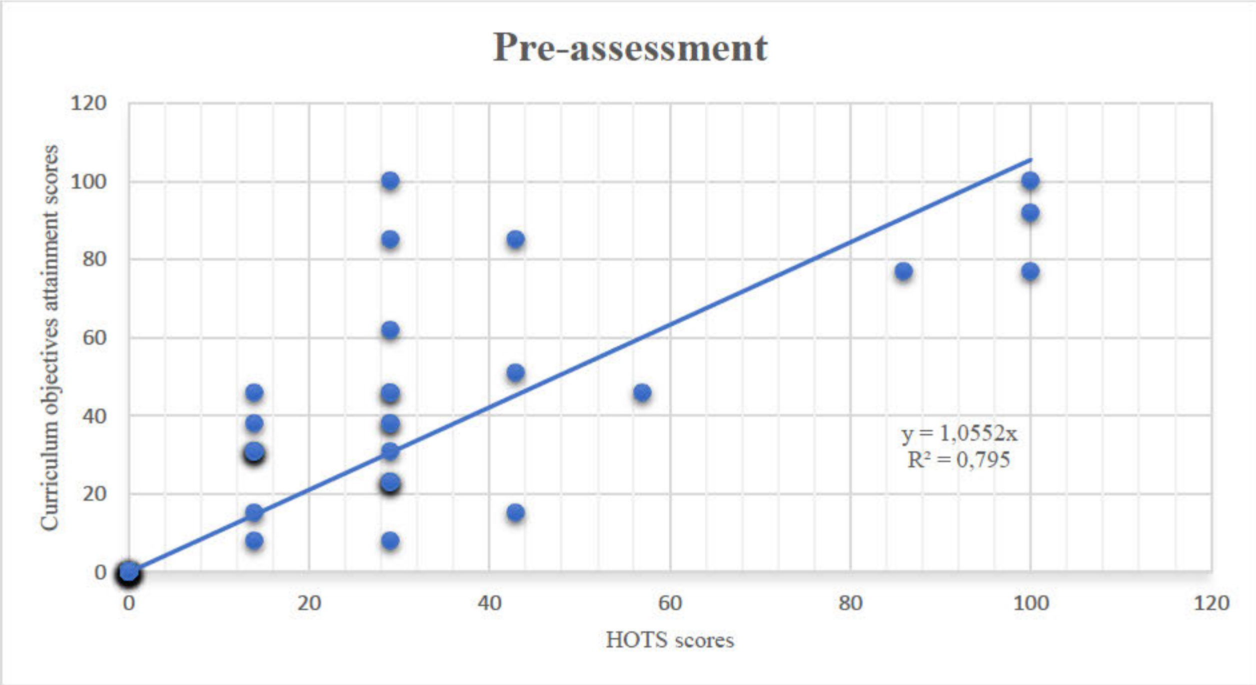

Figure 5.1: Relationship between the students' objectives attainment and HOTS in the pre-assessment.

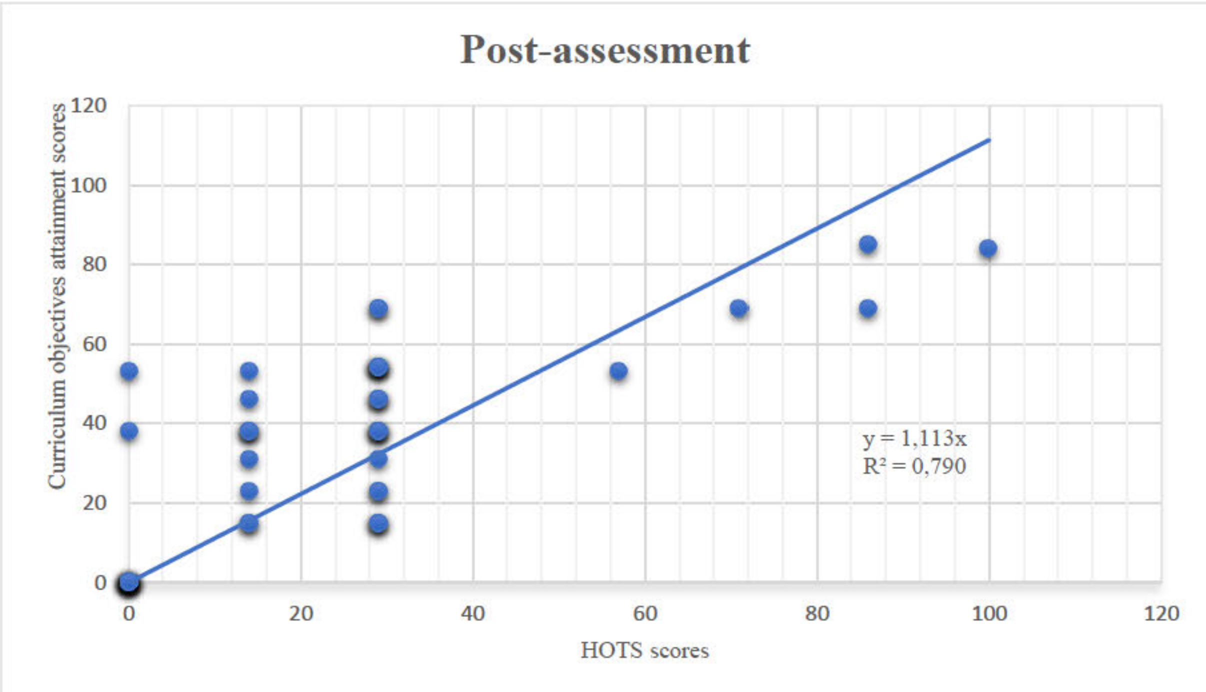

Figure 5.2: Relationship between the students' objectives attainment and HOTS in the post-assessment.

HOTS scores of both the pre-assessment and post-assessment, illustrated in *Figure 5.1* and *Figure 5.2* respectively.

According to *Figure 5.1*, in the pre-assessment the curriculum objectives attainment and HOTS students' scores have a linear relationship. The correlation coefficient between them was found to be:

$$R = \sqrt{0.795} = 0.892 \tag{5.2}$$

This correlation coefficient indicates a strong relationship between the curriculum objectives attainment and HOTS students' scores of the pre-assessment.

Again, *Figure 5.2* shows that in the post-assessment the curriculum objectives attainment and HOTS students' scores have a linear relationship. Their correlation coefficient was found to be:

$$R = \sqrt{0.790} = 0.889 \tag{5.3}$$

This correlation coefficient indicates a strong relationship between the curriculum objectives attainment and HOTS students' scores of the post-assessment.

In that regard, the objectives attainment and HOTS scores showed to be directly proportional for the current actual data. Hence, we deduce that eMnambithi TVET College should focus on fully equipping students with HOTS to facilitate better curriculum objectives attainment. In 2017 a particular study was conducted, it was found that there was a strong linear relationship between HOTS and student's academic achievement *[Tanujaya et al, 2017]*. Our view is that academic achievement also includes curriculum objectives attainment. Hence, we perceive the results of *Tanujaya et al (2017)* to be similar to that of the current study.

5.6. Conclusion

The two models namely Tyler's Objectives and SVHIR model were successfully applied to the actual data in this chapter. It was found that the curriculum at eMnambithi TVET College is most likely to be incapable of equipping the student for attainment of the curriculum objectives. Further on, it was also found that the curriculum at eMnambithi TVET College is most likely to be incapable of equipping the students with HOTS. Lastly, a strong linear relationship was identified between the curriculum objectives attainment and HOTS students' scores.

Chapter 6: Conclusions, limitations, and recommended themes for the further studies

6.1. Introduction

The TVET College data set under investigation in this study, forms a sample of the overall KZN TVET Colleges' data set. Hence, it has a potential to suggest the real characteristics of the total KZN TVET Colleges' data. In chapter 5, the sample data (eMnambithi TVET College) findings have already been explained, and what they imply about eMnambithi TVET College. Therefore, in this chapter other than answering the current study's questions and showing achievement of the aim and objectives of the study, we attempt to use those findings of the sample data to infer the characteristics of the overall data (KZN TVET Colleges). In that regard, this chapter consist of the conclusions on answers to the main research questions and sub-questions, limitations, and recommended themes for the further studies.

6.2. Answers to main research question and sub-questions

This section presents a summary of the findings of the current study, which are based on the development and application of the two-curriculum evaluation model on the TVET NATED mathematics curriculum in operation at the eMnambithi TVET Colleges in KZN. The current study consisted of two segments, namely, model development and model application. The authenticity of the second segment (model application) highly depends on the first segment (model development), given there is no proper application without a well-developed accurate model. Nevertheless, our initial intention was to give less attention to the first segment and focus more on the second segment, to produce a conclusion about TVET College's N1 to N2 mathematics curriculum based on the data set of all the TVET Colleges in KZN. That turned to be not the case, the reason was that the current study took an innovative approach for curriculum evaluation. Hence, the first segment had to be well pioneered not elsewhere except in this study. That caused the first segment to spontaneously became a centre of attention than the second segment. Also, the issue of the limited data became the second cause of less attention to the second segment. In that regard, the study remained limited to conclude holistically on the N1 to N2 mathematics curriculum as implemented at the TVET Colleges in KZN. Hence in our deductions we use “most likely”, which say this is a highly possible case in the overall data of the N1 to N2 mathematics curriculum in KZN. Nonetheless, this does not make the current study’s contribution minor. Given, out of the two segments of the study, the first segment was successfully executed. For the second segment, half of it if not more was also executed, given the only remaining item in the second segment is data collection in a larger scale and repetition of what has already been executed in this study to conclude on the curriculum under investigation.

Chapter 4 presented the solution of the main question of the current study which is: How can the TVET mathematics curriculum from N1 to N2 be evaluated with a focus on students’ attainment of the curriculum objectives, and the equipping of students with the necessary HOTS? Tyler’s objectives model was identified as a suitable model to evaluate the capability of the curriculum to equip student for the attainment of the curriculum objectives. In the current study, Tyler’s contribution began from the pre-evaluation of the curriculum up to the construction of the pre- and post-assessments. As to how to use the two assessments in the actual evaluation, he left that to everyone to innovate a suitable method for that purpose. Therefore, this study successfully formulated the average difference ($\bar{X}$) concept in sub-section 4.2.2, which was directly applied to the objective students’ scores in *Appendix 5*, and we produced a conclusion about the capability of the curriculum to equip students for objectives attainment. Further on, the SVHIR model was successfully developed for the evaluation of the capability of the curriculum to equip students with HOTS. That model was directly applied to the HOTS students’ scores in *Appendix 5*. To summaries the answer to the main question, the study found Tyler’s objectives model and SVHIR model as suitable models to evaluate the attainment of the curriculum objectives and curriculum capability to equip students with the necessary HOTS respectively.

Chapter 5 answers the two sub-questions of the current study which are: i) Is the TVET mathematics curriculum from N1 to N2 capable enough to equip the students for the attainment of the curriculum objectives? ii) Is the TVET mathematics curriculum from N1 to N2 capable to equip students with the necessary HOTS? That was done through the application of the two models namely Tyler’s objectives and SVHIR model to the collected data in *Appendix 5* as mentioned in the above paragraph. With the Tyler’s model, it was found that at the research site the current selected learning experience to operate in N1 to N2 mathematics curriculum classes focused on acquiring information and the thinking skills. The selected learning experience was found most likely to be ineffective to equip students for the curriculum objectives attainment, hence the curriculum is incapable too. Through the application of the SVHIR model, the curriculum was also found most likely to be incapability of equipping students with HOTS. As mentioned in the literature review that the Tyler's model mainly focuses on curriculum strengths and weaknesses, rather than on the performance of students as individuals *[Tyler, 1949]*. To summarise the answers to the sub-questions, the current study found the public South African TVETs Colleges' N1 to N2 mathematics curriculum most likely to be incapable of equipping students with the curriculum objectives attainment. Also, the curriculum was found to be most likely to be not equipping students with necessary HOTS based on the available data from the current study. Further, the student ability to attain curriculum objectives and their HOTS were found to have a strong linear relationship. Which implied, fully equipping students with HOTS will cause them to better attain the curriculum objectives. In year 2017, similar relationship was obtained between HOTS and mathematics students' performance *[Tanujaya et al, 2017]*. They found that there is a strong linear relationship between HOTS and student's academic achievement.

In the above paragraphs, beside answering the questions of the current study we also achieved the aim and objectives of the study. Given, the two optimal evaluation models the study aimed to find were shown to be the Tyler's objectives model and SVHIR model in the above paragraphs. So, the attaining of those two models fulfilled the objectives of the study.

Considering that the available data set in the current study is part of the overall TVET Colleges in KZN and it also contains balanced data influential elements that varies across TVET Colleges in KZN, we are of the view that it has a potential to some extent to reflect the real characteristics of overall data. One of the data influential elements is the background of the students. That can impact the data set negatively or positively depending on whether the majority has an advantaged or disadvantaged background. In our data both the backgrounds were fairly represented, given the TVET College we collected data from was located between the rural areas (Qinisa, Embizeni, KwamThandi, etc) and urban areas (Colenso, Ezakheni, Steadville, etc.). Hence, we found a balanced combination of students coming from both advantaged and disadvantaged schools. The other data influential element is the incapabilities and shortage of lecturers, which was also balanced in our case. Firstly, we found that each class was allocated a lecturer. Which was not going to be the case if there was a shortage of lecturers. Secondly, according to the students in response to question number 10 in *Appendix 3*, the majority confirmed the familiarity of the content from teaching and learning (refer to chapter 5). Which meant lecturers were also capable to deliver the content. Moreover, there were other data influential elements that were found to be balance such as availability of equipment, study material, proper campus leadership and many more. In that regard, the current available data set in this study is viewed as one of the suitable sub-data sets to reflect to some extent the characteristics of the overall KZN TVET Colleges data. Hence, from the available data we deduce that the actual overall KZN TVET Colleges' data set is most likely to confirm that the public South African TVET Colleges' N1 to N2 mathematics curriculum is incapable to equip students with HOTS. In addition, looking at the syllabuses separately, N1 was last amended in the year of 1994 and N2 in the year of 1997 *[Department of high Education, 1994 and 1997]*. In general, one may assume the syllabuses are outdated given the tremendous evolution of the world that has happened from the amendment dates of the syllabuses up to now. With those facts, one may consider the current study's results from the models to be holistically in KZN. However, in this study we base our conclusions on the actual data. Hence another study should be conducted in the future, where data ideally should be collected across the TVET Colleges in KZN. This should be done to further confirm what was implied by the current study about the KZN TVET Colleges' data set.

6.3. Limitations

During the period of the current study execution, we have identified a common challenge in most KZN TVET Colleges at a distance. There is lack of proper leadership, which ultimately affect many areas including teaching and learning within each TVET College. Some of the gatekeepers they are aware of the challenge, but instead of rectifying they protect the challenge. Hence, they prevent any research focusing on teaching and learning to take place in their institutions, which make studies like the current study to have limited data of TVET Colleges in some cases.

The initial intention of the current study was to collect data across all KZN TVET Colleges, which was not successful. The study was only able to access the eMnambithi TVET College data set, which in some extent limited the study to fully conclude on the N1 to N2 mathematics curriculum based on the KZN TVET Colleges data set. Hence in the future, application of the two models in the current study to the overall KZN TVET Colleges data set is the priority. In the meantime, we will escalate the matter of the KZN TVET Colleges' gatekeepers to the national DHET office. Hopefully by the time of the future study's execution, the matter would have been resolved.

6.4. Recommended themes for further studies.

- Application of the Tyler's objective model and SVHIR model in the TVET Colleges in KwaZulu-Natal.
- Advancement of the SVHIR model for the inclusion of the zero compartments at TVET Colleges in KwaZulu-Natal.
- Stability analysis of the SVHIR model in the TVET Colleges across the nine Provinces of South Africa.
- Investigation of the content delivery variable in the SVHIR model in the TVET Colleges of South Africa.
- Investigation of the variation of the application of the SVHIR model from basic to higher education.

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APPENDICES

APPENDIX 1: Pre-evaluation questionnaires.

As a lecturer:

1) When you give examples, scenarios or explanations during your teaching sessions; which of the following is <u>mostly</u> your priority?

   a) Using unfamiliar problems is my priority.

   b) Intensity and variety of expression is my priority.

   c) Using less technical terms as possible is my priority.

   d) Using different schemes of information organization is my priority.

   e) Using what interests them is my priority.

   f) None of the above.

2) If opportunities allow you, what kinds of equipment or instruments do you or would you <u>mostly</u> use for demonstration and explanation during your teaching sessions?

   a) I'm not using or would not use any equipment or instruments for demonstration and explanation purposes.

   b) Equipment or instruments that will develop the student's thinking skills.

   c) Equipment or instruments that will increase students' information or knowledge.

   d) Equipment or instruments that will develop the student's interest.

   e) I do or would use other equipment or instruments not mentioned above.

3) What kinds of questions do you <u>mostly</u> ask your students during your class sessions or in a form of homework, tests, Activities, and examinations?

   a) I ask unfamiliar questions that require a student to relate various facts and ideas.

   b) I ask questions that require a student to use the acquired information.

   c) I ask questions that require student's interest.

d) I ask other kinds of questions not mentioned above.

4) If you get a chance, what kinds of questions do you or would you influence your students to ask you during your teaching session or outside you teaching session?

a) Unfamiliar questions.

b) Questions that cannot be immediately found in the textbook or any other publication.

c) Question that are in line with your interest.

d) Other kinds of questions not mentioned above.

APPENDIX 2: Evaluation of Objectives and HOTS questionnaire (pre-assessments)

Part 1 (Objectives of the syllabus)

Application of mathematical principles mastered

1. The mass of a red car plus the mass of a black car is 900kg. Also, three times the weight of the red car plus the weight of the black car is 2300kg. What is the weight of the red car and the black car?

Use of the correct mathematical terminology and to identify the appropriate formulae

2. Fill in the missing words by choosing the correct combination bellow.

   The value $\frac{2}{3}$ can be also called a ……… (i)…………….. of the value $\frac{3}{2}$. The SI unit for the velocity (m/s) is derived from the formula ………… (ii)……..

   a) (i) Fraction and (ii) $\frac{distance}{time}$
   b) (i) Denominator and (ii) $\frac{kilogram}{time}$
   c) (i) Reciprocal and (ii) $\frac{displacement}{time}$
   d) (i) Exponent and (ii) $\frac{accelaration}{time}$

Use of the correct SI units

3. Identify the S.I unit for mass in the following:

   a) Meters per second (m/s)
   b) Meters (m)
   c) Kilograms (kg)
   d) Meters per second square (m/s^2)

Logical reasoning

4. If you work for one day you get paid R4, if you work for two days you get paid R7 and if you work for three days you get R10. What would you be paid when you work for 5 days?
   Choose the correct answer from the following:

   a) R9
   b) R13
   c) R16
   d) R8

Part 2 (HOTS)

Transfer - work systematically through cases in an exhaustive way

5. Simplify the following:

$$\sqrt{\frac{(x^2)^2}{xy^3} \times \frac{y}{x}}$$

Choose the correct option from the following

a) $\frac{x^2}{y^2}$

b) $\sqrt{\frac{x^2}{y^2}}$

c) $\frac{x}{y}$

d) $\sqrt{\frac{x^4}{x^2y^4}}$

e) None of these

Critical Thinking - interpret and extend solutions of problems

6. The following equation has not more than two roots/solutions:

$$x^2 + 2x - 1 = 0$$

6.1. Why the above equation has not more than two roots/solutions?

(a) It is a cubic equation.
(b) It has no solution.
(c) It is a quadratic equation.
(d) It has a constant number 1.

6.2. Elaborate further what is meant by something being a root/solution of a particular equation?

(a) It any integer number.

(b) It a number when substituted in a given equation satisfy it.

(c) It a number when substituted in a given equation result undefined.

(d) It any constant number found in the equation.

Transfer - identify possible applications of mathematics in their surroundings

7. One chocolate and one apple cost a total amount of R50 while four chocolates and three apples cost a total amount of R190. How much is each chocolate and each apple?

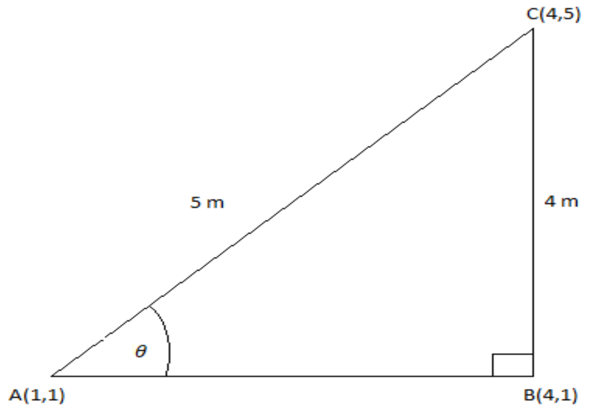
Transfer - translate a worded or graphically represented situation to relevant mathematical formalisms

8. Write down the following sentences/statements in a form of mathematical equations:

   One red car together with a black bicycle costs R150 000. Also the price of three red cars and a black bicycle is R430 000.

Problem Solving - use with reasonable skill available tools for mathematical exploration

9. Given the following diagram:



9.1. Mention the method/s that can be used to find the distance AB.

9.2. Use the above-mentioned method/s to calculate the distance AB (e.g. If you mentioned two methods, find the distance of AB by the first method and after that use the second method)

APPENDIX 3: Evaluation of Objectives and HOTS questionnaire (post assessments)

Part 1 (Objectives of the syllabus)

Application of mathematical principles mastered

1. The mass of a red car plus the mass of a black car is 600kg. Also, three times the weight of the red car plus the weight of the black car is 1400kg. What is the weight of each red car and each black car?

Use of the correct mathematical terminology and to identify the appropriate formulae

2. Fill in the missing words by choosing the correct combination bellow.

   The value $\frac{10}{3}$ can be also called a ……… (i)…………….. of the value $\frac{3}{10}$. The SI unit for the velocity (m/s) is derived from the formula ………… (ii)……..

   e) (i) Fraction and (ii) $\frac{distance}{time}$
   f) (i) Denominator and (ii) $\frac{kilogram}{time}$
   g) (i) Reciprocal and (ii) $\frac{displacement}{time}$
   h) (i) Exponent and (ii) $\frac{accelaration}{time}$

Use of the correct SI units

3. Identify the S.I unit for distance in the following:

   e) Meters per second (m/s)
   f) Meters (m)
   g) Kilograms (kg)
   h) Meters per second square (m/s^2)

Logical reasoning

4. If you work for one day you get paid R2, if you work for two days you get paid R5 and if you work for three days you get R8. What would you be paid when you work for 5 days?
   Choose the correct answer from the following:

   e) R9
   f) R11
   g) R14
   h) R10

Part 2 (For HOTS)

Transfer - work systematically through cases in an exhaustive way

5. Simplify the following:

$$\sqrt{\frac{(x^2)^2}{xy^5} \times \frac{y}{x}}$$

Choose the correct answer in the following

f) $\frac{x^2}{y^2}$

g) $\sqrt{\frac{x^2}{y^2}}$

h) $\frac{x}{y^2}$

i) $\sqrt{\frac{x^4}{x^2y^4}}$

Critical Thinking - interpret and extend solutions of problems

6. The following equation has not more than two roots/solutions:

$$x^2 + 3x - 10 = 0$$

6.1 Why the above equation has not more than two roots/solutions?

(a) It is a cubic equation.
(b) It has no solution.
(c) It is a quadratic equation.
(d) It has a constant number 10.

6.2 Elaborate further what is meant by something being a root/solution of a particular equation?

(a) It any integer number.

(b) It a number when substituted in a given equation satisfies it.

(c) It a number when substituted in a given equation result undefined.

(d) It any constant number found in the equation.

Transfer - identify possible applications of mathematics in their surroundings

7. One chocolate and one apple cost a total amount of R40, while four chocolates and three apples cost a total amount of R150. How much is each chocolate and each apple?

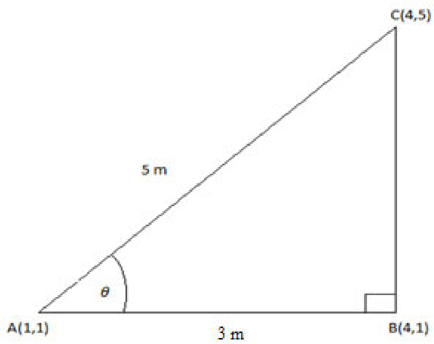
Transfer - translate a worded or graphically represented situation to relevant mathematical formalisms

8. Write down the following sentences/statements in a form of mathematical equations:

One red car together with a black bicycle costs R200 000. Also, the cost of three red cars and a black bicycle is R580 000.

Problem Solving - use with reasonable skill available tools for mathematical exploration

9. Given the following diagram:



9.1 Mention the method/s that can be used to find the distance BC.

9.2 Use the above-mentioned method/s to calculate the distance AC (e.g., If you mentioned two methods, find the distance of AC by the first method and after that use the second method)

Evaluation instrument validation

10. Do you think all the above questions from 1 – 9 are familiar or relevant to what you have learnt from the N1 to N2 curriculum and in class lessons?

(a) YES

(b) NO

APPENDIX 4: Marks scoring grid

PART 1 (OBJECTIVES OF THE SYLLABUS)	
Questions	**Marks Allocation**
Question 1: Application of mathematical principles mastered.	• Formulation of the first equation. [1 mark] • Formulation of the second equation. [1 mark] • Solving for variable 1 (red car mass/ black car mass). [1 mark] • Solving for variable 2 (black car mass/ red car mass). [1 mark] **Total [4 marks]**
Question 2: Use of the correct mathematical terminology and to identify the appropriate formulae.	• Knowing the first correct term. $\left[\frac{1}{2}\ mark\right]$ • Knowing the second correct term. $\left[\frac{1}{2}\ mark\right]$ **Total [1 mark]**
Question 3: Use of the correct SI units.	• Knowing the correct IS unit. $\left[\frac{1}{2}\ mark\right]$ **Total [1 mark]**
Question 4: Logical reasoning.	• Correct answer. [1 mark] **Total [1 mark]**
PART 2 (HOTS)	
Question 5: Transfer - work systematically through cases in an exhaustive way.	• Correct answer. [1 mark] **Total [1 mark]**
Question 6.1.: Critical Thinking - interpret and extend solutions of problems.	• Correct answer. [1 mark] **Total [1 mark]**
Question 6.2.: Critical Thinking - interpret and extend solutions of problems.	• Correct answer. [1 mark] **Total [1 mark]**
Question 7: Transfer - identify possible applications of mathematics in their surroundings.	• Formulation of the first equation. [1 mark] • Formulation of the second equation. [1 mark]

	• Solving for variable 1 (chocolate/ apple). [1 mark] • Solving for variable 2 (apple/ chocolate). [1 mark] **Total [4 marks]**
Question 8: Transfer - translate a worded or graphically represented situation to relevant mathematical formalisms.	• Labelling the variables. [1 mark] • Formulation of the first equation. [1 mark] • Formulation of the second equation. [1 mark] **Total [3 marks]**
Question 9.1.: Problem Solving - use with reasonable skill available tools for mathematical exploration.	• Identifying the required method. [1 mark] **Total [1 mark]**
Question 9.2.: Problem Solving - use with reasonable skill available tools for mathematical exploration.	• Stating Pythagoras theorem formula. [1 mark] • Solving for AB. [1 mark] **Total [2 marks]**

APPENDIX 5: Pre- and Post-assessment student's scores for curriculum objectives and HOTS

*Any negative percentage difference is set to be zero ($-x_i = 0$)

Student Order (i)	Pre-assessment scores		Post-assessment scores		Percentage difference (x_i)	SVHIR Model compartment
	Part 1: Objectives (a_i)	Part 2: HOTS (M_i)	Part 1: Objectives (b_i)	Part 2: HOTS (M_i)		
1	0	0	29	23	29	Infected
2	29	69	0	0	0	Infected
3	29	54	0	0	0	Infected
4	14	53	100	77	100	Healthy
5	0	53	43	15	43	Infected
6	29	46	0	0	0	Infected
7	14	38	29	46	17,44186	Infected
8	29	54	0	0	0	Infected
9	29	38	0	0	0	Infected
10	29	38	0	0	0	Infected
11	29	38	0	0	0	Infected
12	29	46	0	0	0	Infected
13	14	38	0	0	0	Infected
14	0	0	29	0	29	Susceptible
15	14	15	0	0	0	Infected
16	29	69	43	51	19,71831	Healthy
17	0	23	14	0	0	Susceptible
18	29	23	29	8	0	Infected
19	29	15	0	0	0	Infected
20	14	15	14	31	0	Infected
21	14	31	14	15	0	Infected
22	14	38	0	0	0	Infected
23	0	38	14	31	14	Infected
24	14	46	14	31	0	Infected
25	29	54	14	31	0	Infected
26	29	23	29	31	0	Infected
27	71	69	29	62	0	Healthy
28	29	15	29	23	0	Infected
29	100	84	100	100	0	Healthy
30	86	69	100	92	100	Healthy
31	29	15	14	31	0	Infected
32	29	54	86	77	80,28169	Healthy
33	29	38	0	0	0	Infected
34	57	53	0	0	0	Infected
35	29	46	0	0	0	Infected

36	29	54	29	23	0	Infected
37	86	85	0	0	0	Infected
38	29	31	0	0	0	Infected
39	0	0	29	85	29	Recovered
40	0	0	29	100	29	Recovered
41	0	0	29	46	29	Infected
42	0	0	29	23	29	Infected
43	0	0	43	85	43	Recovered
44	0	0	29	38	29	Infected
45	0	0	14	38	14	Infected
46	0	0	57	46	57	Infected
47	0	0	14	46	14	Infected